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THE PERFORMANCE OF THERMAL AND VELOCITY WALL LAWS

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Abstract. *The main goal of this work is evaluate the computational performance of thermal end velocity laws of the wall. The results obtained are compared to experimental data, to the numerical results of three different velocity wall laws and one temperature wall function. The test cases used are a termal backward facing step and an asymmetric plane diffuser turbulent flows. The turbulence model adopted is the classical $\kappa - \varepsilon$ adapted to dilatable flows. Spacial discretization is done by P1/isoP2 finite element method and temporal discretization is implemented using a semi-implicit sequential scheme of finite difference. The coupling pressure-velocity is numerically solved by a variation of Uzawa's algorithm. To filter the numerical noises, originated by the symmetric treatment used by Galerkin method to the convective fluxes, a balancing dissipation method is adopted. The remaining non-linearities, due to laws of wall, are treated by minimum residual method. The hardware used was a dual-processor station Pentium III Xeon at 933MHz each processor.*

keywords: *turbulence, velocity wall laws, thermal wall laws, $\kappa - \varepsilon$ model, finite elements.*

1. Introduction

The conferences AFORS-HTTM about complex turbulent flows, in the years 1980 and 1981 at Stanford, are important historical marks of the studies of parietal turbulence in industry applications, from which resulted the proposition of using the flow over a backward facing step as a standard case to provide analysis of numerical simulation of turbulence models. Since then, the main problems encountered are still linked to representing the flow that occurs in the inner region of the boundary layer, where the intensity of local gradients and the consequent effects of molecular viscosity and turbulent diffusivity of momentum are very difficult to be treated simultaneously.

The high gradients encountered in the inner region of the turbulent boundary layer makes the calculation of the flow quite expensive. In the other hand, the simultaneous performance of viscous and turbulent effects is a hard challenge for turbulence models. As solution for these special characteristics of parietal turbulence two options exist: the use of wall laws, capable to represent the dynamic behavior of the inner region of the boundary layer, and the use of low Reynolds turbulence models, having each one of these solutions advantages and disadvantages.

The low-Reynolds models inconveniences are the need of very refined meshes to discretize the viscous sub-layer, transition region and beginning of the logarithmic region, and the low degree of generality of this kind of model, very dependent of the calibration of its constants, which vary with the geometry and with other characteristics of the flow.

Contrarily, the use of laws of the wall substantially reduce the need of very refined meshes in the near wall region, and they are not restricted to a specific geometry or other flow conditions. The greatest inconvenience of the laws of the wall is the numeric instability induced by its use. The flow modelling in the proximity of solid contours of the calculation domain, through laws of the wall, introduces additional instabilities in the system of equations, caused by the explicit method that evaluates the wall functions.

In algorithms that adopt procedures of temporal integration of the governing equations, as a way of minimizing the need for knowledge of the initial conditions of the flow, the instability associated to the use of wall laws is amplified, demanding the adoption of special methods of numeric stabilization, specifically designed for this function.

In the simulation of parietal turbulent flows in which the density is a function of of temperature gradients, is necessary the simultaneous resolution of the conservative equations of mass, momentum and energy, as well as equations of state capable of representing the variation of some thermodynamic properties of the fluid with temperature gradients. When the use of wall functions is the choice to model the near wall region, it is also needed the simultaneous employment of wall functions for velocity and temperature in the system of equations to be solved.

The objective of this work is to test the numerical performance of the wall laws proposed by Cruz and Silva Freire (1998), for velocity and for temperature, in cases of confined flows under adverse pressure gradient and with thermal variation due to the presence of a heated wall.

The algorithm to be tested, Turbo 2D, is a combination of the numerical simulation methodology using finite elements of strongly heated wall flows, proposed by Brun (1988), with an error minimization method adapted to a finite elements, for the simulation of turbulent wall flows with non-linear boundary conditions, proposed by Fontoura Rodrigues (1991), using the classic $\kappa - \varepsilon$ turbulence model of Jones and Launder (1972). By applying Galerkin's method for finite elements to the calculation of convection dominant flows, numerical oscillations without physical meaning can appear. This fact occurs due to the usage of Galerkin's method, that gives a symmetric treatment to the flow modelling, which is a non symmetric physical phenomenon. To lower the tendency of numerical oscillation, a balancing dissipation method, proposed by Huges and Brooks (1979) and Kelly et al. (1980) and implemented by Brun (1988), is used in Turbo 2D.

To accomplish the test of performance of laws of the wall, two cases were selected: the asymmetric plane diffuser of Buice and Eaton (1996), to test the capability to predict the separation of the flow under a smooth adverse pressure gradient, and the test case of the thermal backward facing step of Vogel and Eaton (1984), to evaluate the performance of different temperature laws of the wall in a separating turbulent flow, and also to verify the relation between the thermal and velocity fields of this separating thermal turbulent flow.

The results obtained are compared to the respective experimental data available from the literature, with other simulations with Turbo 2D, using different laws of the wall, and with some numerical results from other simulations, provided by the research community of the ERCOFTAC and NPARC Alliance from NASA.

2. Governing equations

The turbulent one phase flows analyzed in the present work are homogeneous, at low Mach number, and buoyancy forces are though small compared to convective effects. This way, the conservation equations of mass, momentum and energy, which describe the phenomena, are respectively represented, in Einstein's notation, by the relations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \ , \tag{1}$$

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \rho g_i + \frac{\partial \tau_{ji}}{\partial x_j} \ , \tag{2}$$

$$\rho C_p \frac{DT}{Dt} = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right) + q''' + \beta T \frac{Dp}{Dt} + \tau_{ij} \frac{\partial u_i}{\partial x_j} \ , \tag{3}$$

where ρ is the fluid density, t represents time, x_i a cartesian coordinate, R is the constant for the air, u_i is the velocity field, p is the total pressure, Cp represents the specific heat coefficient at constant pressure, considered constant with temperature in the present work, T is the temperature, k is the thermal conductibility, g_i is the acceleration of gravity, β is the thermal expansion coefficient, q''' represents a internal source term of heat and τ_{ij} is the shear stress tensor, with its constitutive relation for a newtonian fluid given by:

$$\tau_{ij} = \mu \left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial u_l}{\partial x_l} \delta_{ij} \right] \ . \tag{4}$$

where δ_{ij} is the delta of Kronecker operand. In equation (4), μ represents the fluid dynamic viscosity, considered here to be independent of the pressure, being a function of temperature only. The empiric relation adopted in the present work and the ideal gas relation are, in order to complete the system of equations:

$$\mu = \mu(T) = aT^n \ , \tag{5}$$

$$\rho = \frac{p}{RT} \ , \tag{6}$$

where R, a and n are material constants for the air, and they are, respectively, 287 $\frac{J}{kg\,K}$, 3,68x10-7 $\frac{m^2}{s\,K}$ and 0,685.

2.1. Dimensionless governing equations

In order to simplify the notation adopted, the variables in their dimensionless form have the same representation as the dimensional variables, being these last indicated when necessary.

The process of obtaining the system of equations in a dimensionless form, equations (1), (2), (3) e (6), is conventional, but a special attention is given to the energy equation. In dimensionless form, the energy equation is given by:

$$\rho\frac{DT}{Dt} = \frac{1}{Re\ Pr}\frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right) + (\gamma-1)M_0^2\frac{Dp}{Dt} + M_0^2\frac{1}{Re}\Phi\ , \tag{7}$$

with γ being the relation between the specific heat coefficients, at constant pressure, C_p, and volume, C_v, Φ represents the mechanical energy dissipation rate per fluid volume, due to shear viscosity effects, and M_0 is the Mach number, defined as a function of the reference values of velocity and temperature:

$$\gamma = \frac{C_p}{C_v}\ , \tag{8}$$

$$M_0 = \frac{U_0}{\sqrt{\gamma R T_0}}\ . \tag{9}$$

Considering that the flows of the test cases considered in this work are at low Mach numbers ($M_0 \leq 0,3$), the terms multiplied by a second order Mach number can be neglected. The resulting system of equations in dimensionless form, that describe the flows considered in the present work, is:

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u_i}{\partial x_i} = 0\ , \tag{10}$$

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial}{\partial x_j}\left\{\mu\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_l}{\partial x_l}\delta_{ij}\right]\right\} + \frac{1}{Fr}\rho g_i\ , \tag{11}$$

$$\rho\left(\frac{\partial T}{\partial t} + u_i\frac{\partial T}{\partial x_i}\right) = \frac{1}{Re\ Pr}\frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right)\ , \tag{12}$$

$$\rho = \frac{1}{1+T}\ , \tag{13}$$

with the numbers of Reynolds, Prandtl and Froude, represented by Re, Pr and Fr respectively, defined as functions of the reference values as follows:

$$Re = \frac{\rho_0 U_0 L_0}{\mu_0}\quad ,\quad Pr = \frac{C_p\ \mu_0}{k} = \frac{\nu_0}{\alpha}\quad \text{and}\quad Fr = \frac{U_0^2}{g_0\ L_0}\ . \tag{14}$$

In the present work, C_p, k, ν, g_i and the thermal diffusivity α are considered constants, and the reference values, indicated by the sub-index $_0$, are taken from the free flow or environmental conditions.

2.2. The turbulence model

As long as computational power today is not capable of representing all turbulent scales of the flow, the methodology adopted is a transformation of the system of instantaneous dimensionless governing equations, relations (10), (11), (12), (13), into a system of mean equations, obtained using a statistical treatment in the above equations.

In cases where the flow does not show significant variations on density, the Reynolds averaging is used. But when the variations in density are significant, the Reynolds averaging becomes inconvenient, due to the high number of unknown correlations resulted, leaving the closure problem of the system of governing equations without solution. The solution given by Favre (1965), for flows with considerable variation of density, uses the Reynolds averaging only for density and pressure, while for velocity and temperature, a mass-weighted averaging is adopted, called the Favre averaging (1965).

The closure of the mean equations is based on Boussinesq's (1877) hypothesis of eddy viscosity, adapted to variable density flows, by Jones and McGuirk (1979). It is important to note that the Favre averaging is equivalent to the Reynolds averaging in flows that does not have variations in density. For the velocity fluctuation correlation tensor, the Reynolds Stress Tensor, the closure takes the form

$$\bar{\rho}\overline{u''_i u''_j} = \frac{2}{3}\left(\bar{\rho}\kappa + \mu_t\frac{\partial\tilde{u}_l}{\partial x_l}\right)\delta_{ij} - \mu_t\left(\frac{\partial\tilde{u}_i}{\partial x_j} + \frac{\partial\tilde{u}_j}{\partial x_i}\right)\ , \tag{15}$$

where μ_t is the eddy viscosity and κ is the turbulence kinetic energy. For the velocity and temperature fluctuations correlation vector, interpreted as the turbulent flux of temperature, the closure adopted is the turbulent diffusivity hypotheses

$$\bar{\rho}\overline{u_i''\theta} = -\alpha_t \frac{\partial \widetilde{T}}{\partial x_i} = -\frac{\mu_t}{Pr_t}\frac{\partial \widetilde{T}}{\partial x_i} \ , \tag{16}$$

where $\widetilde{T}$ is the mean temperature from Favre's averaging, θ represents the temperature fluctuations, α_t is the turbulent thermal diffusivity and Pr_t is the turbulent Prandtl number, considered as a constant of value equal to 0.9 in the present work. In order to equations (15) and (16) turn possible to solve the closure problem of the system of mean equations, it is necessary to determine the value of the eddy viscosity μ_t. The form adopted in this work, to express the eddy viscosity μ_t as a function of the turbulence kinetic energy κ and the dissipation rate of turbulence kinetic energy ε , is using the Prandtl - Kolmogorov relation

$$\mu_t = C_\mu \bar{\rho}\frac{\kappa^2}{\varepsilon} = \frac{1}{Re_t} \ , \tag{17}$$

where C_μ is a constant of value 0,09. With the adoption of relation (17), the $\kappa - \varepsilon$ turbulence model relation imposes the necessity of two supplementary transport equations to the system of mean equations, destined to evaluation of variables κ and ε. Once defined the closure of the system of mean equations, the direction proposed by Brun (1988) produces the following system of equations:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\widetilde{u}_i}{\partial x_i} = 0 \ , \tag{18}$$

$$\bar{\rho}\left(\frac{\partial \widetilde{u}_i}{\partial t} + \widetilde{u}_j\frac{\partial \widetilde{u}_i}{\partial x_j}\right) = -\frac{\partial \bar{p}^*}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\left(\frac{1}{Re}+\frac{1}{Re_t}\right)\left(\frac{\partial \widetilde{u}_i}{\partial x_j}+\frac{\partial \widetilde{u}_j}{\partial x_i}\right)\right] + \frac{1}{Fr}\bar{\rho}g_i \ , \tag{19}$$

$$\bar{\rho}\left(\frac{\partial \widetilde{T}}{\partial t} + \widetilde{u}_j\frac{\partial \widetilde{T}}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left[\left(\frac{1}{Re\,Pr}+\frac{1}{Re_t\,Pr_t}\right)\frac{\partial \widetilde{T}}{\partial x_j}\right] \ , \tag{20}$$

$$\bar{\rho}\left(\frac{\partial \kappa}{\partial t} + \widetilde{u}_i\frac{\partial \kappa}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{Re}+\frac{1}{Re_t\,\sigma_\kappa}\right)\frac{\partial \kappa}{\partial x_i}\right] + \Pi - \bar{\rho}\varepsilon + \frac{\bar{\rho}\beta g_i}{Re_t\,Pr_t}\frac{\partial \widetilde{T}}{\partial x_i} \ , \tag{21}$$

$$\bar{\rho}\left(\frac{\partial \varepsilon}{\partial t} + \widetilde{u}_i\frac{\partial \varepsilon}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{Re}+\frac{1}{Re_t\,\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right] + \frac{\varepsilon}{\kappa}\left(C_{\varepsilon 1}\Pi - C_{\varepsilon 2}\bar{\rho}\varepsilon + C_{\varepsilon 3}\frac{\bar{\rho}\beta g_i}{Re_t\,Pr_t}\frac{\partial \widetilde{T}}{\partial x_i}\right) \ , \tag{22}$$

$$\bar{\rho} = \frac{1}{1+\widetilde{T}} \ , \tag{23}$$

where:

$$\frac{1}{Re_t} = C_\mu \bar{\rho}\frac{\kappa^2}{\varepsilon} \ , \tag{24}$$

$$\Pi = \left[\left(\frac{1}{Re_t}\right)\left(\frac{\partial \widetilde{u}_i}{\partial x_j}+\frac{\partial \widetilde{u}_j}{\partial x_i}\right) - \frac{2}{3}\left(\bar{\rho}\kappa + \frac{1}{Re_t}\frac{\partial \widetilde{u}_l}{\partial x_l}\right)\delta_{ij}\right]\frac{\partial \widetilde{u}_i}{\partial x_j} \ , \tag{25}$$

$$p^* = \bar{p} + \frac{2}{3}\left[\left(\frac{1}{Re}+\frac{1}{Re_t}\right)\frac{\partial \widetilde{u}_l}{\partial x_l} + \bar{\rho}\kappa\right] \ , \tag{26}$$

and the constants of the model, according to Launder and Spalding (1974), are given:

$$C_\mu = 0,09 \ , \ C_{\varepsilon 1} = 1,44 \ , \ C_{\varepsilon 2} = 1,92 \ , \ C_{\varepsilon 3} = 0,288 \ , \ \sigma_\kappa = 1 \ , \ \sigma_\varepsilon = 1,3 \ , \ Pr_t = 0,9 \ . \tag{27}$$

2.3. Near wall treatment

The $\kappa - \varepsilon$ turbulence model is incapable of properly representing the laminar sub-layer and the transition regions of the turbulent boundary layer. To solve this inconvenience, the solution adopted in this work is the use of laws of the wall for temperature and for velocity, capable of properly representing the flow in the inner region of the turbulent boundary layer.

There are four velocity and two temperature laws of he wall implemented on Turbo 2D. To confront the results of simulations using the wall functions proposed by Cruz and Silva Freire (1998), the classic log-law for velocity and for temperature, the velocity law of the wall of Mellor (1966) and the velocity law of the wall proposed by Nakayama and Koyama (1984) were also used, and these results were then compared to the experimental data and with results from algorithms that use other methodologies, like $\kappa - \omega$ and Large Eddy Simulation, made available in the literature.

2.3.1. Velocity law of the wall of Mellor(1966)

Deduced from the equation of Prandtl for the boundary layer flow, considering the pressure gradient term for integration, this wall function is a primary approach to flows that suffer influence of adverse pressure gradients. Its equations are, respectively, for the laminar and turbulent region

$$u^* = y^* + \frac{1}{2} p^* y^{*2} \ , \tag{28}$$

$$u^* = \frac{2}{K}\left(\sqrt{1+p^*y^*} - 1\right) + \frac{1}{K}\left(\frac{4y^*}{2+p^*y^* + 2\sqrt{1+p^*y^*}}\right) + \xi_{p^*} \ , \tag{29}$$

where the asterisk super-index indicates dimensionless quantities of velocity u^*, pressure gradient p^* and distance to the wall y^*, as functions of scaling parameters to the near wall region, K is the Von Karman constant, and ξ_{p^*} is Mellor's integration constant, function of the near-wall dimensionless pressure gradient, determined in his work of (1966).

The intersection of both regions is considered to be the same as the log law expressions, with $y^* = 11,64$. The relations between the dimensionless near wall properties and the friction velocity u_f are:

$$y^* = \frac{y\, u_f}{\nu} \quad , \quad u^* = \frac{\widetilde{u}_x}{u_f} \quad \text{and} \quad p^* = \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x}\frac{\nu}{{u_f}^3} \ . \tag{30}$$

In equation (29) the term ξ_{p^*} is a value obtained from the integration process proposed by Mellor (1966) and is a function of the dimensionless pressure gradient. Its values are obtained through interpolation of those obtained experimentally by Mellor, shown in table (1).

Table 1: Mellor's integration constant (1966)

p^*	$-0,01$	$0,00$	$0,02$	$0,05$	$0,10$	$0,20$	$0,25$	$0,33$	$0,50$	$1,00$	$2,00$	$10,00$
ξ_{p^*}	$4,92$	$4,90$	$4,94$	$5,06$	$5,26$	$5,63$	$5,78$	$6,03$	$6,44$	$7,34$	$8,49$	$12,13$

2.3.2. Velocity law of the wall of Nakayama and Koyama (1984)

In their work, Nakayama and Koyama (1984) proposed a derivation of the mean turbulent kinetic energy equation, that resulted in an expression to evaluate the velocity near solid boundaries. Using experimental results and those obtained by Strattford (1959), the derived equation is

$$u^* = \frac{1}{K^*}\left[3(t - t_s) + ln\left(\frac{t_s+1}{t_s-1}\frac{t-1}{t+1}\right)\right] \ , \tag{31}$$

with

$$t = \sqrt{\frac{1+2\tau^*}{3}} \quad , \quad \tau^* = 1 + p^*y^* \quad , \quad K^* = \frac{0,419 + 0,539p^*}{1+p^*} \quad and \quad {y^*}_s = \frac{e^{K\ C}}{1+p^{*0,34}} \ , \tag{32}$$

where K^* is the expression for the Von Karman constant modified by the presence of adverse pressure gradients, τ^* is a dimensionless shear stress, $C = 5,445$ is the log-law constant and t, ${y^*}_s$ and t_s, a value of t at position ${y^*}_s$, are parameters of the function.

2.3.3. Velocity law of the wall of Cruz and Silva Freire (1998)

Analyzing the asymptotic behavior of the boundary layer flow under adverse pressure gradients, Cruz and Silva Freire (1998) derived an expression for the velocity. The solution of the asymptotic approach is

$$u = \frac{\tau_w}{|\tau_w|}\frac{2}{K}\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dp_w}{dx}y} + \frac{\tau_w}{|\tau_w|}\frac{u_f}{K} ln\left(\frac{y}{L_c}\right) \quad with \quad L_c = \frac{\sqrt{\left(\frac{\tau_w}{\rho}\right)^2 + 2\frac{\nu}{\rho}\frac{dp_w}{dx}u_R} - \frac{\tau_w}{\rho}}{\frac{1}{\rho}\frac{dp_w}{dx}} \ , \tag{33}$$

where the sub-index $_w$ indicates the properties at the wall, K is the Von Karman constant, equal to 0.4, L_c is a length scale parameter and u_R is a reference velocity, evaluated from the highest positive root of the asymptotic relation:

$${u_R}^3 - \frac{\tau_w}{\rho}u_R - \frac{\nu}{\rho}\frac{\partial p}{\partial x} = 0 \ . \tag{34}$$

The proposed equation for the velocity (33) has a behavior similar to the log law far from the separation and reattachment points, but close to the adverse pressure gradient, it gradually tends to Stratford's equation (1959). The same process was used to derive the temperature law of the wall by Cruz and Silva Freire (1998).

2.3.4. Temperature log law of the wall of Cheng and Ng (1982)

For the calculation of the temperature, Cheng and Ng (1982) derived an expression for the near wall temperature similar to the log law of the wall for velocity. For the laminar and turbulent regions, the equations are respectively

$$\frac{(T_0 - T)_y}{T_f} = y^* \, Pr \quad \text{and} \quad \frac{(T_0 - T)_y}{T_f} = \frac{1}{K_{Ng}} ln(y^*) + C_{Ng} \ , \tag{35}$$

where T_0 is the environmental temperature and T_f is the friction temperature. The intersection of these regions are at $y^* = 15,96$, and the constants K_{Ng} and C_{Ng} are, respectively, 0,8 and 12,5.

2.3.5. Temperature law of the wall of Cruz and Silva Freire (1998)

Using the same arguments of the velocity law of the wall, Cruz and Silva Freire (1998) derived an expression for the friction temperature for thermal flows under adverse pressure gradients. The expression is:

$$\frac{T_w - T}{Q_w} = \frac{Pr_t}{K\rho C_p u_f} \, ln \, \frac{\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dp_w}{dx} y} - \sqrt{\frac{\tau_w}{\rho}}}{\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dp_w}{dx} y} + \sqrt{\frac{\tau_w}{\rho}}} + \frac{Pr_t}{K\rho C_p u_R} \, ln \, \frac{4Eu_R^3}{\nu |\frac{dp_w}{dx}|} + AJ \ , \tag{36}$$

with the parameters defined as:

$$AJ = 1.11 Pr_t \sqrt{\frac{AX}{K}} \left(\frac{Pr}{Pr_t} - 1 \right) \left(\frac{Pr}{Pr_t} \right)^{0,25} \quad and \quad AX = 26 \frac{|\frac{\tau_w}{\rho}|^{\frac{1}{2}}}{u_R} \ . \tag{37}$$

In the above equations, Q_w is the wall heat flux and E is a constant of the equation, with value equal to 9.8. The friction temperature is defined as:

$$T^* = \frac{Q_w}{\rho C_p u_f} \tag{38}$$

3. Numerical methodology

The numerical solution of the proposed system of governing equations, of a dilatable turbulent flow, has as main difficulties: the coupling between all equations; the non-linear behavior resulting of the simultaneous action of convective and eddy viscosity terms; the explicit calculations of boundary conditions in the solid boundary; the methodology of use the continuity equation as a manner to link the coupling fields of velocity and pressure.

The solution proposed in the present work suggests a temporal discretization of the system of governing equations with a sequential semi-implicit finite difference algorithm proposed by Brun (1988) and a spatial discretization using finite elements of the type P1-isoP2. For more, on temporal and spatial discretization implemented in Turbo 2D, the reader should consult Soares and Fontoura Rodrigues (2003).

4. Numerical results

Two cases were simulated to test the performances of the velocity and thermal laws of the wall. To test the performance of the velocity wall function, the asymmetric plane difuser of Buice and Eaton (1996) was chosen. To evaluate the influence of a thermal field in a separating flow, the thermal backward facing step of Vogel and Eaton (1984) was chosen, for it is the first serious experimental case of thermal separating flow intended to serve as a comparison for numerical simulation. For both cases, the boundary conditions were fully developed turbulent flow in the inlet, environment values at the outlet and the wall functions to perform the solid boundaries conditions.

4.1. Assimetric plane diffuser of Buice and Eaton (1996)

In order to test the performance of the velocity law of the wall proposed by Cruz and Silva Freire (1998) in detecting the presence of smooth adverse pressure gradients, but capable of generating a separation of the boundary layer, the case of the two-dimensional flow in the asymmetric plane diffuser was chosen.

This case has as main characteristics the precise reference values and experimental measurements, the separation of the boundary layer is exclusively due to the presence of a smooth adverse pressure gradient and the possibility to study with great trustability the detachment and the redeveloping processes of the turbulent boundary layer. The difficulty of obtaining these results from numerical simulation, based on the results obtained by Ataídes and Fontoura Rodrigues (2002), justify the recommendation made at the 8^{th} ERCOFTAC (1999) to use this case as a standard in the simulation of parietal turbulent flows.

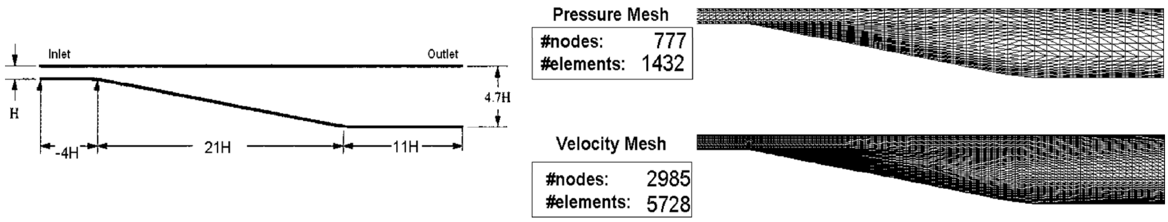

Figure 1: The assymetric plane diffuser of Buice and Eaton (1996) and the finite element meshes

The turbulent flow developed in the diffuser section is isothermal, with a Reynolds number equal to 20.000, based on the height of the inlet section. The reference velocity is equal to 20,8 m/s. The spatial discretization can be seen in figure (1).

In figures (2) and (3), the velocity profiles obtained with the simulation with four different wall functions implemented on Turbo 2D are shown, the log law and the laws of the wall proposed by Mellor (1965), Nakayama and Koyama (1984) and Cruz and Silva Freire (1998). In the velocity axis, on the bottom of those figures, the Δ is a parameter to correct the indicated values of velocity, equal to the bottom axis value that corresponds to the location of the profile, indicated in the upper axis. For example, the second profile of figure (2) is located at 2.59H. The corresponding value in the bottom axis is "1", so $\Delta = 1$ and the maximum relative velocity is approximately 0.90.

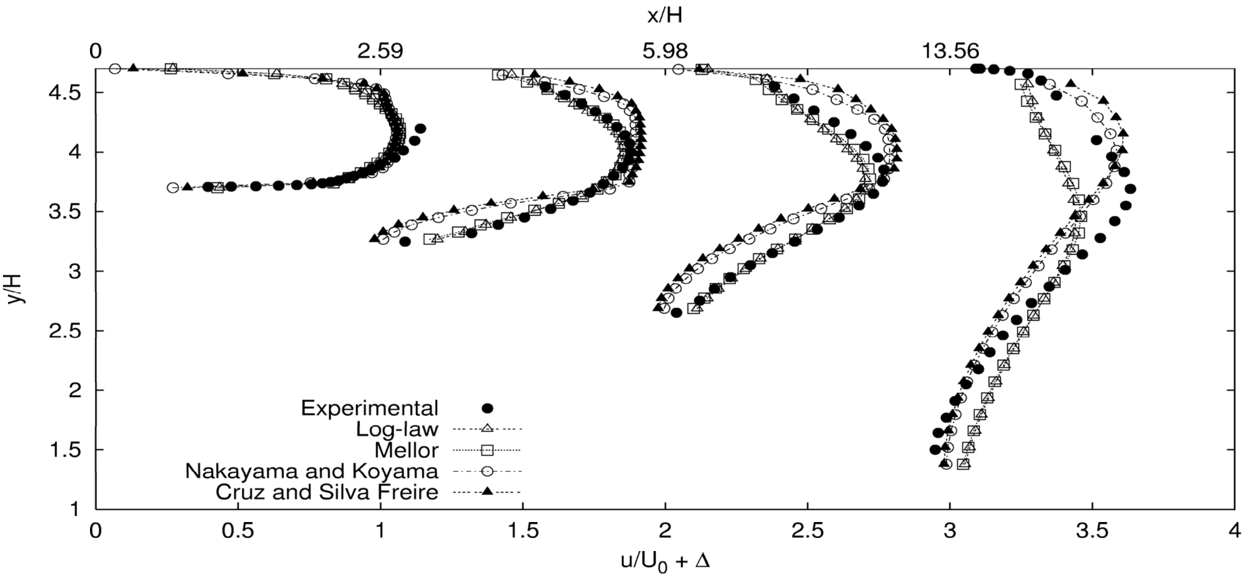

Figure 2: Velocity profiles in the asymmetric plane diffuser

These results show that only the laws of the wall of Nakayama and Koyama (1984) and Cruz and Silva Freire (1998) are able to perceive the detachment of the turbulent boundary layer. Comparing these two profiles, a slight advantage from Cruz and Silva Freire can be seen, for it reaches better values at the near wall and the maximum velocity regions. The results obtained with the other wall functions show that they cannot detect the detachment of the boundary layer, induced by the smooth adverse pressure gradient formed in the diffuser section.

The profiles of the pressure coefficient C_p, figure (4), show that the wall laws of Nakayama and Koyama (1984) and Cruz and Silva Freire (1998) could predict more precisely the pressure field, and as a consequence, the detachment of the boundary layer.

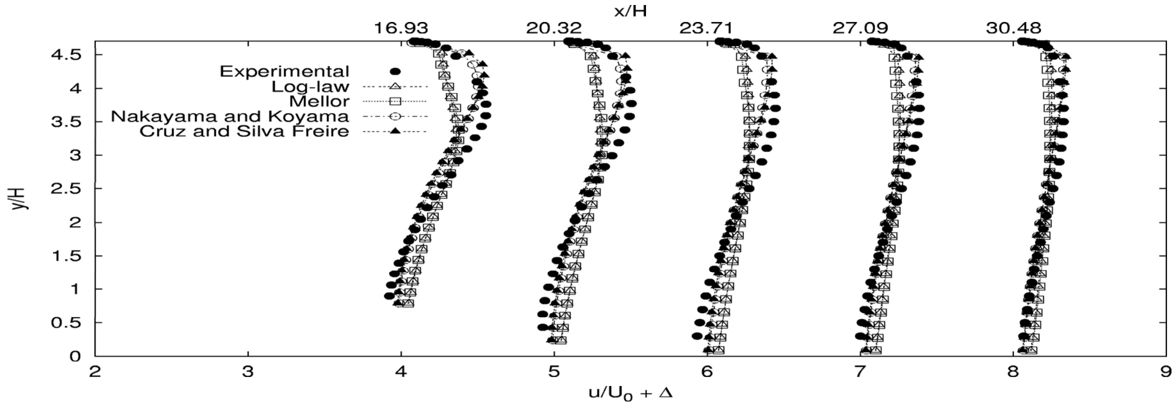

Figure 3: Velocity profiles in the asymmetric plane diffuser

A comparison with results achieved by the code WIND of NASA's NPARC Alliance CFD Group can be seen at table (2), where x_d and x_r are respectively the location where the detachment and the reattachment of the recirculating zone occur, and H is the height of the inlet. These results, presented in table (2), show that the length of the recirculating zone obtained with wall functions are closer to experimental data than those obtained with other models. Also, the detachment point obtained with all models presented are underestimated comparing to experimental data, being the result of Spallart-Almaras (1992) the closest the experimental data.

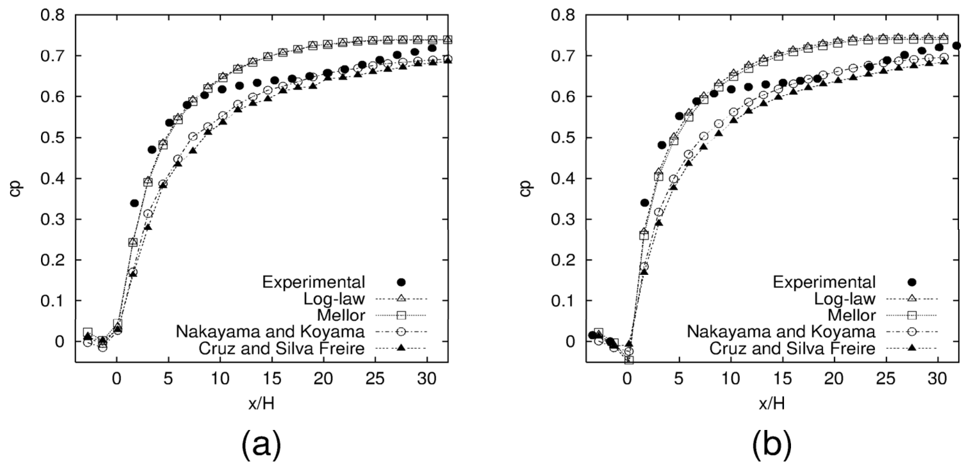

Figure 4: Dimensionless pressure coefficient at (a) upper wall and (b) lower wall

Table 2: Comparison with other models from NASA's WIND

Model	**Detachment point** $(\mathbf{x_d/H})$	**Reattachment point** $(\mathbf{x_r/H})$	**Recirculating zone** $((\mathbf{x_r - x_d})/\mathbf{H})$
WIND Chien $\kappa - \varepsilon$	0,7583	20,943	20,185
WIND SST	1,9168	30,6793	28,76
WIND Spalart-Allmaras	4,0377	32,680	28,64
TURBO 2D - Nakayama and Koyama	2,04	22,2	20,16
TURBO 2D - Cruz and Silva Freire	1,06	24,1	23,04
Experimental	**6,00**	**29,5**	**23,5**

4.2. Thermal Backward facing step of Vogel and Eaton (1984)

The objective of the wall function for temperature of Cruz and Silva Freire (1998) is to represent the influence of the pressure gradient on the temperature field, established by forced convection, in a turbulent flow. To evaluate the intended behavior of the formulation proposed, its performance is compared to those obtained with the temperature law of the wall proposed by Cheng and Ng (1982), that does not consider any effects of the pressure gradient directly on the temperature field. The test case chosen to accomplish this comparison, of Vogel and Eaton (1984), displays the flow through a backward facing step with the lower wall heated by a specified heat flux.

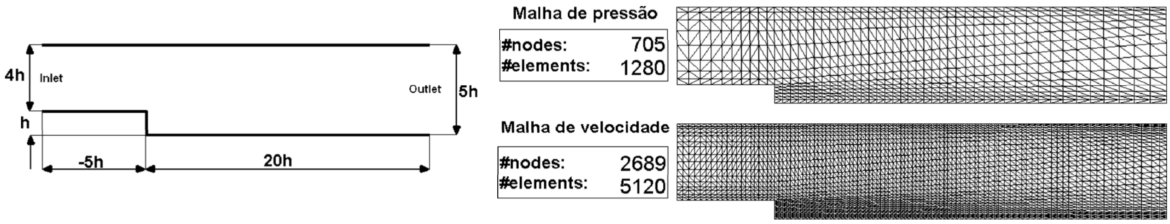

Figure 5: The thermal backward facing step of Vogel and Eaton (1984)

The Reynolds number of this case is 27.600, based on the reference velocity equal to 11.3 m/s and on the backward step height of 0,0381 m, both at the inlet. Figure (5) shows a diagram of the case and the finite element meshes used in the simulation. The heat flux is inserted in the domain through the bottom horizontal wall, right after the step, and its value is 270 W/m^2. At the inlet, an isothermal fully developed turbulent flow is imposed, and at the outlet, zero pressure condition is specified.

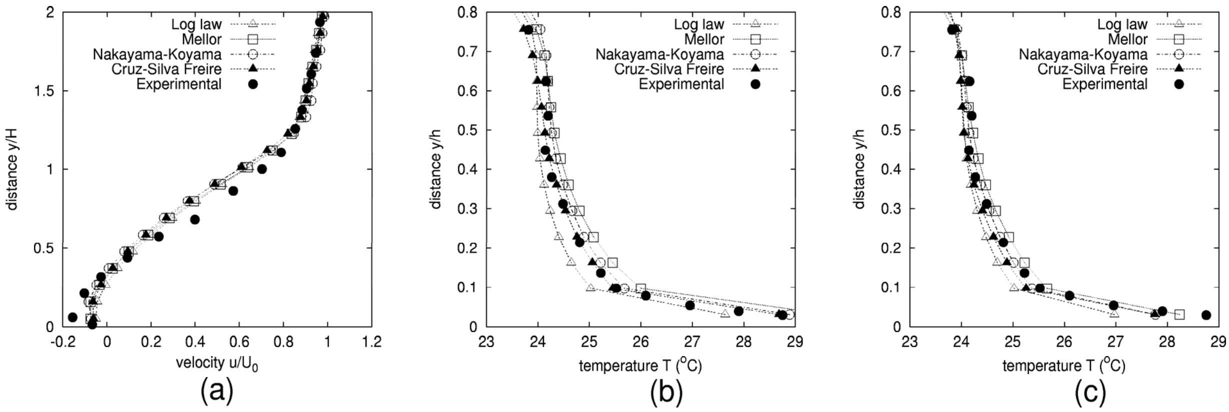

Figure 6: (a) Velocity profiles at x=4,66h; (b) Temperature profiles with each velocity wall function and the thermal log-law (c) Temperature profiles with each velocity wall function and the thermal law by Cruz and Silva Freire (1998)

In order to perform the test of the thermal law of the wall proposed by Cruz and Silva Freire (1998), several profiles were taken, as shown in figure (6). Figure (6a) shows the velocity profile at 4,66 step heights from the location of the backward facing step, where the velocity profiles achieved with all laws of the wall are almost identical. It must be marked that once the convective forces are predominant over the buoyancy effects, there was no noticeable difference in the velocity field by changing the thermal wall function, and as these velocity profiles are almost the same, this is a good case to test the performance of the temperature wall laws.

Table 3: Reattachment length for the thermal backward facing step of Vogel and Eaton (1984)

Model	Reattachment length $\mathbf{x_r/h}$
TURBO 2D - Log	5,15
TURBO 2D - Mellor	5,65
TURBO 2D - Nakayama e Koyama	5,95
TURBO 2D - Cruz e Silva Freire	6,05
Experimental	**6,66**

In figures (6b) and (6c) it is made clear the effects of the velocity field over the temperature profile, and it is noticeable that for the same velocity wall function used, there are two different temperature profiles, clearly not proportional one to the other. It is noted that, by using the temperature law of the wall proposed by Cruz and Silva Freire (1998), there was an improvement to all temperature profiles.

Table (3) concludes the analysis presented in this work by showing that the reattachment length obtained wall function proposed by Cruz and Silva Freire (1998) is a little closer to the experimental data than the length obtained with the one proposed by Nakayama and Koyama.

5. Conclusions

The velocity law of the wall proposed by Cruz and Silva Freire (1998) is very robust, as it provided the closer results from the experimental data on both cases analyzed in this work. It was able not only to predict the separation in the diffuser, but also to get the best result for the length of the recirculating zone of the diffuser. Even with the improvements of other turbulence models, like the SST of Menter (1993) and the model of Spalart Allmaras (1992), the use of wall laws is a way of avoiding the use of expensive turbulence models while still providing good agreement with the experimental data. The expression proposed by Cruz and Silva Freire (1998) for the thermal law of the wall not only shows good results, but instigate further research on thermal boundary layers subjected to adverse pressure gradients. A more detailed investigation will be the topic of further works, associated with studies of models for the transport of temperature fluctuations.

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7. References

Ataídes, R.S.C. & Fontoura Rodrigues J.L.A., 2002. "Estudo computacional de escoamentos turbulentos com descolamento de camada limite". 9th ENCIT, Caxambu, Brasil.

Brun, G., 1988. "Developpement et application d'une methode d'elements finis pour le calcul des ecoulements turbulents fortement chauffes ". Tese de Doutorado, Laboratoire de Mécanique des Fluides, E.C. Lyon.

Buffat, M. 1981. "Formulation moindres carrés adaptées au traitement des effets convectifs dans les équations de Navier-Stokes". PhD thesis, Université Claude Bernard, Lyon, France.

Buice, C. & Eaton, J., 1996. "Experimental investigationof flow through an asymmetric plane diffuser". Annual Research Briefs - 1995, Center of Turbulence Research, Stanford University/ NASA Ames. 117-120.

Cheng, R.K. & Ng, T.T., 1982. "Some aspects of strongly heated turbulent boundary layer flow". Physics of Fluids, vol. 25(8).

Cruz D.O.A. & A.P. Silva Freire. 1998, "On single limits and the asymptotic behaviour of separating turbulent boundary layers". International Journal of Heat and Mass transfer. Vol. 41 , no. 14, pp. 2097-2111.

Cruz D.O.A. & A.P. Silva Freire. 2002, "Note on a thermal law of the wall for separating and recirculating flows". International Journal of Heat and Mass transfer. Vol. 45 , pp. 1459-1465.

DalBello, T., 2003, "NPARC Alliance - Buice 2D Diffuser", NASA John H. Glenn Research Center.

Favre, A., 1965. "Equations de gaz turbulents compressibles". Journal de Mecanique, vol. 3 e vol. 4.

Fontoura Rodrigues, J.L.A., 1991. "Um método de mínimo resíduo adaptado ao cálculo de condições de contorno não-lineares no escoamento turbulento bidimensional". XI COBEM, pp 465-468.

Launder, B.E., Spalding, D.B., 1974. "The numerical computation of turbulent flows". Computational Methods Applied to Mechanical Engineering, vol. 3, pp. 269-289.

Mellor, G.L., 1966. "The effects of pressure gradients on turbulent flow near a smooth wall". Journal of Fluid Mechanics, vol. 24, part 2.

Menter, F.R., 1993. "Zona two equation $\kappa - \omega$ turbulence models for aerodynamic flows". AIAA, vol. 93-2906.

Nakayama, A. & Koyama, H., 1984. "A wall law for turbulent boundary layers in adverse pressure gradients". AIAA Journal, vol. 22, no 10.

Soares, D. V. , Ataídes, R. S. C. & Fontoura Rodrigues, J. L. A., 2003. "Numerical simulation of a turbulent flow over a heated wall". 17th COBEM, São Paulo, Brasil

Spalart, P.R. and Allmaras, S.R., 1992, "A one-equation turbulence model for aerodynamic fows", AIAA Paper, 92-0439.

Stratford, B.S., 1959. "The prediction of separation of turbulent boundary layer". Journal of Fluid Mechanics, vol. 5.

Vogel, J. C. & Eaton, J.K., 1985. "Combined heat transfer and fluid dynamic measurements downstream of a backward facing step". Journal of Heat transfer. Vol. 107, pp. 922-929.

Zandonade, P. S. K. & Fontoura Rodrigues, J.L.A., 2000. "Nonlinear models for the Reynolds tensor in the simulation of wall-bounded flows". 8th ENCIT - Porto Alegre- Brasil.