

EDUCATIONAL EXPERIMENTAL PLATFORM FOR STUDIES OF NONLINEAR DYNAMICAL SYSTEMS

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Abstract. *A great number of researchers have studied and taught about non-linear phenomena exhibiting non-periodic and bounded solutions in several areas of science and engineering. However, in most cases, these concepts are not explored experimentally in order to facilitate thoroughly understanding of the student. In this paper we have examined the dynamical behaviour of an elastic wide plate induced by two electromagnets and submitted to external excitation. This study have technical applications but, in addition, the main goal is to serving as a convenient educational experimental platform model providing subsidies to understand better the non-linear and chaotic phenomena by undergraduate and graduate students .*

Keywords: *Experimental education, nonlinear phenomena, nonlinear dynamics, chaos*

1. INTRODUCTION

Many technical devices such a motors, generators, transformers, fusion reactors, employ elastic wide plate in magnetic fields. It has also been shown that flexible structures either ferromagnetic material that carrying large currents can buckle in magnetic fields.

An experimental apparatus is designed reasoned as follows: a slender steel beam is clamped in a rigid framework. Two permanent magnets at the base pull the beam in opposite directions.

The magnets are so strong that the beam buckles to one side or the other. This configuration is locally stable and these buckled states are separated by a energy barrier, corresponding to the unstable equilibrium in which the beam is straight and poised halfway between the magnets.

To drive the system out of its stable equilibrium, the whole apparatus is shaken from side to side with an electromagnetic vibration generator. For weak forcing, the beam is observed to vibrate slightly while staying near one or the other magnet, but as the forcing is slowly increased, there is a sudden point at which the beam begins whipping back and forth erratically. The irregular motion is sustained and can be observed for a long time.

The main goal of this work is, by means of the proposed platform, provide a convenient educational experimental apparatus to better understanding about non-linear and perhaps chaotic phenomena for undergraduate and graduate students.

2. THE THEORETICAL MODEL AND THE EXPERIENTAL PLATFORM

The mechanical system whose theoretical model is developed is shown in Figure 1. A flexible rod is clamped in a rigid base. The electromagnets pull the beam in opposite directions and intensity of the field is strong enough to deflect the rod from one side to another.

The electromagnets generate a magnetic field that induces a magnetization \mathbf{M} per unit volume in the solid. The rod can be modelled as a soft magnetic material where \mathbf{M} is proportional to the local magnetic field in the solid [Reitz, 1980]; i.e.,

$$\vec{M} = \left[\frac{\chi}{(\chi + 1)} \right] \frac{\vec{B}}{\mu_0} \quad (1)$$

where:

μ_0 is the magnetic permeability of a vacuum;

χ is the magnetic susceptibility

The field \mathbf{B} can be written in terms of the field \mathbf{B}^0 produced by external magnets, and a field produced by the magnetization itself, \mathbf{B}^1 . If self-forces on the rod are neglected then the external magnets produce both a force and moment distribution on the beam given by [Feynman, 1965]

$$\vec{F} = \vec{M} \cdot \nabla B^0 \quad (2)$$

$$\vec{C} = \vec{M} \times \vec{B}^0 \quad (3)$$

These forces can be derived from a magnetic potential and therefore they are conservative. Thus,

$$W = -\frac{1}{2} \int \vec{M} \cdot \vec{B}^0 dv \quad (4)$$

The existence of this potential - expressed as a volume integral over the rod - is important for the dynamics and stability of the rod.

The non-linearities included in the analysis reflect the inhomogeneous nature of the magnetic field \mathbf{B}^0 and the magnetic force and couple.

To determine the magnetization is assumed that the rod is wide plate. It is further assumed that locally the magnetic field \mathbf{B}^0 can be treated as uniform and that the dependence of \mathbf{M} on the beam curvature is small. Under these assumptions it can be shown that \mathbf{M} is given by the expression

$$\vec{M} = \frac{\chi B_0(x, y)}{\mu_0} \left(\frac{\cos \beta}{\mu_r} \hat{n} + \sin \beta \hat{a} \right), \quad (5)$$

where:

\hat{n} is a unit vector normal to the rod plate;

\hat{a} is a unit vector tangent to the rod plate;

β is the local angle between the external field \mathbf{B}^0 and the plate normal \hat{n} ;

$\mu_r = \chi + 1$ is the relative permeability of the rod.

The field shape \mathbf{B}^0 is to be evaluated in terms of the local x, y coordinates of the plate neutral surface.

If the x,y components of \mathbf{B}_0 are introduced, defined by $B_{0x}=B_0\cos\alpha$ and $B_{0y}=B_0\sin\alpha$ and the local slope of the rod with the x axis by θ , then one can rewrite equation and the magnetic energy potential then takes the form

$$W = -\frac{\chi}{4\mu_0\mu_r} \int_0^L (B_1 + B_2 \sin 2\theta + B_3 \cos 2\theta) ds, \quad (6)$$

where:

$$B_1 = (\mu_r + 1)(B_{0x}^2 + B_{0y}^2); \quad (7)$$

$$B_2 = 2(\mu_r - 1)B_{0x} B_{0y}; \quad (8)$$

$$B_3 = (\mu_r - 1)(B_{0x}^2 - B_{0y}^2). \quad (9)$$

Here the integration is carried out over the original length of the rod and B_1 , B_2 and B_3 are functions of the rod displacement.

The nonlinear elastic forces are small even for the large displacements of the rod tip. Moreover, if a single mode approximation is made for the beam deformation, the elastic energy can be write in the form

$$P = \frac{1}{2} k y^2 + (\text{higher order terms}) \quad (10)$$

Thus, the potential for the elastic and magnetic forces will be

$$V = W + P \quad (11)$$

With the usage Lagrangian's formalism we can write

$$L = T - (W + P) \quad (12)$$

where T , W and P are kinetic, magnetic and elastic energies respectively.

Beside this, in Galerkin's method a suitable set of (orthogonal) basis function $\phi_j(x)$, which satisfy the boundary conditions, is chosen and the unknown displacement, expressed as

$$v(x, t) = \sum_{j=1}^{\infty} a_j(t) \phi_j(x) \quad (13)$$

A typical choice for the ϕ_j in vibration problems are the normal modes of the associated linear problem.

One then substitutes expression (13) into the equation of motion and takes the inner product (i.e., integrates over the rod length) with $\phi_k(x)$, $k = 1, 2, 3, \dots$, thus obtaining an infinite set of second order ordinary differential equations for the unknown modal coefficients $a_j(t)$.

Since it is considered that the lowest mode is dominant in the motion of interest here, a single mode approximation

$$v(x, t) = a(t) \phi(x) \quad (14)$$

can be chosen where $\phi(x)$ is required to satisfy

$$\phi(0) = \phi'(0) = \phi''(L) = 0, \quad D\phi''(L) + k\phi'(L) = 0, \quad \int_0^L \phi^2 dx = 1.$$

Finally, we can write the equation of motion this system in the known and with the dimensionless form

$$\ddot{x} + \delta \dot{x} - x + x^3 = F \cos \omega t \quad (15)$$

where

$\delta > 0$ is the damping constant;
 F is the forcing strength
 ω is the forcing frequency

The experiment consists of a system in which a flexible rod has one end attached to a rigid base while the other end remains free. Near the free end, there is a mobile base that is driven by a stepper motor.

On the mobile base, which moves horizontally left and right, set up two electromagnets that generate electromagnetic forces that deflect the rod, a ruler potentiometric varying signal 4 to 20 mA and a fiber optic sensor is capable of detecting the positioning of the rod during the motion.

Sensors that indicate positions of centrality and final course in the mobile base are also used.

A driver is used for the stepper motor, allowing control over the different positions of the mobile base and a speed control. We use a data acquisition system from National InstrumentsTM, and thus account with a high sensitivity for monitoring signals from various sensors, allowing reliable results.

It is also used a programmable logic controller (PLC) that manages the stepper motor driver, manages the Lab View software, receives signals from the sensors in their digital inputs and, depending on data programmed by the user can "toggle" inputs digital stepper motor. Then, the driver recognizes the program and performs the proper movements related to the programmed sequence. Next are shown two figures, one that illustrates the mechanical device mounted and another representing a flow chart seeks to clarify the interrelationship between the devices used in the test rig.

The stepper motor M1 illustrated in Figure 1 is capable of 4 Nm of torque and resolution of 25000 pulses per revolution. This gives a precise movement of the mobile base of 0.014 degrees every step. This engine is connected to a pulley through a timing belt, and the function of this coupling, is move the mobile base.

It can also be observed in Figure 1, illustrations of the sensors S1, S2, S3 and S4 and inductors B1 and B2. The functions of these elements are described below:

S1- final course's sensor of mobile base on the left side;

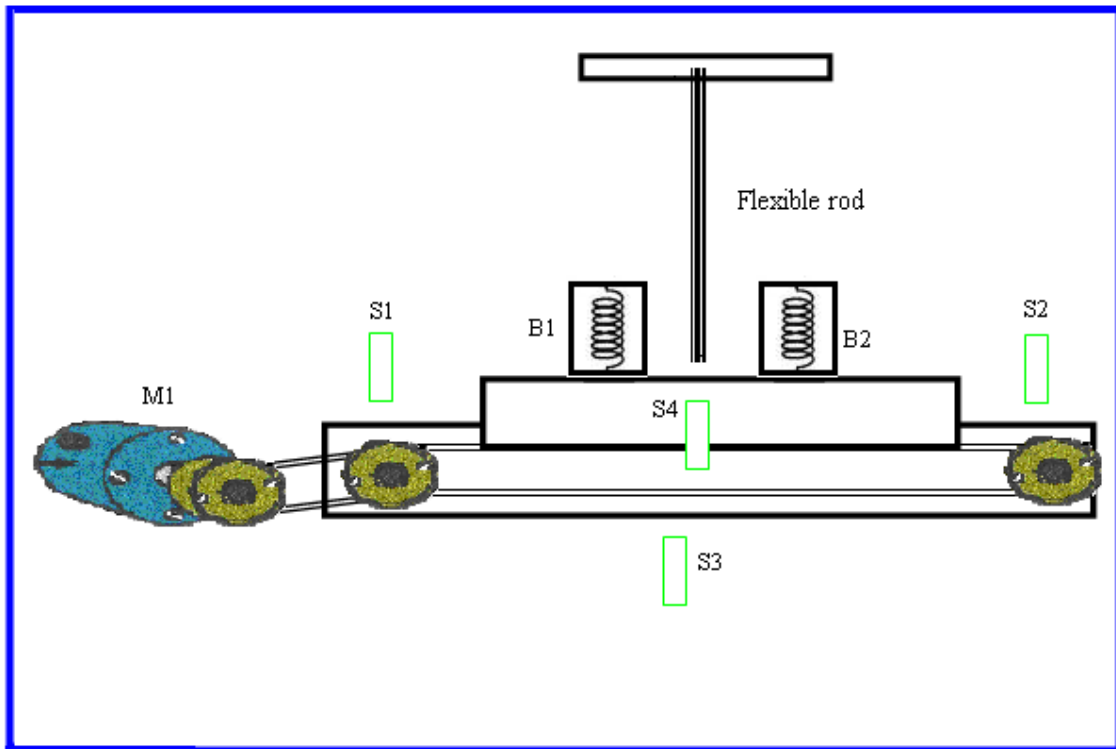


Figure 1: Experimental Device

- S2 - final course's sensor of mobile base on the right side;
- S3 - sensor that indicates the central position of the mobile base;
- S4 - sensor that monitors the position of the flexible shaft;
- B1- Inductor of the left side, generating an electromagnetic field;
- B2 - Inductor of the right side, which generates electromagnetic field.

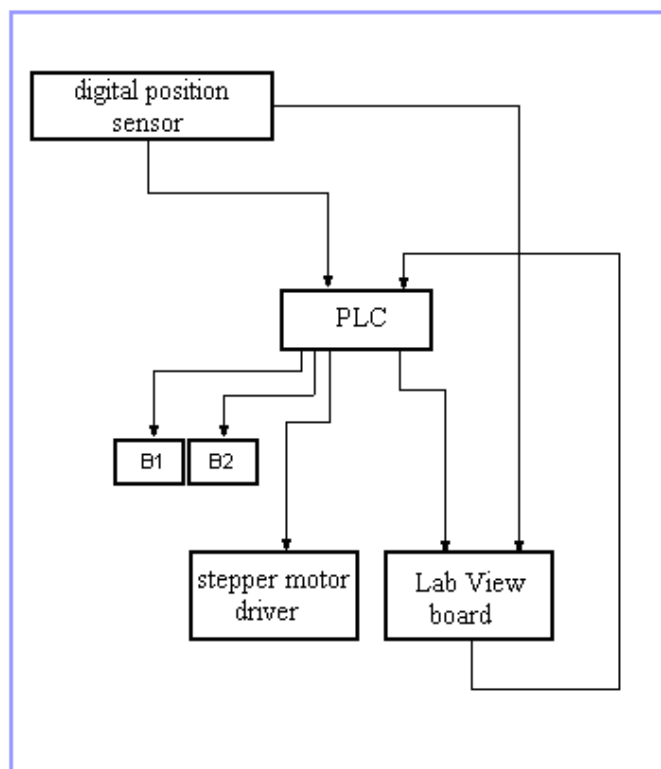


Figure 2: Flow chart of project

2.1. Programmable Logic Controller (P.L.C.)

Is used in this application a PLC with 16 inputs and 11 digital outputs of the manufacturer Rochweel Automation. The software of the PLC consists of a programming language dedicated and stipulated by the equipment manufacturer. Basically it is conceptualized in a language known as Ladder, and events happen sequentially, that is, if an event occurs it is necessary that the previous event had occurred.

The PLC has the function of the switching of various types of programs in the driver of the stepper motor to perform the various movements and also receive input signals from sensors that allow LabView, for example, terminate the test.

It is noteworthy that this device accepts digital signals from the voltage level of 24 Volts. In this experiment boards and LabView conduct the stepper motor work with level of recognition of input signals 5volts. For this reason, in order that the elements can communicate, using the resources of each terminal relays opto-couplers - electronic devices that are compact - with voltage levels ranging from 24 volts to 5 volts and vice versa.

2.2. Stepper Motor

This electronic device is critical to the smooth operation of the platform because with their help we can control the position, velocity and acceleration of the mobile base. Furthermore, it allows flexibility of changing variables in the tests, if necessary.

The programming software for this driver is called XWARE, was built in ASCII language and its programming is relatively simple.

In this program there are several commands, one of which allows - if there is a need - make the communication among the relevant different devices via serial port 232 or DH-485. This prevents, for example, accumulations of wireless connections in optimizing the design.

This driver also allows one to work with a loop feedback signal. Through a door in that it is possible to connect an encoder - sensors that read the different speeds and angular positions.

(2)

2.3. Signal Acquisition System

Is used in this experiment an electronic board signal acquisition from National Instruments SCXI 1200. This card is designed to receive 08 input signals and digital inputs or 08 analog outputs, ranging from the "range" between -10 volts and +10 volts.

The signals from sensors and / or potentiometric ruler are sent to an entry addressed in the board. These signals are processed by a filter before being sent to the channel and the goal of this filter is to absorb noises coming from some other equipment that are not part of the platform.

The board has 32 bit and has a capacity to collect 300,000 points per second. Therefore, the recorded reading is extremely representative of the physical quantity of the phenomenon analyzed.

The signals are received by the Lab View software on a PC equipment and the aim of this software is effecting the conversion of electronic signals in values such as position, velocity, acceleration, time, etc.

2.4. Platform's Operation Modes

2.4.1 Start Mode

When the button is operated related to this mode, the mobile base is sent to the center of the device. After each test, the mobile base can make his stop in any position. So the way that it start allows each new test may have its beginning from a predetermined point, or from a reference.

2.4.2 Calibration Mode

In this mode of operation permits that we will know the maximum amplitudes in the system that causes the detachment of the stem of each electromagnet. The mobile base moves with constant velocity to the right of the central region until the end of the rod comes loose from the electromagnet that is stuck due to electromagnetic interaction.

At this moment, gathering up the size of the amplitude at the time of detachment of the rod and it can serve as reference for further testing or allowing additional observations about what happens (or might occur) right there. The same procedure can be done on the opposite side, that is, the left side.

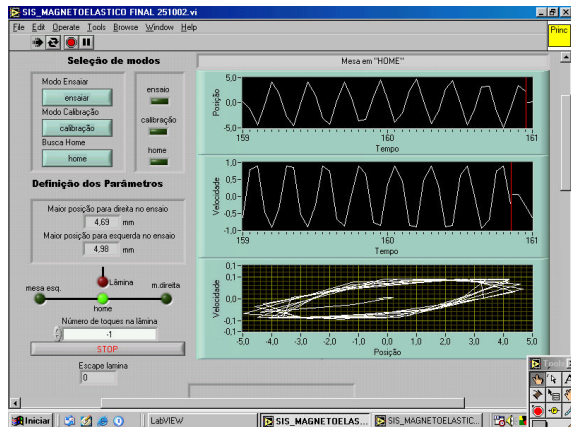
2.4.3 Practice Mode

When the practice button is pressed, the counter starts the test while maintaining or increasing the frequency and amplitude of displacement of the mobile base, drives the coils generating an electromagnetic field that interacts with

the flexible stem. The optical sensor monitors the position of the rod and the data are sent to the Lab View which can plot the graphs recording the phenomena.

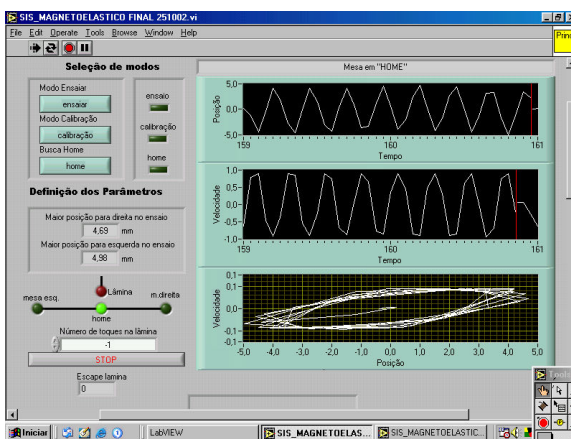
3. GRAPHICAL INTERFACES

We would like to illustrate the graphical interfaces that were implemented by the software acquisition and signal processing Lab View.



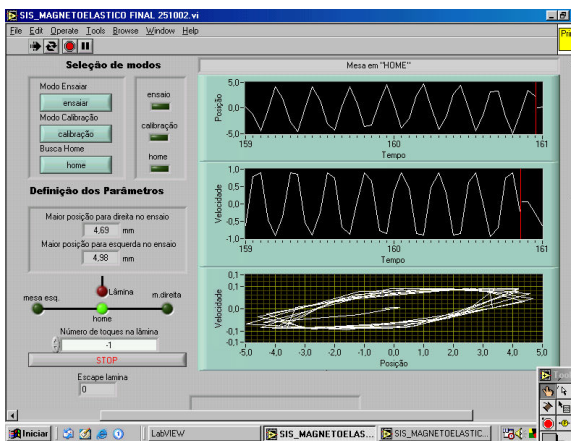
**Indicator
Position x Time**
*Provides the evolution of the position of
the rod end with time.*

Figure 3: Position x Time Indicator



**Indicator
Velocity x Position**
Provides the evolution of velocity with position

Figure 4: Velocity x Position Indicator



**Indicator
Velocity x Time**
Provides the evolution of velocity with time

Figure 5: Velocity x Time Indicator

4. FINAL CONSIDERATIONS

The apparatus allows obtain experimental verification of results, in most cases are explored and presented in terms of undergraduate and graduate education only in theoretical or computational simulations.

Thus, periodic solutions, chaotic and transient solutions indicative of chaos can be observed experimentally by making appropriate choices of initial conditions.

5. REFERENCES

- Arrowsmith, D. K., Place, C. M. "An introduction to dynamical systems", Cambridge University Press. (1990).
- Chen, Y. C., Naughton, J. M. ,” An undergraduate laboratory platform for control system design, simulation, and Implementation”. IEEE Control Systems Magazine, p 12-20 (2000).
- Cox, A . "Magnetostratigraphic time scale", Cambridge University Press (1982).
- David, S. A. ; Rosario, J. M. . Investigation about nonlinearities in a robot with elastic members. Computational Fluid and Solid Mechanics - Solid and Structures ^{JCR}, v. 1, K.J.Bathe editor, pp. 137-139, Elsevier Science Ltd., England(2001.)
- David, S. A. ; Rosario, J. M. ; Machado, J. . Investigation about chaos in a magneto-elastic dynamical system. Computational Fluid and Solid Mechanics - Solids and Structures ^{JCR}, Multi-Physics, vol.2, K.J.Bathe editor, pp. 1120-1123, Elsevier Science Ltd., England,ISBN 0-08-043944-6. (2001).
- David, S.A.; Rosário, J.M. Modeling, Simulation and Control of Flexible Robots Nonlinear Dynamics, Chaos, Control and Their Applications to Engineering Sciences, Chapter 3: Control, Robotics, Neural Networks and Optimization Engineering, vol. 1, Published by ABCM and AAM, pp. 353-358, ISBN: 85-900351-2-3, 1999.
- Feynman, R. P., Leighton, R. B., Sands, M. "The Feynman Lectures on Physics", Addison-Wesley, Reading, M.A. (1965).
- Goldstein, H. "Classical Mechanics", Reading Mass., Addison Wesley Publishing Company, Second Edition (1981).
- Guckenheimer, J., Holmes, P. "Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields ", Springer - New York (1983).
- Hagedorn, P. Oscilações Não Lineares. São Paulo: Editora Edgard Blucher LTDA, 1984.
- Moon, F. C. "Chaotic and Fractal Dynamics: An Introduction for Applied Scientists and Engineers", Wiley - New York (1992).
- Moon, F. C. "Problems in Magneto-Solid Mechanics", chapter V in Mechanics Today - S. Nemat-Nasser Editor. (1978).
- Nayfeh, A. H. , Mook, D. T., "Nonlinear Oscillations", Wiley & Sons - New York (1979).
- Reitz, J.R., Milford, F. J., Christy, R. W., "Fundamentos da Teoria Eletromagnética", Editora Campus, (1980).
- Steidel, R. F., "An Introduction to Mechanical Vibrations" John Wiley & Sons, (1989).