NUMERICAL ANALYSIS OF A SPECIFIC HEAT AND THERMAL CONDUCTIVITY MEASUREMENT CELL

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***Abstract**. In this paper we present and discuss the design of an experimental apparatus for measuring thermal conductivity and specific heat of liquids. The apparatus consists essentially of a long annular cavity, filled with the liquid sample. The theory of the cell employs the hypotheses of purely radial heat flux and no natural convection inside the annular space. Thermal conductivity is calculated from data obtained during the steady state, while for specific heat evaluation, data pertaining to the transient regime are needed. Numerical solutions for the natural-convection flow and heat transfer inside the cell were obtained with the aid of the finite volume method. The results obtained showed that thermal conductivity measurement data are good for Grashof numbers less than 10^4 only.*

***Keywords:** Thermal conductivity measurement, Annular cavity, Natural convection.*

1. INTRODUCTION

The importance of thermophysical properties in the analysis of flow and heat transfer processes is evident. The particular interest that motivated this work was the measurement of thermal conductivity and specific heat of drilling muds. These properties are essential in order to study the heat transfer and flow of these viscoplastic materials during the drilling process (Souza Mendes et al., 1999). In this work, the design and construction of an experimental apparatus for measuring thermal conductivity and specific heat of fluids is described. We called the apparatus the ***k-c cell.*** The theory used in the cell design is discussed and validated through comparisons with numerical solutions for the natural-convection flow and heat transfer inside the apparatus. One of the hypothesis used in the development of the *k-c* cell is that heat transfer inside the annular space is due to conduction only. Therefore, the geometry and range of operation must be such that natural convection does not affect the temperature field significantly. The numerical solutions served as a design tool to guide in the choice of the appropriate geometry and other relevant parameters.

Several papers can be found in the literature that are concerned with the thermal conductivity for non-Newtonian fluids (Lee *et al*., 1981; Cocci and Picot, 1973; Chitrangad and Picot, 1981; Wallace *et al*., 1985; Loulou *et al.*, 1992; Chaliche *et al*., 1994; Lee and Irvine,1997). Lee et al. (1981) made measurements with a parallel-plate apparatus in a static condition (no forced flow of the sample). Cocci and Picot, 1973 used an axial flow cell and showed that the thermal conductivity increases with shear rate for a polymeric fluid. Chitrangad and Picot (1981) and Wallace *et al*. (1985) used Couette flow cell. Chitrangad and Picot (1981) also obtained thermal conductivities increasing with the shear rate. However, Wallace *et al*. (1985) observed that this behavior can change depending on the polymer molecular weight.

Loulou *et al.* (1992) and Chaliche *et al*. (1994) used a cone-and-plate cell, and also obtained shear-rate dependent thermal conductivities for polymeric fluids. Lee and Irvine (1997) developed a coaxial cylinder apparatus with a rotating outer cylinder, to measure thermal conductivity of non-Newtonian fluids as a function of shear rate. It was also found that thermal conductivity increases with the shear rate for the fluids analyzed. The increase is more significant for lower-concentration solutions. These results indicate that the effect of shear rate should be considered for viscoelastic liquids.

We failed to find in the literature a discussion regarding the measurement of thermal conductivity of viscoplastic materials. The conductivity for this type of material may also be shear-rate dependent. For the sake of simplicity, however, in these first steps towards developing an experimental procedure for measuring thermal conductivity of viscoplastic materials, we decided not to consider shear rate effects, and hence we design our first cell for static samples only.

2. THE *k-c* MEASUREMENT CELL

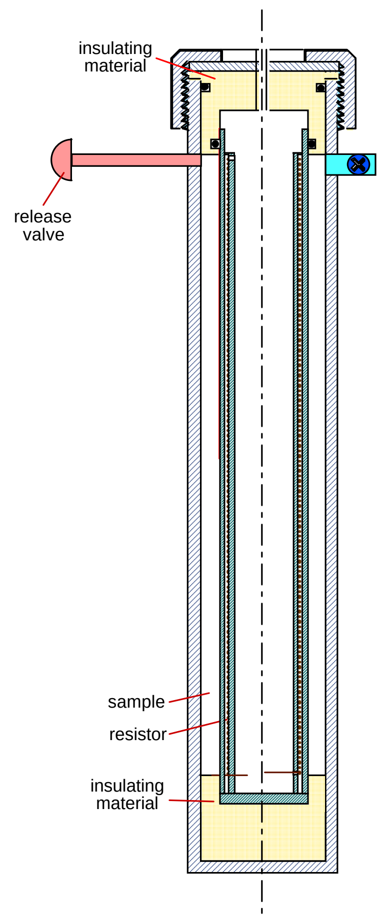


Figure 1: Sketch of the the k-c cell. The sample is place in the annular space between the inner and outer cylinder.

<u>*Experimental apparatus:*</u> Souza Mendes *et al.* (1999) developed an experimental apparatus for measuring thermal conductivity and specific heat of fluids. An schematic of the *k-c* cell is shown in Fig. 1. The liquid sample is injected inside the annular, which is kept hermetically closed during the measurements. An electrical resistance provides a known heat flux at the inner wall of the annular space. Two teflon layers at the top and bottom ends of the cell prevent extraneous heat losses. Four thermocouples are mounted flush with the inner

cylinder wall, to allow temperature measurements at this radial location. During the measurements, the *k-c* cell is positioned vertically, and is totally imersed in a constant temperature bath. The thermophysical properties of the sample are determined using the measurement data for bath temperature, electrical power dissipated at the resistor, and inner cylinder wall temperature.

Theory of the k-c cell:

The energy equation, considering only radial conduction, is given by:

$$\rho c_p \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left[rk \frac{\partial T}{\partial r} \right] = 0 . \quad (1)$$

In the above equation, ρ is the fluid density, c_p is the specific heat , k is the thermal conductivity, r is the radial coordinate, t is the time and T is the temperature. The initial and boundary conditions are:

$$\begin{aligned} T(r,0) &= T_0 \\ T(R_0,t) &= T_0 \\ \frac{\partial T}{\partial r}(R_i,t) &= -\frac{Q}{2\cdot\pi\cdot R_i\cdot L\cdot k} \end{aligned} \quad (2)$$

where Q is the wall heat flux, R_i is the inner cylinder radius and L is the cylinder length. The analytical solution of the above equation is given by:

$$\theta(r,t) = \frac{Q}{2\cdot\pi\cdot L\cdot k}\cdot\left\{ \ln\left(\frac{R_0}{r}\right) + \frac{\pi}{R_i}\cdot\sum_{n=1}^{\infty}\exp\left(-\alpha\cdot\lambda_n^2\cdot t\right)\cdot\frac{J_0^2\cdot(\lambda_n\cdot R_0)\cdot\left[J_0\cdot(\lambda_n\cdot r)\cdot Y_1\cdot(\lambda_n\cdot R_i) - Y_0\cdot(\lambda_n\cdot r)\cdot J_1\cdot(\lambda_n\cdot R_i)\right]}{\lambda_n\cdot\left[J_1^2\cdot(\lambda_n\cdot R_i) - J_0^2\cdot(\lambda_n\cdot R_0)\right]} \right\} \quad (3)$$

In this equation, $\theta = T - T_0$ and λ_n are the eigenvalues, given by:

$$J_1\cdot(\lambda_n\cdot R_i)\cdot Y_0\cdot(\lambda_n\cdot R_0) - Y_1\cdot(\lambda_n\cdot R_i)\cdot J_0\cdot(\lambda_n\cdot R_0) = 0 \quad (4)$$

The first five roots of the above equation for R_i = 0.011 m and R_o = 0.015 m are shown in Table 1.

Table 1: Eigenvalues of equation (3) for R_i = 0.011 m and R_o = 0.015 m

n	λ_n	A_n
1	4.17645549295105E+02	-8.48127536219224E-04
2	1.18701129765937E+03	-1.11873496398581E-04
3	1.96887761683018E+03	-4.09121923733676E-05
4	2.75274477116606E+03	-2.09654504814640E-05
5	3.53728928952131E+03	-1.27058524769007E-05

The temperature at the inner wall ($r = R_i$) is given by:

$$\theta(R_i,t) = \frac{Q}{2\cdot\pi\cdot L\cdot k}\left\{\ln\left(\frac{R_0}{R_i}\right) + \frac{1}{R_i}\cdot\sum_{n=1}^{\infty} A_n\cdot e^{-\alpha\cdot\lambda_n^2\cdot t}\right\} \tag{5}$$

where

$$A_n = \frac{J_0^2\cdot(\lambda_n\cdot R_0)\left[J_o\cdot(\lambda_n\cdot R_i)\cdot Y_0\cdot(\lambda_n\cdot R_i)\cdot J_1\cdot(\lambda_n\cdot R_i)\right]}{\lambda_n\cdot\left[J_1^2\cdot(\lambda_n\cdot R_i) - J_0^2\cdot(\lambda_n\cdot R_0)\right]} \tag{6}$$

An expression for the thermal conductivity is obtained by using the solution of the steady state, for which the temperature field is given by:

$$\theta(r) = \frac{Q}{2\cdot\pi\cdot L\cdot k}\cdot\ln\left(\frac{R_0}{r}\right) \tag{7}$$

Thus, by evaluating the above equation at $r = R_i$, and solving for the thermal conductivity, we obtain

$$k = \frac{Q}{2\cdot\pi\cdot L\cdot(T(R_i) - T_0)}\cdot\ln\left(\frac{R_0}{R_i}\right). \tag{8}$$

3. **NUMERICAL SOLUTIONS**

To analyze the flow and heat transfer inside the *k-c* cell, a numerical procedure is employed. The flow was assumed to be steady and axisymmetric. The computational domain is shown in Fig. 2, where some geometrical simplifications adopted are indicated.

Conservation equations

The mass conservation equation is given by:

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rv) = 0 \tag{9}$$

where u and v are the axial and radial velocity components, respectively.

The conservation equations of momentum are given by:

$$\begin{gathered}\rho\cdot\left(\frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial r} + v\cdot\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\cdot\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\cdot\frac{\partial(r\cdot u)}{\partial r}\right] + \frac{\partial^2 u}{\partial z^2}\right\} \\ \rho\cdot\left(\frac{\partial v}{\partial t} + u\cdot\frac{\partial v}{\partial r} + v\cdot\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial z} - \rho\cdot g\cdot(1 - \beta\cdot(T - T_0)) + \mu\cdot\left[\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial v}{\partial r}\right) + \frac{\partial^2 v}{\partial z^2}\right]\end{gathered} \tag{10}$$

In the above equations, p is the pressure, μ is the viscosity, g is the gravity acceleration, β is the volume expansion coefficient, and T_0 is a reference temperature.

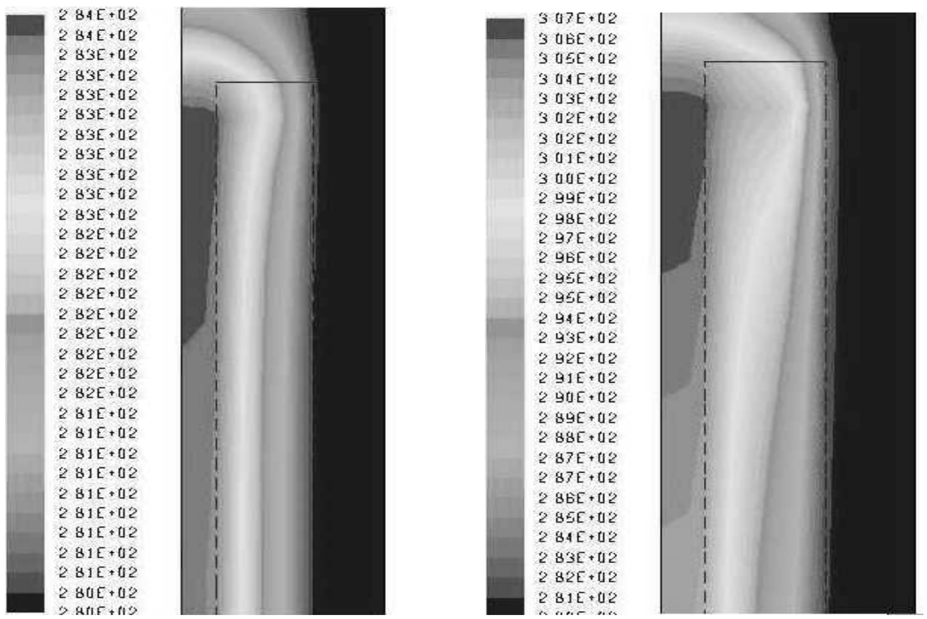
The energy conservation equation is given by:

$$\rho \cdot Cp \cdot \left(\frac{\partial T}{\partial t} + u \cdot \frac{\partial T}{\partial r} + v \cdot \frac{\partial T}{\partial y} \right) = \frac{1}{r} \cdot \frac{\partial}{\partial r} \cdot \left(r \cdot k \cdot \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial y} \left(k \cdot \frac{\partial T}{\partial y} \right) \tag{11}$$

The momentum boundary conditions are the no-slip condition and impermeability conditions at the solid boundaries. For the energy equation, a constant heat flux is imposed at the inner wall, while the outer wall is subject to a uniform temperature. The upper and lower walls are considered adiabatic.

4. NUMERICAL RESULTS

The numerical solution of the governing equations was obtained via finite volume method, with the aid of the FLUENT package (Fluent Inc.). The effect of natural convection was investigated for several cases.

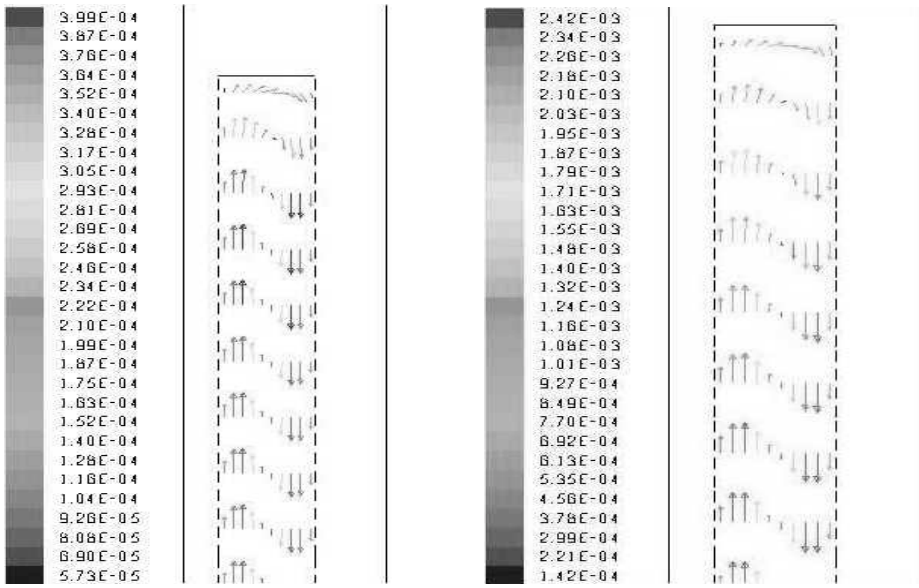


(a) **(b)**

Figure 2: Isotherms for water, T_o = 280.35 K , (a) Q = 4.33 W and (b) Q = 39 W.

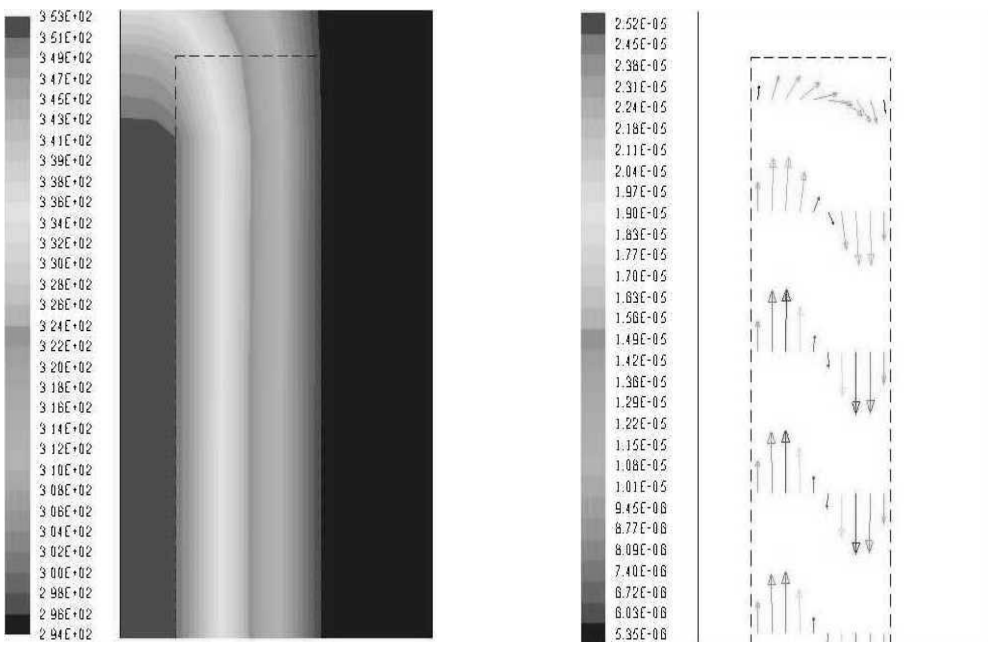
Figures 2 and 3 show the isotherms and velocity vectors for water (k=0.62 W/mK), for two different heat flux values at the inner wall. The temperature at outer wall was maintained at 280.35 K. It can be observed that at the larger heat flux case (i.e., larger Grashof number, defined as $Gr = \rho^2 g \beta QS^3/2k\pi R_i \mu^2$), there is an important distortion of the isotherms at the upper region of the annular region, due to the natural convection flow. This influence can be better observed with the aid of the velocity vectors. It can be noted that, at the case of larger Grashof number, the average recirculation velocity is one order of magnitude larger than for the lower Grashof number.

Fig. 4 shows the isotherms and velocity vectors for the flow of glycerin (k=0.29 W/mK), heat power at the inner wall equal to 39.01 W and outer wall temperature equal to 294.15 K. In this case, Grashof number is lower than that obtained with the water cases. It can be noted that natural convection is negligible, because there is rather little distorsion of the isotherms, and the velocities are quite small.



(a) **(b)**

Figure 3: Velocity vectors for water, T_o = 280.35 K , (a) Q = 4.33 W and (b) Q = 39 W.



(a) **(b)**

Figure 4: (a) Isotherms and (b) Velocity vectors for gliceryn, T_o = 294.15 K and Q = 39 W.

In Fig. 5, the percent deviation between values the thermal conductivity values employed in the numerical simulation and the ones obtained with eq. (8) is shown. It can be observed that for Grashof numbers lower than 10^4, the deviation is about 5%. For Grashof numbers larger than this value, the deviation increases, showing the importance of the effect of natural convection.

Experimental measurements performed using the proposed *k-c* cell with water and glycerin were compared with thermal conductivity values found in the literature. The complete set of data is presented by Souza Mendes et al, (1999). In Fig. 6, the percent deviation is shown as a function of the Grashof number. The different Grashof numbers for the same liquid and heat flux were obtained by changing the average temperature of the sample and therefore its viscosity. It can be observed that for Grashof numbers lower than 10^4, the deviation is below 10%. For larger values of *Gr*, the deviation increases. The trend is the same one predicted by the theoretical calculation. As expected, the errors associated with the experimental measurements are larger.

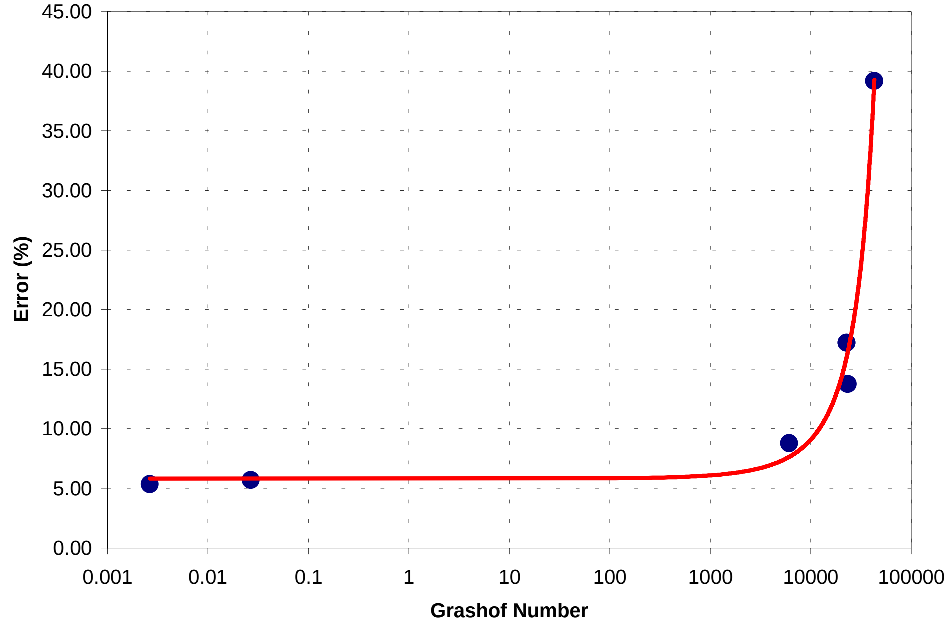


Figure 5: Deviation between input and calculated thermal conductivity values *versus* Grashof number.

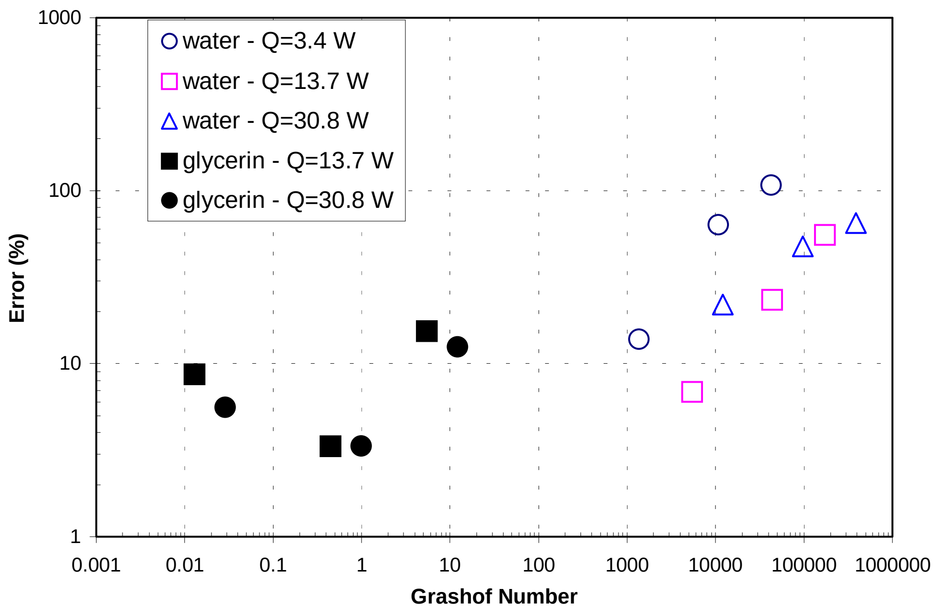


Figure 6: Deviation of measured thermal conductivity values *versus* Grashof number.

5. FINAL REMARKS

Numerical solutions of the flow inside a thermal conductivity measurement cell was performed. The main goal was to evaluate the hypothesis pure radial conduction, adopted in the cell theory. The conservation equations of mass, momentum and energy were solved numerically to obtain the velocity and temperature fields via the finite volume method. The numerical and experimental results indicate that natural convection is harmful for Grashof numbers larger than 10^4.

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