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LDA MEASUREMENTS OF THE FLOW BEHIND A HYDRAULIC STRUCTURE

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Abstract. *The energy dissipation behind a hydraulic structure was experimentally studied with the aid of Laser-Doppler anemometry. Special attention has been given to the investigation of the behaviour of the rock protection bed in response to a variety of flow conditions. The flow field over the rip-rap was studied in order to asses its efficiency in preventing damage to the foundations of the hydraulic structure as well as in protecting the river bed from erosion. To conduct this study, measurements of mean velocity and turbulence intensities were carried out using laser-Doppler anemometry. A detailed description of the experimental apparatus and instrumentation is given in a specific section. The results obtained are compared with the empirical laws available in literature. Discrepancies between the theoretical predictions and the measured scour profiles were observed.*

Key-words: Laser-Doppler anemometry, turbulence, channel flow, experimental simulation, scouring.

1. Introduction

The flow behaviour behind a hydraulic structure is undeniably a very complex phenomenon and has been a source of interest and questioning since ancient times. Despite all the background knowledge furnished by the classical laws of Hydrodynamics, some difficult tasks such as energy dissipation or the influence of turbulence effects are not yet fully understood.

In the present study, the physical problem of interest is the energy dissipation behind a dam located on an alluvial bed. Considering this specific situation, it is a common practice to use stilling basins as the energy dissipator device, followed by a rock protection bed. One of the main concerns with regard to stilling basins is the fact that, under adverse flow conditions, the hydraulic jump might be dislocated from the basin downstream of the rock protection, thereby causing erosion and possible damage to the structure itself.

Therefore, the objective of this work is to study the process involved in the energy dissipation behind a hydraulic structure, namely, a dam located on a river bed. Special attention has been given to the investigation of the scour process under two different discharge flow conditions. Measurements of the flow field over the rock protection were conducted in order to study its response in preventing damage to the foundations of the structure as well as in protecting the river bed from erosion. The dimensions of the scour hole generated by the most critical flow condition were compared with the theoretical predictions, showing some discrepancies. To accomplish these activities, a physical model of the Crestuma-Lever dam, located at Douro River, Portugal, was built. All the experiments were conducted in a water channel located at the Hydraulics Laboratory of the University of Porto. The choice for the physical model was taken based on two main reasons: firstly, the Crestuma-Lever dam is a presently operational one, so that there is a future possibility of field data being obtained; secondly, specific experimental data are available for comparison since previous studies have already been conducted on models of this prototype. Actually, the present study was encouraged by earlier investigations which results were not conclusive about the prediction of the scouring on rock-protection beds.

Consequently, one point of particular concern to many researchers is to generate reliable experimental data that would shed some light on the development of theories capable of predicting the response of the rock protection. Furthermore, guidelines or criteria could be devised for the proper selection of the grain-size particles and for the design of the whole rip-rap, which would improve the capacity of assuring the integrity of both the structure and the river bed.

Hence, a preliminary laboratory investigation was performed for a variety of flow conditions occurring on the rip-rap and the results obtained are presented herein with the hope that this data might be useful for the purpose above mentioned.

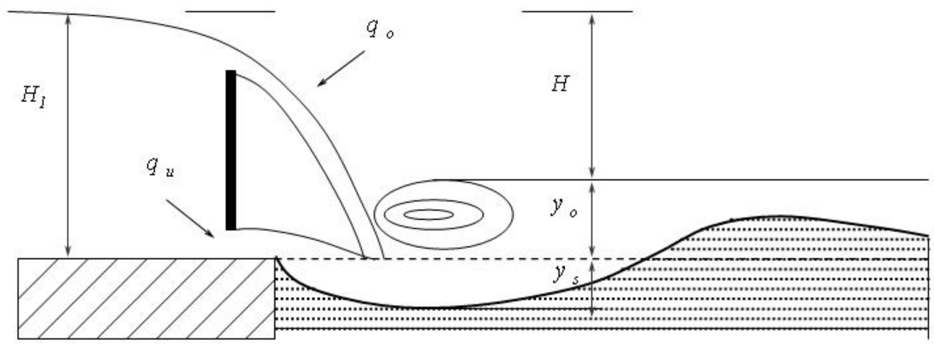

Figure 1: An illustration of the physical problem of interest.

2. A brief literature review

In this section, a short literature review is presented. An effort has been made to select, from the literature of the field, the most relevant studies where new formulas for calculating the scour depths have been proposed. The results obtained in this work will then be compared with these available predictions of the rock protection behaviour.

Stilling basins are normally used to increase the protection of the alluvial bed from intensive scour. However, little generally applicable information is available on the dimensions of the scour holes downstream such structures. Novak (1955, 1961) stated that the use of a stilling basin which is sufficiently long to contain the hydraulic jump will reduce the scour in approximately 45 to 65%. Catakli et al. (1973) conducted an important series of small-scale experiments on a spillway having a stilling basin with length $5y_{0.}$ Considering as a reference the equations for scour without a stilling basin, and taking into account his experimental results, the autors proposed the following equation:

$$y_s + y_0 = 1.6 H_1^{0.2} q^{0.6} d_{90}^{-0.1}, \tag{1}$$

where y_s represents the depth of scour below the original level bed level, y_0 is the downstream water depth and H_1 is the upstream water level, all dimensions are considered in meters. q denotes the flow rate per unit width (m^2/s) and d_{90} represents the characteristic size of the bed particles. In all the formulas the grain size d is expressed in millimeters unless otherwise specified. Figure (1) gives an illustration of the variables of interest, where q_o and q_u denotes flow over the gates and flow under the gates respectively. Despite obtaining good results, the study developed by Catakli et al. (1973) was limited in the sense that the author only considered the cases where the hydraulic jump was completely situated inside the stilling basin. However, in most practical applications, variations in the flow rate may lead the hydraulic jump downstream, i.e., outwards of the stilling basin.

When the jump manage to escape the basin and reach the rock protection, Eq. (1) deduced by Catakli et al. (1973) does not apply any more. Now, equations for jumps impinging directly on the rip-rap should be used. As explained by Ghetti and Zanovello(1954), the local scour is a very complex phenomenon, especially in terms of estimating the maximum erosion depths. During the formative phase of the scour profile, the local sediment transport is rather active, while approaching the equilibrium condition the phenomenon tends to a purely hydraulic mechanism, in which the hole profile is the result of a mass balance between removed and deposited particles inside the pool.

A comprehensive literature review and data analysis carried out by Mason and Arumugan (1985) proved that the concept of an equilibrium scour depth is actually valid. This result justifies the neglecting of transient effects on the equations for the prediction of the final scour.

Schoklitsch (1932) evaluated model tests for the specific case of underflow alone with a short horizontal sill, and proposed the following equation:

$$y_s = 0.378\, H_1^{0.5} q^{0.35} + 2.15a\,, \tag{2}$$

where a is the level of the downstream river bed below the sill level. The effect of grain-size on the scour was negligible for the range d = 1.5 to 12 mm in the model tests; the difference H_1 between the upstream water level and sill level varied from 0.3 to 1.0 m in his experiments. The tail water depth was not varied independently. Schoklitsch (1932) also discusses the effect of sill shape and form of the hydraulic structure, but most of the information is restricted to the specific shapes used.

Veronese (1937) considers the equation below as the most appropriate to describe the behaviour of plunging jets:

$$y_s + y_0 = 3.68 H_1^{0.225} q^{0.54} d_{90}^{-0.42}\,, \tag{3}$$

for d ranging from 9 to 36 mm and q between 0.01 and 0.07 m^2/s.

Jager (1939) reanalyzed the data obtained by Veronese (1937) and gave the relationship:

$$y_s + y_0 = 6H^{0.25} q^{0.5} \left(y_0 / d_{90}\right)^{0.33}. \tag{4}$$

Eggenberger (1944) conducted tests in a laboratory flume and proposed the equation:

$$y_s + y_0 = 22.9 H^{0.5} q^{0.6} d_{90}^{-0.4}\,, \tag{5}$$

for d_{90} ranging from 1.2 to 7.5 mm, q_0 between 0.006 and 0.024 m^2/s and H between 0.19 and 0.35 m.

Müller (1944) performed tests for two different conditions, namely, (i) flow under the gates and (ii) flow over and underneath the gates simultaneously. For the condition (i), Müller reported two patterns of flow with different quantities of scour, which he describes as submerged wavy jet and a free wavy jet. For the two cases the first coefficient in Eq. (5) was modified and the new values proposed are 10.35 for the submerged wavy jet and 15.4 the case of free wavy jet. Then, for a free wavy jet we have:

$$y_s + y_0 = 15.4 H^{0.5} q^{0.6} d_{90}^{-0.4}. \tag{6}$$

Hartung (1957) repeated the tests of Veronese using q = 0.0036-0.21 m^3/s, grain size from 2 to 15 mm and obtained:

$$y_s + y_0 = 12.4 H^{0.36} q^{0.64} d_{85}^{-0.32}. \tag{7}$$

However, Katoulas (1967) remarks that values obtained from Eggenberger and Müller's relationships are usually reduced by approximately 30 to 50 % for practical engineering applications. The uncertainties associated to the evaluated scour depths from the above equations are yet quite large. As highlighted by Mason and Arumugan (1985), those formulas which give good agreement for model purposes may not be directly applicable to prototype applications.

3. Considerations about stilling basins

Stilling basins where hydraulic jumps develop are often used by engineers as a mean of dissipation of the flow energy during flood discharges. It is a common practice to protect the downstream structure of a dam with a concrete slab, which characteristics depend on the hydraulic jump development. The river bed located downstream of the concrete apron is generally protected by rock material specially designed for this purpose.

The hydraulic jump can take place in a wide range of conditions, leading to different rates of energy dissipation. In general, at least four distinct types of hydraulic jumps can occur on a horizontal apron, which are commonly classified by the Froude number value.

Assuming a hydraulic jump with conjugate depths D_1 and D_2, the Froude number can be defined as:

$$F_1 = \frac{U_1}{\sqrt{gD_1}}\,, \tag{8}$$

where g is the acceleration due to gravity. U_1 and D_1 are, respectively, the velocity and depth of the flow entering the jumps.

For F_1 values between 1.0 and 1.7 a slight difference in the conjugate depths D_1 and D_2 is noticed, which increases with increasing Froude numbers. F_1 values from 2.5 to 4.5 originate a hydraulic jump with pulsating features. The entering jet oscillates from the bottom to the water surface, inducing high turbulence levels on the flow field. For F_1 values between 4.5 and 9 a stable jump is expected to occur. High differences between the conjugate depths D_1 and D_2 occur for Froude numbers higher than 9. Naturally, these limits above referred are not strict values, and overlapping may occur.

In addition, the downstream water level has a strong influence on the development of the hydraulic jump. It will develop inside the stilling basin if the downstream water level approaches the second conjugate depth D_2. Smaller y_0 values will cause the jump to leave the basin. Another characteristic that plays an important role on this process is the length of the jump. It is generally related to the Froude number value and for the case of a horizontal apron, for example, it can reach the maximum value of six times the downstream conjugated depth D_2. Special devices, such as end-sills and aprons are built on the stilling basin slab in order to decrease the length of the jump and therefore to diminish the distance where the energy dissipation occurs.

Consequently, the scour problems are bound to arise when the hydraulic jumps sweep out form the stilling basin into the rock protection bed.

4. Experimental set up

All the experiments were conducted at the Hydraulics Laboratory of the Civil Engineering Department of the University of Porto. In the following two sections the facilities and instrumentation used are thoroughly described.

4.1. Experimental facilities

The Hydraulics Laboratory has two different water channels which work in a closed loop system. The water channel used in the present work is the longest one, which dimensions are 1.33 m high, 1 m wide and 40 m long. Its side walls along the last 15 m are made of glass windows of 1 m high and 2 m long, supported by a concrete frame. These windows allow a visual inspection of the test section as well as the use laser-Doppler anemometry to investigate the flow field. The whole recirculation system is consisted by two underground reservoirs, four pumps with a maximum capacity of 150 l/s and one superior stabilizing reservoir. An illustration of the system is shown in Figure (2).

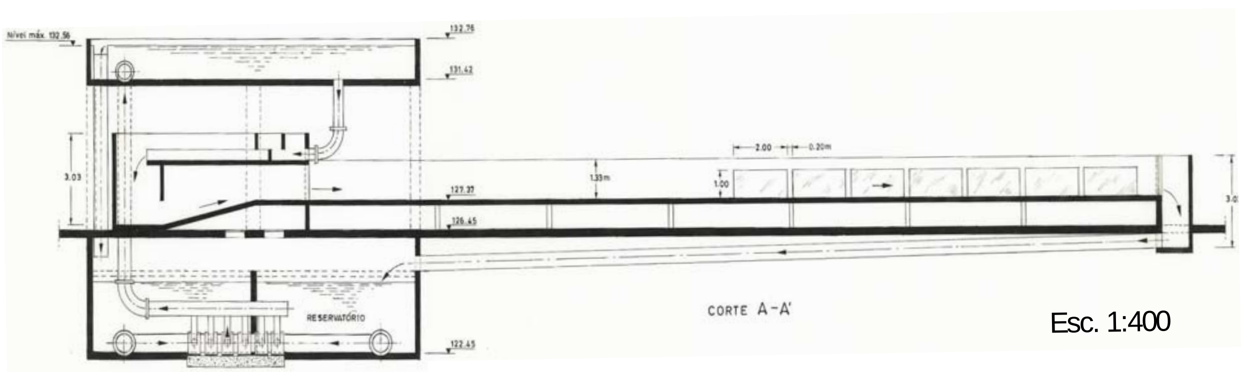

Figure 2: General view of the water channel.

During a conventional run, it was enough to use just two pumps to keep the whole system in equilibrium, with a maximum discharge variation of 0.8 %. Those two pumps were used to supply the head tank feeding the water channel at a discharge of 90 l/s. Two magnetic flow meters were used during all set of experiments. One is located in the supply line in parallel with a valve, which together allow a discharge flow control of 0.1 l/s at the inlet of the channel. The second flow meter has an uncertainty of 0.4 % and was positioned at the outlet of the channel, downstream of the rip-rap model. At the end of the channel there is a vertical steel gate operated by a gear box, which was used to control the water depth downstream the model of the dam.

At the entrance of the channel, the water was made to pass through a series of screens and filters to stabilize, uniform and suppress any excessive turbulence as well as to control the grain-size of the particles in suspension in the water.

4.2. Model characteristics

The model of the Crestuma dam was made in Perspex using a scale of 1:80, and respecting the natural proportions of the prototype. Considering that the gravitational forces are predominant, the Froude number similarity was used for the model construction. Using the scale above defined and taking into account the width of the channel, 1m, the portion of Crestuma represented on this study includes one central span and correspondent piles and two incomplete spans (half of the central span).

Its gates are composed by two separate double-slicing structures, which moved independently and allowed flow discharges over or underneath it, or yet both conditions simultaneously. The rock protection bed was situated downstream of the stilling basin, occupying an extension of 1m long. The top layer of the rock protection bed was divided in four horizontally distributed sections of 25 cm long each. The bed-grain size distribution used for this set of experiments is shown in Table (1), where the fourth section consisted of sand and section one corresponds to the location nearest to the stilling basin. Figure (3) gives a schematic view of the gate, stilling basin and end sill of the model used.

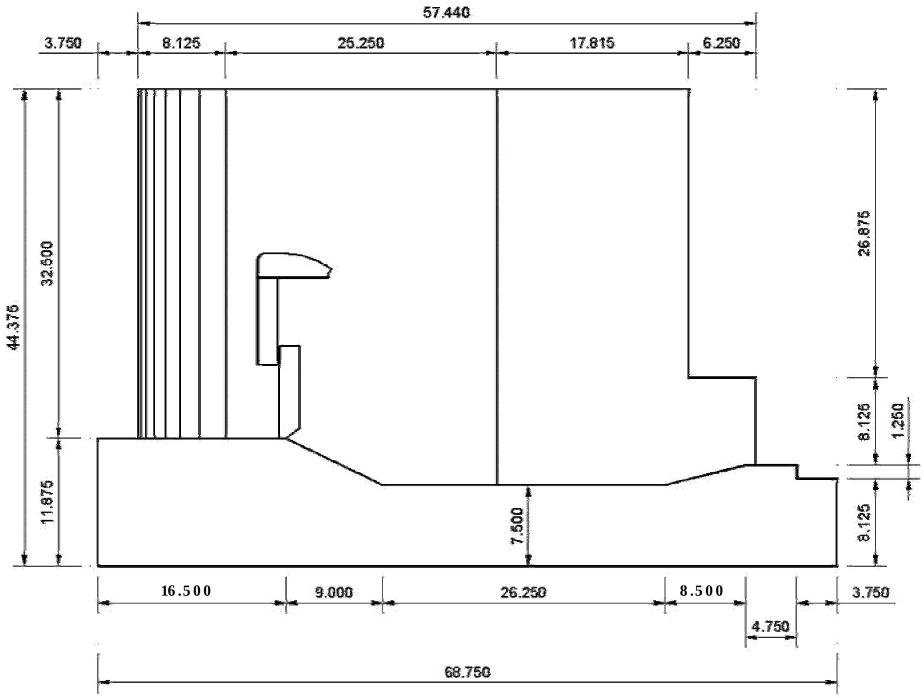

Figure 3: Lateral view of the model. Dimensions in centimeters.

Table 1. Description of the grain-size particles. Dimensions in mm.

	d_{35}	d_{50}	d_{65}	d_{75}	d_{84}	d_{85}	d_{90}
Section 1	13.2	15.0	15.5	16.0	17.6	17.7	18.1
Section 2	7.2	8.0	8.2	9.0	13.6	13.7	14.1
Section 3	6.6	7.0	7.5	8.0	11.2	11.7	12.0

4.3 Instrumentation

A one component Dantec laser-Doppler anemometry system was used in the forward scatter mode to conduct the measurements of the mean and fluctuating velocity field. The laser source used to form the beams is a 100 mW Ar-ion, operating in multi-mode. A Bragg cell unit was used to introduce an electronic shift of 0.6 MHz, allowing the resolution of the direction of the flow field and the correct measurement of near-zero mean velocities. The beams were made to pass through a series of conditioning optical elements, namely, a pin-hole section and a beam expander, with an expansion ratio of 1.95, in order to achieve a small measurement volume and to improve the optical alignment. Front lenses of 600 mm focus length were mounted on the probe in order to accurately position the measurement volume on the centerline of the water channel. Before being collected by the photomultiplier, the scattered light was made to pass through an interference filter of 514.5 nm, so that only the green light is acquired. The signal from the photomultiplier was band-pass filtered and processed by a TSI 1990C Counter, operating in the single measurement per burst mode. A series of LDA biases were avoided by adjusting the strictest parameters on the data processor. For this set of experiments, a frequency validation setting of 1% was used with 2^4 cycles of comparison. A 1400 Dostek card interfaced the counter with a 80486 based computer, which was used to provide the statistical results. For each point measured, a sample size of 10,000 values has been considered. Table (2) lists the main characteristics of the laser-Doppler system used. Figure (4) presents a typical experimental set up.

Table 2. Main characteristics of the laser-Doppler anemometer in air at e^{-2} intensity.

Laser wavelength	514.5 nm
Measured half angle of the beams	3.65º
Fringe spacing	4.041 mm
Frequency shift	0.60 MHz
Dimensions of the measurement volume:	
Major axis	2.53 mm
Minor axis	162.0 mm

This whole system was used to measure both the longitudinal and the vertical velocity components. This could be done simply by turning the probe around its axis, so that, on both conditions, the fringe distribution was perpendicular to the measured velocity component. Typical uncertainties associated to the mean and fluctuating velocity data were estimated in 1 % and 4 %, respectively.

Figure 4: General view of the sensor probe and whole experimental set up.

4.4. Measurement conditions

Firstly, two conditions of flow discharge were used, namely, 11.5 l/s and 12.2 l/s. The operation of those gates allows the flow to pass underneath them, above them or both simultaneously. So, for the flow discharge of 11.5 l/s, it has been investigated the cases of flow above the gates and underneath the gates. As for the condition of 12.2 l/s, only the condition of flow underneath the gates was studied. The general features of the experimental simulations are summarized in Table (3).

Table 3. Main features of the experimental measurements.

q (l/s)	Path of the flow	H_1 (mm)	y_0 (mm)	Froude	Gate opening (mm)
11.5	over	165.0	58.5	1.4	-
11.5	under	165.0	58.5	5.81	9.0
12.2	under	165.0	60.5	5.79	12.5

5. Results

5.1. Mean and turbulent velocity data

Figure (5) shows the mean longitudinal velocity component measured along the rock protection bed for the flow rate of 11.5 l/s. A comparison is made between the behaviour of the flow over (red arrows) and under the gates (green arrows). For all the graphics shown in this section, the following convention is used: the vertical coordinate denotes the vertical distance from the rip-rap nondimensionalized by y_0, the downstream water level, and the longitudinal coordinate represents horizontal distance from the beginning of the rip-rap nondimensionalized by the length of the rock protection bed. It can be observed that the underflow situation produces higher mean velocities in the vicinity of the stilling basin, where a consequent reduction on the water depth can also be noticed. Figures (6) and (7) represent the turbulent kinetic energy for the cases above mentioned, respectively. From those pictures it can be observed that the values for the underflow situation are almost two times higher than those for the overflow case in the region above the first section of the rip-rap. The evaluation of these pictures leads to the conclusion that these high values of mean and fluctuating velocities exerted a mild influence and a low level of instability on the first section of the rock protection bed.

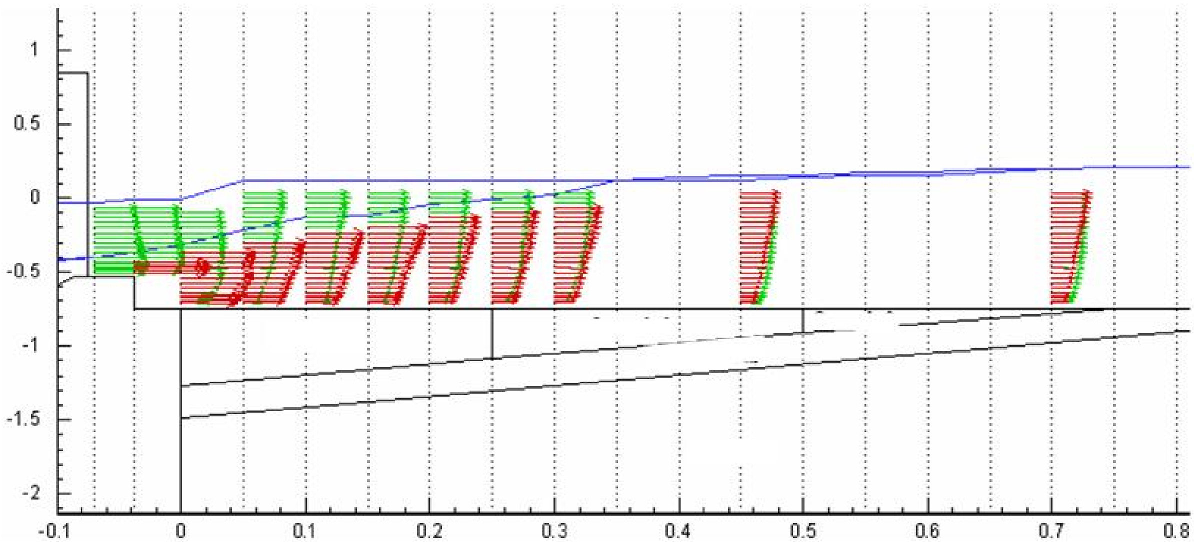

Figure 5: Mean velocity measurements of the flow field.
(Q = 11.5 l/s, red arrows: flow under the gates, green arrows: flow over the gates)

The flow field produced by the flow rate 12.2 l/s is shown in Figure (8). In this picture, a comparison is made between this case (green arrows) and the case of underflow for the flow rate of 11.5 l/s (red arrows). Considerable higher mean velocities occur in the region of the first and second horizontal sections of the rock protection bed. As it was expected, in this case the hydraulic jump was dislocated from the stilling basin downstream into the rock protection, generating a large scour hole, which is illustrated in Figure (9). It was observed that, at the point of impact on the rip-rap, the hydraulic jump acts as a jet and dissipates energy by excavating a scour hole. However, the degree to which the hole will be developed is still very much uncertain. The results for the turbulent kinetic energy are presented in Figure (10). Since the measurements were conducted after the scour hole had reached an equilibrium state, we can observe that the energy along the scoured rip-rap is quite low. Actually, when the final equilibrium has been reached, the jet has no longer sufficient energy at the point of impact to remove or raise any further particle.

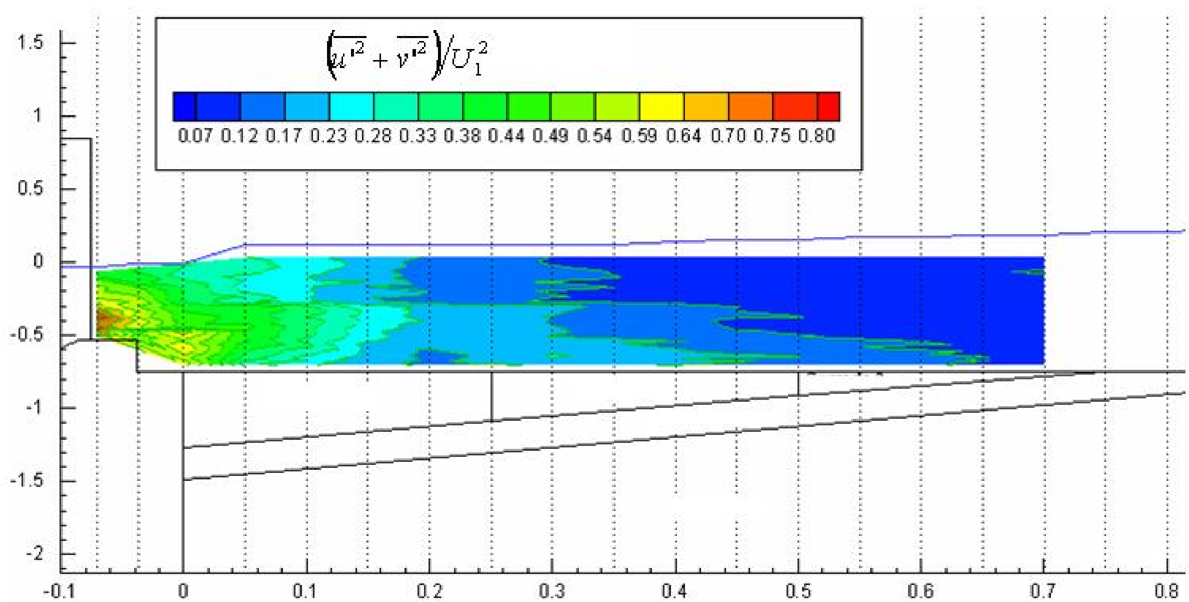

Figure 6: Turbulent kinetic energy for the flow above the gates. (Q = 11.5 l/s)

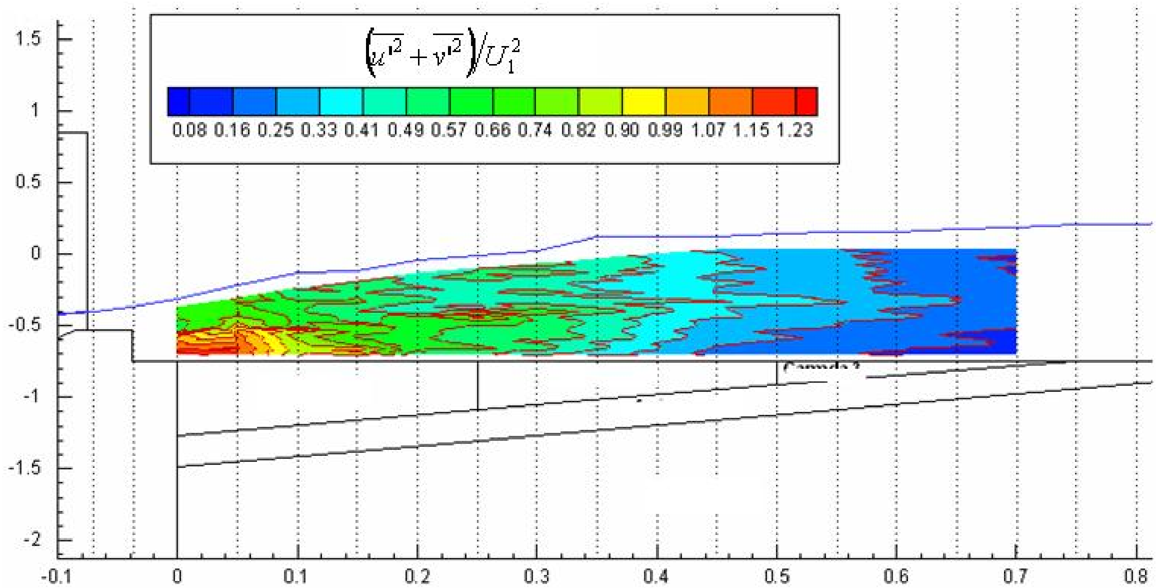

Figure 7: Turbulent kinetic energy for the flow under the gates. (Q = 11.5 l/s)

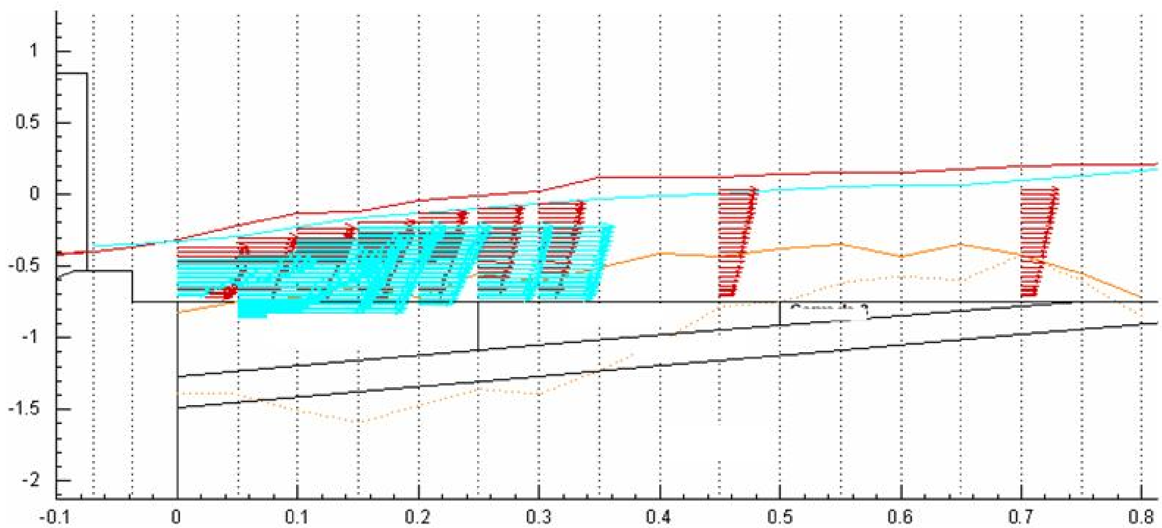

Figure 8: Mean velocity measurements of the flow field.
(Flow under the gates, red arrows: Q = 11.5 l/s, green arrows: Q = 12.2 l/s)

Figure 9: Illustration of the rock protection bed and the observed scour hole.

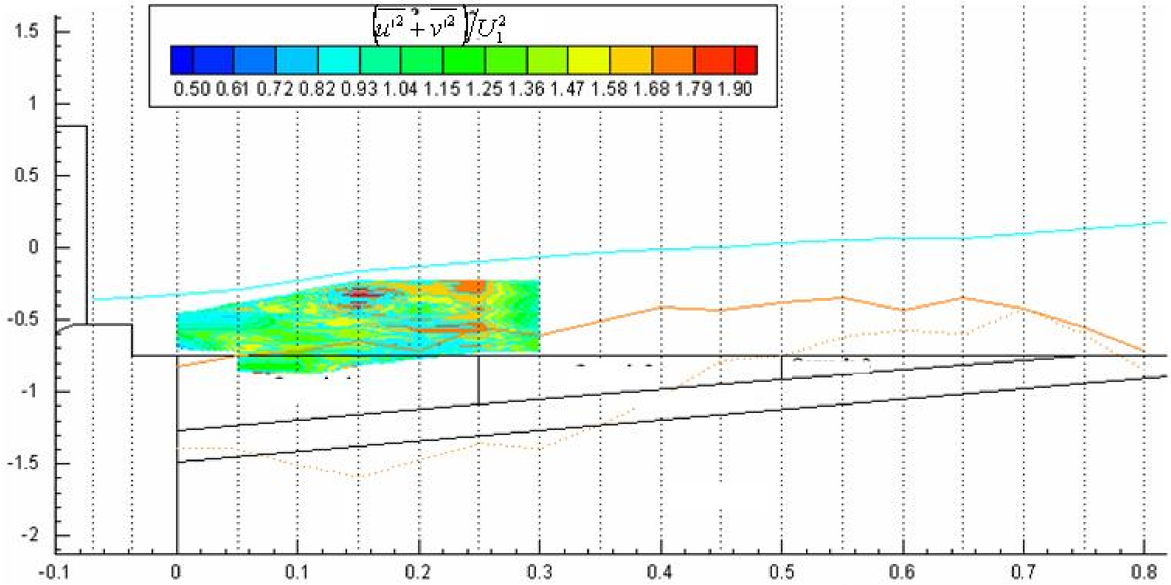

Figure 10: Turbulent kinetic energy for the flow under the gates. (Q = 12.2 l/s)

5.2. Analysis of the scour process

Concerning the flow situations studied, only the flow rate equal to 12.2 l/s caused scour on the rock protection. Flow discharge of 11.5 l/s over the gate system caused a hydraulic jump situated inside the stilling basin. A slight difference between the conjugate depths could be observed. The same flow rate value underneath the gates lead to a hydraulic jump which entering section was located approximately at the beginning of the horizontal part of the stilling basin. Although no erosion was registered on the rock protection, there was an incipient scour process occurring. The hydraulic jump developing within the stilling basin started to show a certain oscillating behaviour which induced slight movements on the bed particles, but indeed not strong enough to initiate the scour process.

The oscillating behaviour clearly increased for the 12.2 l/s flow rate, the entering section was moved downstream to approximately the end of the horizontal part of the stilling basin. The scouring process occurred leading to a considerable scour hole.

Table 4. Comparison between predicted and observed scour depths. (* These equations were not capable of predicting any scour depth.)

Author	Equation	y_p predicted (mm)	$(y_p - y_s / y_p)$ (%)
Catakli (1973)	Eq. (1)	1.21	-
Schoklisch (1932)	Eq. (2)	59.64	17.84
Veronese (1937)	Eq. (3)	0.23	*
Jager (1939)	Eq. (4)	-3.15	*
Eggenberger (1944)	Eq. (5)	-17.8	*
Müller (1947)	Eq. (6)	50.57	3.0
Hartung (1957)	Eq. (7)	70.15	30.1

Having studied the flow field for the conditions of integrity and scouring of the rock protection, the maximum scour depth observed, y_s = 49 mm, is now compared with the theoretical predictions available in literature. These results are presented in Table (4). The equations shown in Section 2 were evaluated using the data described in Tables (1) and (3). The values of q took into account the effective width of the dam. On evaluating Eq. (2), the dimension of the model's sill, a, is 12.5mm. The equation proposed by Catakli et al. was evaluated for the flow rate of 11.5 l/s, when the hydraulic jump was located inside the stilling basin. The calculation presents a good result since no scour was observed during the experiments. The best agreements were obtained for the equations proposed by Schoklisch and Müller, and it might be explained by the specific character of these formulas. The first one takes into account the influence of the sill, while the second is specific for underflow conditions. The extreme failure of Eggenberger's equation in predicting the observed scour might be attributed to the difference between the upstream and downstream water levels, H. The value he used in his experiments is about twice the one used in the present study. In fact, the three other equations evaluated also showed discrepancies around 80%. The equations proposed by Veronese and Jagger couldn't predict the scour maximum depth as well.

6. Conclusion

The present work has experimentally studied the behaviour of the rock protection bed located behind a dam, in response to two different discharge conditions. Mean velocity and fluctuations profiles obtained with the aid of laser-Doppler anemometry were shown for region over the rip-rap. The results presented herein intended to compare the influence of two different flow rates, on the generation of scour holes on the rip-rap. The instability region which generates a significant scour profile on the rip-rap was clearly identified for the critical condition. From this set of experiments it can be concluded that the situations where exists a great difference between the upstream and downstream water levels are to be avoided, since more potencial energy has to be dissipated. It was observed that higher mean velocities where present along the rip-rap for the case of higher potential energy. Finally, the scour depth observed has been compared to the theoretical predictions available in literature. In general, the formulas failed to predict the correct scour depth, and the more specific equations were the ones which provided the best agreements. In conclusion, the complexity of energy dissipation and scouring process has made the theoretical analysis of such problem very difficult. Therefore, research to date has concentrated on developing empirical formulas for equilibrium depth based fundamentally in experiment and observation.

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