A COUPLED INTEGRAL EQUATIONS APPROACH FOR THE ANALYSIS OF DRYING IN TWO-DIMENSIONAL CAPILLARY-POROUS MEDIA

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Abstract*. This work deals with the solution of the heat and mass transfer problem during drying of capillary porous media. The physical problem considered here is described by the linear Luikov's equations in cylindrical coordinates. Such equations are integrated along the radial direction in order to reduce the number of independent spatial variables. Two different approaches used to represent the dependent variables at the cylinder surface include: (i) The Lumped Approach and (ii) The Coupled Integral Equations Approach (CIEA). The use of the CIEA results on a formulation as simple as that for the lumped approach but much more accurate, since the gradients along the radial direction are not neglected.*

Keywords: Luikov's equations, Heat and mass transfer, Drying, Coupled Integral Equations Approach.

1. INTRODUCTION

The phenomena of heat and mass transfer in capillary porous media has practical applications in several different areas including, among others, drying and the study of moisture migration in soils and construction materials (Luikov, 1966, Mikhailov and Ozisik, 1984, Ribeiro and Lobo, 1998). For the mathematical modeling of such phenomena, Luikov (1966) has proposed his widely known formulation, based on a system of coupled partial differential equations, which takes into account the effects of the temperature gradient on the moisture migration.

Different approaches have been used for the solution of Luikov's equations in one-dimensional and multi-dimensional problems (Comini and Lewis, 1976, Mikhailov and Ozisik, 1984, Lobo et al., 1987, Lobo et al., 1995, Guigon et al., 1999, Ribeiro et al., 1993, Cotta, 1993, Ribeiro and Cotta, 1995, Ribeiro and Lobo, 1998, Duarte and Ribeiro, 1998). The use of the *Generalized Integral Transform Technique* with simple eigenvalue problems involving analytical eigenfunctions, can avoid the calculation of complex eigenvalues for the drying problem based on Luikov's formulation. For more details on the use of such hybrid numerical-analytical technique, the reader is referred to the works of Ribeiro et al. (1993), Cotta (1993), Ribeiro and Cotta (1995) and Ribeiro and Lobo(1998).

Several multidimensional heat transfer problems involve small gradient along a specific spatial direction or even inside the whole body. A common engineering approach in such cases is to integrate the governing equations in the directions with small gradients. An approximate *Lumped Formulation* results when the gradients are fully neglected. However, an improved approximate formulation, which takes into account the gradients effects, can be obtained by using the so-called *Coupled Integral Equations Approach* (CIEA) (Cotta et al., 1990, Cotta and Mikhailov, 1997, Traiano et al., 1997, Cheroto et al., 1997). In the CIEA, Hermite integrals (Mennig et al., 1983) are used in order to approximate temperatures at the surfaces of the body after the integration of the governing equations. The use of the CIEA can result in formulations as simple as those obtained with the lumped approach, but much more accurate, because the gradients are not neglected.

In this paper we examine the solution of a two-dimensional drying problem in cylindrical coordinates. The coupled heat and mass transfer in the capillary-porous body is formulated with Luikov's equations. Temperature and moisture content gradients along the radial direction are supposed small, so that the governing equations are integrated in this direction. Both the lumped and the CIEA approximations are considered in this work to approximate the dependent variables at the surface of the cylinder. The resulting one-dimensional problem is solved with the Generalized Integral Transform Technique (GITT). The effects of lateral heat losses are addressed in the paper.

2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem under picture in this work involves a cylindrical capillary porous medium of radius R_0 and length l, initially at uniform temperature and uniform moisture content. One of the boundaries, which is impervious to moisture transfer, is put in contact with a heater. The other boundary is put in contact with the dry surrounding air, thus resulting in a convective boundary condition for both the temperature and the moisture content. The lateral surface of the cylinder is also supposed to be impervious to mass transfer, but heat losses at this boundary are taken into account through a convective boundary condition. The linear system of equations proposed by Luikov (1966), for the modeling of such physical problem involving the drying of a capillary porous media, can be written in dimensionless form as (Luikov, 1966, Mikhailov and Özisik, 1984, Cotta, 1993, Ribeiro, 1993, Ribeiro and Lobo, 1998):

$$\frac{\partial\theta(R,Z,\tau)}{\partial\tau}=\alpha\frac{\partial^2\theta(R,Z,\tau)}{\partial Z^2}-\beta\frac{\partial^2\phi(R,Z,\tau)}{\partial Z^2}+\frac{r_a^2\alpha}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Z,\tau)}{\partial R}\right]-\frac{r_a^2\beta}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\phi(R,Z,\tau)}{\partial R}\right]$$

$$\frac{\partial\phi(R,Z,\tau)}{\partial\tau}=Lu\frac{\partial^2\phi(R,Z,\tau)}{\partial Z^2}-LuPn\frac{\partial^2\theta(R,Z,\tau)}{\partial Z^2}+ \\ +\frac{r_a^2Lu}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\phi(R,Z,\tau)}{\partial R}\right]-\frac{r_a^2LuPn}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Z,\tau)}{\partial R}\right]$$

in 0<R<1 and 0<Z<1, for τ>0 (1.a,b)

$$\theta(R,Z,0)=0,\quad \phi(R,Z,0)=0,$$

for $\tau=0$, in 0<R<1 and 0<Z<1(1.c,d)

$$\frac{\partial\theta(0,Z,\tau)}{\partial R}=0, \quad \frac{\partial\phi(0,Z,\tau)}{\partial R}=0, \qquad \text{at } R=0 \text{ and } Z=0 \text{ for } \tau>0 \tag{1.e,f}$$

$$\frac{\partial\theta(1,Z,\tau)}{\partial R}-Bi_{qr}\,[1-\theta(1,Z,\tau)]=0, \qquad \text{at } R=1 \text{ and } 0<Z<1 \text{ for } \tau>0 \tag{1.g}$$

$$\frac{\partial\phi(1,Z,\tau)}{\partial R}=Pn\frac{\partial\theta(1,Z,\tau)}{\partial R}, \qquad \text{at } R=1 \text{ and } 0<Z<1 \text{ for } \tau>0 \tag{1.h}$$

$$\frac{\partial\theta(R,1,\tau)}{\partial Z}-Bi_q\,[1-\theta(R,1,\tau)]+(1-\varepsilon)KoLuBi_m\,[1-\phi(R,1,\tau)]=0,$$

$$\frac{\partial\phi(R,1,\tau)}{\partial Z}+Bi_m^{*}\phi(R,1,\tau)=Bi_m^{*}-PnBi_q\left[\theta(R,1,\tau)-1\right], \text{ at } Z=1 \text{ and } 0<R<1, \text{ for } \tau>0 \tag{1.i,j}$$

The various dimensionless groups appearing above are defined as

$$\theta(R,Z,\tau)=\frac{T(r,z,t)-T_0}{T_s-T_0}, \quad \phi(R,Z,\tau)=\frac{u_0-u(r,z,t)}{u_0-u_s}, \quad Q=\frac{ql}{k(T_s-T_0)}, \quad \tau=\frac{at}{l^2}, \tag{2.a-d}$$

$$Lu=\frac{a_m}{a}, \quad Pn=\delta\frac{T_s-T_0}{u_0-u_s}, \quad Bi_q=\frac{hl}{k}, \quad Bi_m=\frac{h_m l}{k_m}, \quad Ko=\frac{\lambda}{c}\frac{u_0-u_s}{T_s-T_0}, \quad Bi_{qr}=\frac{h_r R_0}{k}, \tag{2.e-j}$$

$$r_a=\frac{l}{r}, \qquad R=\frac{r}{R_0}, \qquad Z=\frac{z}{l}, \qquad Bi_m^{*}=Bi_m\,[1-(1-\varepsilon)\,Pn\,Ko\,Lu\,] \tag{2.k-n}$$

$$\alpha=1+\varepsilon KoLuPn, \qquad \beta=\varepsilon KoLu \tag{2.o,p}$$

where a is the thermal diffusivity of the porous medium, a_m is the moisture diffusivity in the porous medium, c is the specific heat of porous medium, h *and* h_r are the heat transfer coefficients at the top and lateral surfaces, respectively, h_m is the mass transfer coefficient, k is the thermal conductivity, k_m is the moisture conductivity, l is the thickness of porous medium, q is the prescribed heat flux, λ is the latent heat of evaporation of water, T_s is the temperature of the surrounding air, T_o is the uniform initial temperature in the medium, u_s is the moisture content of the surrounding air, u_o is the uniform initial moisture content in the medium, δ is the thermogradient coefficient and ε is the phase conversion factor. *Lu, Pn* and *Ko* denote the Luikov, Posnov and Kossovitch numbers, respectively.

3. APPROXIMATE FORMULATIONS

In order to derive the approximate formulations addressed in this work, we integrate equations (1) along the radial direction. By substituting into the resulting expressions the boundary conditions (1.e,f,g,h) and by using the following definitions of average temperature and moisture content at each cross section:

$$\hat{\theta}(Z,\tau)=2\int_0^1 R\,\theta(R,Z,\tau)\,dR \qquad \hat{\phi}(Z,\tau)=2\int_0^1 R\,\phi(R,Z,\tau)\,dR \tag{3.a,b}$$

we can write the formulation for the coupled heat and mass transfer problem under picture here as:

$$\frac{\partial\hat{\theta}(Z,\tau)}{\partial\tau}=\alpha\frac{\partial^2\hat{\theta}(Z,\tau)}{\partial Z^2}-\beta\frac{\partial^2\hat{\phi}(Z,\tau)}{\partial Z^2}-2r_a^2Bi_q\theta(1,Z,\tau)+2r_a^2Bi_q$$

$$\frac{\partial\hat{\phi}(Z,\tau)}{\partial\tau}=Lu\frac{\partial^2\hat{\phi}(Z,\tau)}{\partial Z^2}-LuPn\frac{\partial^2\hat{\theta}(Z,\tau)}{\partial Z^2} \qquad \text{in } 0<Z<1, \text{ for } \tau>0 \qquad (4.a,b)$$

$$\hat{\theta}(Z,0)=0, \quad \hat{\phi}(Z,0)=0, \qquad \text{for } \tau=0, \text{ in } 0<Z<1 \qquad (4.c,d)$$

$$\frac{\partial\hat{\theta}(0,\tau)}{\partial Z}=-Q, \quad \frac{\partial\hat{\phi}(0,\tau)}{\partial Z}=-PnQ, \qquad \text{at } Z=0 \text{ for } \tau>0 \qquad (4.e,f)$$

$$\frac{\partial\hat{\theta}(1,\tau)}{\partial Z}-Bi_q[1-\hat{\theta}(1,\tau)]+(1-\varepsilon)KoLuBi_m[1-\hat{\phi}(1,\tau)]=0, \quad \text{at } Z=1 \text{ for } \tau>0 \qquad (4.g)$$

$$\frac{\partial\hat{\phi}(1,\tau)}{\partial Z}+Bi_m^*\hat{\phi}(1,\tau)=Bi_m^*-PnBi_q\left[\hat{\theta}(1,\tau)-1\right], \qquad \text{at } Z=1 \text{ for } \tau>0 \qquad (4.h)$$

We note in equation (4.a) that such a formulation for the problem involves the temperature at the lateral surface of the body, besides the average temperature and average moisture content at each cross section. Two different approaches are used here to approximate $\theta(1,Z,\tau)$, as described next.

3.1 Lumped Approach

In the traditional lumped approach, gradients inside the body along the radial direction are completely neglected. Therefore, we can approximate the temperature at the lateral surface by the average temperature, that is,

$$\theta(1,Z,\tau)=\hat{\theta}(Z,\tau) \qquad (5)$$

With the use of the approximation given by equation (5), problem (4) only involves as dependent variables the average temperature and average moisture content at each cross section.

3.2 Coupled Integral Equations Approach

An improved approximate formulation can be obtained, by using Hermite integrals (Mennig et al., 1983) to write the temperature at the lateral surface of the body in terms of the average temperature at each cross section.

The so-called $H_{0,0}$ and $H_{1,1}$ Hermite approximations for the integration of a function f(x) are given respectively by

$$\int_a^b f(x)dx \cong \frac{b-a}{2}[f(a)+f(b)] \qquad (H_{0,0}) \qquad (6.a)$$

$$\int_a^b f(x)dx \cong \frac{b-a}{2}[f(a)+f(b)]+\frac{(b-a)^2}{12}[f'(a)-f'(b)] \qquad (H_{1,1}) \qquad (6.b)$$

which correspond to the trapezoidal and corrected-trapezoidal integration rules, respectively.

In this work, we use the $H_{1,1}$ expression in order to approximate the average temperature defined by equation (3.a) and the $H_{0,0}$ expression to approximate the integral of the temperature gradient along the radial direction, that is,

$$\hat{\theta}(Z,\tau)=2\int_0^1 R\theta(R,Z,\tau)dR \cong \theta(1,Z,\tau)+\frac{1}{6}\left[\frac{\partial}{\partial R}\left[R\theta(R,Z,\tau)\right]\Big|_{R=0}-\frac{\partial}{\partial R}\left[R\theta(R,Z,\tau)\right]\Big|_{R=1}\right]$$

$$\int_0^1\frac{\partial\theta(R,Z,\tau)}{\partial R}dR=\left[\theta(1,Z,\tau)-\theta(0,Z,\tau)\right]\cong\frac{1}{2}\left[\frac{\partial\theta(1,Z,\tau)}{\partial R}+\frac{\partial\theta(0,Z,\tau)}{\partial R}\right] \qquad (7.a,b)$$

Equations (7.a,b) are then solved in order to obtain the following expression for the temperature at the lateral surface of the body

$$\theta(1,Z,\tau)=\frac{4}{4+Bi_{qr}}\left(\hat{\theta}(Z,\tau)+\frac{Bi_{qr}}{4}\right) \qquad (8)$$

By substituting equation (8) into equation (4.a) and rearranging, we obtain a formulation for the present problem similar to that obtained with the lumped approach, except for the Biot number at the lateral surface. For the CIEA, a modified Biot number appears in the formulation. It is defined as:

$$Bi_{qr}^{*}=\frac{4Bi_{qr}}{4+Bi_{qr}} \qquad (9)$$

Therefore, we can write the approximate formulation for problem (1) as

$$\frac{\partial\hat{\theta}(Z,\tau)}{\partial\tau}=\alpha\frac{\partial^2\hat{\theta}(Z,\tau)}{\partial Z^2}-\beta\frac{\partial^2\hat{\phi}(Z,\tau)}{\partial Z^2}-\eta\,\hat{\theta}(Z,\tau)+\eta \qquad \text{in } 0<Z<1, \text{ for } \tau>0 \qquad (10.a)$$

$$\frac{\partial\hat{\phi}(Z,\tau)}{\partial\tau}=Lu\frac{\partial^2\hat{\phi}(Z,\tau)}{\partial Z^2}-LuPn\frac{\partial^2\hat{\theta}(Z,\tau)}{\partial Z^2} \qquad \text{in } 0<Z<1, \text{ for } \tau>0 \qquad (10.a)$$

$$\hat{\theta}(Z,0)=0,\quad \hat{\phi}(Z,0)=0, \qquad \text{for } \tau=0, \text{ in } 0<Z<1 \qquad (10.c,d)$$

$$\frac{\partial\hat{\theta}(0,\tau)}{\partial Z}=-Q,\qquad \frac{\partial\hat{\phi}(0,\tau)}{\partial Z}=-PnQ, \qquad \text{at } Z=0, \text{ for } \tau>0 \qquad (10.e,f)$$

$$\frac{\partial\hat{\theta}(1,\tau)}{\partial Z}-Bi_q\left[1-\hat{\theta}(1,\tau)\right]+(1-\varepsilon)KoLuBi_m\left[1-\hat{\phi}(1,\tau)\right]=0, \text{at } Z=1, \text{ for } \tau>0 \qquad (10.g)$$

$$\frac{\partial\hat{\phi}(1,\tau)}{\partial Z}+Bi_m^{*}\hat{\phi}(1,\tau)=Bi_m^{*}-PnBi_q\left[\hat{\theta}(1,\tau)-1\right], \qquad \text{at } Z=1, \text{ for } \tau>0 \qquad (10.h)$$

where, for the Lumped Approach we have

$$\eta=2r_a^2Bi_{qr}, \qquad (11.a)$$

and for the CIEA based on the $H_{1,1}$, $H_{0,0}$ approximations given by equations (7.a,b) we have:

$$\eta=2r_a^2Bi_{qr}^{*} \qquad (11.b)$$

Therefore, the use of the approximation for θ(1,Z,τ) obtained with the CIEA does not introduce any additional complexity into the formulation for the problem, as compared to the lumped approach. However, more accurate results are expected with the CIEA instead of the lumped approach, since the radial gradients in the body are now taken care of, through the approximation given by equation (8), instead of being neglected.

We note that the use of the $H_{0,0}$ approximation for the average temperature results in a formulation identical to that obtained with the lumped analysis and, hence, is not discussed here.

4. SOLUTION OF THE APPROXIMATE FORMULATION FOR THE PROBLEM

We use in this work the GITT for the solution of the one-dimensional problem (10) (Ribeiro et al., 1993, Cotta, 1993, Ribeiro and Cotta, 1995, Ribeiro and Lobo, 1998). In order to reduce the effects of the non-homogeneties on the convergence of the series solution, we filter problem (10) by writing its solution as

$$\widehat{\theta}(Z,\tau)=\widehat{\theta}_s(Z)+\widehat{\theta}_h(Z,\tau) \qquad \widehat{\phi}(Z,\tau)=\widehat{\phi}_s(Z)+\widehat{\phi}_h(Z,\tau) \tag{12.a,b}$$

where the following steady-state filter problem is used in the analysis:

$$\alpha\frac{d^2\widehat{\theta}_s(Z)}{dZ^2}=\beta\frac{d^2\widehat{\phi}_s(Z)}{dZ^2}+\eta\,\widehat{\theta}_s(Z)-\eta \qquad \text{in } 0<Z<1, \tag{13.a}$$

$$\frac{d^2\widehat{\phi}_s(Z)}{dZ^2}=Pn\frac{d^2\widehat{\theta}_s(Z)}{dZ^2} \qquad \text{in } 0<Z<1 \tag{13.b}$$

$$\frac{d\theta_s(0)}{\partial Z}=-Q, \quad \frac{d\phi_s(0)}{dZ}=-PnQ, \qquad \text{at } Z=0 \tag{13.c,d}$$

$$\frac{d\theta_s(1)}{dZ}+Bi_q\,\theta_s(1)=Bi_q-(1-\varepsilon)\,KoLuBi_m\,[1-\phi_s(1)], \qquad \text{at } Z=1 \tag{13.e}$$

$$\frac{d\phi_s(1)}{dZ}+Bi_m^*\phi_s(1)=Bi_m^*-PnBi_q\,[\theta_s(1)-1], \qquad \text{at } Z=1 \tag{13.f}$$

By substituting equations (12.a,b) into equations (10) and using equations (13), we obtain the homogeneous problem as:

$$\frac{\partial\widehat{\theta}_h(Z,\tau)}{\partial\tau}=\alpha\frac{\partial^2\widehat{\theta}_h(Z,\tau)}{\partial Z^2}-\beta\frac{\partial^2\widehat{\phi}_h(Z,\tau)}{\partial Z^2}-\eta\,\widehat{\theta}_h(Z,\tau) \qquad \text{in } 0<Z<1, \text{ for } \tau>0 \tag{14.a}$$

$$\frac{\partial\widehat{\phi}_h(Z,\tau)}{\partial\tau}=Lu\frac{\partial^2\widehat{\phi}_h(Z,\tau)}{\partial Z^2}-LuPn\frac{\partial^2\widehat{\theta}_h(Z,\tau)}{\partial Z^2} \qquad \text{in } 0<Z<1, \text{ for } \tau>0 \tag{14.b}$$

$$\widehat{\theta}_h(Z,0)=-\widehat{\theta}_s(Z), \quad \widehat{\phi}_h(Z,0)=-\widehat{\phi}_s(Z), \qquad \text{for } \tau=0, \text{ in } 0<Z<1 \tag{14.c,d}$$

$$\frac{\partial\widehat{\theta}_h(0,\tau)}{\partial Z}=\frac{\partial\widehat{\phi}_h(0,\tau)}{\partial Z}=0, \qquad \text{at } Z=0, \text{ for } \tau>0 \tag{14.e,f}$$

$$\frac{\partial\widehat{\theta}_h(1,\tau)}{\partial Z}+Bi_q\,\widehat{\theta}_h(1,\tau)=(1-\varepsilon)\,KoLuBi_m\widehat{\phi}_h(1,\tau), \qquad \text{at } Z=1, \text{ for } \tau>0 \tag{14.g}$$

$$\frac{\partial\widehat{\phi}_h(1,\tau)}{\partial Z}+Bi_m^*\widehat{\phi}_h(1,\tau)=-PnBi_q\widehat{\theta}_h(1,\tau), \qquad \text{at } Z=1, \text{ for } \tau>0 \tag{14.h}$$

The integral transform / inverse formula pairs for temperature and moisture content are defined, respectively, as

$$\bar{\bar{\theta}}_i(\tau)=\frac{1}{N_i^{1/2}}\int_0^1 \varphi_i(Z)\hat{\theta}_h(Z,\tau)dZ \tag{15.a}$$

$$\hat{\theta}_h(Z,\tau)=\sum_{i=1}^{\infty}\frac{1}{N_i^{1/2}}\varphi_i(Z)\bar{\bar{\theta}}_i(\tau) \tag{15.b}$$

and

$$\bar{\bar{\phi}}_i(\tau)=\frac{1}{P_i^{1/2}}\int_0^1 \Gamma_i(Z)\hat{\phi}_h(Z,\tau)dZ \tag{16.a}$$

$$\hat{\phi}_h(Z,\tau)=\sum_{i=1}^{\infty}\frac{1}{P_i^{1/2}}\Gamma_i(Z)\bar{\bar{\phi}}_i(\tau) \tag{16.b}$$

The eigenfunctions and normalization integral for temperature are given, respectively, by:

$$\varphi_i(Z)=cos(\gamma_i Z) \qquad N_i=\frac{1}{2}\left[1+\frac{Bi_q}{\gamma_i^2+Bi_q^2}\right] \tag{17.a,b}$$

where the eigenvalues are the positive roots of

$$(\gamma_i)tan(\gamma_i)=Bi_q \tag{17.c}$$

Similarly, we have for the moisture content

$$\Gamma_i(Z)=cos(\xi_i Z) \qquad P_i=\frac{1}{2}\left[1+\frac{Bi_m^*}{\xi_i^2+Bi_m^{*2}}\right] \tag{18.a,b}$$

with eigenvalues given by the positive roots of

$$(\xi_i)tan(\xi_i)=Bi_m^* \tag{18.c}$$

The integral transformation of problem (14) results on the following system of coupled ordinary differential equations:

$$\frac{d\bar{\bar{\theta}}_i(\tau)}{d\tau}+(\alpha\gamma_i^2+\eta)\bar{\bar{\theta}}_i(\tau)-\beta\sum_{j=1}^{\infty}A_{ij}^*\bar{\bar{\phi}}_j(\tau)=\frac{\varphi_i(1)}{N_i^{1/2}}KoLu\left[(Bi_m-\varepsilon Bi_q)\hat{\phi}_h(1,\tau)+\varepsilon PnBi_q\hat{\theta}_h(1,\tau)\right]$$

$$\frac{d\bar{\bar{\phi}}_i(\tau)}{d\tau}+Lu\,\xi_i^2\bar{\bar{\phi}}_i(\tau)-LuPn\sum_{j=1}^{\infty}B_{ij}^*\bar{\bar{\theta}}_j(\tau)=-\frac{\Gamma_i(1)}{P_i^{1/2}}LuPn\left[Bi_m^*\hat{\theta}_h(1,\tau)+(1-\varepsilon)KoLuBi_m\hat{\phi}_h(1,\tau)\right]$$

(19.a,b)

where

$$\hat{\theta}_h(1,\tau)=-\frac{1}{Bi_q}\left[\sum_{j=1}^{\infty}\bar{f}_j\frac{d\bar{\bar{\theta}}_j(\tau)}{d\tau}+Ko\sum_{j=1}^{\infty}\bar{f}_j^*\frac{d\bar{\bar{\phi}}_j(\tau)}{d\tau}\right]-\frac{\eta}{Bi_q}\sum_{j=1}^{\infty}\bar{f}_j\bar{\bar{\theta}}_j(\tau) \tag{19.c}$$

$$\hat{\phi}_h(1,\tau) = -\frac{1}{LuBi_m}\left[\sum_{j=1}^{\infty} \bar{f}_j^* \frac{d\bar{\hat{\phi}}_j(\tau)}{d\tau}\right] \tag{19.d}$$

$$\bar{f}_j = \frac{1}{N_j^{1/2}}\int_0^1 \varphi_j(Z)\, dZ \tag{19.e}$$

$$\bar{f}_j^* = \frac{1}{P_j^{1/2}}\int_0^1 \Gamma_j(Z)\, dZ \tag{19.f}$$

$$A_{ij}^* = \frac{1}{N_i^{1/2}P_j^{1/2}}\int_0^1 \varphi_i(Z)\Gamma_j(Z)dZ \tag{19.g}$$

$$B_{ij}^* = \frac{1}{N_j^{1/2}P_i^{1/2}}\int_0^1 \varphi_j(Z)\Gamma_i(Z)dZ \tag{19.h}$$

The solution for the system (19), truncated to a sufficiently large order to reach convergence, is obtained with the subroutine DIVPAG of the IMSL (1987). Then, the average temperature and moisture content in the region can be computed by using the following expressions, derived with an integral balance approach (Cotta, 1993):

$$\hat{\phi}_h(Z,\tau) = \hat{\phi}_h(1,\tau) - Pn\sum_{j=1}^{\infty} \bar{P}_j(Z)\frac{d\bar{\hat{\theta}}_j(\tau)}{d\tau} - \frac{\alpha}{Lu}\sum_{j=1}^{\infty} \bar{P}_j^*(Z)\frac{d\bar{\hat{\phi}}_j(\tau)}{d\tau} - \eta Pn\sum_{j=1}^{\infty} \bar{P}_j(Z)\bar{\hat{\theta}}_j(\tau) \tag{20.a}$$

$$\hat{\theta}_h(Z,\tau) = \hat{\theta}_h(1,\tau) + \frac{1}{\alpha}\{\beta\left[\hat{\phi}_h(Z,\tau) - \hat{\phi}_h(1,\tau)\right] - \sum_{j=1}^{\infty}\bar{P}_j(Z)\frac{d\bar{\hat{\theta}}_j(\tau)}{d\tau} - \eta\sum_{j=1}^{\infty}\bar{P}_j(Z)\bar{\hat{\theta}}_j(\tau)\} \tag{20.b}$$

where

$$\bar{P}_j = \frac{1}{N_j^{1/2}}\int_Z^1\int_0^{Z'} \varphi_j(Z'')\, dZ''dZ' \tag{20.c}$$

$$\bar{P}_j^* = \frac{1}{P_j^{1/2}}\int_Z^1\int_0^{Z'} \Gamma_j(Z'')\, dZ''dZ' \tag{20.d}$$

5. RESULTS AND DISCUSSION

We examine below the effects of the Biot number in the radial direction on the approximate solutions obtained *via* lumped and $H_{1,1}$,$H_{0,0}$ approaches. Figures 1.a,b show the results for $Bi_{qr} = 0, 1$ and 10, as well as the results obtained with a one-dimensional solution *via* GITT, for the average temperature and average moisture content at the position $Z=0$. Other parameters of importance for the analysis were taken as: $Lu=0.4$, $Pn=0.6$, $Ko=5.0$, $Bi_q=Bi_m=2.5$, $\varepsilon=0.2$ and $Q=0.9$.

As expected, the lumped and CIEA solutions are in perfect agreement with the 1D solution for $Bi_{qr}=0$. As Bi_{qr} increases, the average temperatures obtained with the approximate solutions tend to be smaller than that for the 1D solution, due to the lateral heat losses. The same behavior is observed for the average moisture content.

By comparing the 2D approximate solutions, we can notice that the average temperatures and the average moisture contents tend to be larger with the $H_{1,1}$,$H_{0,0}$ approximation than with the lumped approach. This is due to the fact that the modified Biot number given by equation (9) for the $H_{1,1}$,$H_{0,0}$ approximation, which takes into account the radial gradients, is smaller than the actual Biot number. This effect is more noticeable for larger Biot numbers in the radial direction, such as $Bi_{qr}=10$.

The solution *via* GITT of the two-dimensional problem given by equations (1) is now under implementation. A comparison of the two approximate solutions proposed here with such a 2D solution will establish their ranges of validity in terms of the radial Biot number, for different values of *Lu, Pn, Ko, Biq, Bim, Q* and *ε*.

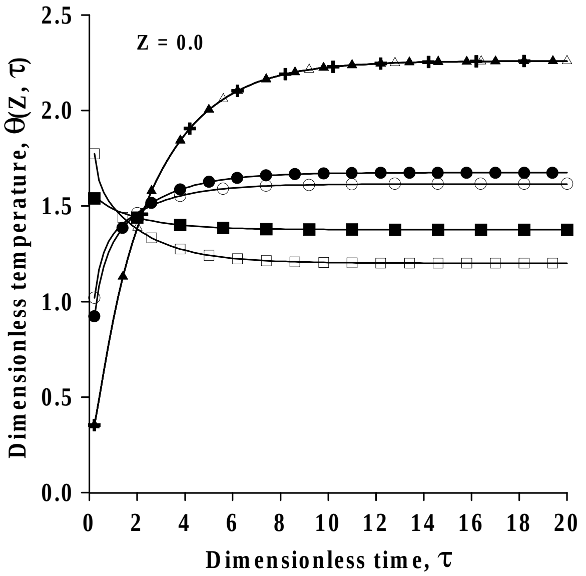

Figure 1.a Dimensionless temperature

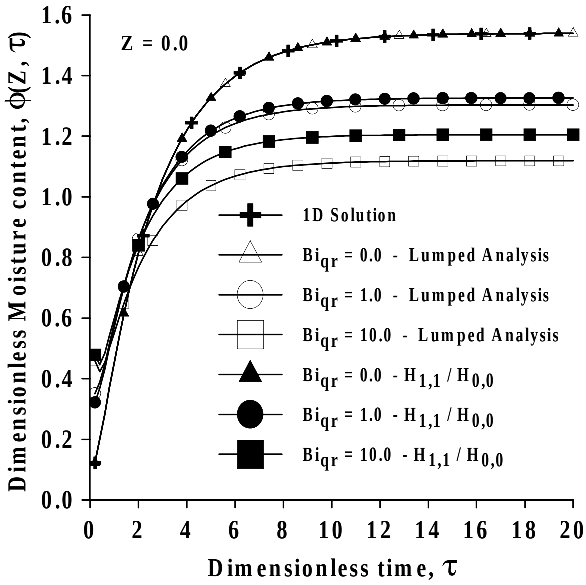

Figure 1.b Dimensionless moisture content

6. CONCLUSIONS

Two approximate solutions for the drying problem in a cylindrical capillary-porous medium were proposed in this paper. They involve the use of the traditional lumped approach and of the so-called Coupled Integral Equations Approach. In this last approach, Hermite $H_{1,1}$ and $H_{0,0}$ expressions were used to approximate the average temperature and the integral of the radial flux in each cross section, respectively. Similar formulations were obtained with these two approaches, resulting in almost identical required analytical and computational works for their implementation. As the radial Biot number increases, larger temperatures and moisture contents were observed with the CIEA solution, because the radial gradients were not neglected in this formulation as they were for the lumped analysis.

REFERENCES

Cheroto, S., Guigon, S. M. S., Ribeiro, J. W., Cotta, R. M., (1997), Lumped-Differential Formulations for Drying in Capillary Porous Media, Drying Technology, 15(3&4), 811-835.

Comini, G., Lewis, R. W., 1976, “A Numerical Solution of Two Dimensional Problems Involving Heat and Mass Transfer”, Int. J. Heat Mass and Transfer, vol. 19, pp. 1387-1392.

Cotta, R. M. and Mikhailov, M. D., 1997, Heat Conduction – Lumped Analysis, Integral Transforms and Symbolic Computation, Wiley, New York

Cotta, R. M., (1993), Integral Transforms in Computational Heat and Fluid Flow, CRC Press, Boca Raton, FL.

Cotta, R. M., Özisik, M. N., Mennig, J., 1990, Coupled Integral Equation Approach for Phase-Change Problem in Two-Region Finite Slab, J. Franklin Institute, 327, 225-234.

Duarte, J. B. F., Ribeiro, J. W., 1998, “Generalized Integral Transform Solutions of 3-D Luikov Problems”. In: Advanced Computational Methods in Heat Transfer V, Computational Mechanics Publications.

Guigon, S. M. S., Dantas, L. B., Scofano Neto, F., Cotta, R. M., 1999, Exact solution of Luikov’s equations for drying in capillary porous media, Hybrid Methods in Engineering, vol. 1, pp. 365-387.

IMSL Library, MATH/LIB, Houston, Texas, 1987.

Luikov, A. V., 1966, Heat and mass transfer in capillary-porous bodies, Pergamon Press, Oxford.

Lobo, P. D. C., Boles, M. A. and Ozisik, M. N., 1995, The effects of geometry for multi-region heat and mass transfer governed by the nonlinear Luikov system, Proceedings of the International Symposium on Two-Phase Flow Modeling and Experimentation, Rome, Italy, pp. 1243-1247.

Lobo, P. D. C., Mikhailov, M. D. and Özisik, M. N., 1987, On the complex eigenvalues of Luikov system of equations, Drying Technology 5(2), 273-286.

Mennig, J., Auerbach, T. and Halg, W., 1983, Two-point Hermite Approximations for the Solution of Linear Initial and Boundary Value Problems, Comput. Meth. Appl. Mech Eng., 39, 199-224.

Mikhailov, M. D. and Özisik, M. N., 1984, Unified Analysis and Solutions of Heat and Mass Diffusion, John Wiley, New York.

Ribeiro, J. W. and Lobo, P. D. C., 1998, In: The Integral Transform Method in Thermal and Fluids Science and Engineering, R. M. Cotta (Ed), Begell House Inc. Publishers, NY, pp. 125-143

Ribeiro, J. W. and Cotta, R. M., 1995, On the Solution of Nonlinear Drying Problems in Capillary Porous Media Through Integral Transformation of Luikov Equations, Int. J. Num. Meth. Eng. 38, 1001-1020.

Ribeiro, J. W., Cotta, R. M. and Mikhailov, M. D., 1993, Integral transform solution of Luikov's equations for heat and mass transfer in capillary porous media, Int. J. Heat and Mass Transfer, 36, 4467-4475.

Traiano, F., Cotta, R. M., Orlande, H. R. B., 1997, Improved Approximate Formulations for Anisotropic Heat Conduction, Int. Comm. Heat Mass Transfer, 24, 869-878.