Proceedings of the 10th Brazilian Congress of Thermal Sciences and Engineering -- ENCIT 2004
Braz. Soc. of Mechanical Sciences and Engineering -- ABCM, Rio de Janeiro, Brazil, Nov. 29 -- Dec. 03, 2004

Paper CIT04-0586

A COMPARATIVE STUDY OF SONIC BOX AND AN EULER/NAVIER-STOKES METHOD FOR OSCILLATING TRAPEZOIDAL WINGS IN SONIC FLOW

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Abstract. *This paper is a continuation of the work of Soviero and Pinto (2000), Cesar and Soviero (2001) and Cesar and Soviero (2002). In the previous works, a computer code for their Sonic Box Method was developed and tested. The results of this code were compared to the ones of several other researchers, such as Landahl (1961), Rodemich and Andrew (1965), and Ruo et al. (1974), showing excellent agreement. The Sonic-Box Method was developed initially for rectangular oscillating wings subjected to a sonic unsteady flow. This method was later modified by Cesar and Soviero for thick delta and rectangular wings. Furthermore, a new approach to perform the integration of the kernel function was developed. The capability of the code to model delta wings was demonstrated in Cesar and Soviero (2001) and for thick wings in Cesar and Soviero (2002), both for oscillations in pitch and plunge motions. The present paper is concerned with trapezoidal wings. Some preliminary results for the AGARD WING 445.6 were obtained and compared with data generated by a Euler/Navier-Stokes solver based on a finite difference method. The movement of the planar wing was restricted to a pitch motion around an axis perpendicular to the mid-chord point at the root. The preliminary results show good agreement among the pressure distributions generated by these two different methods.*

Keywords. *Unsteady aerodynamics, Sonic flow, Panel method, Linearized theory, Finite thickness wing*

1. Introduction

The adequate knowledge of the forces acting on three-dimensional wings in oscillatory motion is very important for the study of flutter and other aeroelastic responses of an aircraft, because the aeroelastic problem is frequently critical in the transonic regime. A nonlinear partial differential equation with nonlinear boundary conditions governs the physical problem. The basic small perturbation equation governing the velocity potential for transonic flow over a thin wing is well known (Landahl, 1961). For low amplitude and high-frequency oscillation, where the unsteady part is considered to be a small disturbance to the steady part, the steady mean flow can be completely uncoupled from the unsteady counterpart of the equation, leading to a linearized unsteady transonic flow governing equation.

In this work we develop an approximate method to take into account effects due to finite wing thickness. The transonic oscillatory aerodynamic parameters are predicted in a range of frequencies where linearization can be considered valid. The present study is limited to non-viscous and shock-free flow around trapezoidal wings.

Ruo *et al.* (1974) mentioned in their work that almost all unsteady transonic flow theoretical work lies within the framework of linearized theory where the thickness effect of the wing is neglected. An important consequence of these linearization is the suppression of deviations in local Mach number from freestream value. These deviations have appreciable effect on the propagation of pressure disturbances over the lifting surface, then significant improvement in the theory may be accomplished by “recoupling” the steady and unsteady flow parameters so that solutions may approximately consider variations in mean local Mach number caused by finite wing thickness. In the present work, this is achieved by considering all of the steady-flow parameters over the wing to be invariant within a small finite region. This latter assumption, equivalent to the concept of local linearization (Spreiter and Alksne, 1958; Rubbert and

Landahl, 1967), permits the nonlinear differential equation for the potential to be reduced to a linear equation with variable coefficients containing the local Mach number. By means of an appropriate coordinate transformation, the equation becomes identical to the linearized transonic unsteady-flow equation with constant coefficients. Numerical results are then obtained through the new sonic-box method presented in (Soviero and Pinto, 2000; Cesar and Soviero, 2001).

The type of wing treated has a swept trailing edge, without control surfaces. Because of the transformation method used, the mean steady flow everywhere over the wing must not be very different from that of the undisturbed stream. Comparisons are made to the new sonic-box method, for cases with and without thickness effects, as well as to results from an Euler finite difference solution.

2. Problem Formulation

Consider a small thickness wing, immersed in an inviscid compressible fluid that translates with the undisturbed flow velocity U close to the speed a_∞, the sound speed of the undisturbed flow, performing a small amplitude oscillation around its zero angle of attack position. The wing is assumed to be smooth and thin enough so that the small-perturbation velocity potential equation for transonic flow can be applied. The physical coordinates and a sample wing geometry are shown in Fig. (1).

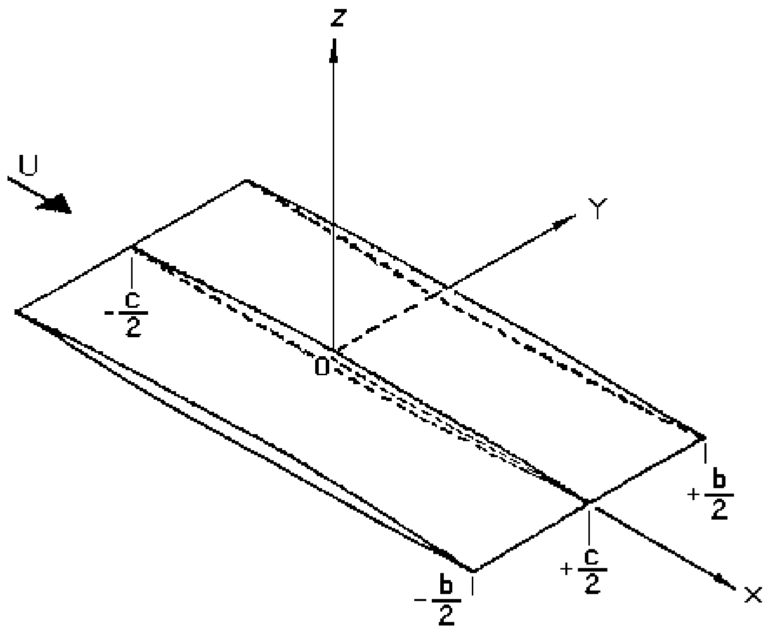

Figure 1. Coordinate system on the physical plane of the wing, *c* is the wing root chord and b is the wing span.

The chosen coordinate system has the *X*-axis in the direction of the flow. The undisturbed position of the wing is in the *XY* plane, with the *X*-axis formed by the straight line passing through the leading edge half span point and the trailing edge half span point, the origin is at the intersection point of the wing semi-span with the wing extended root chord (c*).

2.1. Sonic Aerodynamic model

The physical problem is governed by nonlinear partial differential equations and nonlinear boundary conditions. The complex velocity perturbation potential Φ due to the harmonic small-amplitude motion of a thick wing is described by the nonlinear differencial equation given below:

$$\frac{\partial^2\Phi}{\partial X^2}+\frac{\partial^2\Phi}{\partial Y^2}+\frac{\partial^2\Phi}{\partial Z^2}-\frac{1}{a^2}\left[\frac{\partial^2\Phi}{\partial t^2}+\frac{\partial V^2}{\partial t}+\vec{V}\cdot\vec{\nabla}\left(\frac{V^2}{2}\right)\right]=0 \qquad (1)$$

where a is the local speed of sound, V the local flow velocity lies in the positive X.

Equation (1) is linearized assuming that:

(a) The local velocity vector differs only slightly in direction and magnitude from the free-stream velocity vector. This is the basic assumption of the small disturbance theory, that is equivalent to assume that the local Mach number M is close to the value of the free stream Mach number M_∞;

(b) All steady-flow parameters are considered to be invariant within small finite regions of the wing surface, equivalent to the local linearization, Rubbert and Landahl (1967).

The linearized partial differential equation corresponding to the velocity potential for a small perturbation unsteady transonic flow over a small thickness wing is

$$\frac{\partial^2\overline{\phi}}{\partial Y^2}+\frac{\partial^2\overline{\phi}}{\partial Z^2}-\frac{1}{a^2}\frac{\partial^2\overline{\phi}}{\partial t^2}-\frac{2M}{a^2}\frac{\partial^2\overline{\phi}}{\partial X\partial t}=0 \tag{2}$$

where $\overline{\phi}$ is made nondimensional relative to U and a reference length L.

Assuming that the wing describes a harmonic motion then the small perturbation potential can be written as

$$\hat{\phi}(X,Y,Z,t)=\overline{\phi}(X,Y,Z)\exp[-i\omega t] \tag{3}$$

where ω is the angular frequency of the motion.

The complex velocity perturbation potential $\hat{\phi}$ due to the harmonic small-amplitude motion of a thick wing is described in the frequency domain by a linear differential equation, Eq. (4), similar to the one given by Landahl (1961) for a thin wing shown below

$$\frac{\partial^2\hat{\phi}}{\partial Y^2}+\frac{\partial^2\hat{\phi}}{\partial Z^2}-\frac{2Vi\omega}{a^2}\frac{\partial\hat{\phi}}{\partial X}+\frac{\omega^2}{a^2}\hat{\phi}=0 \tag{4}$$

Defining a new complex potential as

$$\hat{\phi}=\phi\exp[-(i\omega X)/(2aM)], \tag{5}$$

now, using the transformation :

$$\mathrm{x}=X/\mathrm{L},\quad \mathrm{y}=\mathrm{M\,Y/L},\quad \mathrm{z}=\mathrm{M\,Z/L}, \tag{6}$$

the governing differential equation of the problem, *i.e*., Eq. (4), can be rewritten as the classical diffusion uation (Morse and Feshbach 1953). This equation is analogous to the subsonic (Soviero and Bortolus, 1992), to the supersonic (Soviero and Resende, 1997) and the sonic (Soviero and Pinto, 2000; Cesar and Soviero, 2001) formulations, which reads:

$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-2ik\frac{\partial\phi}{\partial x}=0 \tag{7}$$

where $k = \omega L/U$ is the reduced frequency and the mean steady local Mach number M is assumed to be known.

Equation (7) is a parabolic differential equation and possesses source and doublet elementary solutions. The present procedure here is similar to the one by Ruo *et al*. (1974), it is assumed that the physical state is adequately described within a limited region by related linear equations in which all parameters involved have their local values taken as being invariant. This is the underlying assumption of the local linearization concept described in Spreiter and Alksne (1958), and also in Rubbert and Landahl (1967). This approach suggests that, in the case of unsteady flow, the calculations can be carried out with sufficient accuracy, using the linearized equations which contain the local values of the steady flow parameters. Landahl (1963) cites evidence for the validity of applying the concept of local linearization to the case of unsteady flow.

The approach here is to stretch the physical wing in order to work on a transformed plane with a transformed wing, which can be considered a thin wing, and whose local Mach numbers over its surface are known and has the same value of the corresponding Mach numbers over the surface of the physical wing. The free stream flow is sonic for both wings.

The computation of the unsteady flow is then performed transforming back the flow properties obtained in the transformed plane applying the similarity law for unsteady transonic flow Eq. (8), Cesar (2004), given by:

$$\phi(x,y,z;\sigma;k;M;\varepsilon)=M^{-1}\phi(x,y,z;M\sigma;k;M;\tilde{\varepsilon}), \tag{8}$$

where σ is the semi-span to semi-chord ratio, ε is the thickness ratio defined by the thick to semi-chord ratio, $\tilde{\varepsilon}$ is the transformed wing thickness ratio, and $\tilde{\varepsilon} << 1$.

The linearized boundary condition on the wing surface is written in the transformed plane as

$$\omega(x,y) = \frac{\partial \phi}{\partial z} = \frac{\exp(ikx/2)}{M}\left[\frac{\partial h}{\partial x} + ikh\right] \tag{9}$$

where $h(x,y)$ represents the wing surface non-dimensional vertical displacement. The complex pressure coefficient is written as

$$C_p = -\frac{2}{UL}\exp\left(-\frac{ikx}{2}\right)\left[\frac{\partial \phi}{\partial x} + i\frac{k}{2}\phi\right] \tag{10}$$

and pressure continuity is ensured if Eq. (10) is applied to both sides of the wake, that is,

$$\delta C_p = 0 = \frac{\partial \delta\phi}{\partial x} + i\frac{k}{2}\delta\phi \tag{11}$$

where $\delta\phi$ and δC_p are the complex velocity potential and pressure coefficient jump between the lower and upper surfaces of the wake, respectively. It is important to stress that the same condition applies whenever the trailing edges are subsonic. The treatment of the three-dimensional thick wing problem is similar, though not the same, to the one performed by Ruo *et al.* (1974), where it was used the classical sonic box method reported in (Rodemich and Andrew, 1965; Olsen, 1966). Here it is used the new sonic box method reported in (Soviero and Pinto, 2000; Cesar and Soviero, 2001).

The solution of problem just described is obtained from the integral equation that relates the potential jump across the lifting surface (and wake) to the downwash. For a planar configuration this integral is written as

$$\omega(x,y) = \frac{1}{4\pi}\iint \delta\phi \frac{ik}{(x-x_0)^2}\exp\left(-\frac{ik(y-y_0)^2}{2(x-x_0)}\right)dx_0 dy_0 \tag{12}$$

The integrand represents a doublet at (x_0, y_0) inducing a normal velocity in the wing plane (z=0) at the receiving point (x,y). The integral sign must be taken in its usual way along y_0 and in the sense of the finite part integration (Hadamard, 1928; Heaslet and Lomax, 1957) along x_0. The general formulation relative to integral Eq. (12) can be found in Morse and Feshbach (1953), and is a result of the application of Green's theorem to the diffusion equation (Eq. (7)). Its kernel function, the induced doublet velocity, comes from the unitary strength source velocity potential,

$$\phi(x,y,z) = \frac{1}{4\pi(x-x_0)}\exp\left(-ik\frac{(y-y_0)^2+(z-z_0)^2}{2(x-x_0)}\right) \tag{13}$$

for a point source placed at (x_0, y_0, z_0), with $x > x_0$, which must be differentiated twice along the z direction, and for $x \le x_0$, $\phi = 0$. In order to have the correspondent values for the physical wing we apply Eq. (8) to the results obtained by Eq. (13).

2.2. Euler aerodynamic model

The Euler solver used in the present work is a modified version of a code developed by Sankar *et al.*(1990). The vector form of the 3-D Euler equations based on an arbitrary curvilinear coordinate system can be written in non-dimensional form as:

$$\mathbf{Q}_\tau + \mathbf{E}_\xi + \mathbf{F}_\eta + \mathbf{G}_\varsigma = 0 \tag{14}$$

where $\boldsymbol{Q}$ is the vector of unknown flow properties; and $\boldsymbol{E}$, $\boldsymbol{F}$, $\boldsymbol{G}$ are the inviscid flux vectors.

The time derivative, $\boldsymbol{Q}_\tau$, of equation (1) is approximated using two-point backward difference at the new time level $n+1$. All spatial derivatives are approximated by standard second-order central differences and are represented by the difference operators δ. With the above described time and space discretizations, Eq. (14) becomes:

$$\Delta \mathbf{Q}^{n+1} = -\Delta \tau \left(\delta_\xi \mathbf{E}^{n+1} + \delta_\eta \mathbf{F}^{n,n+1} + \delta_\varsigma \mathbf{G}^{n+1} \right) \tag{15}$$

Application of Eq. (15) to the grid points leads to a system of non-linear, block penta-diagonal matrix equations for the unknown $\Delta \mathbf{Q}^{n+1} = \mathbf{Q}^{n+1} - \mathbf{Q}^n$, Eq. (15), since the convection fluxes ***E***, ***F***, ***G*** are non-linear functions of the vector of unknown flow properties ***Q***. Eq. (15) is then linearized using the Jacobean matrices $\mathbf{A} = \partial \mathbf{E} / \partial \mathbf{Q}$ and $\mathbf{C} = \partial \mathbf{G} / \partial \mathbf{Q}$. This results in a system of linear, block penta-diagonal matrix equations, which is considerably expensive to solve. The approach used here is to employ an approximate factorization and the diagonal algorithm of Pulliam and Chaussee[11], to diagonalize **A** and **C**. This approach yields:

$$\mathbf{T}_\xi^n \left[\mathbf{I} + \Delta \tau \delta_\xi \mathbf{\Lambda}_\xi^n \right] \mathbf{N}^n \left[\mathbf{I} + \Delta \tau \delta_\varsigma \mathbf{\Lambda}_\varsigma^n \right] \left(\mathbf{T}_\tau^{-1} \right)^n \Delta \mathbf{Q}^{n+1} = \Delta \tau \left(\delta_\xi \mathbf{E}^n + \delta_\eta \mathbf{F}^{n,n+1} + \delta_\varsigma \mathbf{G}^n \right) \tag{16}$$

The solution of Eq. (16) involves two block-tridiagonal systems where the blocks are diagonal matrices. The use of standard central differences to approximate the spatial derivatives can give rise to the growth of high frequency errors in the numerical solution with time. To control this growth, a set of $2^{nd}/4^{th}$ difference non-linear, spectral radius based, explicit artificial dissipation terms are added to the discretized equations.

Oscillations were simulated around a steady state angle $\alpha_o = 0.0^0$. For the test cases presented here, the Mach number was 1.0. The reduced frequency k is defined as the ratio of the circular frequency ω times a reference chord c by the speed of the sound a_∞. In order to avoid any transient contributions to the unsteady results, acquisition of the unsteady pressures was performed only in the second cycle of oscillation. That is, from a steady state converged solution the unsteady simulation is run for two cycles of wing movement, and the unsteady data is stored after the end of the first cycle. Considering the non-dimensionalization of the Navier-Stokes formulation, the non-dimensional time is given as $\tau = a_\infty t / c$. As the unsteady pressures are acquired, a Fourier transformation is used to obtain the frequency-domain components, which can be written as a discrete transformation as:

$$\mathrm{Re}(\boldsymbol{Cpi}) = \frac{\boldsymbol{k}\Delta\tau}{2\pi\Delta\alpha} \sum_{m=m_1}^{m_1+m_T} \boldsymbol{Cp_m \, sin}(\boldsymbol{km}\Delta\tau) \tag{17}$$

$$\mathrm{Im}(\boldsymbol{Cpi}) = \frac{\boldsymbol{k}\Delta\tau}{2\pi\Delta\alpha} \sum_{m=m_1}^{m_1+m_T} \boldsymbol{Cp_m} \cos(\boldsymbol{km}\Delta\tau) \tag{18}$$

3. Numerical Model

The physical wing is approximated by a region composed of *nx* boxes along the root chord and *ny* boxes along the wing semi-span Fig. (2). The chords of these panels equals to *c/nx* and its spans are equal to *b/2ny*.

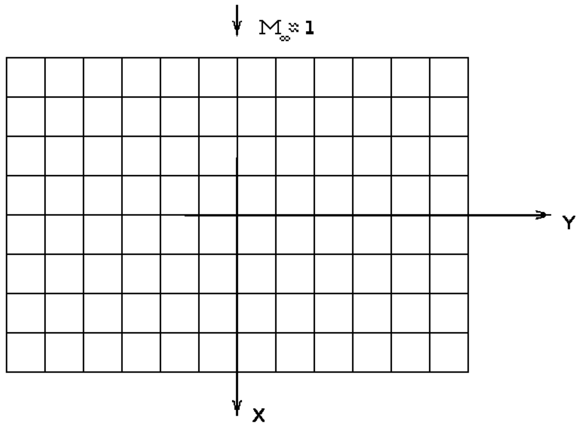

Figure 2. Mapping the wing by *nx* panels in the root chord direction and *ny* panels in the wing semi-span direction.

Using the Euler finite difference solver described in 2.2 to calculate the local Mach number, we apply a type of Prandtl-Glauert transformation (Eq. 6) to obtain a transformed thin wing that has the similarity transonic law (Eq. 8) relating its velocity potential in the transformed plane to that of the physical wing.

The transformed wing is a stretched wing mapped by *nx* rows, having *ny* columns in each row. The local Mach number is used to stretch each panel of a row and since *M* presents small differences between the panels. Therefore, they will have different span in the transformed plane. In order to circumvent the problem of having different span for each panel the transformed wing surface is discretized by *nx* panels along its root chord and *ny* panels along its semi-span. The transformed wing has panels with chords given by *c*/*nx* and spans given by $b/2ny$, where b is the span of the transformed wing. This transformed span is found considering it equal to the maximum value encountered comparing the sum of each transformed panel that forms the *nx* rows in which it was divided the transformed wing. The resulting transformed wing could have a different number of panels in each row, if this is the case; the wing is treated as a swept wing, if not; as an unswept wing.

Once the number of panels per row could be different from the physical to the transformed wing, the following criteria is adopted: Each time that the centroid of the transformed wing panel is correspondent to a point inside a certain panel of the physical wing, the parameters of that transformed panel will be related to the parameters of the physical panel. The panels formed by this approximation of the transformed wing are rectangular and have a constant distribution of normal doublet over its surface. This distribution is given by the value of the normal doublet superficial density.

The flow tangency condition is satisfied in the control points of each panel, which coincide with the geometric center of the panels, by assuming a non-penetrability condition written as a boundary condition relation in the small disturbance context. Thus, the boundary conditions are enforced at control points located at each panel geometrical center. The solution of the problem is obtained by solving integral Eq. (12) for $\delta\phi$ by using the boundary conditions Eqs. (9) and (11) for the wing and wake surfaces, respectively. Both surfaces are discretized through the use of small rectangular elements of unknown constant density doublets, as for the case of the sonic Mach box formulation, Rodemich and Andrew (1965).

One can identify four kinds of integration domain, Fig. (3), in order to obtain the influence coefficients at P. Region I is completely upstream of the limiting Mach lines drawn from P, region II is only partially upstream of the limiting Mach lines, and region III is completely downstream of the limiting Mach lines, corresponding to zero influence. Region IV corresponds to the self-induced influence coefficient. In region I numerical integration is straightforward because the integrand is never singular.

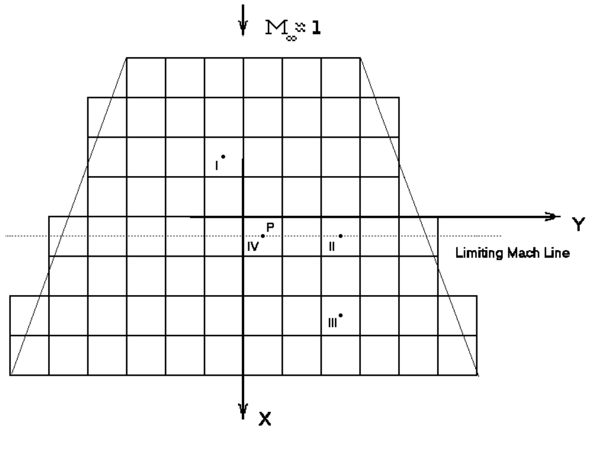

Figure 3. Transformed wing with aspect ratio Λ and thickness ratio $\tilde{\varepsilon} = 0$ at $M_\infty = 1.0$, mapped with *nx* panel along root chord and *ny* panels along its semi-span.

The influence coefficient for regions II and IV is, Soviero and Pinto (2000),

$$F(x,y) = -\frac{1}{2\pi}\left[\frac{\exp(\lambda_4)}{(s-y)} + \frac{\exp(\lambda_3)}{(s+y)}\right] + \frac{1}{2}\sqrt{\frac{ik}{2\pi x_p}}\left[erf(\lambda_2) - erf(\lambda_1)\right], \quad (19)$$

where

$$s = b/4ny \qquad \lambda_{1,2} = (y \pm s)\sqrt{\frac{ik}{2x_p}}, \quad \lambda_{3,4} = -\frac{ik(s \pm y)}{2x_p}, \quad (20)$$

Denoting W_i as the induced velocity over the panel i, F_{ij} as the influence coefficient of panel j over panel i and $\delta\phi_j$ the superficial doublet density of panel j, follows the system that represents the problem

$$\lfloor F_{ij} \rfloor \cdot \{\delta\phi_j\} = \{W_i\}. \quad (21)$$

Solving the system we find the values of $\delta\phi_j$ which belong to the transformed wing to have the correspondent values for the physical wing we apply Eq. (8).

5. Results

The test case to be investigated is the AGARD wing 445.6 weakened model #3 (Yates, 1988), in sonic flow condition. The test case under investigation, is well known as a standard aeroelastic configuration, and it has an aspect ratio of 4 and a NACA 65A004 airfoil section. The flow Reynolds number is $0{,}627\times10^6$ and the density of the test medium (air) is 0.0634 kg/m^3 , at low pressure condition.

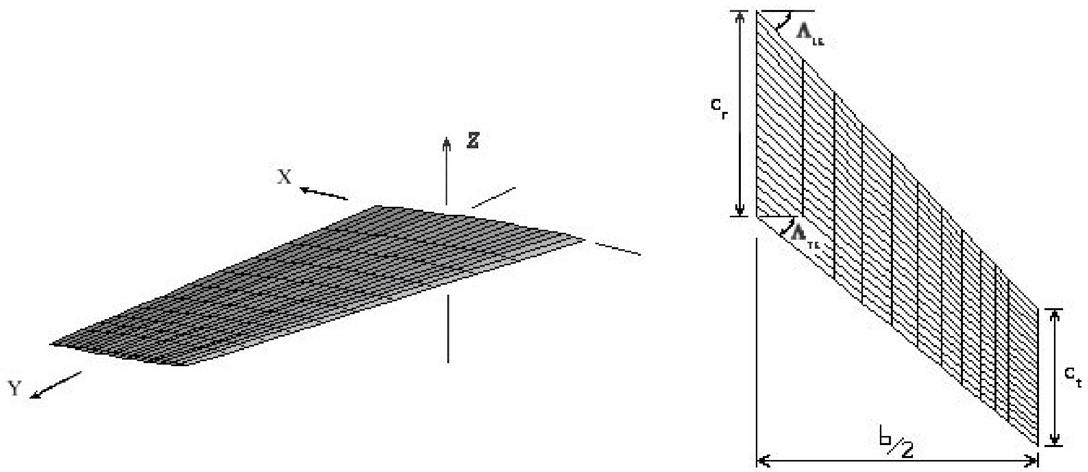

Figure 4 : Sketch of the AGARD 445.6 wing.

The computation of the nonlinear unsteady pressures is performed from the finite differences solution of Euler equations, using the numerical method described in section 2.2. The computational mesh surrounding the wing is an algebraic generated "C" type topology, with 141 points in the ξ direction, tangent to the lifting surface boundary, where 121 points are over the lifting surface solid surface. In the η direction, normal to the wing surface, there are 41 points between the solid surface and the limit of the computational mesh. And finally, in the ζ direction there are 25 points aligned with the spanwise direction, where 17 points are over the lifting surface, and the remaining are between the wing tip and the computational domain limit.

The finite difference Euler simulation were performed for the wing undergoing a rigid body pitch harmonic oscillation around an axis perpendicular to the root chord, at its 50% location. The reduced frequency for the finite difference solution of the Euler equations is defined as $k_r = \omega c / a_\infty = 0.28252$, in conformity with the Euler equations nondimensionalization.

The sonic box results for the thick wing case were obtained for a discretization of the same wing in 45 boxes along its extended root chord (c*), which is defined by the distance from the wing apex to its most afterward point, that is from the apex to the trailing edge of the wing tip, and with 90 boxes along the wing half span. The thickness effect is represented herein by the consideration of the computed local Mach number distribution, interpolated in the centroid of each of the boxes. These Mach number are computed using same Euler numerical method, for steady state conditions at zero angle of attack.

The reduced frequency used in the sonic box computation is a different value from the one employed in the Euler formulation, since it is nondimensionalized by the extended half root chord and the undisturbed flow speed. Therefore, for the case of the sonic box method, its definition is $k = \omega c^* / 2U$, leading to *k =0. 52673.*

A calculation using the sonic box procedure was conduced for the thin wing case in order to evaluate the influence of the thickness on the pressure distribution jump over the wing. It was noticed that the pressure jump was diminished when the wing thickness is taken into account.

The resulting pressure coefficient difference distribution along the wing chord nondimensionalized for the wing extended root chord (c*) are presented in Figures 5, 6 and 7.

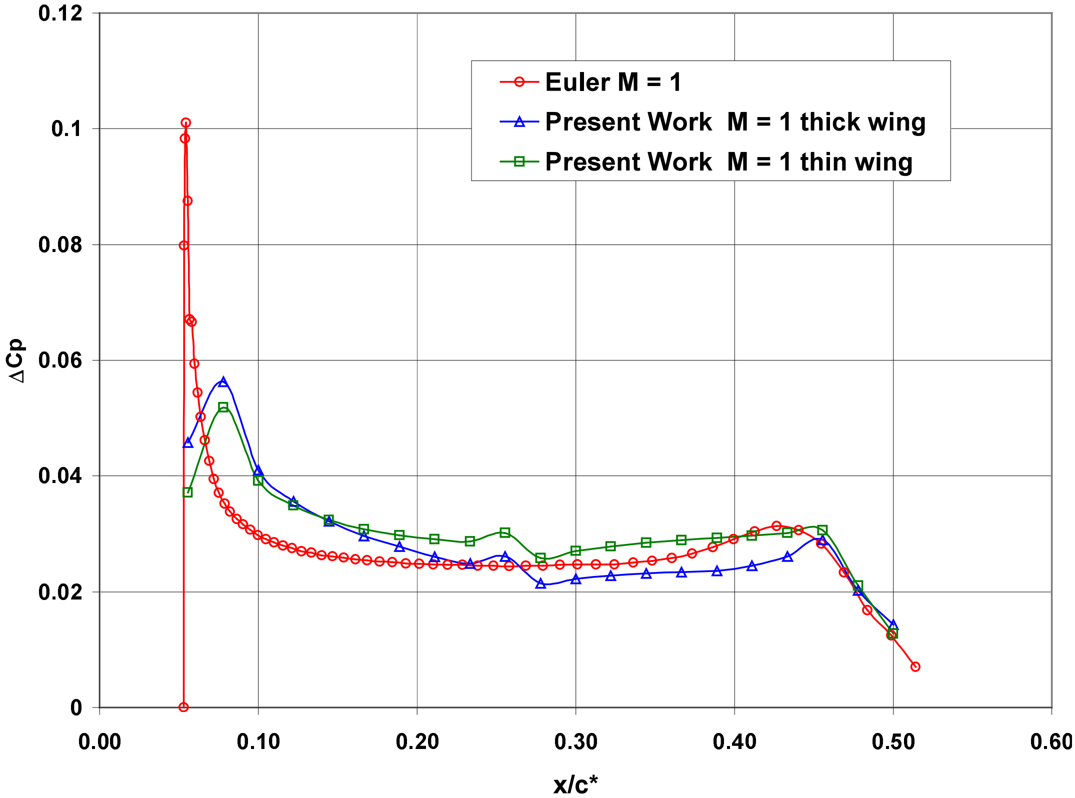

Figure 5 – Pressure distribution along chord length y/s = 7.7%.

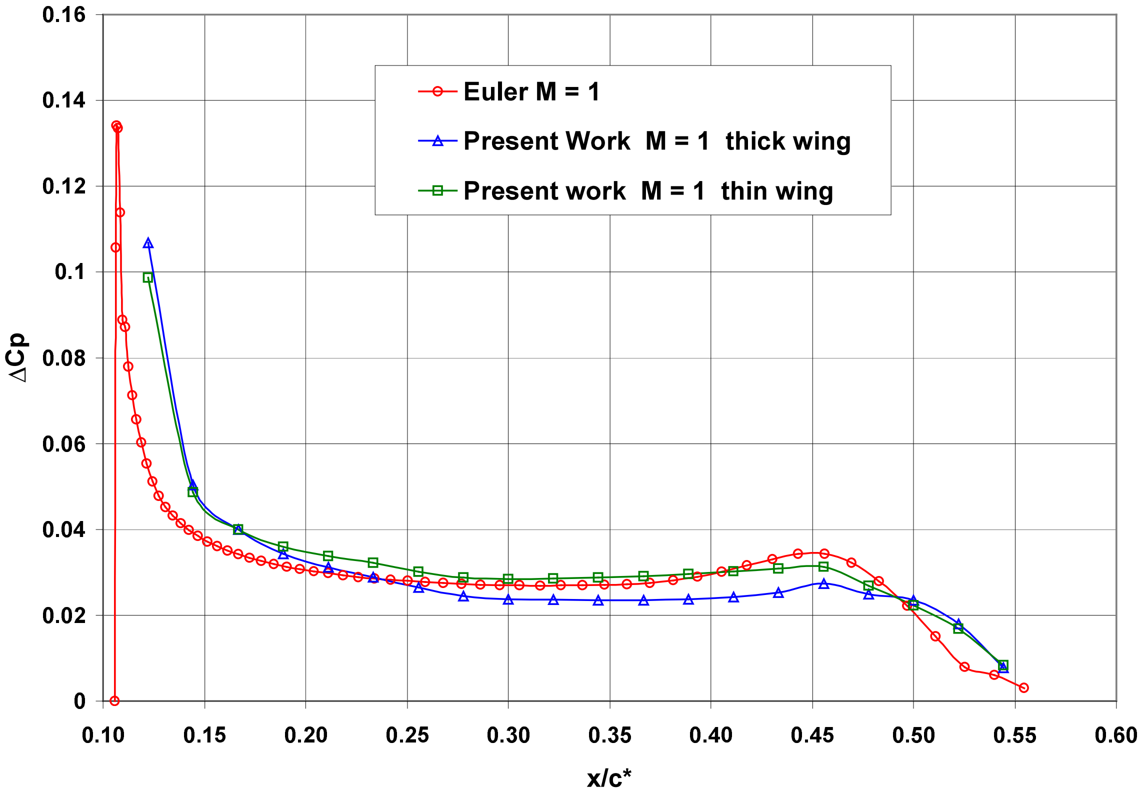

Figure 6 – Pressure distribution along chord length y/s = 15.4%.

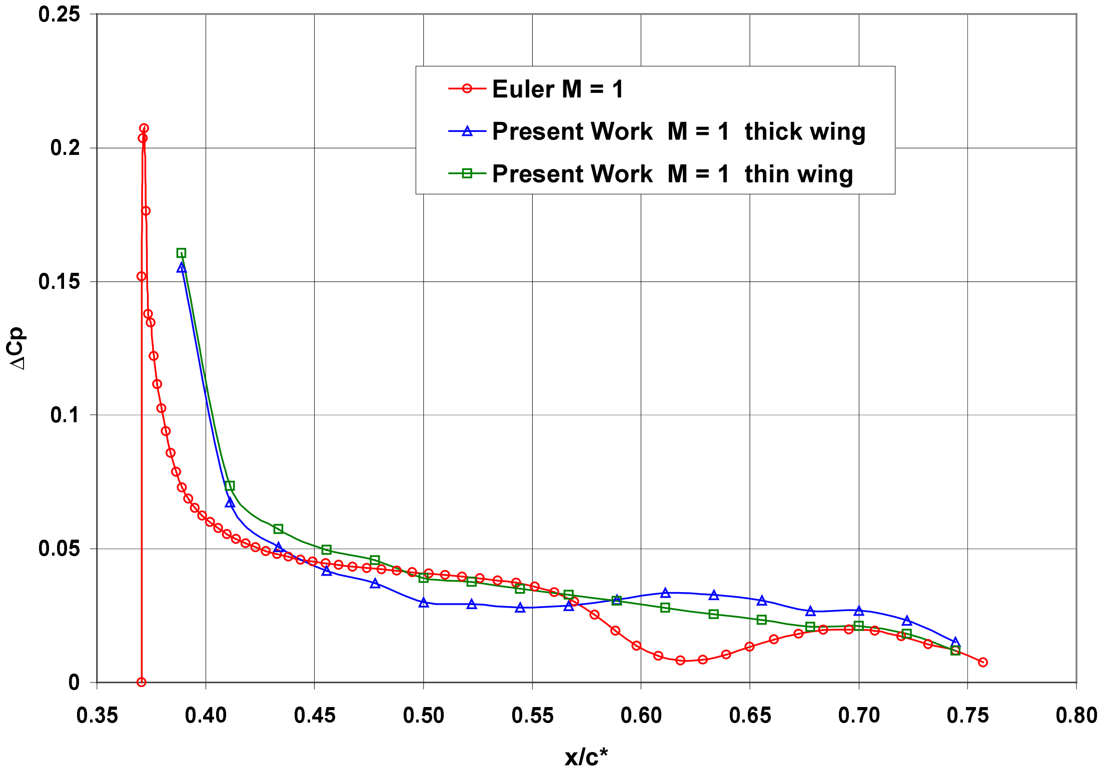

Figure 7 – Pressure distribution along chord length y/s = 53.9%.

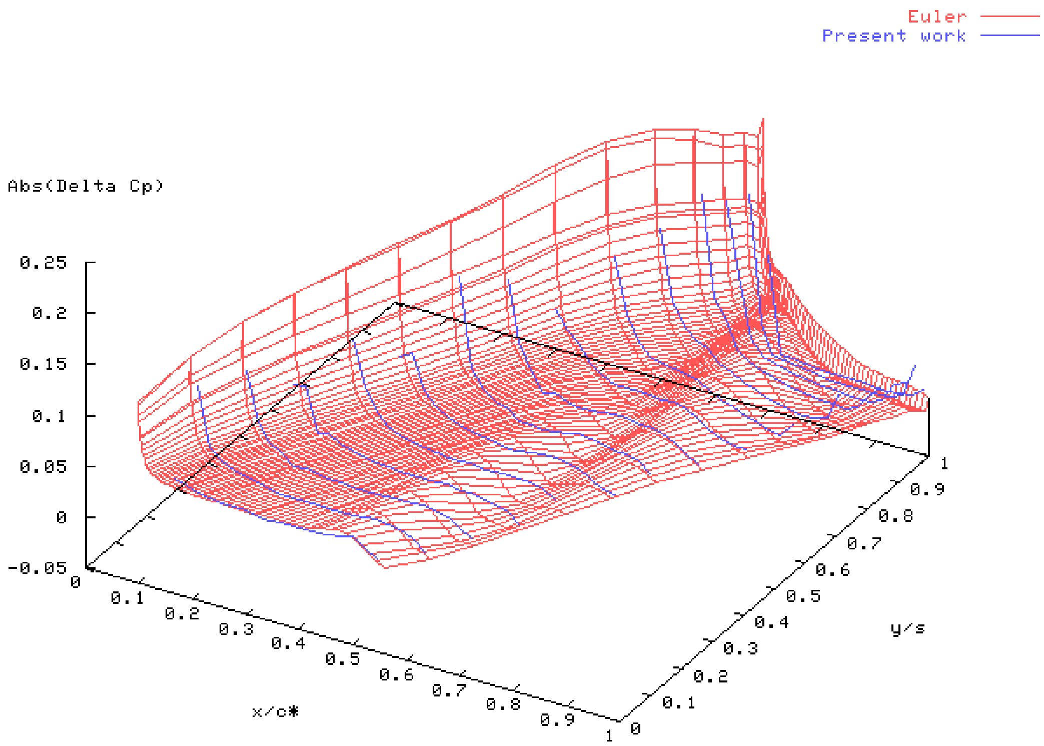

Figure 8 – Surface pressure distribution over AGARD 445.6 wing

In Figure 8 is presented the computed pressure distribution along the wing surface for the thick wing case. The Mach cone line, here is a straight line perpendicular to the *X*-axis, have its origin at the leading and at the trailing edge points of the wing root and at the wing tip chords, are noted in the sonic box formulation results as well as in the Euler results. The Mach cone line explains the peak that is seen near the point $x/c^* = 0.45$ at Fig. 5. One should observe that the results obtained from the Sonic Box formulation presents a good agreement with those computed from the nonlinear Euler simulation.

The Euler solution, expended approximately 8.2 CPU hours for the simulation of two cycles of pitch motion, in a Silicon Graphics Octane II workstation, with two R12000 Risc processor, while the time expended by the sonic box method was less than seven minutes on a computer with a single 2.8 GHz Pentim IV processor.

6. Conclusions

The method show good agreement with a nonlinear solution obtained by the finite difference Euler equation simulation. method, used here for comparison. In spite of the fact that further investigation is necessary to evaluate the limits of the method, the quality of the results obtained, till now, suggests that the present Method could be an useful tool for unsteady pressures computation for a preliminary transonic aeroelastic analysis.

7. Acknowledgement

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brasília, DF, under Grant 300.682/93-0.

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