UNCERTAINTY ANALYSIS IN FLIGHT THRUST DETERMINATION

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Abstract. *Flight tests for determination of the propulsion system net thrust are of utmost importance because the aircraft carrying capacity is dependent on the calculated thrust. Relative uncertainty of 2% or more can undermine the profitability of the aircraft. Therefore, one must determine the actual propulsion system thrust with the least uncertainty possible. Propulsion systems in the range of 75 kN may be underestimated, say 1%, causing 2 passengers less in the payload list. This work sets forth to define an appropriate uncertainty analysis of the measurements taken during the flight tests, aiming at a pre-defined uncertainty. Although the approach in this work is the thrust and its uncertainty calculation from flight test data, the procedure is applicable to other measurements, in the laboratory or elsewhere. Indication of how to get the information needed for the calculations is given.*

Keywords: *Uncertainty, IFTD, Thrust, Gas turbine, Performance*

1. INTRODUCTION

The precise determination of the aircraft drag polar is of utmost importance because the aircraft characteristics like cruise speed, cruise altitude, rate of climb, maximum take-off weight and so on, are related to the lift and drag coefficients. Since it is impractical to experimentally determine such coefficients due to the impossibility of testing the full scale aircraft, data taken from wind tunnel model tests have to be confirmed through flight tests measurements.

The aircraft drag polar must be determined with low uncertainty, requiring therefore that the aircraft drag be determined very accurately. Usually one uses the leveled and stabilized flight condition at which drag equals thrust. Parizi-Negrão et al. (1998) proposed themselves to better study the methods for the in-flight thrust determination, having in mind the uncertainty involved in the measurements during the aircraft flight tests. This work deals with the establishment of procedures to calculate the aircraft drag following the recommendations of the SAE AIR-1703 report (1986) and analyses the calculated thrust uncertainties associated to the different methods for the determination of in-flight thrust, giving clues to how to select the most adequate method as far as uncertainty associated to the calculate thrust is concerned. A study of each parameter, used for thrust determination, is made aiming at the method selection. This work is considering that the instrumentation has already been defined and its features are known.

2. IN-FLIGHT THRUST DETERMINATION

There are several methods for the determination of thrust in flight. SAE AIR-1703 (1986) lists many possible methods. Guimarães (1997) and Guimarães et al. (1998) suggest that CFD may also be used, in addition to those methods. In this work a method from the group of the *gas path/nozzle methods* is chosen. The method uses measurements taken from the propulsion nozzles, hot and cold nozzles, from an unmixed turbofan engine. Different ways of measuring some of the parameters give rise to variants of the chosen method, to which different uncertainties are associated, both to the instruments and the calculated thrust.

The schematic shown in Fig. 1 represents the intake and discharge flows in a gas turbine. In this case as unmixed turbofan is considered, without loss of generality. Similar reasoning may be applied to other jet engines. In this engine, the air flow is compressed in the fan (C), after which two streams are originated. One stream continues through the remaining engine rotating components (hot stream), exiting at the hot nozzle, and the other (cold stream) is directed straight to the cold nozzle. The cold and the hot jets do not mix inside the engine. The hot stream, after leaving the fan, goes through the remaining compression stages, entering the combustion chamber (CC) where it is heated by burning fuel in the air stream. The hot gases are expanded in the turbines to produce the exact amount of power to drive the compressors. The propelling nozzles accelerate the streams to the ambient. Thrust is generated due to the momentum variation across the engine and due to the atmospheric and nozzle discharge sections static pressure difference (AIR-1703, 1986).

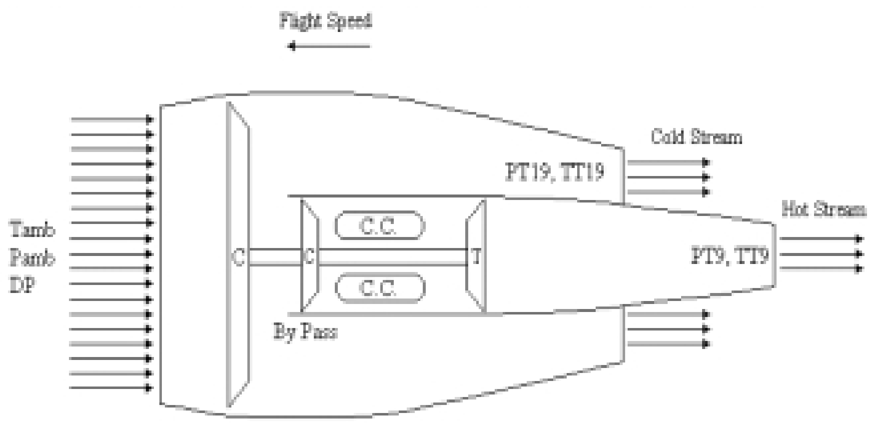

Figure 1 - Unmixed turbofan engine schematics

In Fig. 1 are indicated all measured parameters that are used in the thrust calculation. PT and TT mean stagnation (or total) pressure and temperature, respectively, Pamb and Tamb are the ambient pressure and temperature, respectively, and DP is the difference between the nozzle inlet stagnation pressure and the ambient pressure.

Let F_G be the gross thrust delivered by the gas turbine, W_{in} the air mass flow entering the engine, V_∞ the flight speed, W_{out} the air or gas mass flow leaving the gas turbine and W_f the burnt fuel mass flow.

From the continuity equation one has:

$$W_{in} = W_{out} - W_f \quad (1)$$

The mass flows through the nozzles can be calculated from their flow coefficients experimentally determined, and considering constant flow properties without loss of generality (SAE AIR-1703, 1986).

For unchoked nozzle one has:

$$W_{OUT} = C_V \frac{A\,P_T}{\sqrt{R\,T_T}} \left(\frac{P_{amb}}{P_T}\right)^{\frac{1}{\gamma}} \sqrt{\frac{2\,\gamma}{\gamma - 1}\left[1 - \left(\frac{P_{amb}}{P_T}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (2a)$$

For choke nozzle:

$$W_{OUT} = C_V \frac{A\,P_T}{\sqrt{R\,T_T}} \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{1-\gamma}} \sqrt{\frac{2\,\gamma}{\gamma - 1}\left[1 - \left[\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{1-\gamma}}\right]^{\frac{\gamma-1}{\gamma}}\right]} \quad (2b)$$

Equation 2b comes directly from Eq. 2a for unit Mach number (M=1) at the nozzle discharge section.

The flow coefficient C_V is defined as the quotient of the actual mass flow through the nozzle and the mass flow passing through the same nozzle if the flow were ideal and full expanded.

The gross thrust F_G can be calculated from the nozzle coefficient C_T, defined as the quotient of the actual nozzle gross thrust and the one derived from the full expansion through an ideal nozzle that passes the same mass flow. Thus, using Eq. 2a or 2b it follows that

$$F_G = C_T W_{OUT} \sqrt{RT_t} \sqrt{\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{P_{amb}}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (3)$$

The coefficients C_V and C_T are experimentally determined so that they must be measured in test beds. There are specialized laboratories that do this work. Nevertheless they are not always equipped to test full scale nozzles of the sizes encountered in gas turbines. Therefore there is the need to use reduced scale models, whose test data are correlated to the data taken from the actual nozzles installed on the engines. The actual nozzles and respective engines are bench tested at GLTB (*Ground Level Test Bed*) and/or at ATF (A*ltitude Test Facility*). In this work such coefficients are set do unity, without masking the results of the study.

The net thrust F_N (propulsive force F_N equals drag) is obtained subtracting the aerodynamic drag X_R (Eq. 4) from the gross thrust F_G:

$$X_R = W_{in}\, V_\infty \quad (4a)$$

$$F_N = F_{G9} + F_{G19} - X_R \quad (4b)$$

where F_{G9} and F_{G19} are, respectively, the gross thrusts due to the hot and cold streams.

The flight speed V_∞ is calculated from the pressure difference ΔP between the total and static pressures measured during the flight, using the isentropic equation and the Mach number definition, following Eqs. 5.

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{\Delta P}{P_{amb}} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \qquad \text{and} \qquad V_\infty = M\sqrt{\gamma R T_{amb}} \tag{5}$$

3. UNCERTAINTY IN THE DETERMINATION OF THRUST

The uncertainty calculation method used in this work is the one recommended by the SAE AIR-1678 (1986) report. Although this recommendation does not follow the more recent ISO Guide (1993), experience has shown that it is adequate for the studies like the one in this paper. The instruments used for measurement of the parameters that enter in the thrust calculation are complex equipments, usually incorporating sensors that emit electrical signals dependent on physical properties of the medium at which the measurement is made. Such sensors are affected by several factors like vibration, temperature, heat conduction, dilatation and, depending on the sensor type, of many other factors. These instruments are laboratory calibrated and installed on the engine. Therefore, it is not correct to take measurements without consideration of uncertainty in calibration. Quality instruments are accompanied by the manufacturer information on such uncertainty.

The uncertainty in a measurement is due to a random part (instrument characteristic standard deviation) and a bias (systematic) part, as considered by the SAE AIR-1678 (1986) report, being calculated by

$$U = \sqrt{B^2 + (2S)^2} \tag{6}$$

The multiplicative factor 2 in Eq. 6 is used for the confidence level of 95,45% in the result.

Let F be the adopted model for the thrust calculation, depending on n variables. For instance, F can be given by Eq. 3.

In the following a procedure for the uncertainty determination of a quantity F, function of n parameters, that is, $F = F(X_1, X_2, ..., X_n)$ is developed.

The uncertainty of F, given by SAE AIR-1678 (1986) is

$$U_F^2 = \sum_{i=1}^{n}\left[\left(\frac{\partial F}{\partial X_i}\right)^2 U(X_i)\right] \tag{7}$$

The term $\dfrac{\partial F}{\partial X_i}$ is the sensitivity coefficient, in this work denoted by θ_i. It is usual to give the relative uncertainty of the instruments in percentage. In this case, θ_i is defined as the relative sensitivity coefficient and is define by

$$\theta_i = \frac{\partial F}{\partial X_i}\frac{X_i}{F} \tag{8}$$

If the function F is given by an analytical expression, the uncertainty coefficients are easily directly obtained from the derivatives of F, calculated at the specified flight conditions. Such coefficients may also be obtained by numerically calculating the variation of F at small increments of each of the considered parameters. In this work the numerical calculation of the coefficients was adopted, since the in-flight thrust is expressed, in many cases, by a very complex system of equations. The gas turbine is simulated by a computer program (deck) like the one developed by Bringhenti (1999) and Bringhenti and Barbosa (1999).

4. MODELS FOR THE DETERMINATION OF IN-FLIGHT THRUST

To simplify the explanation, the methods considered in this work are represented by blocks like the one shown in Fig. 2. The input parameters are the ones that are measured in flight. It is beyond the scope of this work to analyze the processes utilized for the measurements, their difficulties and, in some cases, the impossibility of measurements. The reader might refer to Benedict (1972) to start with the fundamentals of temperature, pressure and flow measurements in general. Specifically for these parameters measurement in gas turbines, at the test benches or installed in aircrafts, the reader might work together with the engine manufacturers, since it is not a straight select-install-measure procedure, requiring the manufacturer intervention in most cases. The measured parameters are converted in the thrust F_G by the operations that are represented by the rectangle in Fig. 2, where F1 indicates the functional that transforms the input parameter values in the value of parameter F_G.

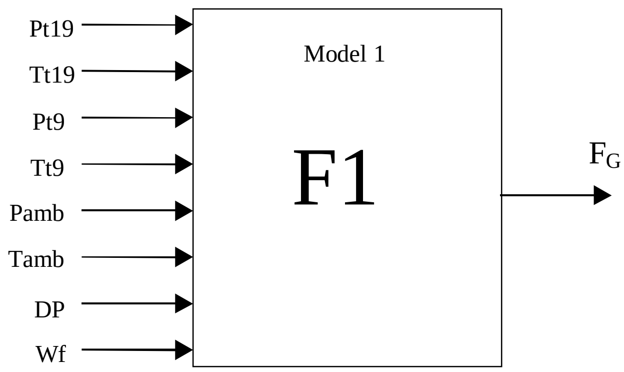

Figure 2 - Representation of a method for the determination of in-flight thrust - Method 1

The thrust $F_G = F_1(P_{t19}, T_{t19}, P_{t9}, T_{t9}, P_{amb}, T_{amb}, DP, W_f)$ calculation procedures were implemented in a computer program and used for the calculation of the sensitivity coefficients for each method. The methods that were used are similar to each other. In fact they differ only by the way some of the parameters are acquired, as will be shown below. Emphasis must be given to the fact that these methods are actually modifications to the chosen method and are being considered to investigate if it is possible to decrease the uncertainty of the calculated thrust, given the difficulties associated with the measurement of some of the parameters like the temperature of the outcoming nozzle flows, that can be indirectly obtained. In this case it is necessary that the fuel properties be available in addition to the list of properties indicated above.

The methods studied in this work are the following:

a. Method 1 - all parameters indicated in Fig. 1 are measured.
b. Method 2 - parameter T_{t9} is not measured; instead, it is calculated iteratively from the energy balance across the engine, considering all processes adiabatic:

$$W_{IN} c_{Pamb} T_{tamb} + \eta_{CC} W_f Q = W_{19} T_{t19} c_{P19} + W_9 T_{t9} c_{P9} \tag{9}$$

To do that it is required the fuel lower heating value, Q, as well as the combustor efficiency, η_{CC}. In the case of gas turbines, such efficiency is high, in the order of 99%

so that it was set to 100%, without loss of generality. Fig. 3 shows schematically this model.

c. Method 3 - similar to Method 2, but calculating parameter T_{t19} (cold flow total temperature) and measuring the remaining parameters. Fig. 4 shows schematically this model.

d. Method 4 - temperature T_{t9} is measured and T_{t19} is calculated from the temperature increase across the fan, admitting the fan isentropic efficiency η_{Fan} of 85%.

$$T_{t19} = T_{amb}\left(1+\frac{\gamma-1}{\gamma}M^2\right)\left[1+\left(\left(\frac{P_{t19}}{P_{amb}+\Delta P}\right)^{\frac{\gamma-1}{\gamma}}-1\right)/\eta_{Fan}\right] \tag{10}$$

e. Method 5 - temperature T_{t19} is calculated as in Model 4 and T_{t9} is calculated from energy balance, as in Model 2.

Attention must be paid to the fact that all these methods are equivalent in terms of thrust calculation but distinct in terms of how the values of the parameters are obtained.

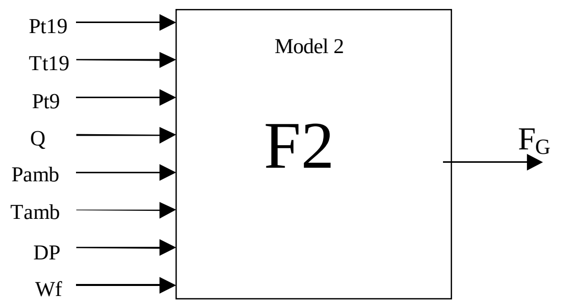

Figure 3 - Representation of Model 2

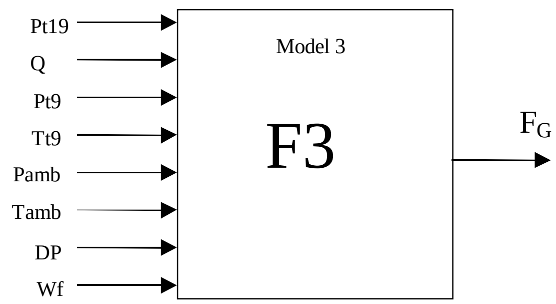

Figure 4 - Representation of Model 3

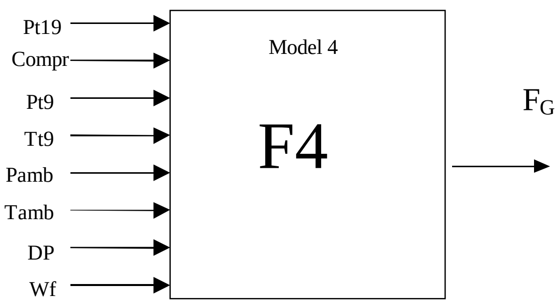

Figure 5 – Representation of Model 4

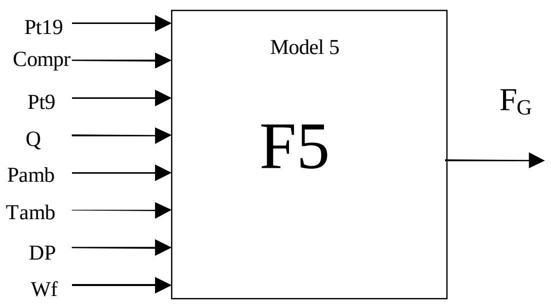

Figure 6 – Representation of Model 5

5. CALCULATIONS

For each one of the 5 methods described above the calculation of thrust and sensitivity coefficients were performed, considering as the point of reference one specific flight condition. To choose the most adequate method to determinate the thrust, the 5 selected methods were applied firstly considering uncertainties of 1% (Eq. 6), to both random and bias components. Actually such figures are not always equal because they are associated to the aircraft test instrumentation to measure pressures, temperatures, flows and so forth, with distinct calibrations.

For each Method the sensitivity coefficients were calculated having in mind Eq. 8, in the following way:

a. Calculation of the engine thrust F at the specified flight condition;
b. With increment ΔX_i for each parameter at a time, calculation of the corresponding thrust F_i;
c. The sensitivity coefficient corresponding to each parameter is obtained by division of the variation of F by ΔX_i , $(F_i - F)/ \Delta X_i$;
d. The relative sensitivity coefficient is calculates by $[(F_i - F)/\Delta X_i][X_i/F]$;
e. Repetition of items b, c and d for each of the remaining parameters.

Table 1 lists the parameters of interest and the equations in which they are involved.

Table 1 - Parameters for the calculation of thrust

Parameters	Equations	Parameters	Equations
P_{t9}	2, 3		
T_{t9}	2, 3, 10	P_{amb}	2, 3, 5, 10
P_{t19}	2, 3, 10	T_{amb}	2, 3, 5, 9, 10
T_{t19}	2, 3, 9, 10	ΔP	5, 10
W_f	1	Q	9

Table 2 was set from the list of parameters involved in the calculation of thrust, for the 5 considered methods.

If the parameter is not explicitly used for the thrust calculation, the corresponding influence coefficient is set to zero.

Table 2 - Results of the simulation for the 5 different methods

		Method 1	Method 2	Method 3	Method 4	Method 5
Parameters		Sensitivity Coefficients				
P_{t9}	$5{,}72\ 10^4$ Pa	0,627	0,534	0,382	0,627	0,534
T_{t9}	803 K	0,110	0	-0,182	0,110	0
P_{t19}	$6{,}43\ 10^4$ Pa	1,398	1,362	1,302	1,573	1,471
T_{t19}	300 K	0,524	0,326	0	0	0
W_f	0,274 kg/s	0,004	0,133	0,344	0,004	0,132
P_{amb}	$2{,}38\ 10^4$ Pa	-0,481	-0,502	-0,537	-0,648	-0,606
T_{amb}	219 K	-0,633	-0,406	-0,033	-0,108	-0,079
ΔP	$1{,}25\ 10^4$ Pa	-0,548	-0,527	-0,492	-0,557	-0,532
Q	43,1 MJ/kg	0	0,133	0,352	0	0,133
Final Uncertainty (%)		± 4,225	± 3,859	± 3,641	± 4,258	± 3,965

Note that in this work a four-place uncertainty figure was adopted for the numerical analyses, for the sake of comparison only. The recommended practice is to use one- or two-place figure, bearing in mind that one is dealing with uncertainties.

In order to prepare Table 2 it were taken in consideration uncertainties of 1%, usually accepted for test instrumentation. Data that are specific to the engine, like fan isentropic efficiency (Eq. 5), are not always available but may be inferred from the technology level of the gas turbine. For instance, it was used the figure of 85% but it was checked that a variation of plus or minus 1% on that value did not affect significantly the results.

6. DISCUSSION

Analyzing the sensitivity coefficients shown in Table 2 one sees that the calculated uncertainties suffer great influence of variation of parameters associated to the pressures: P_{t9} (greatest influence), P_{amb}, P_{t9} and ΔP. Therefore, it is recommended that special attention is paid to the instrumentation for the measurement of pressures, selecting best quality instruments. The best choice of instruments will certainly cause change in the values indicated in Table 2, possibly requiring the selection of new method. For the sake of information, the calculations were repeated setting the uncertainties of the pressure instruments to 0,5%. Table 3 shows the new calculated uncertainties. Comparing the uncertainties of Tables 2 and 3, the methods to be selected based exclusively on the criterion of lower calculated thrust uncertainty would be Methods 3 and Method 5. They point also that Method 3 has good results at both analyzed conditions.

Table 3 - Calculated thrust uncertainties for 0,5% in the pressure measurements

	Method 1	Method 2	Method 3	Method 4	Method 5
Final Uncertainty (%)	± 2,654	± 2,208	± 2,087	± 2,160	± 2,026

The uncertainties had their values reduced considerably. The same is not achieved when values of 0,5% are selected for the temperature measurement instruments: as expected they do influence the final calculated uncertainty but very little. Table 4 shows the results.

Table 4 - Calculated thrust uncertainties for 0,5% in the temperature measurements

	Method 1	Method 2	Method 3	Method 4	Method 5
Final Uncertainty (%)	± 3,907	± 3,725	± 3,624	± 4,247	± 3,963

Being the goal the determination of thrust within 1% uncertainty, certainly the considered instrumentation is not adequate. Several simulations were carried out trying different values for the uncertainties, bearing in mind the greatest influence of the pressure measurement instruments uncertainties on the calculated thrust uncertainty. The uncertainties associated to the other instruments were kept at 1% for temperature and 0,5% for the remaining, while for pressure the figure of 0,25% had to be fixed. The results are summarized in Table 5.

Table 5 - Calculated uncertainties for 0,25% in the pressure measurements

	Method 1	Method 2	Method 3	Method 4	Method 5
Final uncertainty (%)	± 2,084	± 1,540	± 1,460	± 1,116	± 1,085

7. CONCLUSIONS

Despite the instrumentation used during the flight test had not been analyzed with regard to the verification if it is adequate or not, the adoption of the uncertainty figures for the calculation and preparation of Tables 2, 3 and 4 allowed the conclusion that the selection of instruments for pressure measurements is highly important because the related uncertainties significantly influence the thrust uncertainty. It is seen that the thrust relative uncertainty of the order of 1% requires instruments whose uncertainties are less than 0,5%. Table 3, for instance, with relative uncertainties of 0,5%, the lowest thrust relative uncertainty is beyond. To achieve the figure of 1% for the measured thrust relative uncertainty, the simulation indicates that the instruments for temperature measurement may have 1% uncertainty but the ones for pressure must be 1% or lower. Table 5, calculated with 1% uncertainty for temperature, 0,25% for pressure and 0,5% for the others, point to the Method 5 as the appropriate to achieve the engine thrust uncertainty in the order of 1%, followed by Method 4, with 1,12% uncertainty. The other methods, to be adequate, must be associated to much better instruments quality.

For a given uncertainty in thrust calculation, an appropriate method might be chosen. This method will require a certain number of measurements and, therefore, instruments. Associated to this fact, the viability of installation of the probes in the engine/aircraft must be accessed. There are cases in which the engine manufacturer would not be happy with intrusions in the engine. The cost of measuring all the parameters (instruments, installation, maintenance, availability, quality of the instruments, etc.) must also be weighted for the final decision to adopt the method.

REFERENCES

Benedict, R. P., 1972, Fundamentals of Temperature, Pressure and Flow Measurements, 2nd. edition, John Wyley & Sons, New York.

Bringhenti, C., 1999, Análise de desempenho de turbinas a gás em regime permanente, MSc. Thesis, ITA, São José dos Campos, SP, Brazil.

Bringhenti, C. e Barbosa, J.R., 1999, Simulação numérica do desempenho de turbinas a gás, COBEM99, paper AAAEIF, November 21-26, Águas de Lindóia, SP, Brazil.

Guimarães, F. R., 1997, Levantamento das características do escoamento ao redor de turbinas a gás instaladas em aeronaves, Graduation Project, ITA, São José dos Campos, SP, Brazil.

Guimarães, F. R., Barbosa, J.R. and Mattos, B., 1998, In-Flight Thrust Determination, LATCYM98, October 5-8, Salta, Argentina.

ISO Guide, 1993, Guide to the expression of uncertainty in measurements, International Organization for Standardization, Switzerland.

Parizi-Negrão, J. R.; Fanton, R. E.; de La Fuente, R. P. and Barbosa, J. R., 1998, In-Flight Thrust Determination, VII International SAE Mobility Technology Conference, November 9-11, São Paulo, Brazil.

SAE Air Information Report AIR 1703, 1986, In-Flight Thrust Determination, Aerospace Technology Conference Exposition, October 13-16, Long Beach, California, USA.

SAE Air Information Report AIR 1678, 1986, Uncertainty of In-Flight Thrust Determination, Aerospace Technology Conference Exposition, Long Beach, October 13-16, California - USA.

ANÁLISE DE INCERTEZA NA DETERMINAÇÃO DE TRAÇÃO EM VÔO

Resumo. *Ensaios em vôo para a determinação da tração líquida do sistema propulsivo de uma aeronave é de grande importância, uma vez que a sua capacidade de carga depende da tração calculada. Incertezas relativas de 2% ou mais podem comprometer a rentabilidade da aeronave. Portanto, deve-se determinar a tração real do sistema propulsivo com a menor incerteza possível. Sistemas propulsivos com tração na faixa dos 75 kN, se tiverem seus empuxos subestimados em, por exemplo, 1%, acarretam a diminuição de 2 passageiros na sua capacidade de transporte. Este trabalho tem a finalidade de definir um procedimento de cálculo de incerteza para avaliar a influência de cada parâmetro na determinação da tração em vôo,* *possibilitando a seleção de um método adequado. Embora o enfoque deste trabalho seja a tração em vôo, o procedimento pode ser aplicado a outras medições, em laboratório ou em outros ambientes. É dada informação de como serem obtidas as informações necessárias para a determinação da incerteza.*

Palavras-chave: Incerteza, IFTD, Tração, Empuxo, Turbina a gás, Desempenho