SIMULATION OF NONLINEAR DYNAMICS OF A PWR CORE BY AN IMPROVED LUMPED FORMULATION FOR FUEL HEAT TRANSFER

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Abstract. *In this work, thermohydraulic behaviour of PWR, during reactivity insertion and partial loss-of-flow, is simulated by using a simplified mathematical model of reactor core and primary coolant. An improved lumped parameter formulation for transient heat conduction in fuel rod is used for core heat transfer modelling. Transient temperature response of fuel, cladding and coolant is analysed.*

Keywords *Reactor thermohydraulics, Transient heat conduction, Lumped parameter analysis, Point kinetics, Reactor neutronics.*

1. Introduction

Accurate prediction of reactor core behaviour during transients and accidents is of major concern for safe operation of nuclear power plants of Pressurized Water Reactor (PWR). Computer programs such as COBRA are available for analysis of transient core behaviour. Nevertheless, simplified approaches have been paralleled large system codes to provide understanding of the physical phenomena and to show consistency with the large code analysis. In simplified core thermohydraulic analysis lumped parameter model has been used widely to obtain transient temperature behaviour in fuel and cladding (Levy, 1999). Recently, Regis et al. (2000) have proposed an improved lumped parameter formulation for transient heat conduction in nuclear fuel rods that gives more accurate results than classical lumped parameter formulation. Su and Cotta (2000) have developed a higher order lumped parameter formulation that improves further the accuracy of predicted transient temperature history.

In this paper, we present a mathematical model to simulate transient neutronic and thermohydraulic behaviour of the PWR core and the primary coolant of a nuclear power plant.

The point kinetics equations are used to model the core neutronics, and the improved lumped parameter formulation proposed by Regis et al. (2000) and Su and Cotta (2000) is used to model the fuel dynamics.

2. The Mathematical Model

The mathematical model is composed by the core neutronics which gives the power evolution $P(t)$ as a function of the reactivity $\rho(t)$, the fuel dynamics which gives the transient temperature behaviour of fuel and cladding, $T_f(t)$ and $T_c(t)$, and the heat balance of primary coolant that gives the bulk coolant temperature $T_m(t)$.

2.1 Reactor Core Neutronics

The core neutronics is described by the point kinetics equations with six delayed neutron precursor groups. The reactivity in the point kinetics equation depends upon the spatially averaged, time-dependent fuel and coolant temperatures, hence, it couples the core neutronics with the thermal-hydraulics. The point kinetics equations are written as

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{m=1}^{G} \lambda_m C_m(t), \tag{1}$$

$$\frac{dC_m(t)}{dt} = \frac{\beta_m}{\Lambda} P(t) - \lambda_m C_m(t), \quad m = 1, 2, ..., G, \tag{2}$$

where P is the total reactor power, C_m the power equivalent of the mth delayed neutron precursor group, ρ the total reactivity, β_m the fraction of delayed neutrons in the mth group, Λ the neutron generation time, λ_m the decay constant of the mth delayed neutron group. The reactivity is the sum of an externally inserted reactivity $\rho_{ext}(t)$ and the feedback reactivities:

$$\rho(t) = \rho_{ext}(t) + \alpha_F(T_F)\delta T_F(t) + \alpha_B \delta T_B(t).$$

The δT_F and δT_B are deviations of the average fuel temperature, $T_F(t)$, and bulk coolant temperature, $T_B(t)$, from their equilibrium values T_{F0} and T_{B0}, respectively. The fuel temperature of reactivity α_F is given by

$$\alpha_F(T_F) = \frac{\gamma}{2\sqrt{T_F}} \ln P(T_{F0}),$$

where γ is a constant that depends only on fuel composition and geometry. The coolant temperature coefficient is nearly a constant over a fairly wide range of operating conditions and is assumed constant in this work.

2.2 Fuel Dynamics

The heat conduction equation for the fuel and cladding at core averaged condition can be written as

$$\rho_f c_f \frac{\partial T_f}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(k_f r \frac{\partial T_f}{\partial r}) + q'''(t), \quad 0 < r < r_{fo}, \tag{3}$$

$$\rho_c c_c \frac{\partial T_c}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(k_c r \frac{\partial T_c}{\partial r}), \quad r_{ci} < r < r_{co}, \tag{4}$$

with appropriate boundary and initial conditions, where $q'''(t) = P(t)/V_f$ and V_f is the fuel volume. A lumped parameter formulation is developed by Regis et al. (2000), in dimensionless form:

$$\frac{d\theta_{f,av}(\tau)}{d\tau} = -\frac{2Bi_{gf}}{R_{fo}}\big(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)\big) + G(\tau), \tag{5}$$

$$\frac{d\theta_{c,av}(\tau)}{d\tau} = -\frac{2K}{(1-R_{ci}^2)}\Big[Bi_{gc}R_{ci}\big(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)\big) - Bi\theta_c(1,\tau)\Big], \tag{6}$$

where,

$$\theta_f = \frac{T_f - T_{m0}}{T_{f0} - T_{m0}}, \quad \theta_c = \frac{T_c - T_{m0}}{T_{f0} - T_{m0}}, \quad \theta_m = \frac{T_m - T_{m0}}{T_{f0} - T_{m0}}$$

$$R = \frac{r}{r_{co}}, \qquad \tau = \frac{k_f t}{\rho_f c_f r_{co}^2},$$

$$K = \frac{k_f \rho_c c_c}{k_c \rho_f c_f}, \qquad Bi = \frac{h r_{co}}{k_c},$$

$$Bi_{gc} = \frac{h_g r_{co}}{\beta_c k_c}, \qquad Bi_{gf} = \frac{h_g r_{co}}{\beta_f k_f},$$

$$\beta_c = \frac{r_{ci}}{r_{co}}, \qquad \beta_f = \frac{r_{fo}}{r_{ci}}$$

$$G = \frac{r_{co}^2 g}{k_f(T_{f0} - T_{m0})}.$$

The surface temperatures $\theta_f(r_{fo},\tau)$, $\theta_c(r_{ci},\tau)$ and $\theta_c(1,\tau)$ are related to the average temperatures $\theta_{f,av}(\tau)$ and $\theta_{c,av}(\tau)$ through expressions provided by Regis et al. (2000) or by Su and Cotta (2000).

2.3 Primary Coolant Temperature

The primary coolant energy equation can be written as

$$M_c c_p \frac{dT_m(t)}{dt} = hS_c(T_c(r_{co},t) - T_m(t)) - 2\dot{m}c_p(T_m(t) - T_{m0}), \tag{7}$$

where $T_m(t)$ is the average coolant temperature in the core, T_{m0} the inlet coolant temperature, $T_c(r_{co},t)$ the temperature at external surface of cladding, M_c the total mass of primary coolant, c_p the specific heat of the coolant, h the heat transfer coefficient, and S_c the total heat transfer area.

3. Numerical Results and Discussion

The Eqs. (1,2,5-7) form a set of ten coupled ordinary differential equations. With specified initial equilibrium state, the ordinary differential equations are solved numerically as an initial value problem by using an algorithm for stiff systems (Press et al., 1992). In this work, we implement the higher order improved lumped parameter formulation which uses two-side corrected trapezoid rule ($H_{1,1}$) for average fuel and clad

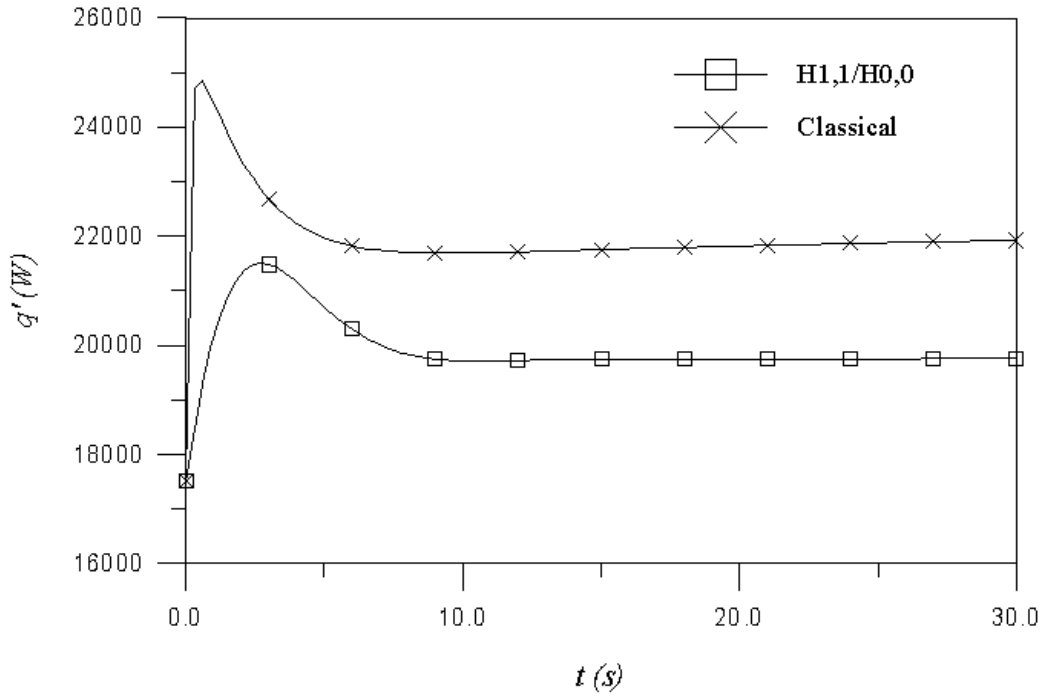

Figure 1 - Transient behaviour of power for step insertion of reactivity.

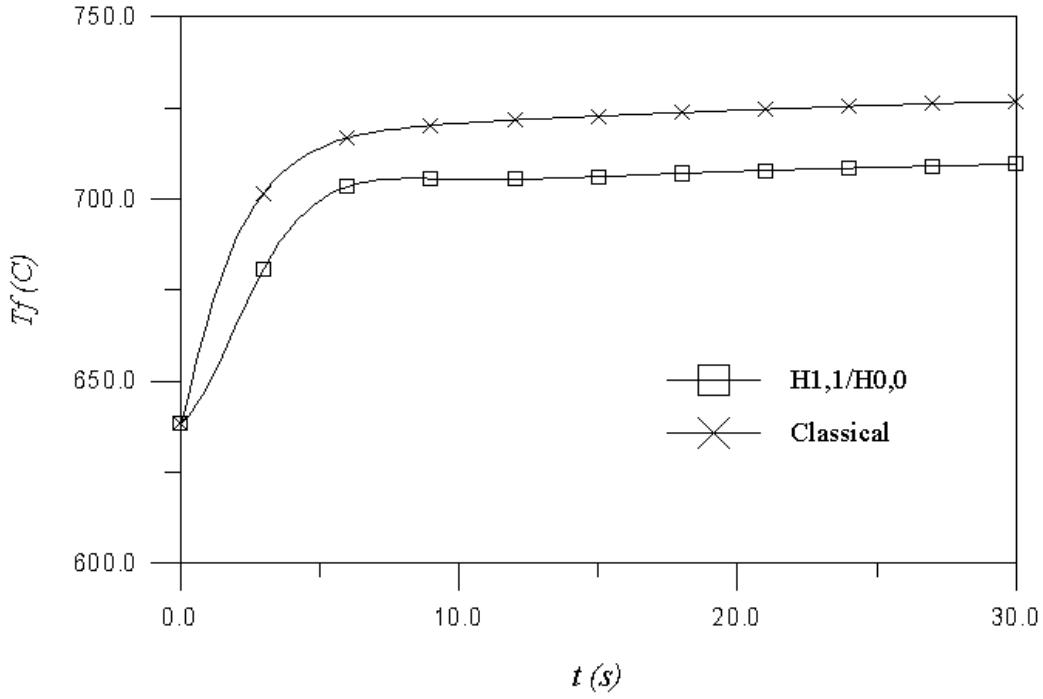

Figure 2 - Transient behaviour of average fuel temperature for step insertion of reactivity.

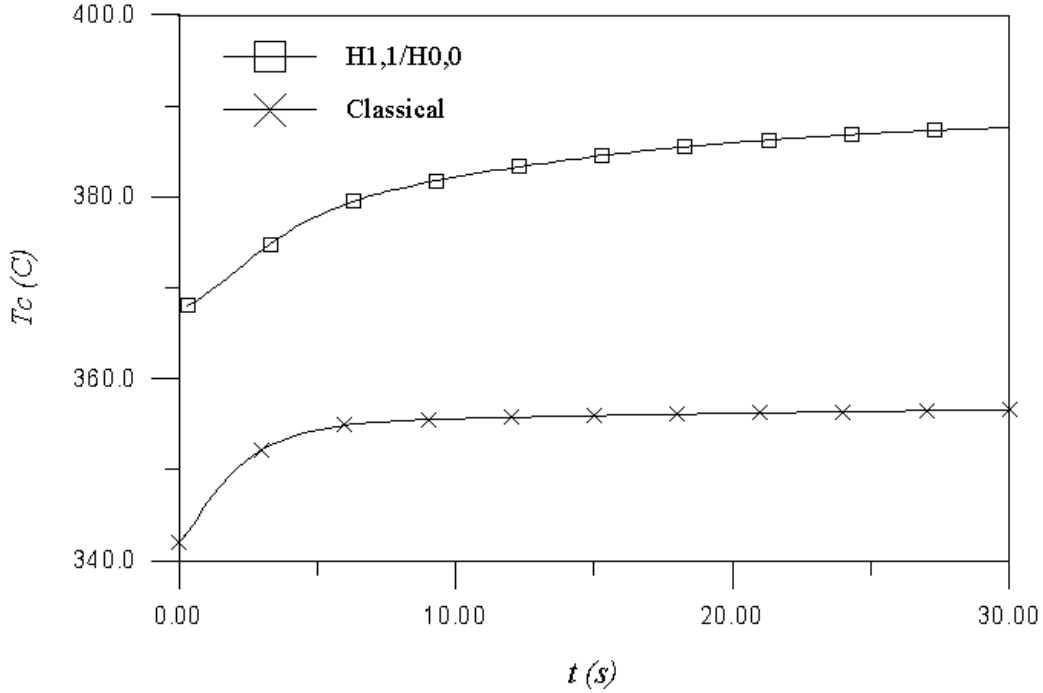

Figure 3 - Transient behaviour of average clad temperature for step insertion of reactivity.

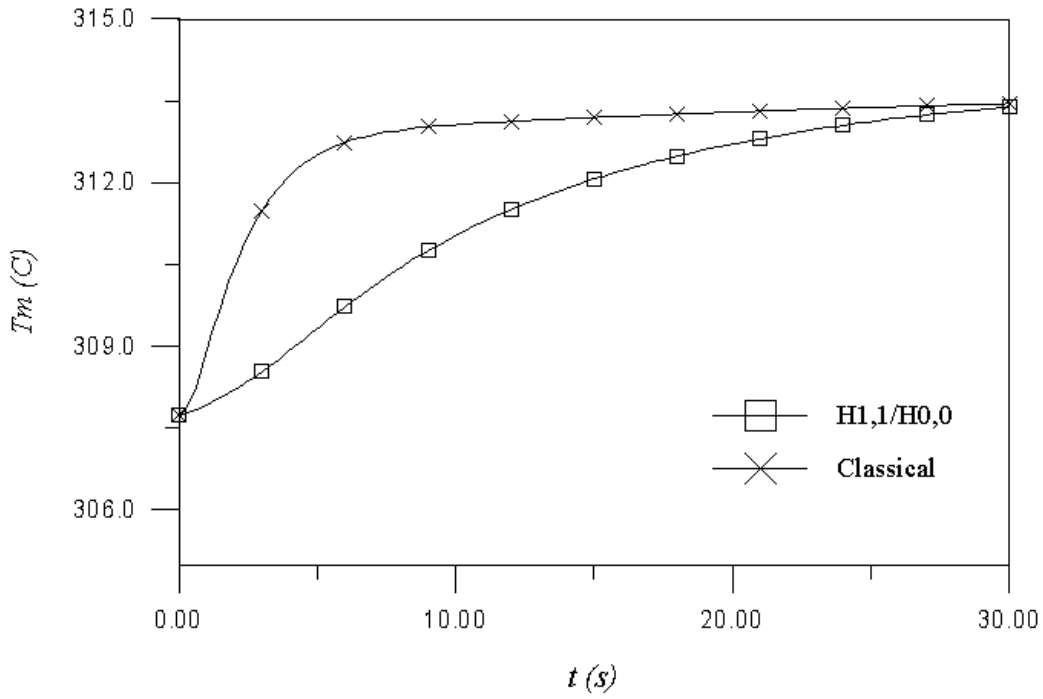

Figure 4 - Transient behaviour of average coolant temperature for step insertion of reactivity.

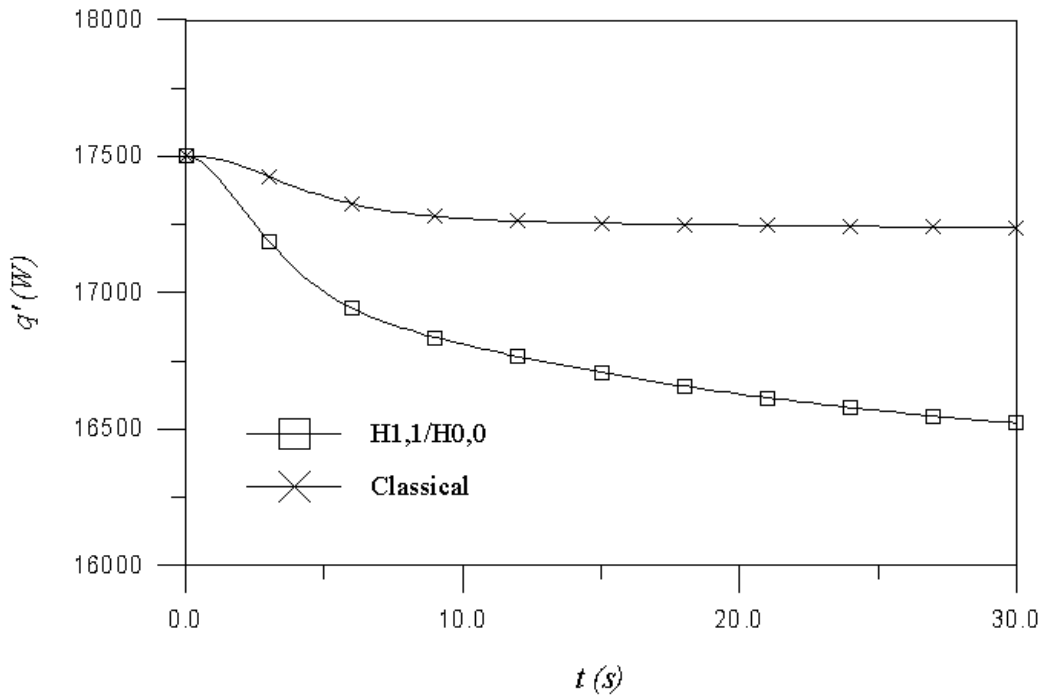

Figure 5 - Transient behaviour of power for partial loss of flow.

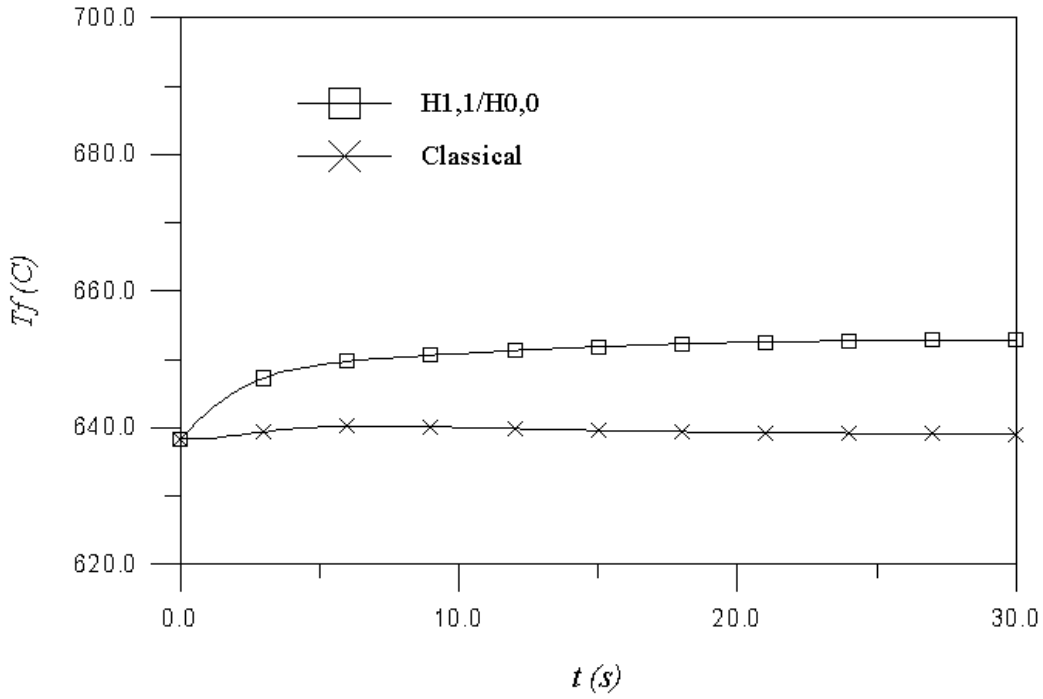

Figure 6 - Transient behaviour of average fuel temperature for partial loss of flow.

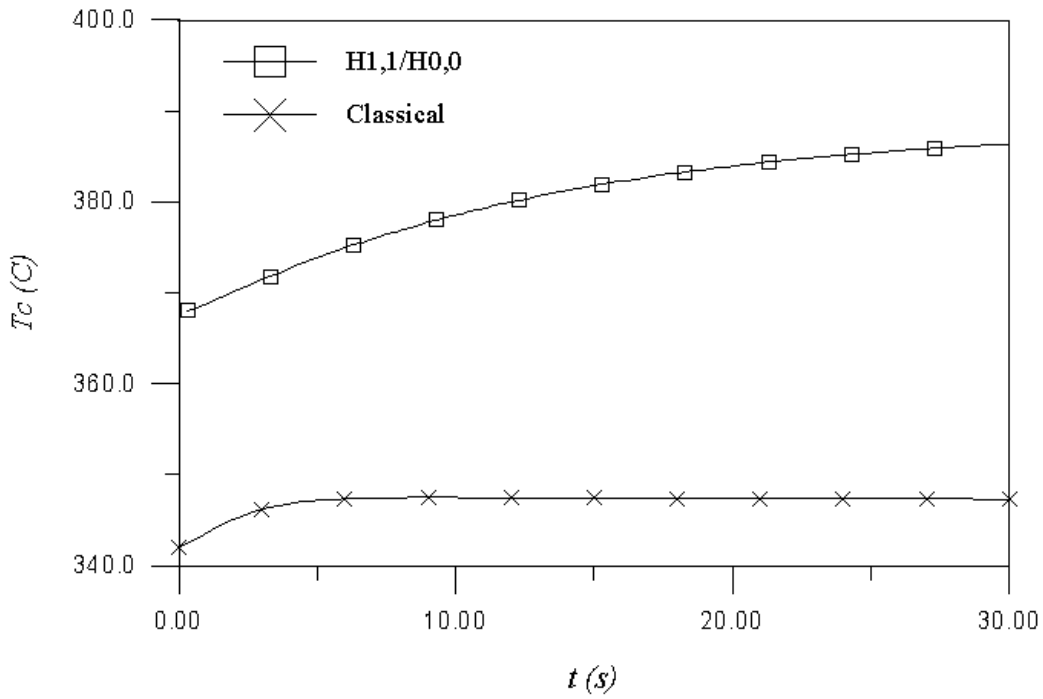

Figure 7 - Transient behaviour of average clad temperature for partial loss of flow.

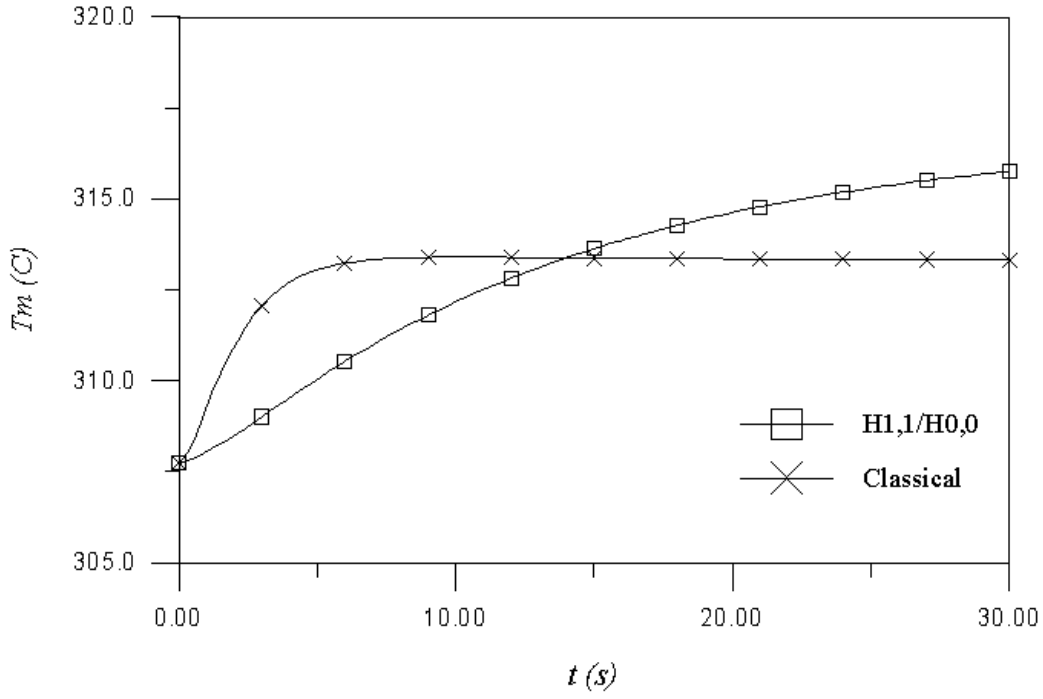

Figure 8 - Transient behaviour of average coolant temperature for partial loss of flow.

temperature integrals and plain trapezoid rule ($H_{0,0}$) for heat flux integrals (Su and Cotta, 2000). For comparison purpose, we also implement a classical lumped formulation for fuel dynamics as that described in Levy (1999).

In Figures 1 to 4 we show numerical results of a simulation for a step insertion of reactivity of 0.3 dollars. Transient behaviours of power, fuel temperature, clad temperature and average coolant temperatures simulated by both methods are given. In Figure 1, we notice that the model with classical formulation for fuel dynamics predicts a larger power overshoot than predicted by the improved lumped formulation. Similarly, the classical formulation gives a higher fuel temperature than that given by the improved formulation as shown in Figure 2. Su and Cotta (2000) have shown by comparison with more accurate solution obtained by finite difference method that the improved formulation gives more accurate fuel temperature than the classical lumped formulation as that described in Levy (1999). On the other hand, the improved lumped model predicts a higher average clad temperature that the classical formulation as can be seen in Figure 3. Although after a period of 30 seconds both formulation predict the same asymptotic value for average coolant temperature, the coolant temperature rising rate predicted by the improved formulation is slower as shown in Figure 4.

Transients induced by a partial loss of one-fourth flow rate are simulated with the numerical results shown in Figures 5 to 8. The improved formulation predicts a large power drop than the classical formulation as seen in Figure 5. While the classical formulation gives a nearly constant fuel temperature, the improved formulation predicts a slight rise in fuel temperature that will induce a negative reactivity to reduce the power level. The improved and classical formulations predict quite different behaviours for the clad and coolant temperatures. The clad and coolant temperatures approach steady state quite rapidly as predicted by the classical formulation, while the improved formulation gives steadily rising clad and coolant temperatures as shown in Figures 7 and 8.

4. Conclusion

In this work, we simulate the nonlinear dynamics of a pressurized water reactor core by using an improved lumped parameter formulation for the fuel dynamics proposed recently by Su and Cotta (2000). The model with improved lumped formulation predicts quite different transient behaviours for a partial loss of flow event. While the proposed model predicts reasonably well the main physical phenomena of the simulated events, its accuracy needs to be assessed through comparison with more accurate computational codes.

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