Proceedings of the 10^{th} Brazilian Congress of Thermal Sciences and Engineering – ENCIT 2004
Braz. Soc. of Mechanical Sciences and Engineering – ABCM, Rio de Janeiro, Brazil, Nov. 29 – Dec. 03, 2004

Paper CIT04-0107

PRESSURE DROP OF NON-NEWTONIAN FLUID FLOWS THROUGH CONVERGING-DIVERGING CHANNELS

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Abstract. *In this work, the flow of viscoelastic fluids through axisimmetric converging-diverging channels is analyzed. The solution of mass and momentum conservation equations is obtained numerically via finite volume technique using the Fluent software. The Generalized Newtonian Fluid constitutive equation was used to model the non-Newtonian fluid behavior, using the Shunk-Scriven model for the viscosity, where a weighted geometric mean between shear and extensional viscosities is assumed. The results of pressure drop are compared to the ones predicted by a previously proposed simplified relation (Souza Mendes and Naccache, 2002) between pressure drop and flow rate, for viscoelastic fluids flow through porous media, in order to analyze its performance.*

keywords: *Viscoelastic fluid, porous media, Darcy's law*

Introduction

Flow of viscoelastic fluids through porous media can be found in several industry processes. One example is the use of water with polymers to enhance oil recovery from reservoirs. Since flows through porous media have a strong extensional kinematic characteristic, with converging-diverging passages, the non-Newtonian fluid mechanical behavior play a significant role on the process.

The mechanical power loss of a liquid due to viscous action while flowing through channels is of the form $Q\Delta p$, where Q is the volume flow rate and Δp the pressure drop. In the classical theories of porous media, it is customary to blame this power loss on the action of shear stresses, so that $Q\Delta p$ would be proportional to an appropriate volume integral of $\tau_{21}\dot{\gamma}$, τ_{21} being the shear stress and $\dot{\gamma}$ the shear rate (the flow is locally in the 1 direction, i. e., $\mathbf{v} = u_1\mathbf{e}_1$, and 2 is the local direction of $\mathrm{grad u_1}$). Examples are the Carman-Kozeny theory and the bundle of capillary tubes models. Most works in the literature use this kind of approach. Pearson and Tardy (2002) present a study of continuum models (such as Darcy's Law for Newtonian fluids) for transport in porous media, giving special attention to the influence of rheological parameters on an effective viscosity, dispersion length and relative permeability in multi-phase flow. They observed that most applications of continuum theory use Darcy's Law or some extension of it. Then, the problem is reduced to prescribe an effective viscosity for single-phase flow or relative permeability in multi-phase flow. However, the complexity of real porous media geometry makes this a very difficult problem. Commit et al. (2000) performed an experimental study to determine the critical Reynolds number for Newtonian and non-Newtoninan purely viscous fluid, which establishes the limit of validity of Darcy's equation. Khuzhayorov et al. (2000) derived a generalised Darcy's Law for transient linear viscoelastic fluid flow in porous media, using the homogenisation theory. It was obtained a dynamic permeability tensor, valid at low Reynolds and Deborah numbers. The results for an Oldroyd fluid flow through a bundle of capillary tubes show that the viscoelastic behavior strongly differs from the Newtonian behavior. However, the comparison of these results to another model obtained in the literature, indicate that there is not yet a conclusive answer for describing the flow of Oldroyd fluids through porous media. Liu and Masliyah (1999) used a volume average approach to derive governing equations for purely viscous flows in porous media, obtaining good agreement with experimental results. Wu and Pruess (1998) present a numerical three-dimensional solution of purely viscous non-Newtonian flow through ideal porous media, using a Darcy's Law with an effective viscosity. Pascal and Pascal (1997) studied the flow of Power-Law fluids through porous media, using a modified Darcy's equation. Some numerical results were compared to exact solutions, presenting good agreement.

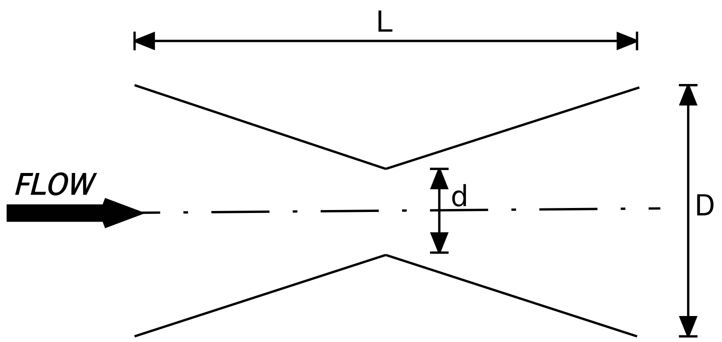

Figure 1: The geometry.

The rate of conversion of mechanical energy into thermal energy in a flow per unit volume is equal to $\mathrm{tr}(\boldsymbol{\tau}\,\mathrm{grad}\,\mathbf{v})$, where $\boldsymbol{\tau}$ is the extra-stress and $\mathrm{grad}\,\mathbf{v}$ is the velocity gradient. In a shear flow this reduces to $2\tau_{21}\dot{\gamma}$, but in general other terms related to extension rates also appear. In a simple but quite illustrative analysis, Durst et al. (1987) showed that these non-shear terms are actually much larger than the shear terms. Their analysis suggests that, for a Newtonian liquid, about 75% of the power loss ($Q\Delta p$) is due to extension. They also argue that the known disparity between the above theories and experimental data is related to the absence of extension in the corresponding model flows.

For Newtonian fluids, the extensional viscosity ($\eta_{\mathrm{E}} \equiv (\tau_{11} - \tau_{22})/\dot{\varepsilon}$, where $\dot{\varepsilon}$ is the extension rate) is just three times the shear viscosity ($\eta_{\mathrm{E}} = 3\mu$). This explains the qualitative success of the simple shear assumption. For viscoelastic liquids, however, the shear and extensional viscosities often behave oppositelly: while the shear viscosity ($\eta_s \equiv \tau_{21}/\dot{\gamma}$) is generally a decreasing function of the shear rate, the extensional viscosity η_{E} increases as the extension rate is increased. Moreover, η_{E} is typically some order of magnitudes larger than η_s, which makes the shear stresses to contribute negligibly to the total power loss.

Chmielewski and Jayaraman (1992) observed that extensional viscosity is the governing material function for flows of non-Newtonian fluids through an array of cylinders, which is a model porous medium. Souza Mendes and Naccache (2002) developed a constitutive relation between the flow rate and pressure drop, which accounts for the fluid's mechanical behavior on extension, using the converging-diverging geometry. In this paper, a numerical simulation of flow of viscoelastic fluids through converging-diverging geometries is performed. The governing mass and momentum conservation equations were solved numerically using finite volume technique and Fluent software. Then, the numerical results of pressure drop were compared to the ones predicted by the new constitutive relation proposed by Souza Mendes and Naccache (2002), in order to analyze its performance.

Mathematical Modeling

The geometry analyzed is shown in Fig. 1. The fluid flows through a converging-diverging axysimmetric channel. A periodically developed flow condition is considered at the flow inlet and outlet. The non-Newtonian fluid behavior is modeled by a modified Generalized Newtonian Fluid constitutive equation, where the extensional-thickening behavior is accounted for. The governing conservation equations are presented bellow, for incompressible fluid, steady flow and negligible external forces.

The mass conservation equation is given by:

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0 \tag{1}$$

where x is the axial coordinate, r is the radial coordinate, u and v are the velocity components on axial and radial directions, respectively. The momentum conservation equations are given by:

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\eta\frac{\partial u}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\eta\frac{\partial u}{\partial r}\right) \tag{2}$$

$$\rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial x}\left(\eta\frac{\partial v}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v}{\partial r}\right) + \rho g \tag{3}$$

where ρ is the density, p is the pressure and g is the gravity.

The Generalized Newtonian fluid constitutive equation is used to model the non-Newtonian fluid behavior, where

$$\boldsymbol{\tau} = 2\eta\mathbf{D} \tag{4}$$

where the viscosity function η is obtained by a weighted geometric mean of shear and extensional viscosities (Souza Mendes *et al.* (1995)):

$$\eta(\dot{\gamma}, R) = \eta_s(\dot{\gamma})^R \eta_u(\dot{\gamma})^{1-R} \tag{5}$$

where $\dot{\gamma} = \sqrt{1/2 \mathrm{tr}\mathbf{D}}$ is the rate-of-strain tensor modulus, $\mathbf{D} \equiv [\nabla\mathbf{v} + (\nabla\mathbf{v})^T]/2)$ is the rate-of-strain tensor, and R is a flow classification parameter, introduced by Astarita (1979), $R \equiv -\frac{\mathrm{tr}\mathbf{W}^2}{\mathrm{tr}\mathbf{D}^2}$. The tensor $\mathbf{W} = \mathbf{W} - \mathbf{\Omega}$ is the relative-rate-of-rotation tensor,$\mathbf{W}$ is the vorticity tensor, defined as $[\nabla\mathbf{v} - (\nabla\mathbf{v})^T]/2$, and $\mathbf{\Omega}$ is a tensor related to the rate of rotation of $\mathbf{D}$ following the motion (Thompson et al., 1999).

The constitutive equation used in this work is in fact a first approximation for the algebraic constitutive equation proposed by Thompson et al. (1999). In this equation, the extra-stress tensor is a function of flow kinematics, through the flow classification parameter R, and is given by:

$$\mathbf{T}(\mathbf{D}, \mathbf{W}) = -p\mathbf{I} + \alpha_1\mathbf{D} + \alpha_2\mathbf{W}^2 + \alpha_3(\mathbf{DW} - \mathbf{WD}) \tag{6}$$

where the coefficients $\alpha_i,\ i = 1, 2, 3$ are of the form $\alpha_i = \alpha_i(II_D, R)$, and are constructed to fit well both shear and extensional data, giving also a reasonable qualitative behavior for other flows ($0 \leq R < \infty$). For the sake of simplicity we assumed $\alpha_2 = \alpha_4 = 0$, and $\alpha_1 = 2\eta$ (Thompson et al. (1999), leading to the Generalized Newtonian model presented above.

In this work, all the results were obtained for a constant shear viscosity and an extensional-thickening viscosity, given by:

$$\eta_u = \eta_0 \left[1 + (\lambda\dot{\gamma})^2\right]^{(n-1)/2} \tag{7}$$

where η_0, λ and n are rheological constants, obtained experimentally.

The Theory: Souza Mendes and Naccache (2002) proposed a simplified relation for viscoelastic flows through porous media, similar to Darcy's law for Newtonian fluids. The relation was developed in two steps. Firstly the pressure drop/flow rate relationship for an ideal pore channel was obtained using a converging-diverging channel and a purely extensional fluid, where the viscosity is given by $\eta = K\dot{\epsilon}^n$, where K and n are rheological parameters and $\dot{\epsilon}$ is the extension rate. Then, a capillaric model theory was applied to obtain the sought-for constitutive equation, which is given by:

$$\frac{Q}{A} = \mathcal{M}\frac{\Delta p}{L} \tag{8}$$

where Q is the flow rate, $\Delta p/L = (\Delta p)_{cycle}/2\mathcal{L}$ is the average pressure gradient (L is the total length of the porous sample), and the mobility $\mathcal{M}$ is

$$\mathcal{M} = \left\{6\phi R_0^2 \mathcal{L}_* \frac{{R_*}^3(1-R_*)^2}{(1-{R_*}^3)}\right\} \cdot \mathcal{K}^{-\frac{1}{n+1}} \tag{9}$$

$$\times \left\{\frac{3(n+1)2^{-(2n+4)}}{\mathcal{L}_* R_0^n (1-R_*)^n \left(1 - {R_*}^{3(n+1)}\right)}\right\}^{\frac{1}{(n+1)}} \qquad \times \left\{\frac{\Delta p}{L}\right\}^{-\frac{n}{n+1}} \tag{10}$$

In this equation, $R_* = d/D$ is the ratio of smallest and largest channel radius, $L_* = L/2/(D-d)$ is a dimensionless channel length, $R_0 = D/2$ is the largest channel radius. The first factor of the RHS of eq. 10 is purely geometrical, while the second is purely rheological. The third and fourth factors are respectively geometry-rheology and pressure gradient-rheology interactions.

Numerical solution

The governing equations presented above have been discretized via the finite volume method described by Patankar (1980), using a central-difference scheme and the SIMPLE algorithm (Patankar, 1980) to couple velocity and pressure. The numerical results were obtained using the Fluent (Fluent Inc.) software. The flow classification parameter and the viscosity function were obtained using a user-defined function of the main program.

In order to validate the numerical solution, some mesh tests have been performed. Results of the velocity at the centerline in two different axial positions (at x=0 and x=L/2) were evaluated for seven different meshes. The error of the values obtained with the worst mesh (20x5, 20 control volumes in axial direction and 5 control volumes in radial direction) in relation to the ones obtained with the finest mesh (140X35) were below 5%. Therefore, it was chosen an intermediate mesh (100x25), which gave a difference of about 0,2% in the velocity values compared to the finest mesh.

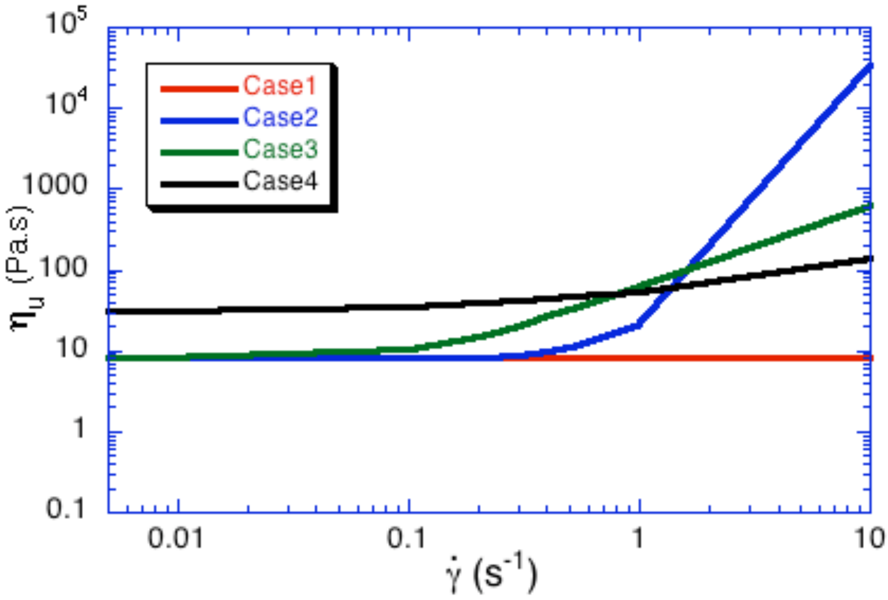

Figure 2: The extensional viscosity function, η_u.

Results and discussion

In this work some previous results were obtained and compared to the ones predicted by the constitutive relation proposed by Souza Mendes and Naccache (2002). The effects of rheological parameters on flow pattern and pressure drop were also analyzed.

First of all, an experimental characterization of a viscoelastic fluid (solution of water, PEG (30%) and PEO (0.05%)) was performed, in order to obtain real values of the shear and extensional viscosities. The fluid analyzed was a Boger fluid, with a constant shear viscosity. The viscosity data were obtained using the ARES rotational rheometer (TA Instruments) and the opposed nozzle rheometer (RFX) (TA Instruments). For this fluid, the shear viscosity is $\eta_s = 0.6$ Pa.s, and the extensional viscosity is constant, $\eta_u = 8$ Pa.s (Case 1). In order to analyze an extensional-thickening behavior, other situations were investigated in the numerical simulations. The viscosity functions of these fluids are shown in Fig.2. For these cases, the extensional viscosity is given by eq. (7), with the following rheological parameters:

- Case 2: $\eta_0 = 8$ Pa.s, $\lambda = 0.8$ s and $n = 5$
- Case 3: $\eta_0 = 8$ Pa.s, $\lambda = 8$ s and $n = 2$
- Case 4: $\eta_0 = 30$ Pa.s, $\lambda = 20$ s and $n = 1.2$

Flow pattern results are shown in Figs. 3-7. Velocity fields for cases 2 and 4 are shown in Figs. 3 and 6. It can be observed that larger values of velocity are obtained at the contraction, as expected. However, for case 4 (higher extensional behavior) the maximum value of velocity is shifted to a point above the centerline. This is due to the fact that for this case higher viscosities are obtained near the centerline, where extensional behavior prevails. The flow classification parameter R is presented in Figs. 4 and 7, for cases 2 and 4. It can be observed that a extensional region $R = 0$ appears near the centerline, and regions near the walls are essencially shear flow regions, with $R = 1$. Accordingly to this behavior it can be viewed, with the aid of Figs. 5 and 8, that viscosity is larger near the centerline.

Finally, the pressure-flow rate results are shown in Figs. 9-12. Figure 9 shows the numerical results of flow rate as a function of pressure drop for all cases analyzed. Figs. 10-12 show flow rate-pressure data obtained in the numerical simulations, and the ones predicted by the constitutive relation proposed by Souza Mendes and Naccache (2002), eq. (10), for cases 2, 3 and 4. The mobility factor $\mathcal{M}$ was obtained using the geometric and rheological parameters for the cases presented above. Therefore, $R_* = d/D = 1/3$, $L_* = 2L/(D-d) = 6$, the porosity $\phi = 0.48$ and $R_0 = 0.075$ m. The rheological parameters that appear in eq. (10) were fitted for the fluids analyzed in cases 2-4 for the range of strain rate investigated. Therefore, for case 2, $K = 20$ Pa.s^n and $n = 1$. For case 3, $K = 64$ Pa.s^n and $n = 1$ and for case4, $K = 54.6$ Pa.s^n and $n = 0.2$. These values lead to $\mathcal{M} = 2.08 \times 10^{-4}$ for case 2, $\mathcal{M} = 1.17 \times 10^{-4}$ for case 3 and $\mathcal{M} = 5.12 \times 10^{-6}$ for case 4. It can be observed that the numerical results are not in good agreement with the ones predicted by the constitutive relation (eq. (10)). Accordingly to the results shown in Souza Mendes and Naccache (2002), the constitutive equation gives good results for a certain range of pressure drop, where the extensional fluid behavior plays

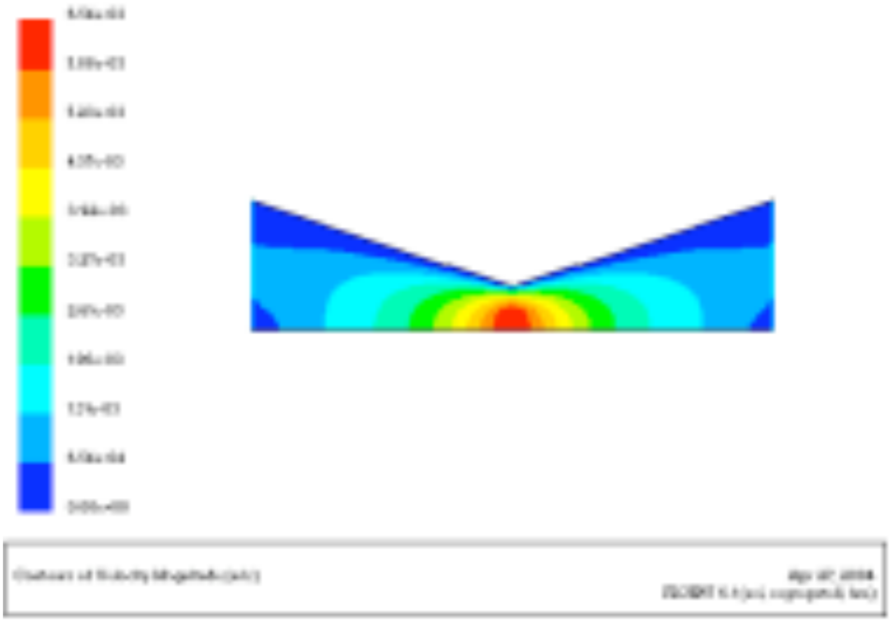

Figure 3: Velocity field for case 2.

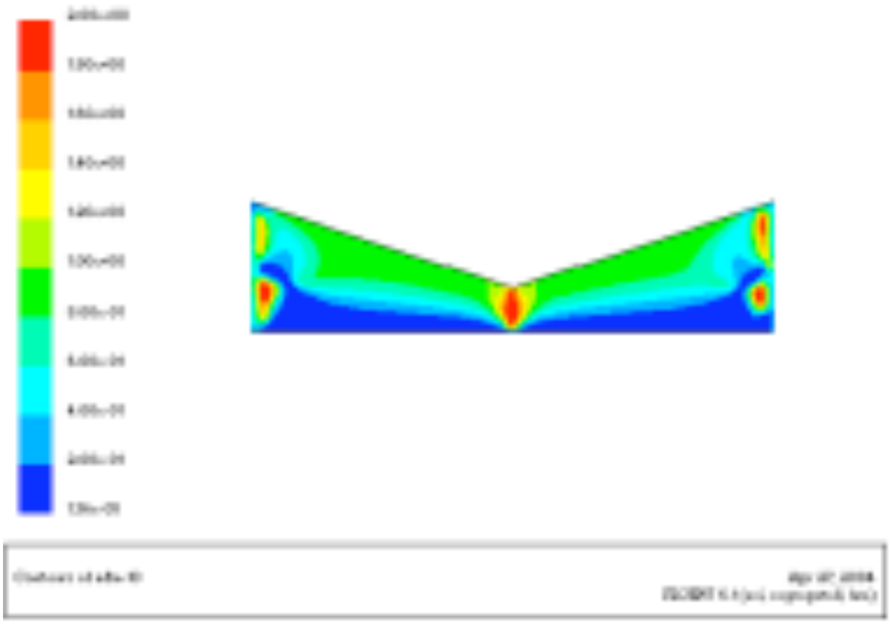

Figure 4: Flow classifier R field for case 2.

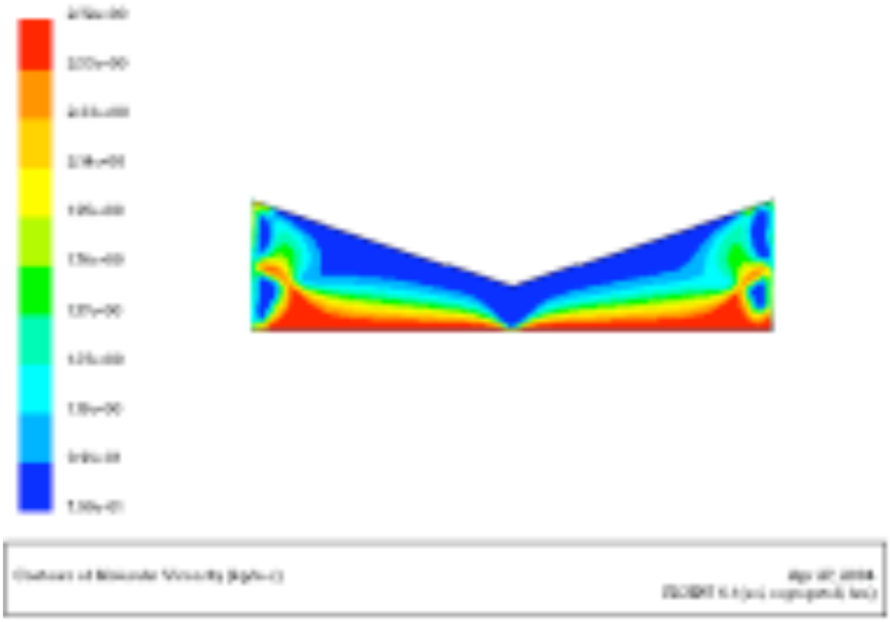

Figure 5: Viscosity field for case 2.

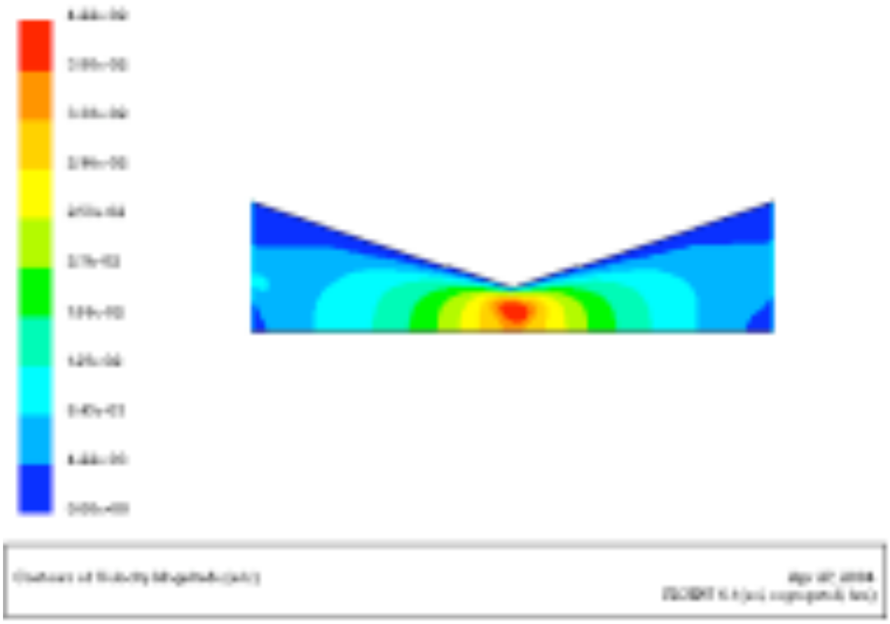

Figure 6: Velocity field for case 4.

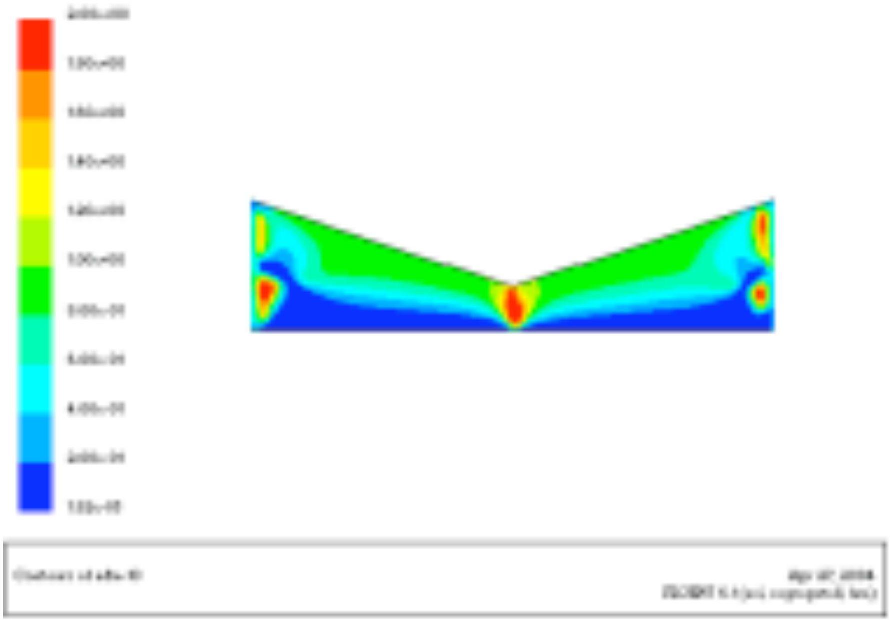

Figure 7: Flow classifier R field for case 4.

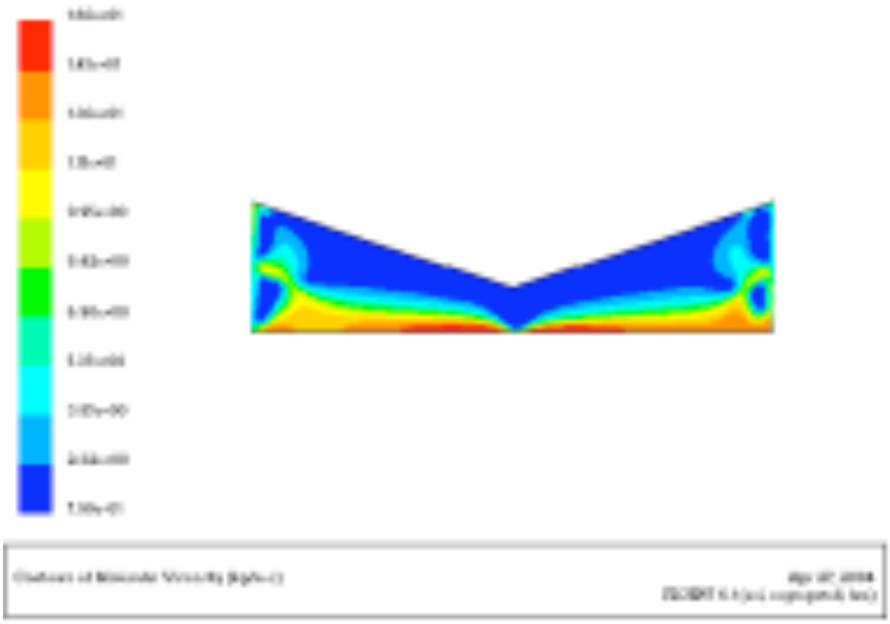

Figure 8: Viscosity field for case 4.

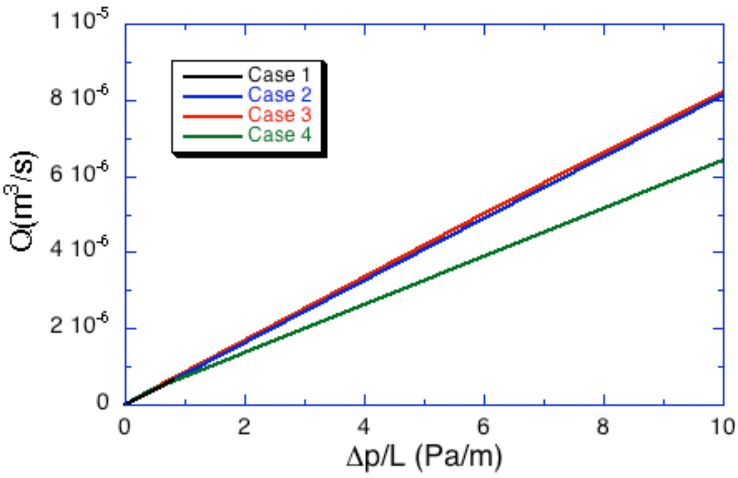

Figure 9: Pressure drop *versus* flow rate for cases 1-4.

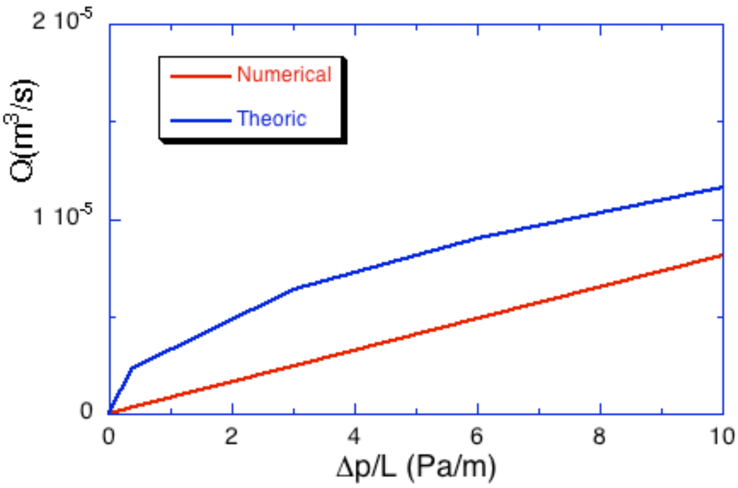

Figure 10: Comparison of numerical and theoretical results for case 2.

an important role due to extensional flow characteristic. As flow rate (or pressure drop) is increased, a gradual change in flow pattern occurs with the fluid flowing more straightly in narrow cores, so that the streamlines get less curved, and the extensional flow characteristics are reduced.

For case 2, the range of flow rate investigated gives a maximum strain rate of 1.03 s^{-1} inside the cannel. For cases 3 and 4 this value is equal to 8.44 s^{-1} and 9.04 s^{-1}, respectively. Analyzing the extensional viscosity function (Fig. 2) for these three cases, it can be observed that up to these points, the extensional viscosity for case 2 is about only twice the value of η_0. The variation of viscosity with strain rate is very smooth for case 4, and case 3 presents a stronger viscosity variation in the range of strain rates investigated. The numerical results obtained for case 2 (Fig. 10) are at the same order of magnitude of the ones predicted by the constitutive relation. However, a linear variation is obtained, probably due to the fact that at that range of flow rate, the extensional viscosity is low and the fluid behaves as a Newtonian fluid. The numerical results obtained for case 3 (Fig. 11) show an almost linear relation between pressure drop and flow rate, In this case, the results are worst for higher flow rates. The comparison of numerical and theoretical results for case 4 is presented in Fig. 12. It can be observed that in this case there is also a slightly non-linear variation of flow rate with pressure drop. However, the results predicted by the constitutive relation are significantly lower than the numerical results. These results are possibly caused by the flow kinematics, meaning that the geometry and rheology considered in the numerical simulations provide only a weak extensional behavior at the range of flow rate analyzed. Another possibly explanation is that the evaluation of the rheological parameters K and n is not good, since they were obtained by a curve fitting ($\eta = K\dot{\epsilon}^n$) of the extensional numerical viscosity. Since in the numerical simulation the resulting viscosity is a weighted average of shear and extensional viscosity, this fitting could not be a good approximation.

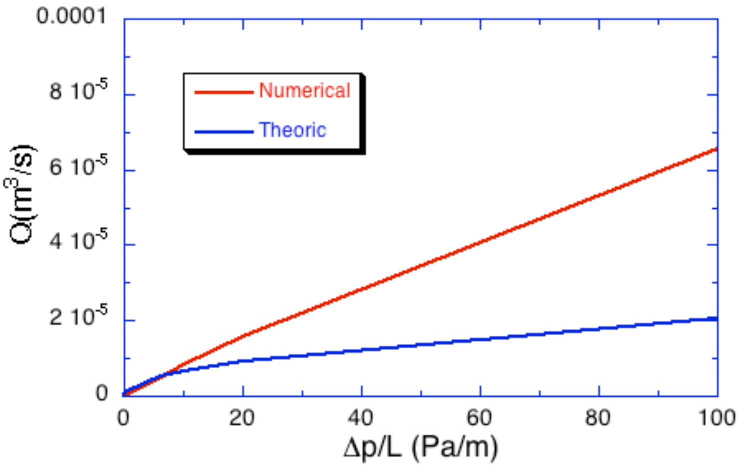

Figure 11: Comparison of numerical and theoretical results for case 3.

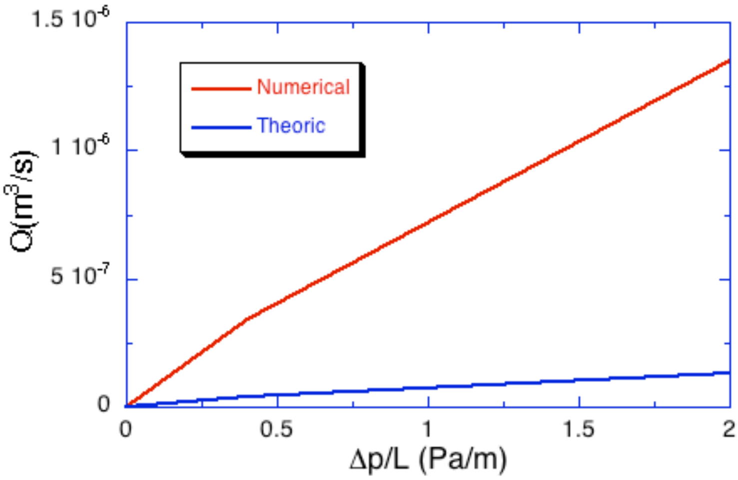

Figure 12: Comparison of numerical and theoretical results for case 4.

Final Remarks

In this work a numerical simulation of viscoelastic fluids through a converging-diverging channel was performed, using the Fluent software. The Generalized Newtonian Fluid constitutive equation was used to model the non-Newtonian fluid behavior, where the viscosity is obtained by a weighted geometric mean between shear and extensional viscosities. The geometry analyzed can be viewed as a model porous media, since they have the same predominantly extensional flow kinematics.

Flow pattern and pressure drop-flow rate data were obtained for different rheological parameters. It could be observed that a purely extensional region appeared near the centerline, while near the walls a shear flow kinematics occured. The results of pressure drop and flow rate were compared to the ones predicted by a previously proposed simplified relation (Souza Mendes and Naccache, 2002) between pressure drop and flow rate, for viscoelastic fluids flow through porous media, in order to analyze its performance. Comparisons showed that the constitutive equation do not reproduce well the pressure drop data in the situations analyzed in this work. One possible explanation is that the range of flow rate investigated did not provide the extensional flow kinematics expected. Another fact that has to be considered is that the mobility factor estimated for the comparisons can carry some error, because of the evaluation of the rheological parameters K and n. Therefore, the tests realized in this work are not conclusive to analyze the performance of the constitutive equation, and more tests should be performed.

Acknowledgements

Financial support for the present research was provided by CNPq and MCT.

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