10o Brazilian Congress of Thermal Sciences and Engineering -- ENCIT 2004
Braz. Soc. of Mechanical Sciences and Engineering -- ABCM, Rio de Janeiro, Brazil, Nov. 29 -- Dec. 03, 2004
Paper Code:CIT04-0619

Radiation Heat Transfer with Ablation in a Finite Plate

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Abstract. The phenomenon of ablation is a process of thermal protection with several applications, mainly, in mechanical and aerospace engineering. Ablative thermal protection is applied using special materials (named ablative materials) externally on the surface of a structure in order to isolate it against thermal effects. The ablative phenomenon is a complex process involving phase changes with partial or total loss of the material. So the position of the boundary it is unknown a priori. The governing equations of the process is a non-linear system of coupled partial differential equations. The uni-dimensional analysis of ablative process on the plate is performed by using the generalized integral transform technique – GITT for solution of the system of governing equations. By application of this technique of solution the system of partial differential equations is transformed in a system of infinite ordinary differential equations that can be solved by numerical techniques available in computational codes after the truncation of that system. The plate of finite thickness at constant properties is subjected to a time-dependent prescribed radiation heat flux at one face, initially with a uniform temperature T_0, and insulated on the other face. After an initial heating period, ablation starts at the heated surface through melting and continuous removal of the plate material. The results of interest are the thickness and the rate of loss of the ablative material. The obtained results are compared with available results of other techniques of solution in the literature.

Word key: Ablation, Heat Transfer, Generalized Integral Transform Technique, Diffusion.

1. Introduction

The interaction between a particular material and the gas boundary layers is, in general, a complicated process. The least interaction occurs for those materials which melt and flow without evaporation; consequently this class of materials is firstly considered. Though melting materials are probably the least intersting from a practical point of view, an understabding of melting is fundamental to an overall underestanding of the ablation problem.

Very little coupling exists between the flowing liquid layer and the external gas boundary layer, especially if the liquid velocity is small compared with the external gas velocity and the melting temperarture is much less than the gas stagnation temperature. The heat transfer, shear and pressure distribution can then be regarded as known and equal to their values for a solid boundary at the melting temperature.

Due to the extreme conditions of aerodynamic heating, the amount of energy involved in such a process is enough to occur the evaporation of the space vehicle. An example of this are the meteorites when they penetrate in the atmosphere and the most recent case of the rocket North American "Columbia", that disintegrated completely in its re-entered in the atmosphere on February, 2003.

The phenomenon of ablation is a process of thermal protection with several applications, mainly, in mechanical and aerospace engineering. Ablative thermal protection is applied using special materials (named ablative materials) externally on the surface of a structure in order to isolate it against thermal effects. The ablative phenomenon is a complex process involving phase changes with partial or total loss of the material. So the position of the boundary it is unknown a priori.

The diagrams shown in Figures 1 and 2 illustrate the ablation phenomenon. In Figure 1 the following proccesses are presented:

1. Convection heat transfer in the boundary layer that represents the main thermal load;
2. Radiation heat transfer;
3. Conduction heat transfer in the virgin material that should satisfy the temperature approach limits in the substructure or in the thermal shield on the structure;
4. Resin decomposition;
5. Fibers decomposition;
6. Passage of the gas produced through of the residuals;
7. Retreat of the surface;
8. Radiation in the wall;
9. Shock in the boundary layer;

10. Combustion in the boundary layer.

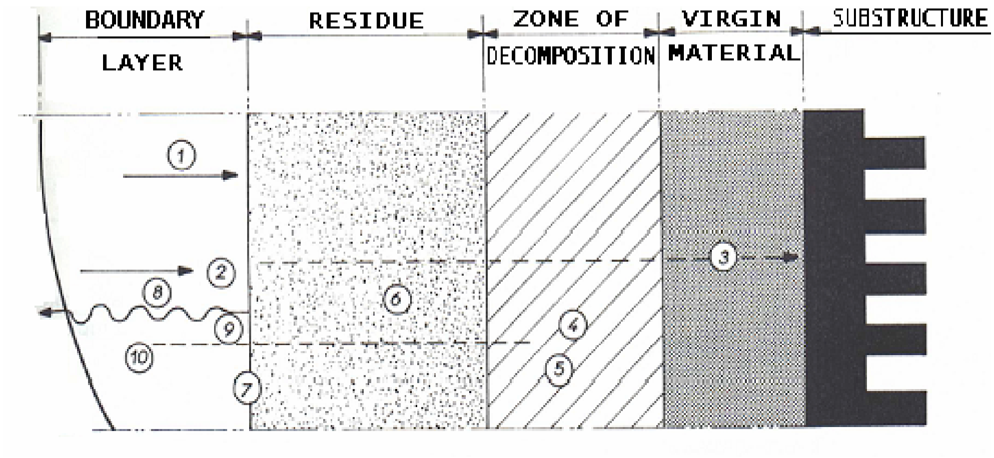

Figure 1: Illustration of ablation phenomenon.

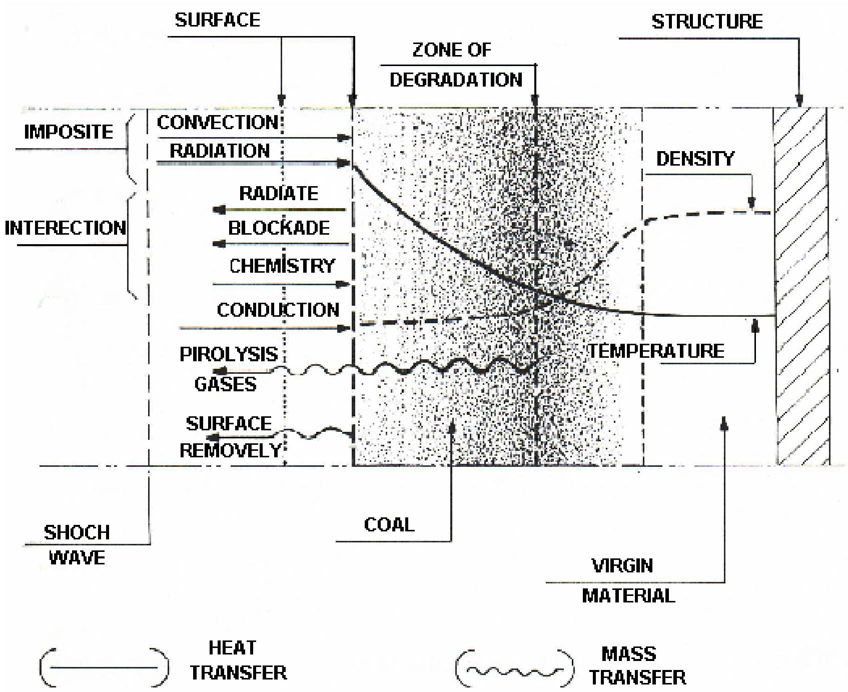

Figure 2: Physical representation of the ablation process involving the melting of the ablative material.

The analytical and numerical, as well as analytical-numerical solutions have been done for this kind of problem. In a two-dimensional, geometry the method of Approximate Integral Balance, was presented by Hsiao & Chung, (1983). The physical and mathematical models of the ablation process have been presented by Lacase (1967). The solution of the diffusion problem with variable coefficients was studied by Cotta & Özisik, (1987). A generalized study of the ablative phenomenon was done by Adams, (1959 and Sutton, (1982). The using of the Generalized Integral Transform Technique (GITT), was presented by Diniz et al., (1990, 1993); Kurokawa et al., (2003); Aparecido Gomes et al., (2004). They solved the uni-dimensional problems of heat diffusion for several geometries. Vallerani (1974) applied the Integral Method for problems of simple classes of ablation. The Classical Integral Transform Technique for linear problems was presented by Mikhailov & Özisik, (1984). The GITT is a generalization of that technique for non-linear problems.

In the present work the uni-dimensional analysis of the ablative process in a Finite Plate has been done using the Generalized Integral Transform Technique as an analytical tool for solution of the differential partial equation, considering the isolated internal side and the external side subject to a prescribed unsteady heat flow considering the radiation heat transfer effect. In the solution of the problem the value of interest is the temperature field in the material.

2. Mathematical Analysis

Here it is considered the radiation heat flux in a Finite Plate of finite thickness with constant physical properties and α (Absorptivity) = ε (Emissivity). Initially, the plate is subjected to a temperature T_0 and its external surface is submitted to prescribed unsteady heat flux and the internal surface is thermally isolated. This problem is solved in two steps: 1) the heating of the material until the phase change temperature is reached (Preablation Period) and 2) the ablation period that start at the heated surface through melting and continuous removal of the plate material (Ablation Period). The Figure 3 shows the plane geometry with a change of coordinate X to coordinated η, accomplished to facilitate the computational implementation. Q(T) it is the prescribed heat flux, S(t) it is the position of the ablative front in X-coordinate and $\eta_b(\tau)$ (shaded material) represents the width of the ablative material that has not yet been consumed.

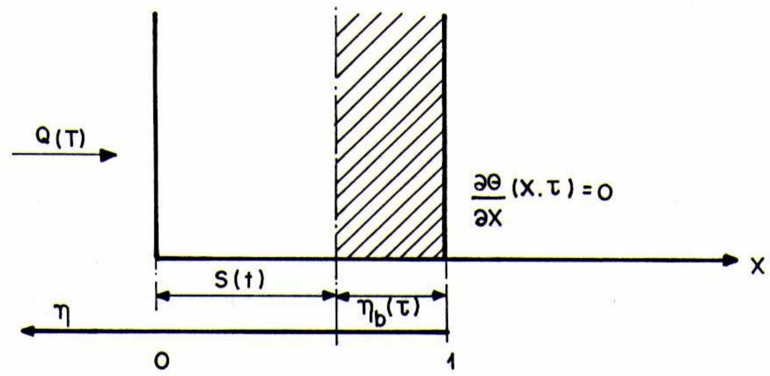

Figure 3.Geometry and coordinates system for ablation in a Finite Plate.

2.1. Description of the problem for the balance of energy

We consider an ablating finite plate which is fed into the plane $x^* = 0$. The Fig. 4 shows a radiation heat flux (q_e^*) directly on the boundary with. An asterisk indicates dimensional variables.

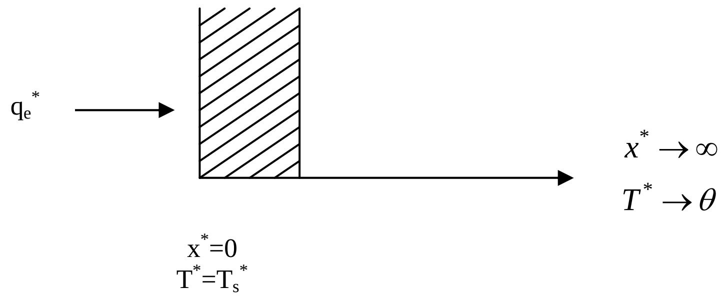

Figure 4. Schematic diagram of an ablating (and/or melting) surface, with the rate of melting controlled by radiant energy input from an external source.

Dimensionless variables are defined in the form:

$$\theta = \frac{\left(T^* - T_0^*\right)}{T_0^*}, \text{ Dimensionless temperature} \tag{1}$$

$$\tau = \frac{\lambda^* t^* \overline{K}^{*2}}{\rho^* c_p^*}, \text{ Dimensionless time} \tag{2}$$

$$Q(\tau) = \frac{q_e^*}{\lambda^* T_0^* \overline{K}^*}, \text{ Dimensionless heat flux} \tag{3}$$

$$X = \overline{K}^* x^*, \text{ Dimensionless coordinate} \tag{4}$$

where $(\overline{K}^*)^{-1}$ is a characteristic length of problem, λ^* is wavelength which radiant energy is emitted, ρ^* is the specific density and c^*_p is the specific heat.

2.2. Preablation Period

In dimensionless form, the heat transfer with effect of radiation ($\alpha = \varepsilon$) for the preablation period is then given, as shown by Penner & Olfe, (1968) as

$$\frac{\partial\theta(X,\tau)}{\partial\tau} = \frac{\partial^2\theta(X,\tau)}{\partial X^2} + 2Q(\tau)E_2(x), \quad 0<X<1, \quad 0<\tau<\tau_m \tag{5}$$

with initial and boundary conditions

$$\theta(X,0) = 0, \; 0 \le X \le 1 \tag{6}$$

$$-\left.\frac{\partial\theta(X,\tau)}{\partial X}\right|_{X=0} = Q(\tau); \tag{7}$$

$$\left.\frac{\partial\theta(X,\tau)}{\partial X}\right|_{X=1} = 0, \tau > 0 \tag{8}$$

and the function $E_2(X)$ has been replaced by the Beer-Bouguer factor exp(-X).

The Equations 5-8 define a standard radiation heat transfer problem, which is readily solved as a special case of the more general formulation presented by Özisik & Mikhailov, (1984).

The following auxiliary problem with τ -dependent quantities is chosen

$$\frac{\partial_2\psi_i(X)}{\partial X^2} + \mu_i^2(\tau)\psi_i(X) \quad 0<X<1, \quad 0<\tau<\tau_m \tag{9}$$

subject to the boundary conditions

$$\frac{\partial\psi(X)}{\partial X} = 0, \; X=0 \tag{10}$$

and

$$\frac{\partial\psi(X)}{\partial X} = 0, \; X=1 \tag{11}$$

which is readily solved to yield

$$\mu_i(\tau) = i\pi \tag{12}$$

$$\psi i(X) = \cos[\mu_i X] \tag{13}$$

where ψ_i are the eigenfunctions and μ_i are the eigenvalues of the Preablation Period.
The Eq. 9 allows the definition of the following integral transform pair

$$\tilde{\theta}(\tau) = \int_{X=0}^{X=1} \psi_i(X)\theta(X,\tau)dX \quad \textit{(Finite integral transform)} \tag{14}$$

$$\theta(X,\tau)=\sum_{i=1}^{\infty}\frac{\psi_i}{N_i}\tilde{\theta}(\tau) \quad \text{(Inversion formula)} \tag{15}$$

$\tilde{\theta}$ and θ are the terms that represent respectively the transformed integral and its inversion formula of the Preablation Period. N_i is the norm of the eigenfunctions, defined as

$$N_i(\tau)=\int_{X=0}^{X=1}\psi_i^{\,2}(X)dX \tag{16}$$

Using the GITT is possible to solve the Eq. 5 together with the boundary conditions, Equations 6, 7 and 8. Thus, Eq. 5 can be solved arriving in the following temperature distribution

$$\theta(X,\tau)=\theta_{AV}(\tau)+\sum_{I=1}^{\infty}\frac{\psi_i(X)}{N_i}\tilde{\theta}_i(\tau) \tag{17}$$

where, θ_{AV} is the average potential, given by

$$\theta_{AV}=\int_0^{\tau}Q(\tau^{'})d\tau \tag{18}$$

The prescribed unsteady heat flux can be,

$$Q(\tau)=A+B\tau+C\tau^2 \text{ or,} \tag{19}$$

$$Q(\tau)=D*e^{\tau/\tau_r} \tag{20}$$

where, τ_r is dimensionless reference time, as shown by Hasiao & Chung, (1983) using the polynomial flux and applying the technique, in Eq. 17, yields

$$\theta_{AV}=F*(A\tau+\frac{B\tau^2}{2}+\frac{C\tau^3}{3}) \tag{21}$$

$$\theta(X,\tau)=F*\left(A\tau+\frac{B\tau^2}{2}+\frac{C\tau^3}{3}\right)+$$

$$\sum_{i=1}^{\infty}\frac{\psi_i(X)}{N_i}\left\{\frac{E}{\mu_i^2}\left[A+\left(B\tau-\frac{1}{\mu_i^2}\right)+C\left(\tau^2-\frac{2\tau}{\mu_i^2}+\frac{2}{\mu_i^4}\right)\right]-\frac{E}{\mu_i^2}\left(A-\frac{B}{\mu_i^2}+\frac{2C}{\mu_i^4}\right)e^{-\mu_i^2\tau}\right\} \tag{22}$$

For the exponentials flux,

$$\theta_{AV}=D*F*\tau_r\left(e^{\tau/\tau_r}-1\right) \tag{23}$$

$$\theta_{AV}=D*F*\tau_r\left(e^{\tau/\tau_r}-1\right)+\sum_{i=1}^{\infty}\frac{\psi_i(X)}{N_i}\left\{\frac{D*E}{\mu_i^2+\frac{1}{\tau_r}}\left(e^{\tau/\tau_r}-e^{-\mu_i^2\tau}\right)\right\} \tag{24}$$

The Equations 14 and 16 represent the temperature distribution for $\tau<\tau_M$ (τ_M is the starting melting time of the material), for each of adopted prescribed heat flux.

The value of τ_M is obtained from solving the transcendental Equation 22 or 24, with

$$\theta(0,\tau_M)\equiv 1 \quad (25)$$

The Equations 22 and 24 represent the temperature distribution of the Preablation period, for the considered flux..

2.3. Ablation Period

For the Ablation Period is used again the Eq. 5, however, in this period the movement of the boundary begins, and the domain now is $S(\tau)< X <1$, and correspondingly $\tau > \tau_m$. The initial and the boundary conditions are

$$\theta(X,\tau_m)=\theta_m(X) \quad , \quad 0\le X \le 1 \quad (26)$$

$$\theta(X,\tau)=1 \quad \text{and} \quad -\frac{\partial\theta(X,\tau)}{\partial X}+\nu\frac{dS(\tau)}{d\tau}=Q(\tau) \text{ , at X = S}(\tau) \quad (27)$$

$$\left.\frac{\partial\theta(X,\tau)}{\partial X}\right|_{x=1}=0, \tau>\tau_m \quad (28)$$

where $\theta_{m(X)}$ is the temperature distribution within the slab when ablation starts, at $\tau=\tau_m$, obtained from the solution of the Equation 5, while the parameter ν in the heat balance for the ablating surface, Eq. 27, is the inverse of the Stefan number and S(τ) is the time-dependent position of the inward moving ablating boundary to be determined from the Eq. (27).

For convenience in the analysis, homogenize of the variables, the following variable transformations are adopted

$$\theta^*(X,\tau)=(X,\tau)-1 \quad (29)$$

$$\eta=1-X \quad (30)$$

With,

$$\eta_b(\tau)=1-S(\tau) \quad (31)$$

With the variable transformations, the Eq. 5 become

$$\frac{\partial\theta(\eta,\tau)}{\partial\tau}=\frac{\partial^2\theta(\eta,\tau)}{\partial X^2}+2Q(\tau)E_2(X)\,, \quad \tau>\tau_M \quad , \quad 0<\eta<\eta_b(\tau) \quad (32)$$

with initial and boundary condition

$$\theta^*(\eta,\tau_m)=\theta(\eta-1,\tau_M)-1 \quad (33)$$

$$\left.\frac{\partial\theta^*(\eta,\tau)}{\partial\eta}\right|_{\eta=0}=0 \quad , \quad \tau>\tau_m \quad (34)$$

$$\theta^*(\eta,\tau)=0 \quad , \quad \tau\le\tau_m \quad , \quad \eta=\eta_b(\tau) \quad (35)$$

with the head balance at $\eta=\eta_b(\tau)$ given by

$$\frac{d\eta_b(\tau)}{d\tau} = \frac{1}{\nu}\frac{\partial\theta^*(\eta,\tau)}{\partial\eta}\bigg|_{\eta=\eta_b(\tau)} - \frac{Q(\tau)}{\nu}, \tau > \tau_m \quad (36)$$

and

$$\eta_b(\tau_m) = 1 \quad (37)$$

The following auxiliary problem for the situation of the Ablation Period with τ -dependent quantities is chosen

$$\frac{\partial_2\phi_i(\eta)}{\partial\eta^2} + \mu_i{}^2(\tau)\psi_i(\eta) \quad 0 < \eta < \eta_b(\tau) \quad (38)$$

$$\frac{\partial\phi_i(\eta)}{\partial\eta}\bigg|_{\eta=0)} = 0; \phi_i[\eta_b(\tau)] = 0 \quad (39)$$

which is readily solved to yield

$$\lambda_i(\tau) = \frac{(2i-1)\pi}{2\eta_b(\tau)} \quad (40)$$

$$\phi i(\eta) = \cos[\mu_i(\tau)\eta_b] \quad (41)$$

where ϕ_i are the eigenfunctions and λ_i are the eigenvalues of the Ablation Period.
The Eq. 38 allows the definition of the following integral transform pair

$$\tilde{\theta}_i^*(\tau) = \int_0^{\eta_{b(\tau)}} Ki(\eta_b,\tau)\theta^*(\eta_b,\tau)d\eta \quad \textit{(Finite integral transform)} \quad (42)$$

$$\theta^*(\eta_b,\tau) = \sum_{i=1}^{\infty} Ki(\eta_b,\tau)\tilde{\theta}_i^*(\tau) \quad \text{(Inversion formula)} \quad (43)$$

$\tilde{\theta}_i^*$ and θ^* are the terms that represent, respectively, transformed her integral and your inversion formula of the Ablation Period, with the symmetric kernel defined as

$$k_i(\eta,\tau) = \frac{\psi_i(\eta,\tau)}{[N_i(\tau)]^{1/2}} \quad (44)$$

and the normalization integral

$$N_i(\tau) = \int_0^{\eta_b}\psi_i^2(\eta)d\eta \quad (45)$$

Application the technique (GITT), with the Eq. 37 and $\tau=\tau_M$, it is possible to solve the ablation period, yielding

$$\frac{d\tilde{\theta}_i^*(\tau)}{d\tau} + \lambda_i^2(\tau)\tilde{\theta}_i^*(\tau) + \sum_{j=1}^{\infty} A_{ij}(\tau)\tilde{\theta}_j^*(\tau) - GQ(\tau)\cdot(-1)^{i+1}\cdot\left[\frac{(2i-1)\pi}{(2i-1)^2\pi^2+4}\right] = 0 \quad (46)$$

$$\frac{d\eta_b(\tau)}{d\tau} = \frac{\sqrt{2}}{2}\frac{\pi}{[\eta_b(\tau)]^{3/2}}\sum_{j=1}^{\infty}(2j-1)\tilde{\theta}_j^*(\tau)\cdot(-1)^j - \frac{Q(\tau)}{\nu} \tag{47}$$

The Eq. 46 and 47 form a system of infinite ordinary differential equations. To solve this system it is necessary the substitution of that infinite system to a finite system of order N. The number of terms in the summation, N is fixed according to the desired accuracy. The truncated system of ordinary differential from Eqs. 46 and 47 is then,

$$\frac{d\tilde{\theta}_i^*(\tau)}{d\tau} + \lambda_i^2(\tau)\tilde{\theta}_i^*(\tau) + \sum_{j=1}^{N} A_{ij}(\tau)\tilde{\theta}_j^*(\tau) - GQ(\tau)\cdot(-1)^{i+1}\cdot\left[\frac{(2i-1)\pi}{(2i-1)^2\pi^2+4}\right] = 0 \tag{48}$$

$$\frac{d\eta_b(\tau)}{d\tau} = \frac{\sqrt{2}}{2}\frac{\pi}{[\eta_b(\tau)]^{3/2}}\sum_{j=1}^{N}(2j-1)\tilde{\theta}_j^*(\tau)\cdot(-1)^j - \frac{Q(\tau)}{\nu} \tag{49}$$

Where E, F and G are constants defined by the analytical solution of the problem using the GITT.

Therefore the solution of the system of ordinary differencial equations, formed by the Eq. 48 and 49, gives the thickness and the rate of loss of the ablative material. These values have not been calculated until this phase of the work. Here, is calculated the temperature distribution by the Equation 36 in the Preablation Period, considering the effect of radiation heat flux on the boundary.

The distribution of temperature of the Preablation Period for radiation heat flux at the boundary is presented in Figure 5. In this case, the temperature at the boundary reached the melting point faster than the case in which radiation is neglected. While, without radiation the time of starting melting is 0.196, with effect of radiation this time is only 0.0357.

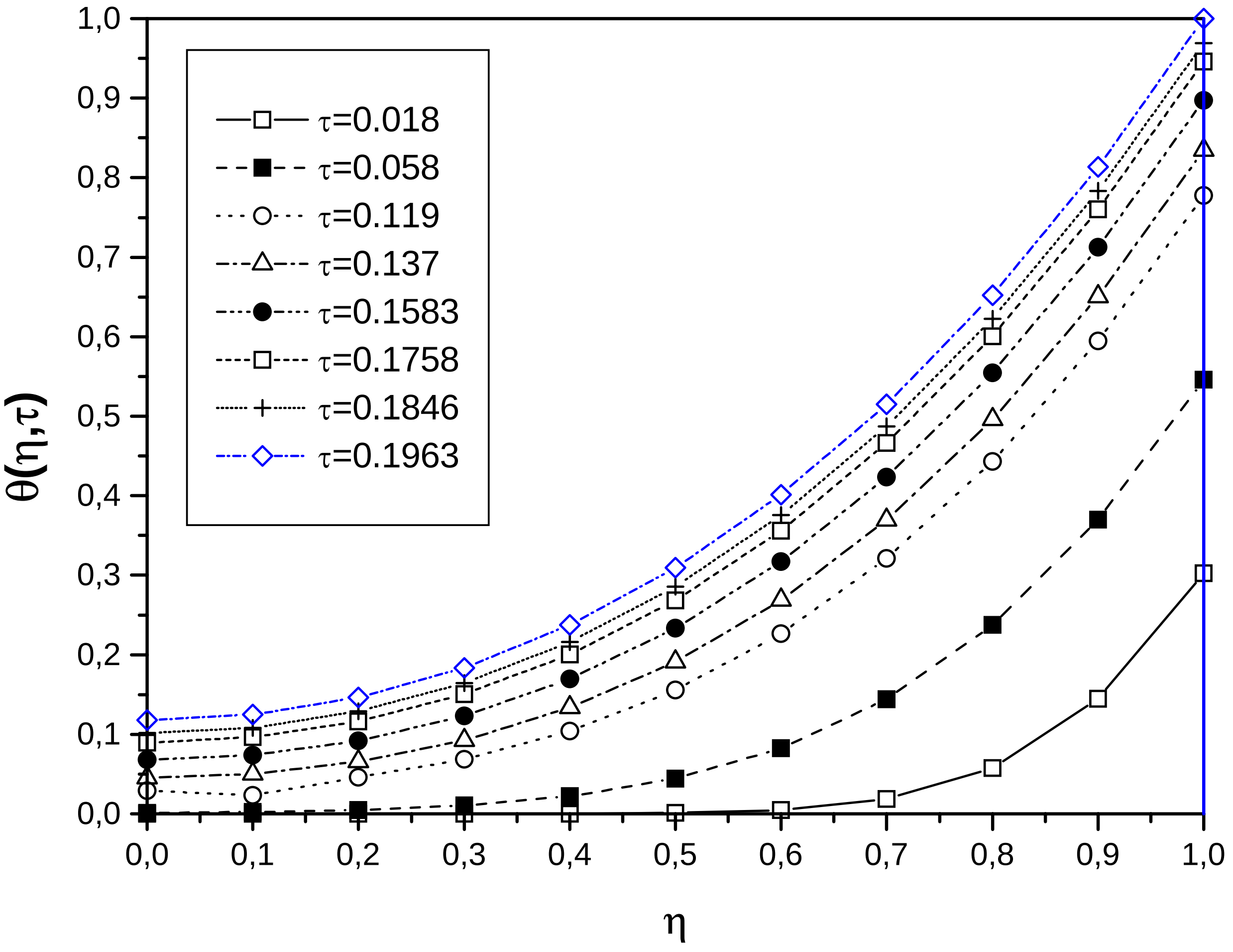

Figure 5. Distribution of temperature of the Preablation Period for radiation heat flux, Q(τ)=2.

Figure 6 shows both the results without radiation and with radiation. In spite of similar behavior between the temperature field, the effect of the radiation is clearly visible. So the material shall be consumed faster in the case of radiation. In next step of this work the width of ablative material and the velocity of the boundary will be calculated to comparison with results available in the literature.

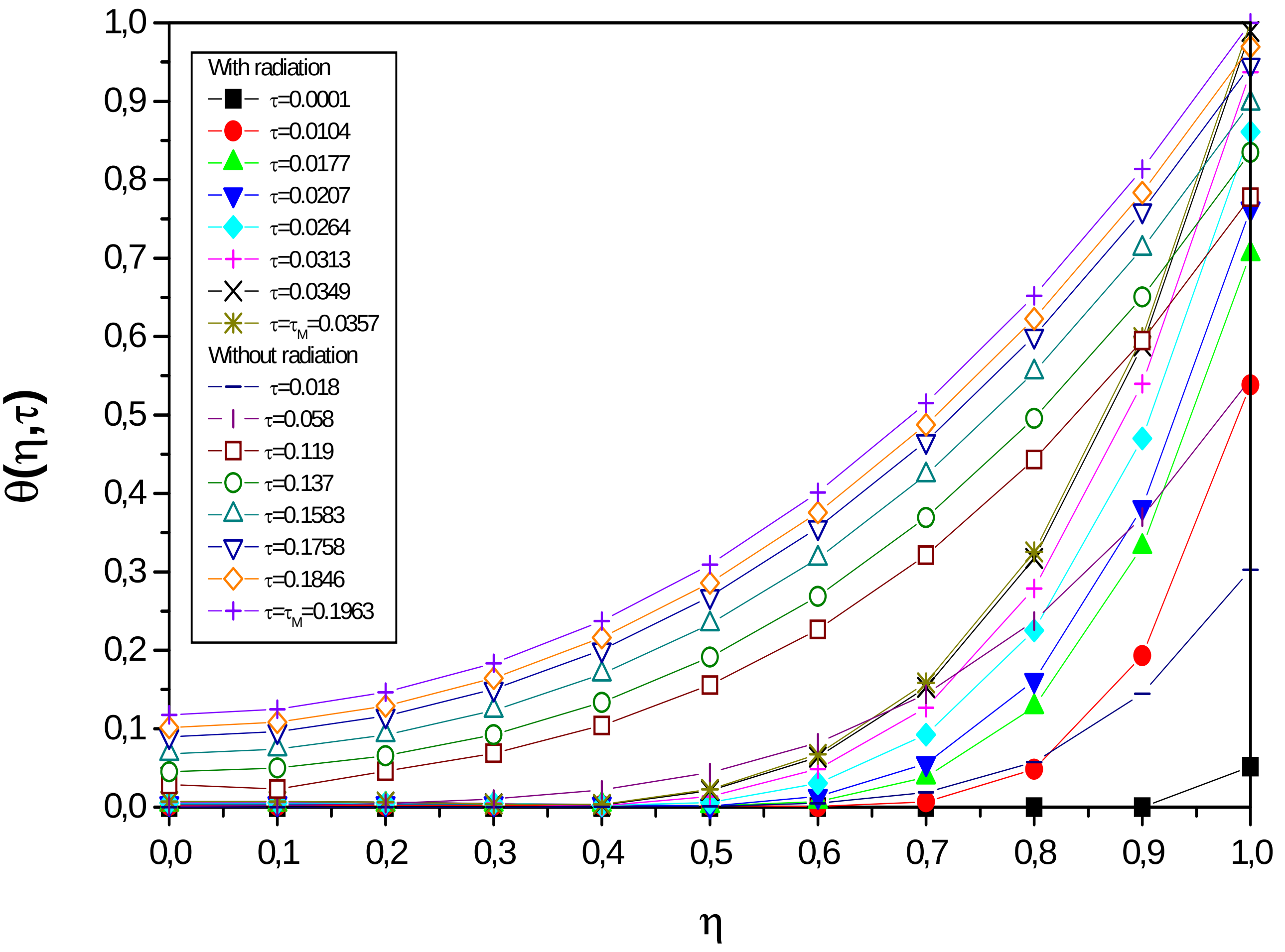

Figure 6: Comparison between the distribution of temperature for the work of Diniz et al., (1990), without radiation, with the case of the radiation heat flux in the boundary, with Q(τ)=2.

3. Discussion and Conclusion

The results of interest are the thickness *S(t)* of the melted material, the velocity of ablation that can be calculated by $dS(t)/dt$ and the temperature field of the Pre-ablation and Ablation periods. Here, it has been calculated just the temperature field of Pre-ablation to show the influence of radiation.

The thickness and the ablative speed are represented by the system of ordinary differencial equations, Eq. 41 and 42, that can be solved by numerical techniques in computational codes.

The temperature distribution with the presence of radiant energy, shown in the Fig. 5, demonstrates a fast increasing of the temperature, as the time increases, in comparison with the work without radiant energy from Diniz et al. (1990), as shown in Fig. 6.

It is also possible to evaluate as the time for the radiant flux is faster than in the radiation absence, fact imposed for the high order of the temperature exponent in the analysis of the heat transfer for radiation. That fact is clearly observed by the form of the growth of the temperature distribution shown in the Fig. 5. The effect of radiation is considered through the last term of the right hand side of the Eq. 5, represented by the term $E_2(X)$. The form as this term is considered is of a exponential, because the phenomenon of the radiation is very fast.

The Generalized Integral Transform Technique has been applied with success in the present work, for the solution of the ablation with radiant energy problem in a finite plate geometry. In the sequence of this work the other results of interest: the depth and the velocity of ablation will be calculated.

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