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INTEGRAL TRANSFORM ANALYSIS OF FLOW DEVELOPMENT IN PARALLEL-PLATES DUCTS

Roseane de Lima Silva
João Nazareno Nonato Quaresma
Chemical and Food Engineering Department, CT, Universidade Federal do Pará, UFPA
Campus Universitário do Guamá, 66075-110, Belém, PA
quaresma@ufpa.br

Carlos Antônio Cabral dos Santos
Solar Energy Laboratory, LES, Mechanical Engineering Dept., DTM, Universidade Federal da Paraíba, UFPB
Cidade Universitária, 58059-900, João Pessoa, PB
cabral@les.ufpb.br

Abstract. *The so-called Generalized Integral Transform Technique (GITT) is employed in the analysis of two-dimensional laminar flow in the entrance region of parallel-plates ducts. A formulation in terms of primitive variables is adopted and, expansions for the velocity field are proposed in such way that the continuity equation is automatically satisfied, after that the integral transformation process leads to a coupled system of ordinary differential equations formulation similar to one when the streamfunction formulation is employed. Results for the velocity and the product of the friction factor-Reynolds number are computed and critically compared with those in the literature.*

Keywords. *Flow development, parallel-plates ducts, Integral transform.*

1. Introduction

The laminar flow and heat transfer inside ducts are of great applications in most of the industrial thermal facilities. Equipment like heat exchangers, condensers and combustible elements of nuclear reactors are typical examples of these applications. For the project and optimization of such devices, information about the flow and heat transfer is of the fundamental importance. The full incompressible Navier-Stokes equations are sometimes employed in the mathematical formulation of such problems, instead of their simplified forms, the boundary-layer equations. Because of this, the velocity field analysis in the developing region of straight channels becomes prohibitive through analytical methodologies.

In this context, purely numerical techniques have been used in such analysis. A literature review reveals contributions of authors that treated of the laminar flow in the inlet region of straight ducts, such the parallel-plates channels, for low Reynolds numbers. The works of Wang and Longwell (1964), Brandt and Gillis (1966) and McDonald et al. (1972) who employed different versions of the finite difference method and Comini and Del Giudice (1982), which used the finite element method are considered classical examples that dealt with this type flow.

More recently, the ideas in the Generalized Integral Transform Technique (GITT) were extended for the solution of the Navier-Stokes equation in incompressible steady state flow within a parallel-plates channel (Pérez Guerrero and Cotta, 1995). The authors considered a streamfunction only formulation with two inlet flow conditions, i.e., uniform parallel flow and uniform irrotational flow and, in addition, the outflow boundary conditions were handled via consideration of a fully developed velocity profile at a sufficiently large truncated duct length.

In this context, the present study is also motivated by using the GITT approach in the solution of the Navier-Stokes equations in the same physical problem analyzed by Pérez Guerrero and Cotta (1995). The differential aspect is that the present work employs a formulation in terms of primitive variables, with eigenfunction expansions for the components of the velocity field, in such a way that the continuity equation is automatically satisfied, and after the integral transformation process, it is obtained a coupled system of ordinary differential equations similar to one when the streamfunction formulation is used. However, this type of formulation can be attractive for the solution of the Navier-Stokes equations in three-dimensional problems, avoiding more involved formulations as the scalar and vector potentials ones (Quaresma and Cotta, 1997). Therefore, the computational algorithm is tested by comparing the present results for the velocity field and for the product of the friction factor-Reynolds number with the available ones in the literature.

2. Mathematical formulation

Developing laminar flow of an incompressible Newtonian fluid within a parallel-plates channel according to Fig. (1) is considered, with the respective inlet and boundary conditions. Thus, the flow is governed by the continuity and Navier-Stokes equations in the primitive variables formulation, which for this problem are written in dimensionless form in the region $-1 < y < 1$ and $x > 0$, respectively, as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1a}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{4}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{1b}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{4}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{1c}$$

with boundary conditions

$$u=1;\quad v=0\quad \text{at}\quad x=0 \tag{2a,b}$$

$$u=u_\infty(y)=\frac{3}{2}\left(1-y^2\right);\quad v=0\quad \text{as}\quad x\to\infty \tag{2c,d}$$

$$u=v=0\quad \text{at}\quad y=-1 \tag{2e,f}$$

$$u=v=0\quad \text{at}\quad y=1 \tag{2g,h}$$

The dimensionless groups employed in equations above are defined as

$$x=x^*/b;\quad y=y^*/b;\quad u=u^*/u_0;\quad v=v^*/u_0;\quad p=p^*/\rho u_0^2;\quad Re=\frac{4u_0 b}{\nu} \tag{3a-f}$$

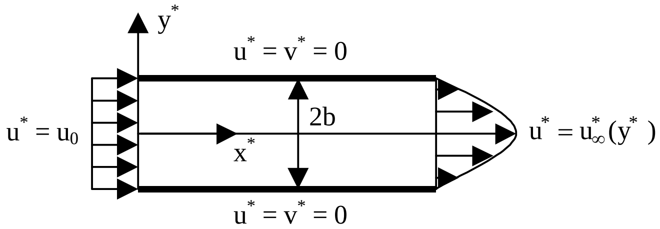

Figure 1. Geometry and coordinate system for hydrodynamic developing rectangular duct flow.

It is taken the derivative of Eq. (1b) in relation to y and of Eq. (1c) in relation to x and the results are subtracted, so that the pressure field is eliminated, to yield

$$u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}-u\frac{\partial^2 v}{\partial x^2}-v\frac{\partial^2 v}{\partial x\partial y}=\frac{4}{Re}\left(\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 u}{\partial y^3}-\frac{\partial^3 v}{\partial x^3}-\frac{\partial^3 v}{\partial x\partial y^2}\right) \tag{4}$$

In order to improve the series convergence, a "filter" is proposed, which reproduces the solution of the fully developed flow and makes homogeneous the boundary conditions in the y direction, one chosen for the process of integral transformation

$$u(x,y)=u_\infty(y)+u_F(x,y) \tag{5}$$

where $u_\infty(y)$ represents the fully developed velocity profile, $u_F(x,y)$ is the velocity potential to be solved, and $u(x,y)$ is the original velocity potential in the longitudinal direction.

The continuity and momentum equations, Eqs. (1a) and (4), respectively, after the application of the separation process for the velocity potential given by Eq. (5), are then rewritten as

$$\frac{\partial u_F}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$u_F\frac{\partial^2 u_F}{\partial x\partial y}+u_\infty\frac{\partial^2 u_F}{\partial x\partial y}+v\frac{\partial^2 u_F}{\partial y^2}+v\frac{d^2 u_\infty}{dy^2}-u_F\frac{\partial^2 v}{\partial x^2}-u_\infty\frac{\partial^2 v}{\partial x^2}-v\frac{\partial^2 v}{\partial x\partial y}=\frac{4}{Re}\left(\frac{\partial^3 u_F}{\partial x^2\partial y}+\frac{\partial^3 u_F}{\partial y^3}-\frac{\partial^3 v}{\partial x^3}-\frac{\partial^3 v}{\partial x\partial y^2}\right) \tag{7}$$

with boundary conditions

$$u_F = 1 - u_\infty; \quad v = 0 \qquad \text{at} \quad x = 0 \tag{8a,b}$$

$$u_F = 0; \quad v = 0 \qquad \text{as} \quad x \to \infty \tag{8c,d}$$

$$u_F = 0; \quad v = 0 \qquad \text{at} \quad y = -1 \tag{8e,f}$$

$$u_F = 0; \quad v = 0 \qquad \text{at} \quad y = 1 \tag{8g,h}$$

3. Solution methodology

In the light of applying the GITT approach in the solution of the PDE system given by Eqs. (6) and (7), due to homogeneous characteristics of the boundary conditions in the y direction, it is more appropriate to choose this direction for the process of integral transformation, and the auxiliary eigenvalue problem is taken as (Pérez Guerrero and Cotta, 1995)

$$\frac{d^4 Y_i(y)}{dy^4} = \mu_i^4 Y_i(y), \quad -1 < y < 1 \tag{9a}$$

with boundary conditions

$$Y_i(-1) = 0; \quad \frac{dY_i(-1)}{dy} = 0\,; \quad Y_i(1) = 0; \quad \frac{dY_i(1)}{dy} = 0 \tag{9b-e}$$

where $Y_i(y)$ and μ_i are, respectively, the eigenfunctions and eigenvalues of the problem (9a), which satisfy the following orthogonality property:

$$\int_{-1}^{1} Y_i(y) Y_j(y) dy = \begin{cases} 0, & i \neq j \\ NV_i, & i = j \end{cases} \tag{10}$$

The normalization integral is defined as

$$NV_i = \int_{-1}^{1} Y_i^2(y) dy \tag{11}$$

Problem (9a) can be analytically solved to yield

$$Y_i(y) = \begin{cases} \dfrac{\cos(\mu_i y)}{\cos(\mu_i)} - \dfrac{\cosh(\mu_i y)}{\cosh(\mu_i)}, & i=1,3,5,... \\[2ex] \dfrac{\sin(\mu_i y)}{\sin(\mu_i)} - \dfrac{\sinh(\mu_i y)}{\sinh(\mu_i)}, & i=2,4,6,... \end{cases} \tag{12a,b}$$

The eigenvalues μ_i are obtained from the following transcendental equation:

$$\tanh \mu_i = \begin{cases} -\tan \mu_i, & i=1,3,5,... \\ \tan \mu_i, & i=2,4,6,... \end{cases} \tag{13a,b}$$

while the normalization integral is computed as

$$NV_i = 2 \tag{13c}$$

The eigenvalue problem defined by Eq. (9a) allows the definition of the following integral transform pair:

$$\frac{d\bar{u}_i(x)}{dx} = \int_{-1}^{1} \tilde{Y}_i(y) v(x,y)\, dy \,, \qquad \text{transform} \tag{14}$$

$$u_F(x,y) = \sum_{i=1}^{\infty} \tilde{Y}_i'(y) \bar{u}_i(x) \,, \qquad \text{inverse for } u_F \tag{15}$$

$$v(x,y) = -\sum_{i=1}^{\infty} \tilde{Y}_i(y) \frac{d\bar{u}_i(x)}{dx} \,, \qquad \text{inverse for v} \tag{16}$$

where $\tilde{Y}_i(y)$ is the normalized eigenfunction

$$\tilde{Y}_i(y) = \frac{Y_i(y)}{\sqrt{NV_i}} \tag{17}$$

Equations (15) and (16) represent the eigenfunction expansions for the components of the velocity field, in such a way that the continuity equation (Eq. (6)) is automatically satisfied (Pérez Guerrero et al., 1998).

The process of integral transformation is made as follows: Eq. (7) is multiplied by $\tilde{Y}_i(y)$, and is then integrated over the domain [-1,1] in y, after that the inverse formulae given by Eqs. (15) and (16) are employed, resulting in

$$\frac{d^4\bar{u}_i}{dx^4} = -\mu_i^4 \bar{u}_i - 2\sum_{j=1}^{\infty} A_{ij} \frac{d^2\bar{u}_j}{dx^2} + \frac{4}{Re}\left\{ \sum_{j=1}^{\infty} \left[\left(B_{ij} - C_{ij} \right) \frac{d\bar{u}_j}{dx} + D_{ij} \frac{d^3\bar{u}_j}{dx^3} \right] \right.$$

$$\left. + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \left(E_{ijk} \bar{u}_j \frac{d\bar{u}_k}{dx} - F_{ijk} \frac{d\bar{u}_j}{dx} \bar{u}_k + G_{ijk} \frac{d^3\bar{u}_j}{dx^3} \bar{u}_k - G_{ijk} \frac{d\bar{u}_j}{dx} \frac{d^2\bar{u}_k}{dx^2} \right) \right\} \tag{18a}$$

where the coefficients in Eq. (18a) are given by

$$A_{ij} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j'' dy \,; \quad B_{ij} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j'' u_\infty dy \,; \quad C_{ij} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j u_\infty'' dy \,; \quad D_{ij} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j u_\infty dy \tag{18b-e}$$

$$E_{ijk} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j' \tilde{Y}_k'' dy \,; \quad F_{ijk} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j \tilde{Y}_k''' dy \,; \quad G_{ijk} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j \tilde{Y}_k' dy \tag{18f-h}$$

The same process of integral transformation is accomplished in the boundary conditions in the direction x, Eqs. (8a-d), to furnish

$$\sum_{j=1}^{\infty} H_{ij} \bar{u}_j(0) = I_i \,; \quad \frac{d\bar{u}_i(0)}{dx} = 0 \,; \quad \sum_{j=1}^{\infty} H_{ij} \bar{u}_j(\infty) = 0; \quad \frac{d\bar{u}_i(\infty)}{dx} = 0 \tag{19a-d}$$

where

$$H_{ij} = \int_{-1}^{1} \tilde{Y}_i \tilde{Y}_j' dy \,; \quad I_i = \int_{-1}^{1} \tilde{Y}_i (1 - u_\infty) dy \tag{20a,b}$$

For the computational implementation of the transformed system given by Eqs. (18) to (20), the summations should be truncated in orders NTV sufficiently high to guarantee results completely converged within a prescribed accuracy.

Therefore to solve the system by efficient numerical algorithms for boundary value problems, such as the subroutine DBVPFD from the IMSL library (1991), which offers an automatic adaptive scheme for local error control of the results for the transformed potentials, it is necessary rewritten the system as a first order one, in the form

$$\bar{u}_i = \chi_i \,, \quad i = 1,2,3,\ldots,NTV \tag{21a}$$

$$\frac{d\bar{u}_i}{dx} = \frac{d\chi_i}{dx} = \chi_{NTV+i} \,; \quad \frac{d^2\bar{u}_i}{dx^2} = \frac{d}{dx}\left(\frac{d\bar{u}_i}{dx}\right) = \frac{d\chi_{NTV+i}}{dx} = \chi_{2NTV+i} \tag{21b,c}$$

$$\frac{d^3\bar{u}_i}{dx^3} = \frac{d}{dx}\left(\frac{d^2\bar{u}_i}{dx^2}\right) = \frac{d\chi_{2NTV+i}}{dx} = \chi_{3NTV+i} \,; \quad \frac{d^4\bar{u}_i}{dx^4} = \frac{d}{dx}\left(\frac{d^3\bar{u}_i}{dx^3}\right) = \frac{d\chi_{3NTV+i}}{dx} \tag{21d,e}$$

Now, introducing Eqs. (21) into Eqs. (18a) and (19), and truncating the infinite summations in the order NTV for the velocity field, we obtain

$$\frac{d\chi_{3NTV+i}}{dx} = -\mu_i^4\chi_i - 2\sum_{j=1}^{NTV} A_{ij}\chi_{2NTV+j} + \frac{Re}{4}\left\{\sum_{j=1}^{NTV}\left[(B_{ij} - C_{ij})\chi_{NTV+j} + D_{ij}\chi_{3NTV+j}\right]\right.$$
$$\left. + \sum_{j=1}^{NTV}\sum_{k=1}^{NTV}\left(E_{ijk}\chi_j\chi_{NTV+k} - F_{ijk}\chi_{NTV+j}\chi_k + G_{ijk}\chi_{3NTV+j}\chi_k - G_{ijk}\chi_{NTV+j}\chi_{2NTV+k}\right)\right\} \tag{22a}$$

with boundary conditions

$$\sum_{j=1}^{NTV} H_{ij}\chi_j(0) = I_i; \quad \chi_{NTV+i}(0) = 0 \tag{22b,c}$$

$$\sum_{j=1}^{NTV} H_{ij}\chi_j(\infty) = 0; \quad \chi_{NTV+i}(\infty) = 0 \tag{22d,e}$$

Instead of working with an approximate formulation, based on the truncation of the channel at a sufficiently large finite length, L, a domain transformation is made in the ordinary differential equations system, mapping the infinite domain into a finite one, through the following simple algebraic transformation:

$$\eta = 1 - e^{-cx} \tag{23}$$

where c is a parameter of scale compression, and $0 \le \eta \le 1$. Therefore, the original boundary conditions of the problem are exactly satisfied.

Through the chain rule, it is possible rewritten the system in terms of the new domain $\eta \in [0,1]$, where $(d\eta/dx)$ is a function depending only on η and of the parameter c, therefore the ordinary differential system is rewritten as

$$\frac{d\chi_i}{d\eta} = \frac{\chi_{NTV+i}}{\left(\frac{d\eta}{dx}\right)}, \quad i = 1, 2, 3, ..., NTV \tag{24a}$$

$$\frac{d\chi_{NTV+i}}{d\eta} = \frac{\chi_{2NTV+i}}{\left(\frac{d\eta}{dx}\right)}; \quad \frac{d\chi_{2NTV+i}}{d\eta} = \frac{\chi_{3NTV+i}}{\left(\frac{d\eta}{dx}\right)} \tag{24b,c}$$

$$\frac{d\chi_{3NTV+i}}{d\eta} = \left\{ -\mu_i^4\chi_i - 2\sum_{j=1}^{NTV} A_{ij}\chi_{2NTV+j} + \frac{Re}{4}\left\{\sum_{j=1}^{NTV}\left[(B_{ij} - C_{ij})\chi_{NTV+j} + D_{ij}\chi_{3NTV+j}\right]\right.\right.$$
$$+ \sum_{j=1}^{NTV}\sum_{k=1}^{NTV}\left[E_{ijk}\chi_j\chi_{NTV+k} - F_{ijk}\chi_{NTV+j}\chi_k + \right.$$
$$\left.\left.\left. G_{ijk}\chi_{3NTV+j}\chi_k - G_{ijk}\chi_{NTV+j}\chi_{2NTV+k}\right]\right\}\right\} \Big/ \left(\frac{d\eta}{dx}\right) \tag{24d}$$

with boundary conditions

$$\sum_{j=1}^{NTV} H_{ij}\chi_j(0) = I_i; \quad \chi_{NTV+i}(0) = 0 \tag{25a,b}$$

$$\sum_{j=1}^{NTV} H_{ij}\chi_j(1) = I_i; \quad \chi_{NTV+i}(1) = 0 \tag{25c,d}$$

In the realm of applications, one is concerned with quantities of practical interest such as the product of the Fanning friction factor-Reynolds number, fRe. Then, the friction factor is defined as

$$f = \frac{\tau_w}{\frac{1}{2}\rho u_0^2} \tag{26}$$

with the introduction of the inverse formula for the velocity field given by Eq. (15), we obtain

$$fRe = \left[24 - 8\sum_{i=1}^{\infty} \tilde{Y}_i''(1)\bar{u}_i(x) \right] \tag{27}$$

4. Results and discussion

The developed computational code was validated using the benchmark results presented by Pérez Guerrero and Cotta (1995) for various Reynolds numbers and different inlet conditions, u = 1 and v = 0, characterizing a parallel flow, and, u = 1 and ω = 0, which characterize an irrotational flow.

Table (1) illustrates the convergence behavior of the longitudinal velocity at the centerline of the duct, for Re = 0, in various axial positions along the channel. It is observed that the velocity is converged to four significant digits for increasing truncation orders (NTV≈18-22) at positions near the duct inlet, where the convergence is slower, and it can be observed that the convergence is improved as the axial position increases. In addition, it is shown a comparison with the results of Pérez Guerrero and Cotta (1995) that also analyzed the problem by employing a streamfunction only formulation and the GITT approach. An excellent agreement between the set of results is observed for this situation.

Table 1. Convergence behavior of the longitudinal velocity component u(x,0) at the centerline of the channel for Re = 0 (inlet conditions: u = 1 and v = 0).

NTV	x = 0.2	x = 0.4	x = 0.6	x = 0.8	$x \to \infty$
6	0.9910	1.187	1.319	1.406	1.500
10	1.043	1.197	1.321	1.406	1.500
14	1.059	1.198	1.321	1.406	1.500
18	1.065	1.198	1.321	1.406	1.500
22	1.066	1.198	1.321	1.406	1.500
26	1.066	1.198	1.321	1.406	1.500
Pérez Guerrero and Cotta (1995)	1.066	1.198	1.321	1.406	——

In Table (2) the convergence behavior is presented for Re = 40. The same observations are verified as in the previous case of Re = 0. Also, the results are validated by comparing them with those ones of Pérez Guerrero and Cotta (1995), and an excellent agreement is observed. In this table comparisons are also shown with solutions obtained by purely numerical techniques (finite difference and finite element methods), where it is observed a good agreement with such results.

Table 2. Convergence behavior of the longitudinal velocity component u(x,0) at the centerline of the channel for Re = 40 (inlet conditions: u = 1 and v = 0).

NTV	x = 0.2	x = 0.4	x = 0.6	x = 0.8	$x \to \infty$
6	0.9253	1.063	1.163	1.249	1.500
10	0.9914	1.082	1.166	1.251	1.500
14	1.013	1.083	1.166	1.251	1.500
18	1.019	1.083	1.166	1.251	1.500
22	1.021	1.083	1.166	1.251	1.500
26	1.022	1.083	1.166	1.251	1.500
Pérez Guerrero and Cotta (1995)	1.022	1.083	1.166	1.251	——
Brandt and Gillis (1966)	1.0223	1.0849	1.1693	1.2535	——
Comini and Del Giudice (1982)	1.0243	1.0884	1.1737	1.2580	——

Tables (3) and (4) show that few terms are required for the convergence of the longitudinal velocity at the position closest to the duct inlet (x = 0.20833). The irrotational inlet flow condition demonstrates a slight influence in the convergence. However, a monotonically convergence is observed for the case of parallel flow, while are required more terms in the series expansions for the irrotational case (around NTV = 38) for a complete convergence of the results. It is observed that the overall convergence rate is not markedly affected when increases the importance of the convective effects, as in this case, but higher truncation orders are needed for the complete convergence in relation to the previous cases of lower Reynolds number. Once again, comparisons with the results of Pérez Guerrero and Cotta (1995) shown an excellent agreement, as well as a good one with those of purely numerical techniques.

Table 3. Convergence behavior of the longitudinal velocity component u(x,0) at the centerline of the channel for Re = 300 (inlet conditions: u = 1 and v = 0).

NTV	x = 0.20833	x = 0.8333	x = 3.3333	x = 7.5	$x \to \infty$
6	0.8293	1.029	1.281	1.425	1.500
10	0.9165	1.071	1.282	1.426	1.500
14	0.9669	1.072	1.281	1.425	1.500
18	0.9915	1.071	1.280	1.425	1.500
22	1.001	1.071	1.280	1.425	1.500
26	1.005	1.071	1.280	1.425	1.500
30	1.006	1.071	1.280	1.425	1.500
34	1.006	1.071	1.280	1.425	1.500
Pérez Guerrero and Cotta (1995)	1.007	1.071	1.280	1.425	——
McDonald et al. (1972)	1.008	1.075	1.283	1.425	——

Table 4. Convergence behavior of the longitudinal velocity component u(x,0) at the centerline of the channel for Re = 300 (inlet conditions u = 1 and ω = 0).

NTV	x = 0.20833	x = 0.8333	x = 3.3333	x = 7.5	$x \to \infty$
10	0.9817	1.137	1.321	1.438	1.500
14	1.023	1.149	1.326	1.440	1.500
18	1.039	1.156	1.330	1.441	1.500
22	1.046	1.161	1.332	1.442	1.500
26	1.049	1.165	1.334	1.443	1.500
30	1.050	1.167	1.335	1.443	1.500
34	1.051	1.169	1.336	1.444	1.500
38	1.052	1.171	1.337	1.444	1.500
Pérez Guerrero and Cotta (1995)	1.052	1.170	1.337	1.444	——
Wang and Longwell (1966)	1.0581	1.1880	1.3572	1.4509	——
McDonald et al. (1972)	1.050	1.170	1.34	1.44	——

Figure (2) shows the comparison of the longitudinal velocity component at the centerline along the channel for different Reynolds numbers with the two types of inlet conditions analyzed (u = 1, v = 0 and u = 1, ω = 0). It can be observed that the solution of the Navier-Stokes equations for the irrotational case in these conditions has better agreement with the boundary layer results. It is also observed that the condition of parallel flow tends more slowly to the approximate results of the boundary layer formulation in relation to the irrotational case, especially for increasing Reynolds numbers.

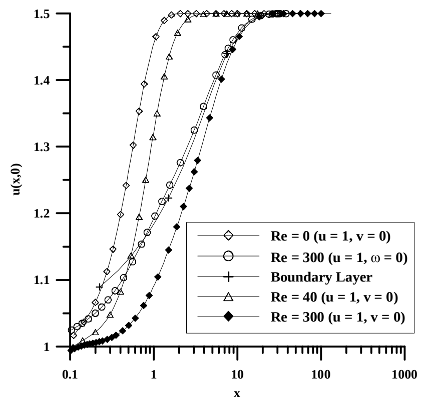

Figure 2. Comparison of the longitudinal velocity component of u(x,0) along the channel for different Reynolds numbers.

Figure (3) shows the development of the velocity profile v(x,y) for Reynolds number 300 with inlet condition (u = 1, v = 0). It is observed an axial increasing and traverse of this velocity component, until to reach the distribution of fully developed flow.

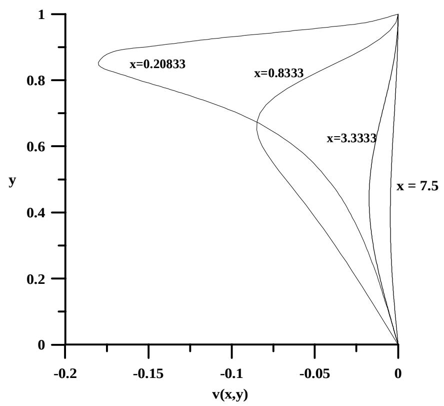

Figure 3. Distribution of the transversal velocity component v(x,y) along the channel length for Re = 300.

Figure (4) brings a comparison of the longitudinal velocity component along the developing region for the case of Re = 300. It is verified that the effect of the concavity of the longitudinal velocity profile is more clearly observable in the situation of parallel flow (u = 1, v = 0). It is also observed that this effect tends to disappear for regions far from the channel entry.

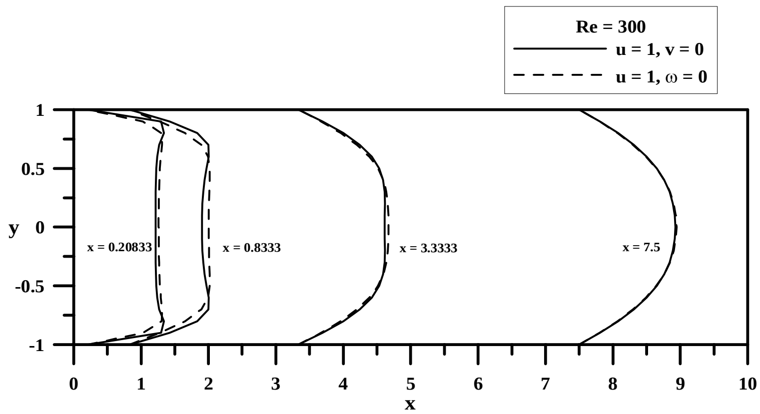

Figure 4. Development of the longitudinal velocity component u(x,y) along the channel for uniform and irrotational inlet conditions (Re = 300).

The graphical analyses of the product of the Fanning friction factor-Reynolds number are shown in Figs. (5) to (7). It can be observed in these figures that the product fRe is strongly dependent of the Reynolds number. One can see that this relationship tends to approximate of fRe = 24, which characterizes the fully developed flow.

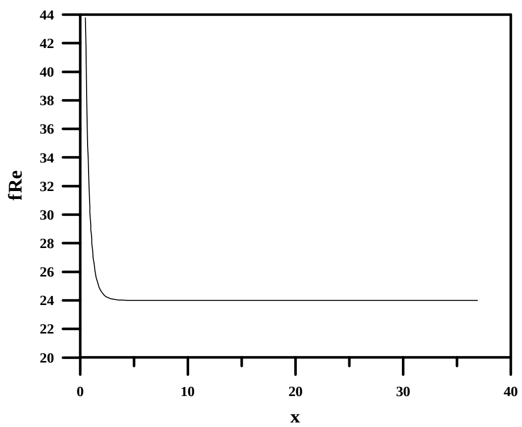

Figure 5. Product fRe along the channel length for Re = 40 (parallel flow).

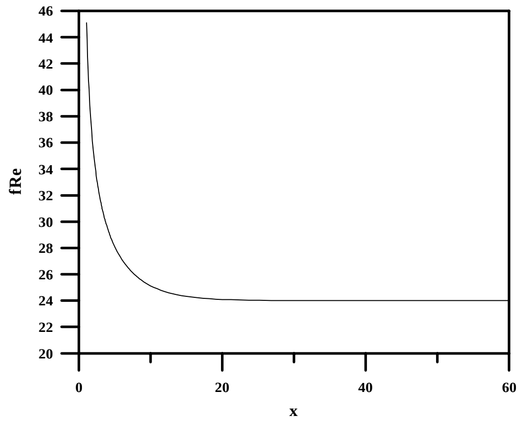

Figure 6. Product fRe along the channel length for Re = 300 (parallel flow).

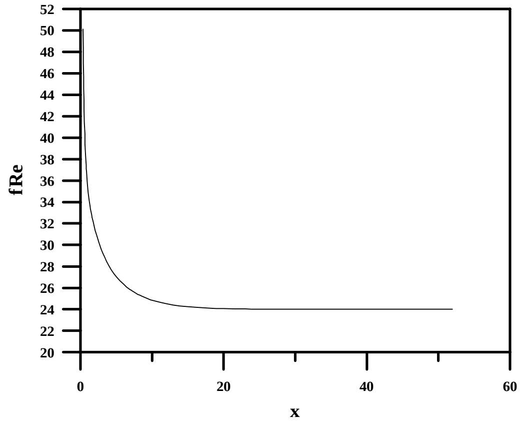

Figure 7. Product fRe along the channel length for Re = 300 (irrotational flow).

Figure (8) presents a comparison of the longitudinal velocity component along the channel for different Reynolds numbers with inlet conditions (u = 1, v = 0) and the solution for the boundary layer formulation. It is verified that the concavity in the velocity profile is clearly observable in the situation for Re = 40, which tends to disappear in the fully developed region, where there is an excellent approximation of this profile with the boundary layer results.

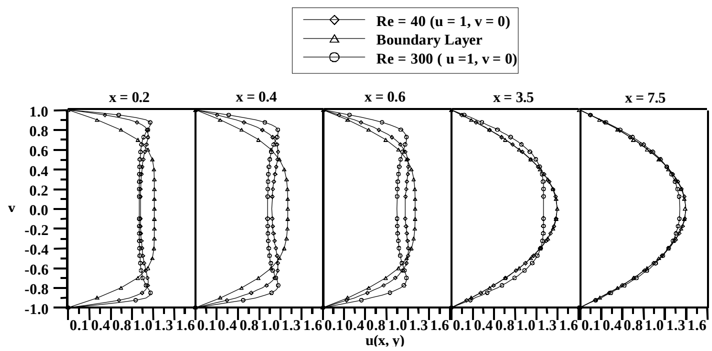

Figure 8. Comparison of the longitudinal velocity component u(x,y) along the channel for different Reynolds numbers.

5. Conclusions

The results demonstrated the applicability of the Generalized Integral Transform Technique (GITT) as an appropriate tool to solve flow problems in parallel plate channels involving the Navier-Stokes equations. The validation of the results for this case of two-dimensional flow suggests that the extension of the present formulation may be more attractive for the solution of the Navier-Stokes equations in three-dimensional problems than those with involved formulations as the scalar and vector potentials ones.

6. References

Brandt, A. and Gillis, J., 1966, "Magnetohydrodynamic Flow in the inlet Region of a Straight Channel", Physics of Fluids, Vol. 9, pp. 690-699.

Comini, G. and Del Giudice, S., 1982, "Finite-Element Solution of the Incompressible Navier-Stokes Equations", Numerical Heat Transfer, Vol. 5, pp. 463-478.

IMSL Library, 1991, MATH/LIB, Houston, TX.

McDonald, J. W., Denny, V. E. and Mills, A. F., 1972, "Numerical Solution of the Navier-Stokes in Inlet Regions", Journal of Applied Mechanics, Vol. 39, pp. 873-878.

Pérez Guerrero, J. S. and Cotta, R. M., 1995, "Integral Transform Solution of Developing Laminar Duct Flow in Navier-Stokes Formulation", International Journal for Numerical Methods in Fluids, Vol. 20, pp.1203-1213.

Pérez Guerrero, J. S., Pereira, L. M. and Santos, C. A. C., 1998, "Uma Alternativa de Solução para as Equações de Navier-Stokes em Variáveis Primitivas Via Transformação Integral", Proceedings of the 5th North-Northeastern Congress of Mechanical Engineering, V CEM-NNE, Vol. 2, Fortaleza, Brazil, pp. 620-626.

Quaresma, J. N. N. and Cotta, R. M., 1997, "Integral Transform Method for the Navier-Stokes Equations in Steady Three-Dimensional Flow", Proceedings of the 10th International Symposium on Transport Phenomena, ISTP10, Vol. 1, Kyoto, Japan, pp. 281-287.

Wang, Y. L. and Longwell, P.A., 1964, "Laminar Flow in the Inlet Section Parallel Plates", AIChE Journal, Vol. 10, pp. 323-329.