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HEAT TRANSFER IN SLUG FLOW IN ELLIPTICAL DUCTS

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Abstract. *This work shows the calculation of the heat transfer parameters for slug flow in the thermal entrance region of elliptical section tubes submitted to a second kind boundary condition. The main difficulty in the application of the boundary conditions in problems with this kind of geometry has been removed by using a suitable coordinate change. The generalized integral transform technique (GITT) has been used to obtain the solution of the energy equation. The mixture temperature and the local and average Nusselt numbers have been calculated for several aspect ratios and the results have been compared with the encountered in the literature.*

Keywords. *Forced convection, slug flow, integral transform, mixture temperature, elliptical ducts.*

1. Introduction

Although many works have been dedicated to forced convection problems in ducts, analytical solutions for complex mass and heat transfer problems are not commonly found in the literature (Shah & London, 1978), Kakaç *et al.* (1998).

The major impediment to derive analytical solutions for that class of problems resides in the impossibility of employing the method of separation of variables. With the development of the Generalized Integral Transform Technique – GITT – this obstacle has been somewhat overcome, making it possible to obtain solutions for the most varied and complex diffusion problems (Aparecido, 1997). Problems such as diffusion ducts with irregular shapes (Aparecido *et al.*, 1990), (Maia *et al.*, 2000), time varying coefficients and boundary conditions (Cotta & Ozisik, 1986), (Cotta & Ozisik, 1987), space dependence for the heat transfer coefficients and boundary conditions (Vick & Wells, 1986), thermally and hydrodynamically developing flows (Silva *et al.*, 1992), diffusion involving moving boundaries (Diniz *et al.*, 1990), and non Newtonian fluids (Maia *et al*, 2002), (Macedo & Quaresma, 2001) have all been provided a suitable analytical solution, albeit not always in a closed expression.

In the present work, heat transfer parameters calculation for the laminar thermally developing flow problem in ducts of elliptical section with boundary condition of second kind, a setup itself not amenable to variable separation, has been presented. Regarding the boundary conditions, an additional difficulty is imposed by the non-regular two-dimensional characteristic of the cross section of the elliptical duct. A suitable change of variables was devised to transform the elliptic profile into a new geometry, simplifying the application of the boundary conditions. The Generalized Integral Transform Technique was applied afterwards to the energy equation to obtain the temperature field in the flow and, consequently, determining the parameters of interest.

2. Analysis

In the formulation of the present problem, thermally and hydrodynamically developing laminar steady state flow has been assumed. The effects of viscous dissipation and axial conduction have been neglected and constant fluid properties have been admitted constant throughout. Therefore, the energy equation, presented in Eq. 1 becomes:

$$\rho c_p w(x,y)\frac{\partial T}{\partial z} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right], \qquad \{(x,y) \in \Omega \ e \ z > 0\}. \tag{1}$$

Eq. 2 gives the velocity profile for fully developed laminar flow inside elliptical tubes:

$$w(x,y) = w_0 \,. \tag{2}$$

In this model, the inlet condition is

$$T(x,y,z) = T_0 \,, \qquad \{(x,y) \in \Omega \;\; e \;\; z > 0\} \,. \tag{3}$$

The condition of constant heat flux at boundary is given by Eq. (4):

$$-k \frac{\partial T\,(x,y,z)}{\partial \eta} \;=\; \dot{q}''_o \;, \qquad \{z > 0, \;\; \eta \perp \Gamma\} \tag{4}$$

Finally, using the symmetry of the problem

$$\left. \frac{\partial T(x,y,z)}{\partial x} \right|_{x=0} = 0 \,, \tag{5}$$

$$\left. \frac{\partial T(x,y,z)}{\partial y} \right|_{y=0} = 0 \,. \tag{6}$$

2.1. Coordinates Transformation

The main difficulty related to the boundary conditions application to elliptical geometries has been removed by employing the following coordinate change, also shown in Fig. 1.

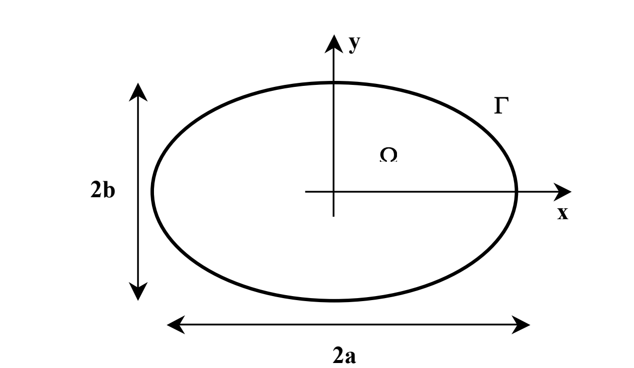

Figure 1. Geometry of the duct cross section and coordinate system.

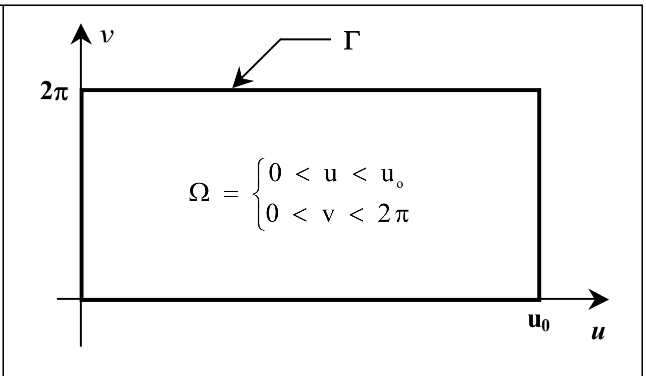

Figure 2. Geometry in the new coordinate system.

The temperature profile and other physical parameters were written in dimensionless form as:

$$\theta(X,Y,Z) = \frac{T_o - T\,(x,y,z)}{\dot{q}''_o \;\, D_h \,/\, k} \,, \tag{7}$$

$$X = \frac{x}{D_h} \,, \qquad Y = \frac{y}{D_h} \,, \qquad Z = \frac{z}{(D_h Pe)} \,, \qquad \eta^* = \frac{\eta}{D_h} \,, \qquad Pe = \frac{\rho c_p w_0 D_h}{k} \,, \tag{8}$$

$$\alpha = \frac{a}{D_h} \,, \qquad \beta = \frac{b}{D_h} \,, \qquad D_h = \frac{4 A_{sc}}{P} \,, \qquad \rho_{asp} = \frac{b}{a} \,, \tag{9}$$

$$A_{sc} = \pi \; ab \,, \qquad P = 4\,a \int_0^{\pi/2} \sqrt{\,1 - \kappa^2 sen^2\theta\,} \;\, d\theta \,, \qquad \kappa = \frac{\sqrt{a^2 - b^2}}{a} \,. \tag{10}$$

Where, A_{sc} is the area of the elliptical section, P the perimeter of the elliptical contour and ρ_{asp} the ellipsis aspect ratio. With these new variables, the energy equation becomes:

$$\frac{\partial\theta(X,Y,Z)}{\partial Z} = \frac{\partial^2\theta(X,Y,Z)}{\partial X^2} + \frac{\partial^2\theta(X,Y,Z)}{\partial Y^2} . \tag{11}$$

The boundary conditions and the entrance condition are as follows

$$\theta(X,Y,Z) = 0 , \quad \{ Z = 0 , \ (X,Y) \in \Omega \} , \tag{12}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial\eta^*} = 1 , \quad \{ Z > 0 , \ \eta^* \perp \Gamma \} , \tag{13}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial X} = 0 , \quad \{ Z > 0 , \ X = 0 \} , \tag{14}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial Y} = 0 , \quad \{ Z > 0 , \ Y = 0 \} . \tag{15}$$

The orthogonal system of elliptic coordinates (u,v) is used to transform the original domain, with elliptical contour in the coordinates (X,Y), into a domain with rectangular contour in the transformed system (u,v)

$$X = \alpha^* cosh(u)\, cos(v), \tag{16}$$

$$Y = \beta^* senh(u)\, sen(v), \tag{17}$$

$$Z = z , \tag{18}$$

With,

$$\alpha^* = \frac{\alpha}{cosh(u_o)} , \tag{19}$$

$$u_o = arctan\,h(\beta/\alpha) , \quad \{0 < \beta < \alpha\} . \tag{20}$$

The coefficients h_u and h_v and the Jacobian J of the transformation of the system of coordinates (X,Y) to the system (u,v) are given by

$$h_u(u,v) = h_v(u,v) = h(u,v) = \alpha^* \sqrt{[sinh^2(u) + sin^2(v)]} , \tag{21}$$

$$J(u,v) = \frac{\partial(X,Y)}{\partial(u,v)} = \alpha^{*2} [sinh^2(u) + sin^2(v)] . \tag{22}$$

With these new variables the equation of the elliptical contour becomes

$$\left[\frac{X}{\alpha^* cosh(u_o)}\right]^2 + \left[\frac{Y}{\alpha^* sinh(u_o)}\right]^2 = 1 , \tag{23}$$

and the energy equation

$$H(u,v) \frac{\partial\theta(u,v,Z)}{\partial Z} = \frac{\partial^2\theta(u,v,Z)}{\partial u^2} + \frac{\partial^2\theta(u,v,Z)}{\partial v^2} , \tag{24}$$

with $H(u,v)$ given by

$$H(u,v) = J(u,v) . \tag{25}$$

The entry and boundary conditions in the new coordinate system become

$$\theta(u,v,Z) = 0 , \quad \{ Z = 0 , \ (u,v) \in \Omega \} , \tag{26}$$

$$\frac{\partial\theta(u,v,Z)}{\partial u} = h(u,v), \quad \{ Z>0, \; u = u_o \}, \tag{27}$$

$$\frac{\partial\theta(u,v,Z)}{\partial u} = 0, \quad \{ Z>0, \; u = 0 \}, \tag{28}$$

$$\frac{\partial\theta(u,v,Z)}{\partial v} = 0, \quad \{ Z>0, \; v = 0, \pi/2, \pi, 3\pi/2 \}. \tag{29}$$

2.2. Homogenization of the Boundary Conditions

For the application of the GITT, it is convenient to implement the homogenization of the boundary conditions to increase the convergence rate of the series that represents the solution. In order to accomplish this, consider the following change of variable

$$\theta(u,v,Z) = \theta^*(u,v,Z) + \frac{u^2}{2\,u_o} h(u_o,v). \tag{30}$$

With this new variable the energy equation is

$$H(u,v)\,\frac{\partial\theta^*(u,v,Z)}{\partial Z} = \frac{\partial^2\theta^*(u,v,Z)}{\partial u^2} + \frac{\partial^2\theta^*(u,v,Z)}{\partial v^2} + G(u,v), \tag{31}$$

with

$$G(u,v) = \frac{h(u_o,v)}{u_o} + \frac{\alpha^* u^2}{2h(u_o,v)\,u_o}\left\{ \cos(2v) - \left[\frac{\alpha^*}{2\,h(u_o,v)}\sin^2(2v)\right]^2 \right\}. \tag{32}$$

The entry and boundary conditions are then redefined

$$\theta^*(u,v,Z) = -\frac{u^2}{2\,u_o} h(u_o,v), \quad \{ Z=0, \; (u,v) \in \Omega \}, \tag{33}$$

$$\frac{\partial\theta^*(u,v,Z)}{\partial u} = 0, \quad \{ Z>0, \; u = u_o \}, \tag{34}$$

$$\frac{\partial\theta^*(u,v,Z)}{\partial u} = 0, \quad \{ Z>0, \; u = 0 \}, \tag{35}$$

$$\frac{\partial\theta^*(u,v,Z)}{\partial v} = 0, \quad \{ Z>0, \; v = 0, \pi/2, \pi, 3\pi/2 \}. \tag{36}$$

2.3. Application of the GITT

To obtain the solution of the diffusion equation in the new coordinate system, Eq.(26), subjected to the conditions given by Eq. (26), (27), (28), (29) the Generalized Integral Transform Technique is applied. In order to accomplish this, consider the following auxiliary eigenvalue problem related to the independent variable v:

$$\frac{d^2\psi(v)}{dv^2} + \mu^2\psi(v) = 0, \quad \{ 0 \le v \le \pi/2 \}, \tag{37}$$

with boundary conditions

$$\frac{d\psi(v)}{dv} = 0, \quad \{ v = 0 \}, \tag{38}$$

$$\frac{d\psi(v)}{dv} = 0, \quad \{ v = \pi/2 \}. \tag{39}$$

The eigenvalues and eigenfunctions associated to this problem are

$$\mu_i = 2\,(i-1), \qquad i = 1,2,3... \tag{40}$$

$$\psi_i(v) = \cos(\mu_i v)\ . \tag{41}$$

The above eigenfunctions are orthogonal allowing the following pair inverse-transform:

$$\tilde{\theta}_i^*(u,Z) = \int_0^{\pi/2} K_i(v)\,\theta^*(u,v,Z)\,dv\,, \tag{42}$$

$$\theta^*(u,v,Z) = \sum_{i=1}^{\infty} K_i(v)\,\tilde{\theta}_i^*(u,Z)\ . \tag{43}$$

Where $K_i(v)$ are the normalized eigenfunctions given by

$$K_i(v) = \frac{\psi_i(v)}{N_i^{1/2}}\,, \tag{44}$$

$$N_i = \int_0^{\pi/2} \psi_i^{\,2}(v)\,dv = \begin{cases} \pi/2 & , \quad i=1 \\ \pi/4 & , \quad i>1 \end{cases} \tag{45}$$

According to the formalism presented by Cotta (1998), Eq. (24) and Eq. (30) are multiplied by operators $\int_0^{\pi/2} K_i(v)\,dv$ and $\int_0^{\pi/2} \theta^*(u,v,Z)\,dv$, respectively. Following this procedure and applying the boundary conditions described by Eq. (26), (27), (28), (29), (33), (34), (35) and (36), the system obtained is

$$\sum_{j=1}^{\infty} A_{ij}(u)\,\frac{\partial \tilde{\theta}_j^*(u,Z)}{\partial Z} + \mu_i^2\,\tilde{\theta}_i^*(u,Z) = \frac{\partial^2 \tilde{\theta}_i^*(u,Z)}{\partial u^2} + C_i(u)\,, \tag{46}$$

where

$$A_{ij}(u) = \int_0^{\pi/2} K_i(v)\,K_j(v)\,H(u,v)\,dv\,, \tag{47}$$

$$C_i(u) = \int_0^{\pi/2} K_i(v)\,G(u,v)\,dv\,. \tag{48}$$

Let us now consider the following eigenvalue problem related to the independent variable u

$$\frac{d^2\phi(u)}{du^2} + \lambda^2\phi(u) = 0\,, \qquad \{0 \le u \le u_o\}\,. \tag{49}$$

Subjected to the boundary conditions

$$\frac{d\,\phi(u)}{d\,u} = 0\,, \qquad \{\,u = 0\,\}\,, \tag{50}$$

$$\frac{d\,\phi(u)}{d\,u} = 0\,, \qquad \{\,u = u_o\,\}\,. \tag{51}$$

The eigenvalues and the eigenfunctions for this new problem are

$$\lambda_m = (m-1)\,\pi\,/\,u_o\,, \tag{52}$$

$$\phi_m(u) = cos(\lambda_m u) . \tag{53}$$

These eigenfunctions are orthogonal and allow the following pair inverse-transform

$$\overline{\tilde{\theta}}^*_{im}(Z) = \int_0^{u_o}\int_0^{\pi/2} K_i(v)\ Z_m(u)\ \theta^*(u,v,Z)\,dv\,du \quad , \tag{54}$$

$$\theta^*(u,v,Z) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} K_i(u)\ Z_m(u)\ \overline{\tilde{\theta}}^*_{im}(Z) . \tag{55}$$

Where $Z_m(u)$ are the normalized eigenfunctions

$$Z_m(u) = \frac{\phi_m(u)}{M_m^{1/2}} , \tag{56}$$

$$M_m = \int_0^{u_o}\phi_m^2(u)\,du = \begin{cases} u_o & , \ m=1 \\ \dfrac{u_o}{2} & , \ m>1 \end{cases} \tag{57}$$

To determine the transformed temperature equation $\overline{\tilde{\theta}}^*_{im}(Z)$, the procedure is similar to that one related to the first eigenvalue problem. Eq. (24) and Eq. (30) are multiplied by operators $\int_0^{u_o} Z_m(u)\,du$ and $\int_0^{u_o}\tilde{\theta}_i^*(u,Z)\,du$, respectively, and using boundary conditions Eq. (26), (27), (28), (29), (33), (34), (35) and (36), the following system of ordinary differential equations is obtained

$$\sum_{n=1}^{\infty}\sum_{j=1}^{\infty} B_{ijmn}\frac{d\overline{\tilde{\theta}}^*_{jn}(Z)}{dZ} + (\mu_i^2+\lambda_m^2)\ \overline{\tilde{\theta}}^*_{im}(Z) + D_{im} = 0 , \tag{58}$$

with

$$B_{ijmn} = \int_0^{u_o} Z_m(u)\ Z_n(u)\ A_{ij}(u)\,du = \int_0^{u_o}\int_0^{\pi/2} K_i(v)\ K_j(v)\ Z_m(u)\ Z_n(u)\ H(u,v)\ dv\,du , \tag{59}$$

$$D_{im} = -\int_0^{u_o} Z_m(u)\ C_i(u)\ du = -\int_0^{u_o}\int_0^{\pi/2} K_i(v)\ Z_m(u)\ G(u,v)\ dv\ du . \tag{60}$$

Parameters B_{ijmn} and D_{im} can be integrated and, therefore, determined. The solution of this system of ordinary differential equations, when subjected to the transformed entry condition, is the following

$$\overline{\tilde{\theta}}^*_{im}(0) = \int_0^{u_o}\int_0^{\pi/2} K_i(v)\,Z_m(u)\theta^*(u,v,0)\,dvdu = -\int_0^{u_o}\int_0^{\pi/2} K_i(v)\,Z_m(u)\frac{u^2}{2u_o}h(u_o,v)\ dv\,du . \tag{61}$$

From system (61) the transformed temperature $\overline{\tilde{\theta}}^*_{im}(Z)$ is obtained. Thus, the dimensionless temperature $\theta(u,v,Z)$ can be numerically evaluated using Eq. (58), just truncating the expansion in series of orthogonal functions, for a given order i = N and $m = M$,

$$\theta(u,v,Z) = \sum_{i=1}^{N}\sum_{m=1}^{M} K_i(v)\ Z_m(u)\ \overline{\tilde{\theta}}^*_{im}(Z) + \frac{u^2}{2u_o}h(u_o,v) . \tag{62}$$

Obviously, larger values of N and M would result in higher accuracy of the numerical results, neglecting round-off errors.

2.4. Calculation of the Bulk Fluid Temperature and the Nusselt Numbers

The bulk fluid temperature for a given duct cross section can be determined by means of a balance of energy between the tube inlet section and the given cross section, at a position *z* along the axis

$$\dot{q}''_o\, P\, z \;=\; \rho\, \bar{u}\, A\, c_p\, (T_{av} - T_e)\,. \tag{63}$$

The dimensionless bulk fluid temperature is expressed by:

$$\theta_{av}(Z) \;=\; 4\, Z\,. \tag{64}$$

The dimensionless bulk fluid temperature can also be determined through the integration of Eq. (65) for the temperature distribution

$$\theta_{av}(Z) \;=\; \frac{D_h^2}{A_{sc}} \int_{\Omega} \theta(X,Y,Z)\; U(X,Y)\, d\Omega\,. \tag{65}$$

In the plane (u,v), θ_{av} is expressed as follows

$$\theta_{av}(Z) = \frac{D_h^2}{A_{sc}} \int_0^{\pi/2}\int_0^{u_o} \{\, [\sum_{i=1}^{N}\sum_{m=1}^{M} K_i(v)\, Z_m(u)\, \overline{\tilde{\theta}}{}^{*}_{im}(Z)\,] + \frac{u^2}{2\,u_o}\, h(u_o,v)\, \} H(u,v)\, du\, dv\;. \tag{66}$$

Eq. (66) and (69) are adequate for the verification of the accuracy of the numerical results when the expansion is truncated in orders $i = N$ and $m = M$.

The average wall temperature is obtained by the integration

$$\theta_{w,av}(Z) \;=\; \frac{4\,D_h}{P} \int_0^{\pi/2} \sum_{i=1}^{N}\sum_{m=1}^{M} K_i(v)\; Z_m(u_o)\, \overline{\tilde{\theta}}{}^{*}_{im}(Z)\; h(u_o,v)\; dv\;. \tag{67}$$

The Nusselt number is defined by $Nu(z) = \dfrac{\mathrm{h}(z)\,D_h}{k}$ with $h(z)$ defined as follows

$$\mathrm{h}(z) \;=\; \frac{q''_o(z)}{T_{w,av}(z) - T_{av}(z)}\,. \tag{68}$$

By using the dimensionless variables

$$Nu(Z) \;=\; \frac{1}{\theta_{w,av} - \theta_{av}}\;. \tag{69}$$

The average Nusselt number is obtained by integrating Eq. (70) along the tube axis

$$\overline{Nu}\,(Z) \;=\; \frac{1}{Z}\int_0^{Z} Nu(Z')\, dZ'\,. \tag{70}$$

Shah & London (1978) define the thermal entry length, L_{th} as the position where the local Nusselt number is 5% higher than the Nusselt number in the region where the fluid is thermally developed. Thus

$$L_{th} \equiv \text{ positive root of } \{1.05\; Nu(\infty) - Nu(Z) = 0\}\,. \tag{71}$$

3. Results and Discussion

In order to determine coefficients $\overline{\overline{\theta}}_{im}(Z)$, the expansion given by Eq. (62) has been truncated to several choices of values for M and N. Parameters B_{ijmn} and D_{im} have been numerically calculated by a Gauss quadrature method (36 points of quadrature) and the equation system, Eq. (62), has been solved by using the routine DIVPAG of the IMSL Library (Visual Numerics, 1994).

It has been noticed that the convergence becomes slower when the aspect ratio $b/a \rightarrow 1$. For truncations to M and N both higher than 25, the values of calculated Nusselt number, for $Z > 0.0001$, converged within around 3 digits or more, for the cases analyzed. Therefore, all calculations in present study truncated the expansion to $M = 25$ and $N = 25$, generating a system of 625 ordinary differential equations.

The difference between the bulk temperature calculated by Eq. (41) and the bulk temperature calculated by Eq.(39) is less than 10^{-5} for $Z > 0.0001$. The results for average wall temperature, local and average Nusselt numbers are presented in Table 1 for aspect ratios $b/a = 0.5$ and $b/a = 0.8$. The behavior of these parameters has been shown in Fig. 3 to 5 for several aspect ratios.

From the results obtained it may be observed that the thermal development is slower when the aspect ratio $b/a \rightarrow 0$. On the other hand, has been noticed that the values of wall temperature and Nusselt number calculated for several eccentricities of ellipsis approximate asymptotically in the region nearby tube entry. In the region where the flow is thermally developed a strong dependence of these parameters with the eccentricity of the ellipsis has been observed, when the aspect ratio $b/a < 0.5$. For the cases where the aspect ratio $b/a > 0.9$, also in fully developed region, an asymptotical approximation of these parameters has been noticed. In particular, the asymptotical local Nusselt number is $Nu = 8.0$ for circular cross section in the duct.

Thermal development occurs farther from the duct entrance when aspect ratio $b/a \rightarrow 0$ and at the limits of $b/a \rightarrow 1.0$ the results for the Nusselt number approximate those obtained for flows in ducts of circular section.

Finally, Table 2 presents the results obtained in this work, corresponding to Nusselt numbers of thermally developed flows.

Table 1. Average wall temperature, local and average Nusselt number along the Z-axis, for several tube aspect ratios.

z	**b/a = 0.50**				**b/a = 0.80**			
	$\theta_{aver.}$	$\theta_{aver.\ wall}$	Nu_{∞}	$Nu_{aver.}$	$\theta_{aver.}$	$\theta_{aver.\ wall}$	Nu_{∞}	$Nu_{aver.}$
0.0001	0.0004	0.0114	91.1	155	0.0004	0.0115	89.8	141
0.0002	0.0008	0.0161	65.2	115	0.0008	0.0162	64.9	108
0.0005	0.0020	0.0257	42.2	76.3	0.0020	0.0258	42.1	73.4
0.0010	0.0040	0.0366	30.7	55.8	0.0040	0.0367	30.6	54.3
0.0020	0.0080	0.0523	22.6	40.6	0.0080	0.0525	22.5	39.8
0.0050	0.0200	0.0845	15.5	27.1	0.0200	0.0852	15.3	26.6
0.0100	0.0400	0.123	12.1	20.2	0.0400	0.124	11.9	19.9
0.0200	0.0800	0.182	9.84	15.4	0.0800	0.183	9.68	15.2
0.0500	0.2000	0.321	8.29	11.4	0.2000	0.321	8.24	11.3
0.1000	0.4000	0.529	7.75	9.69	0.4000	0.525	7.98	9.66
0.2000	0.8000	0.933	7.52	8.64	0.8000	0.926	7.95	8.81
0.5000	2.0000	2.13	7.49	7.95	2.0000	2.13	7.95	8.29
1.0000	4.0000	4.13	7.49	7.72	4.0000	4.13	7.95	8.12
2.0000	8.0000	8.13	7.49	7.60	8.0000	8.13	7.95	8.04
5.0000	20.000	20.1	7.49	7.54	20.000	20.1	7.95	7.98
6.0000	24.000	24.1	7.49	7.53	24.000	24.1	7.95	7.98
7.0000	28.000	28.1	7.49	7.52	28.000	28.1	7.95	7.98
8.0000	32.000	32.1	7.49	7.52	32.000	32.1	7.95	7.98

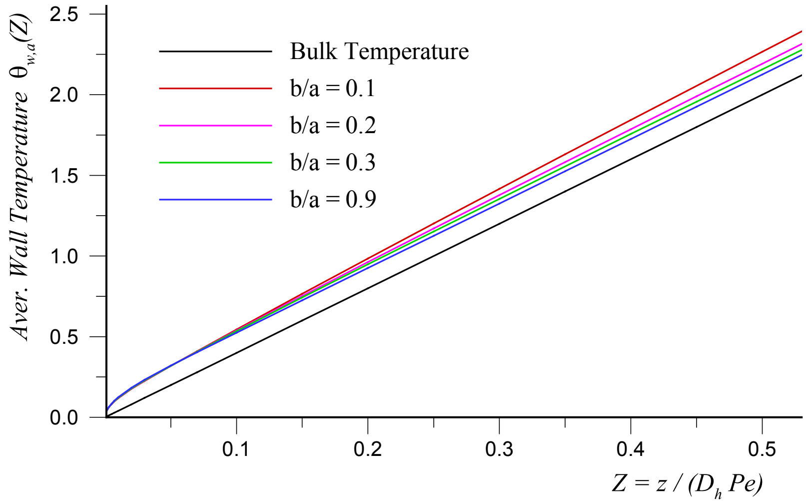

Figure 3. Fluid bulk temperature, $\theta_{av}(Z)$, and average wall temperature, $\theta_{w,av}(Z)$, along the Z-axis, for several aspect ratios b/a = 0.1, 0.3, 0.5, 0.7 and 0.9.

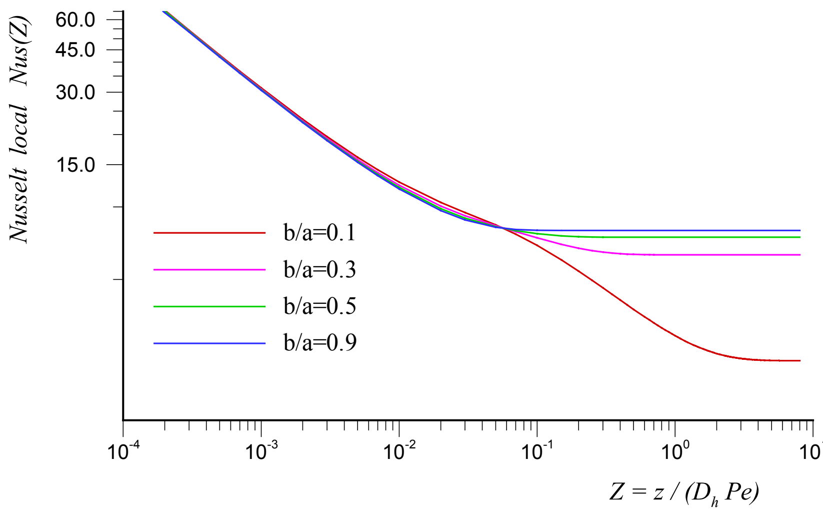

Figure 4. Local Nusselt number along the Z-axis, for several aspect ratios *b/a* = 0.1, 0.3, 0.5, 0.7 and 0.9.

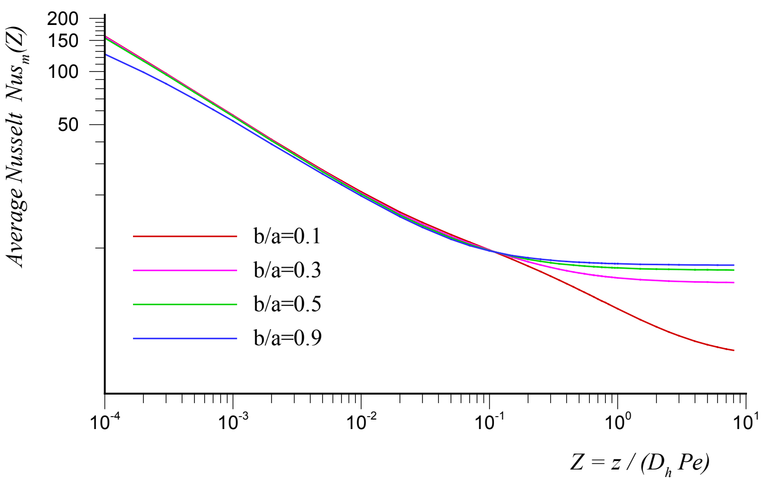

Figure 5. Average Nusselt number along the Z-axis, for several aspect ratios *b/a* = 0.1, 0.3, 0.5, 0.7 and 0.9.

Table 2. Nusselt number for fully developed flow $Nu(\infty)$ and thermal entry length L_{th}, for several tube aspect ratios.

b/a	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.99**
Nu_∞	2.293	4.924	6.336	7.086	7.469	7.695	7.857	7.906	7.929	7.934
L_{th}	2.36	0.522	0.228	0.121	0.0877	0.0647	0.0500	0.0481	0.0471	0.0468

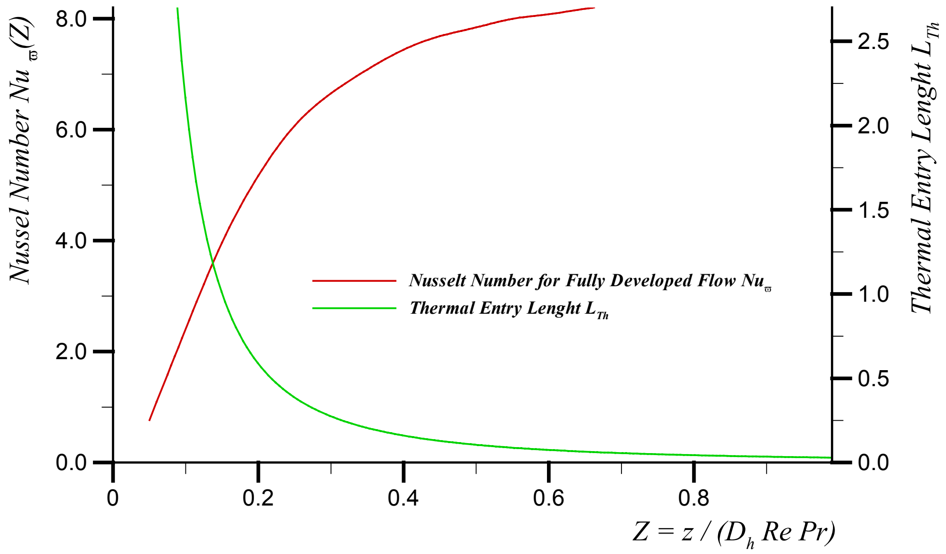

Figure 6. Shows the Limit Nusselt and Thermal Entry Length, for elliptical ducts cross section flow.

4. Conclusion

In the present work, the GITT has been successfully applied to the problem of thermally developing flow inside ducts of elliptical cross section, subjected to second kind boundary conditions. The inherent difficulty of application of the boundary in problems with this geometry has been removed by means of a change of the cartesian variables to a system of orthogonal elliptic coordinates that transforms the contour of elliptical section of the duct to a new domain of rectangular shape. The convergence of the temperature distribution is slow, and it is necessary to truncate M and N at a relatively high order. For regions near to the duct entrance, $Z \approx 0.0001$, has been necessary to truncate the series to $M = N > 25$ to obtain a satisfactory convergence for the Nusselt number calculations, although for values of Z nearby in the region of thermal development, satisfactory convergence has been obtained with fewer terms in the series. In fact, in the fully developed region, five terms in the series have usually been quite sufficient for a good numerical convergence.

The results obtained through GITT allowed the determination of the interest parameters, as well as the wall temperature and the local and average Nusselt number. Moreover, the results for the Nusselt number obtained for the thermally developed region exhibited excellent agreement with those reported in the literature at the limits of $b/a \to 1.0$ (circular section ducts).

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