ERROR ANALYSIS IN A FLAT PLATE TEMPERATURE MEASUREMENT USING THERMOCOUPLE

Valdir Araújo de Souza - email:valdir@mec.ita.cta.Br
Edson Luiz Zaparoli - email:zaparoli@mec.ita.cta.br
CTA – ITA – Departamento de Energia
Pça Mal. Eduardo Gomes, 50 – Vila das Acácias
CEP: 12228-900 – São José dos Campos – SP – Brasil

Abstract. *Flat plates are widely used in a large variety of equipment such as satellites, aircraft, air cooled heat exchangers and etc. When thermocouples are used to measure the temperature of flat plate surfaces, which are exposed to fluids at different temperatures, errors occur because of the heat conduction along the wire. In this work the parameters which affect the error were analysed considering two-dimensional conduction through the plate and the thermocouple wires. This problem was numerically solved employing the finite element method with a triangular non-structured adaptive mesh with six nodes per element. The mesh refinement is automatically done and it is denser in regions, which have sharp curvature, tight geometries and high temperature gradient. Firstly the results from the previously mentioned analysis are compared with the analytical solution where is taken into account a one-dimensional conduction that is suitable for thin flat plates and long thermocouple wires. This comparison allows the validation of the numerical results and also to evaluate the parameters range in which the one-dimensional approach is suitable. The results obtained from this work enable us to quantify the error associated with conduction through the thermocouple wires in the flat plates temperature measurement evaluating the conditions that causes more significant errors.*

Key words: Temperature measurement, Thermocouple conduction error, Finite element.

1. INTRODUCTION

When the thermocouple are used to measure the temperature of surfaces which are exposed to fluids at temperatures different from that, errors occur because of conduction of heat along the thermocouple wires.

The necessity for measuring these surface temperature arises during thermal tests of aircraft heaters, investigation of heated leading-edge system for anti-icing and other types of thermal analysis.

If the thermocouple is mounted on the plate in the hot-fluid side to measure the temperature of the plate, heat flows along the thermocouple wires and thence into the plate increasing the temperature of the plate at the junction so that the temperature recorded by the thermocouple would be higher than the true plate temperature. Conversely, if the thermocouple is placed on the cold-fluid side the temperature recorded by the thermocouple would be lower than the plate.

Concerning this problem Boelter *et al*. (1948) presented an analytical solution for steady-state temperature distribution caused by a thermal source (or sink) in a flat plate surrounded on either side by fluids of different temperature and also to evaluate the thermal error which occurs when thermocouples are used for measuring the plate temperature. Furthermore Boelter and Lockhart (1951) presented experimental results obtained in order to compare with the analytical solution.

Following the same research line Schneider (1955) developed the analytical solution for temperature error estimation in a flat plate surrounded either side by the fluids at different temperatures. In this model all the thermal properties are assumed to be constant with a uniform plate temperature over the thickness and the thermocouple wires heat transfer is taken as one dimensional conduction in a single long cylinder of homogeneous material.

In this work the error introduced by the thermocouple presence is numerically investigated considering the thermocouple as a single cylinder of homogeneous material and a perfect contact between the thermocouple and the body . The conduction error is evaluated as a function of some important parameters such as plate thickness, thermal conductivity ratio (thermocouple and body), the thermocouple radius and the overall convective heat transfer coefficient over the thermocouple wire.

It is essential to stress that in this work the thermocouple was chosen to be placed at the hot fluid side in three configurations: mounted on the surface, embedded half way along the plate thickness and all the way through the plate thickness, as shown by Figures 1, 2 and 3 respectively.

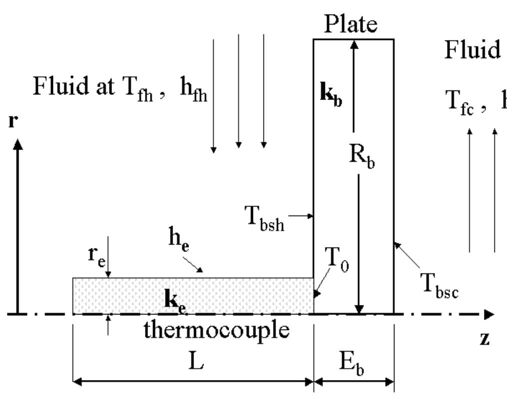


Figure 1 - Thermocouple mounted on the flat plate surface.

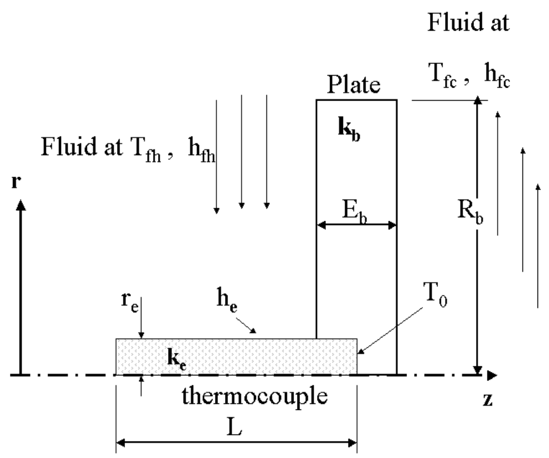


Figure 2 - Thermocouple embedded half way along the flat plate.

In these figures h_{fh} and h_{fc} are the convective heat transfer coefficient over the plate on the hot and cold side, respectively; T_{fh} and T_{fc} are the fluid temperature of the hot and cold fluid; T_{bsh} and T_{bsc} are the surface temperature of the flat plate on the hot and cold side respectively, k_e, is the equivalent thermal conductivity of the thermocouple; T_o is the temperature at the junction or the temperature recorded by the thermocouple; L is the thermocouple length; R_b is the flat plate radius; E_b is the flat plate thickness; h_e is the equivalent convective heat transfer coefficient over the thermocouple and k_b is the flat plate thermal conductivity.

As the thermocouple wires is taken as a single cylinder of homogenous material it is essential to model the replacement of the two wires by one cylinder.

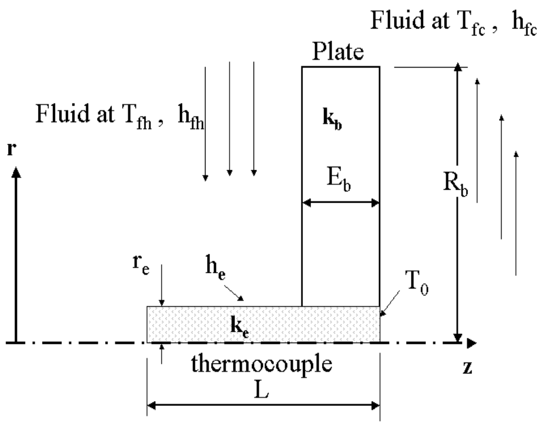


Figure 3 - Thermocouple embedded all the way along the flat plate.

2. MATHEMATICAL MODEL

2.1 Thermocouple equivalent wire

In order to replace the two thermocouple wires by one single cylinder it assumed that both wires have the same radius, that is, $r_{w1}=r_{w2}=r_w$. Then, from this assumption it is possible to obtain the equivalent thermocouple radius, r_e, summing up the frontal area of each wire ($A_{w1}+A_{w2}$) resulting on the frontal area, A_e, of the equivalent wire as it follows.

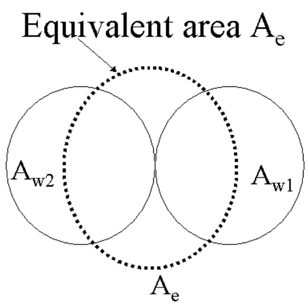


Figure 4 - Equivalent thermocouple frontal area.

From this assumption it becomes that

$$A_e = A_{w1} + A_{w2} \tag{1}$$

$$r_e = r_w \sqrt{2} \tag{2}$$

The next step is to obtain the equivalent thermal conductivity, k_e, and the equivalent convective heat transfer coefficient, h_e.

Before developing the model it is important to show the difference between the heat transfer coefficient over each thermocouple wire when it is a bare or insulated thermocouple.

When the thermocouple is bare the convective heat transfer coefficient, h_{fb}, is equal to the overall heat transfer coefficient, h_f.

If the thermocouple wire is insulated, as shown in Figure 5, the overall heat transfer coefficient, h_f, takes the form as follows:

$$h_f = \frac{1}{\left(\frac{1}{h_{fb}} + \frac{\delta_i}{k_i} + R_c A_c\right)} \tag{3}$$

Where: δ_i and k_i are the thickness and the thermal conductivity of the insulation, respectively; R_c is contact resistance and A_c is the area of contact between the insulation and the thermocouple wire, which product $R_c A_c$ is assumed to be negligible in this work.

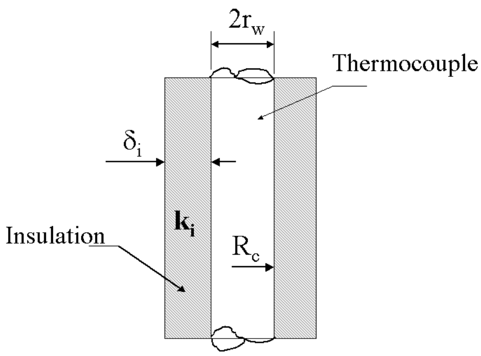


Figure 5 - Insulated thermocouple.

Now it is possible to develop the model for the equivalent thermocouple wires concerning the equivalent thermal conductivity, k_e, and the equivalent heat transfer coefficient, h_e.

Once the equivalent radius, r_e, is obtained from Equation (2), the equivalent convective heat transfer coefficient, h_e, is obtained from the heat loss by the equivalent wire, δQ_e, of length, δL, which is the sum of the heat loss by each individual wire, δQ_{w1}, and, δQ_{w2}, according to Figure 6 it follows that:

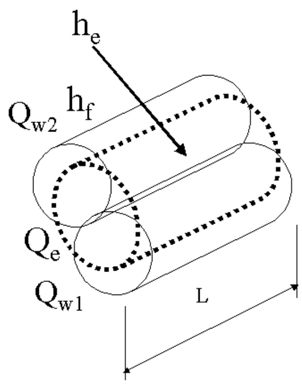


Figure 6 - Equivalent heat loss.

$$\delta Q_e = \delta Q_{w1} + \delta Q_{w2} \tag{4}$$

$$2\pi r_e \delta L h_e (T_w - T_{fh}) = 2\pi r_{w1} \delta L h_f (T_w - T_{fh}) + 2\pi r_{w2} \delta L h_f (T_w - T_{fh}) \tag{5}$$

where T_w represents the surface temperature of the thermocouple.
From Equation (5) it is obtained the following relation

$$h_e = h_f \sqrt{2} \tag{6}$$

To determine the equivalent thermal conductivity, k_e, it is necessary to get the long pin fin equation given by Osizik (1985) for the heat flow rate through the fin as:

$$Q = \theta_o \sqrt{PhkA} \tag{7}$$

Summing up the two wires heat losses (Q_{w1}+Q_{w2}) and equalizing to the equivalent heat loss, Q_e, results:

$$\theta_o \sqrt{P_e h_e k_e A_e} = \theta_o \sqrt{P_1 h_f k_{w1} A_{w1}} + \theta_o \sqrt{P_2 h_f k_{w2} A_{w2}} \tag{8}$$

where: P_e, P_1, P_2 are the perimeters of the equivalent wire and of each thermocouple wire; θ_o=(T_o -T_{fh}) and k_{w1} and k_{w2} are the thermal conductivity of each individual wire.

Then the equivalent thermal conductivity becomes:

$$\sqrt{k_e} = \frac{1}{2}(\sqrt{k_{w1}} + \sqrt{k_{w2}}) \tag{9}$$

2.2 Energy equation

The analytical solution considers the flat plate radius as infinite but, in this work, the flat plate is taken to have large finite radius and the heat transfer process assumed steady-state two-dimensional conduction with constant thermal properties and having no heat generation.

Thus, the energy equation for a cylindrical coordinate system as presented by Osizik (1985) is:

$$\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}\left(\mathrm{k}\frac{\mathrm{r}\partial \mathrm{T}}{\partial \mathrm{r}}\right) + \mathrm{k}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{z}^2} = 0 \tag{10}$$

where k is the thermal conductivity (k=k_e in the equivalent thermocouple wire and k=k_b in the flat plate) and T is the temperature in any region.

2.3 Boundary conditions

In order to solve the problem for the configuration when the thermocouple is placed on the hot-fluid side, the following boundary conditions are considered.

For r=0 and 0<z<L+E_b $\qquad \frac{dT}{dn} = 0$ (11)

For $z=L+E_b$ and $0<r<R_b$ $$k_b \frac{\partial T}{\partial n} = h_{fc}\left(T_{fc} - T\right) \tag{12}$$

For $r=R_b$ and $L<z<E_b+L$ $$\frac{dT}{dn} = 0 \tag{13}$$

For $z=L$ and $r_e<r<R_b$ $$k_b \frac{\partial T}{\partial n} = h_{fh}\left(T_{fh} - T\right) \tag{14}$$

For $r=r_e$ and $0<z<L$ $$k_e \frac{\partial T}{\partial n} = h_e\left(T_{fh} - T\right) \tag{15}$$

For $z=0$ and $0<r<r_e$ $$\frac{dT}{dn} = 0 \tag{16}$$

Where n represents the outward normal unit vector to the domain boundaries.

2.4 Temperature measurement error

In this work the temperature read by the thermocouple or even the temperature of the junction is defined as as T_0 and the temperature measurement error will be presented as a dimensionless temperature error ϕ. The determination of T_0 and ϕ, considering that the thermocouple is mounted on the hot fluid side are given by the following equations:

$$T_0 = \frac{2}{r_e^{\,2}} \int_0^{r_e} T_{(r)} r dr \tag{17}$$

$$\phi = \frac{\left(T_0 - T_p\right)}{\left(T_{fh} - T_p\right)} \tag{18}$$

where T_p is the temperature at the same position as T_0 when the thermocouple is not present.

In this work the effect of E_b/r_e, k_b/k_e and h_{fc}/h_e groups on the dimensionless temperature measurement error are investigated keeping the remaining ones constant.

3. NUMERICAL SOLUTION

This problem was numerically solved employing a program based upon the Galerkin finite element method. This program uses a quadratic interpolation polynomial to convert continuous partial differential equations into discrete nodal equations. The program works with a triangular non structured adaptive mesh with six nodes per element. The mesh refinement is automatically processed and presented more intense refinement in regions which have large curvature that are geometrically small and that are subjected to high temperature gradient. The algebraic equations system has been solved through the iterative conjugategradient method, using the incomplete Cholesky decomposition as a preconditioner, Macsyma Inc. (1996).

4. ANALYTICAL MODEL

Schneider (1955) developed the analytical solution for thermocouple conduction error determination considering a steady state temperature distribution in a uninsulated flat plate with a cylindrical heat source located in the plate as shown in Figure 7.

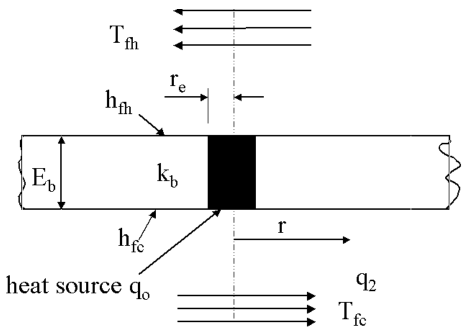


Figure 7 - Flat plate with a cylindrical source.

where the source q_o has a cylindrical shape with volume $\pi r_e^{\,2} E_b$ and at uniform temperature $T_{0.}$

In his analysis Schneider (1955) made some assumptions such as: there is no temperature gradient along the plate thickness, the contact between the body and thermocouple is perfect, thermocouple is considered as a very long single cylinder and the plate has infinite radius.

The final equation for the plate measurement error due to the thermocouple presence is:

$$\frac{\mathrm{T_0} - \zeta}{(\mathrm{T_{fh}} - \zeta)} = \frac{1}{1 + \dfrac{2}{\left(\sqrt{\mathrm{k_{w1}}} + \sqrt{\mathrm{k_{w2}}}\right)} \sqrt{\dfrac{\mathrm{k_b E_b}(\mathrm{h_{fh}} + \mathrm{h_{fc}})}{\mathrm{h_e r_w}}} \dfrac{\mathrm{K_1}(\varepsilon \mathrm{r_e})}{\mathrm{K_0}(\varepsilon \mathrm{r_e})}} \tag{19}$$

and:

$$\varepsilon^2 = \frac{(h_{fh} + h_{fc})}{k_b E_b} \tag{20}$$

where: $K_1(\varepsilon r_e)$ and $K_0(\varepsilon r_e)$ are Bessel functions.

The true plate temperature is obtained for a point very far from the source as:

$$T_{r=\infty} = \frac{(h_{fh} T_{fh} + h_{fc} T_{fc})}{(h_{fh} + h_{fc})} = \zeta \tag{21}$$

5. RESULTS

In this work the following numerical values are employed: h_{fb}=1027.71W/mK, L=3.6x10^{-2} m, R_b=7.18x10^{-2}m , T_{fh}=1174 K, T_{fc}=333.15 K, δ_I=1.27x10^{-4} m, k_i=0.043 W/mK

The following items presents the influence of E_b/r_e, k_b/k_e, h_{fc}/h_e ratios on the dimensionless temperature error.

5.1 Variation of the E_b/r_e

Looking in to Figure 8 it is well observed that the value of ϕ reduces as the ratio E_b/r_e increases and that can be explained through two situations such as : for E_b/r_e <<1 ϕ is high due to the thermocouple radius is relatively big relating to E_b then the heat flux and consequently the isothermal lines are more disturbed at the junction and also the temperature of the wall opposite to the surface where the thermocouple is placed is highly affected. This influence can be observed in Figure 9 (a) which shows the isothermal lines disturbance along the plat thickness for this ratio. On the other hand if the ratio E_b/r_e >>1 ϕ reduce as shown in Fig.8 and that is due to the lower disturbance caused by the thermocouple affecting much less the heat flux at the junction. In Figure 9 (b) it is observed a lower disturbance on the opposite wall when the ratio is equal to 4.

According to Figure 8, the numerically obtained ϕ values reduces as the thermocouple is inserted into the plate.

The difference between the numerical and analytical results are due to: in the analytical model the there is no temperature gradient along the plate thickness and across the thermocouple wire so it is impossible to set a depth of insertion.

5.2 Variation of k_b/k_e

Figure 10 shows the reduction in the values of ϕ as the ratio k_b/k_e increases. The reason why this happen is the fact that when the heat flux coming from the thermocouple hits the plate which has low thermal resistance there is no great disturbance on the isothermal lines at the junction but, on the other hand, if the thermal resistance is high the plate prevent the heat flux from going through causing then a great temperature difference at the junction.

The insertion of the thermocouple in the plate reduces the values of ϕ as can be seen in Figure 10.

Concerning the difference between the analytical and numerical results the considerations are the same as discussed.

5.3 Variation of $h_{fc/}$ h_e

Figure 11 shows the effect of the ratio h_{fc}/h_e on temperature error. It is observed that the temperature measurement error drops when the ratio h_{fc}/h_e increases. That happens because when h_e<<h_{fc} the heat flux rate is small that causes slight temperature disturbance or when h_{fc} is high the heat is transferred mainly to the fluid at the opposite side bringing low alteration in the plate temperature distribution.

From Figure 11 it is also observed the great difference between the numerical and the analytical results turns out due to the same reasons that have been discussed previously.

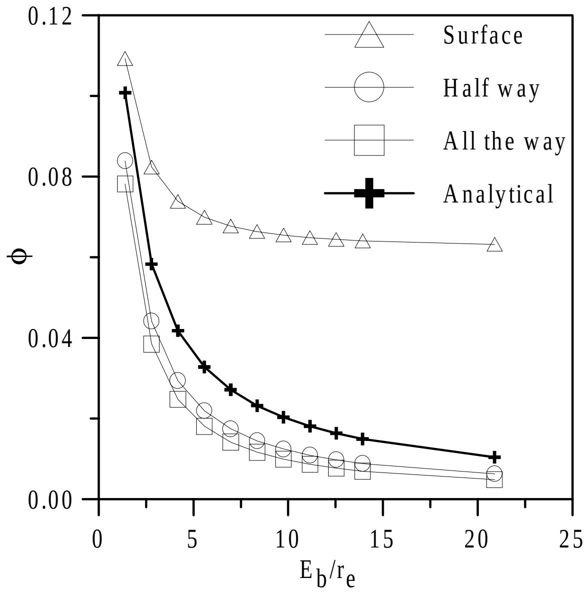
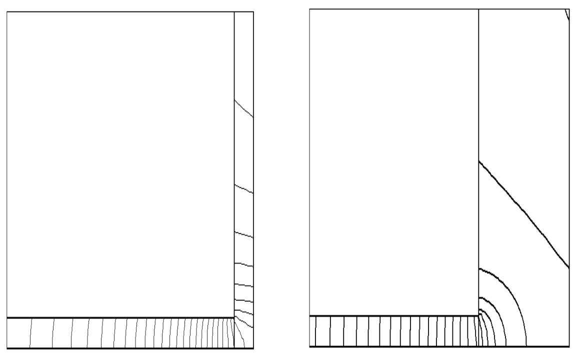


Figure 8 - Dimensionless temperature measurement error as a function of E_b/r_e (k_p/k_e=1.41and h_{fc}/h_e=0.26).



(a) E_b/r_e=0.8 (b) E_b/r_e=4

Figure 9 – Isothermal lines as a function of E_b/r_e.

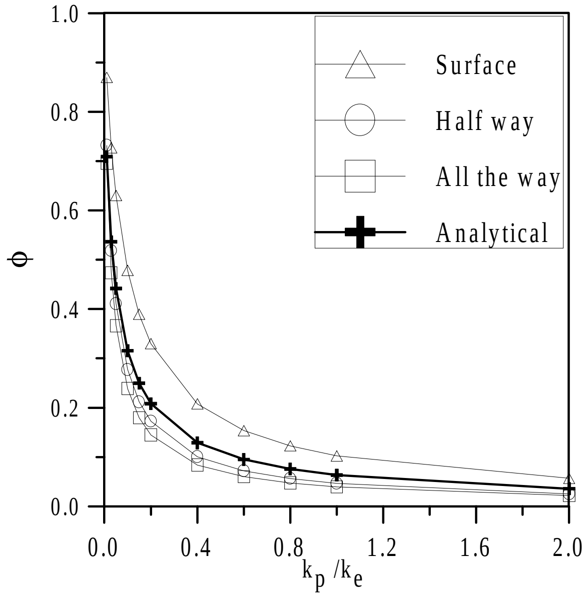


Figure 10 - Analysis of the ratio k_p/k_e (E_b/r_e=3.5 and h_{fc}/ h_e =0.26).

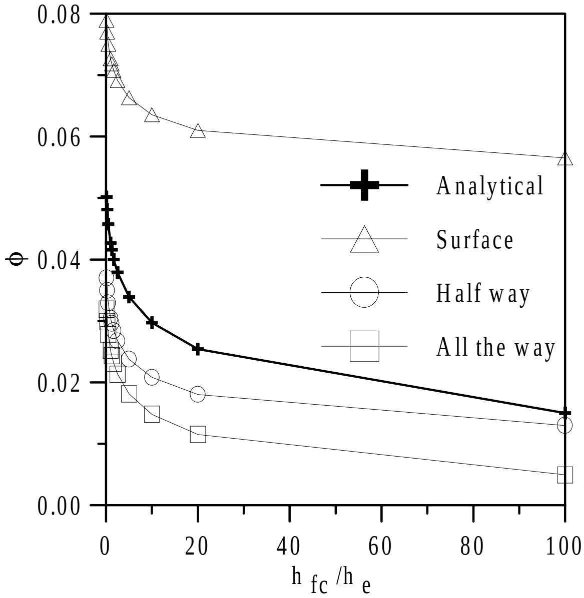


Figure 11 - Influence of the ratio h_{fc}/ h_e (E_b/r_e=3.5 and k_p/k_e=1.41).

6. CONCLUSIONS

The analysis of this problem with the two-dimensional approach allowed the evaluation of the parameters range in which the analytical solution (one-dimensional) is applied.

The two-dimensional analysis provides some insights about this kind of error which can not be obtained through the one-dimensional analysis as for instance the error reduction due to the thermocouple insertion in the plate.

In spite of the difficulty in building up a thermocouple junction inserted in the plate, this kind of assembly causes lower disturbance in the temperature field, resulting in small errors.

However the error trend can be predicted by the one-dimensional analysis. Also it is possible to conclude that in elaborating an experiment the thermocouple employed should have characteristics which minimise the temperature field disturbance of the plate at the junction where the temperature is to be measured.

Acknowledgements

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