SIMULTANEOUS HEAT AND MASS TRANSFER DURING THE DRYING OF PROLATE SPHEROIDAL SOLIDS

Antonio G. B. de Lima - E-mail: gilson@dem.ufpb.br
Departamento de Engenharia Mecânica, CCT, Universidade Federal da Paraíba (UFPB), CEP 58109-970, Cx. Postal 10069, Campina Grande-PB, Brasil.
Silvia A. Nebra - E-mail: sanebra@fem.unicamp.br
Departamento de Energia, FEM, Universidade Estadual de Campinas (UNICAMP), CEP13083-970, Cx. Postal 6122,Campinas-SP, Brasil.

Abstract. *Drying is a simultaneous heat and moisture transport problem. Heat and mass transfer in solids may be described by the energy and mass equations, in conjunction with the initial and boundary conditions. If solid properties, including the diffusion coefficient, are invariant, its possible to treat both heat and mass transfer in a similar manner. This work presents an investigation of the simultaneous heat and mass transfer during the drying of solids with prolate spheroidal shape. A two-dimensional diffusional model applied to a prolate spheroid is presented considering the liquid diffusion as the only mechanism of moisture transport and allowing convective and evaporative boundary conditions at the surface of the solid. The resulting equations are solved numerically using the finite-volume method. To validate the numerical model, results of the mean moisture content and center temperature obtained were compared with experimental data of wheat grain drying and good agreement was obtained. The convective and diffusion coefficients were obtained by fitting the model to experimental data, minimizing the sum of square residuals, in successive trials. The diffusion phenomenon inside a prolate spheroid represents to a higher degree of precision the problem under consideration in comparison to the current approach which uses spherical or cylindrical geometry.*

Keywords: D*rying, Mass transfer, Ellipsoidal geometry, Wheat, Numerical*

1. INTRODUCTION

The removal of moisture from biological products during drying processes has been asubject of study for a long time.

A great number of researchers working in drying process of individual particles has proposed mathematical models of the process using heat and moisture differential equations, and numerical solution technique to solve these equations simultaneously (Young, 1969; Mikhailov & Shishedjiev, 1975; Fortes, 1978; Fortes & Okos, 1981; Fortes *et al.*, 1981; Kameoka *et al.*, 1986; Sokhansanj, 1987; Parti, 1993; Oliveira *et al.*, 1995; Fasina & Sokhansanj, 1995; Irudayaraj & Wu, 1996; Jumah & Mujumdar, 1996; Cheroto *et al.*, 1997)

Many biological products have shape approximately ellipsoidal, and in particular, prolate spheroidal. For example one has rice, wheat, banana, orange, silkworm cocoon, and so on. The problem of heat and mass transfer diffusion on a prolate spheroid is of great interest because, it represents a high degree of precision of the problem in comparison to the current approach which uses spherical or cylindrical geometry.

Heat and mass transfer in spheroidal bodies may be described by energy and mass equations, in conjunction with initial and boundary conditions. If the solid properties, including the diffusion coefficient are invariant, it is possible to treat both heat and mass transfer in a similar manner. This is a relevant formulation and it is unnecessary to repeat the complex derivation for each different body. Recently, theoretical studies of mass diffusion through prolate spheroids has received much attention in the literature considering constant and convective boundary conditions (Lima *et al.*, 1997; Lima & Nebra, 1999a; Lima & Nebra, 1999b; Lima & Nebra, 2000).

The objective of this study is to predict numerically in the two dimensional case, the simultaneous heat and mass transfer during drying of prolate spheroidal solids, using the finite-volume method.

2. MATHEMATICAL MODELLING

For non-steady state, the diffusion equation in any coordinate system is given in short form by:

$$\frac{\partial(\psi\Phi)}{\partial t} = \nabla.\left(\Gamma^{\Phi}\nabla\Phi\right) + \Phi''' \qquad (1)$$

where Φ is the potential, Γ^{Φ} is the transport property, ψ is a thermal property and t is the time.

Consider a prolate spheroid of dimension L_1 and L_2 pictured in Fig. 1. To simplify the problem under consideration, the following assumptions were made:

- change of volume of the body is neglected;
- thermophysical properties are variables
- the body is axi-symmetrical around z-axis;
- the moisture content and temperature field are considered symmetric around z-axis all the time;
- the phenomenon occurs under evaporative and convective boundary conditions including heating of vapor produced at the surface.
- no energy and mass generation occurs.

In many physical problems it is best to use an appropriate orthogonal coordinate system ξ, η, ζ instead of the Cartesian coordinates x, y, z. In the case of a body with ellipsoidal geometric shape, an adequate coordinate system is the prolate spheroidal coordinate system. The relation between Cartesian and prolate spheroidal coordinate systems are given by, Magnus *et al.*(1966):

$$x = L\sqrt{\left(1-\xi^2\right)\left(\eta^2-1\right)}\,\zeta\,, \qquad y = L\sqrt{\left(1-\xi^2\right)\left(\eta^2-1\right)}\sqrt{\left(1-\zeta^2\right)}\,, \qquad z = L\xi\eta \qquad (2)$$

where $\xi=\cosh\mu$, $\eta=\cos\phi$, $\zeta=\cos\omega$ and $L=(L_2^2 - L_1^2)^{1/2}$. In these equations, μ, ϕ, ω are prolate spheroidal coordinates, η, ξ,ζ, angular ($\perp$x), radial and angular ($\perp$z) coordinates, and L_1 and L_2 are the ellipse major and minor semi-axis (see Fig. 1).

Then, utilizing the metric coefficients, the variables ξ, η and ζ, the differentiation's rules, the symmetry around the z-axis, $\partial/\partial\omega=0 \Rightarrow \partial/\partial\zeta=0$ and using the assumptions presented, a mathematical model was derived in the prolate spheroidal coordinate system, and is presented below:

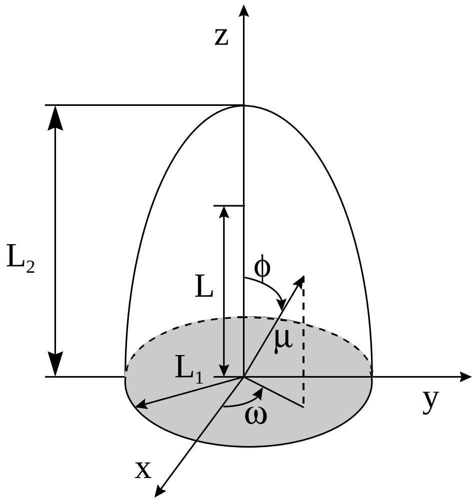

Figure 1- Geometrical parameters of a prolate spheroidal solid

2.1 Mass transfer

To the mass transfer process Φ correspond to moisture content and is denoted by M, $\Gamma^{\Phi} = \rho D$ (density times diffusion coefficient) and $\psi = \rho$ (density). Then Eq. (1) may be re-written as follows:

$$\frac{\partial M}{\partial t} = \left[\frac{1}{L^2(\xi^2-\eta^2)}\frac{\partial}{\partial \xi}\left(D(\xi^2-1)\frac{\partial M}{\partial \xi}\right)\right] + \left[\frac{1}{L^2(\xi^2-\eta^2)}\frac{\partial}{\partial \eta}\left(D(1-\eta^2)\frac{\partial M}{\partial \eta}\right)\right] \qquad (3)$$

with the following boundary conditions:

- Free surface. The mass diffusive flux is equal to the mass convective flux at the surface of the solid.

$$\frac{D}{L}\sqrt{\frac{(\xi^2-1)}{(\xi^2-\eta^2)}}\frac{\partial M}{\partial \xi}\bigg|_{\xi=\xi_f} + h_m\left[M(\xi,\eta,t) - M_e\right]\Big|_{\xi=\xi_f} = 0 \text{ with } \xi_f=L_2/L \text{ in the surface.} \qquad (4)$$

where h_m is the mass transfer coefficient, M_e is the equilibrium moisture content and subscript f refers to the surface of the solid.

- Planes of symmetry. The angular and radial gradients of moisture content are equal to zero at the planes of symmetry.

$$\frac{\partial M(\xi;1;t)}{\partial \eta} = 0, \qquad \frac{\partial M(\xi;0;t)}{\partial \eta} = 0, \qquad \frac{\partial M(1;\eta;t)}{\partial \xi} = 0 \qquad (5)$$

- Constant initial conditions in the interior of the solid.

$$M(\xi;\eta;0)=M_o \qquad (6)$$

The average moisture content $\overline{M}$ of the body is:

$$\overline{M} = \frac{1}{V}\int_V M dV \tag{7}$$

and can be calculated as follows:

$$\overline{M} = \frac{1}{\int_0^1 \int_1^{\frac{L_2}{L}} \left(\xi^2 - \eta^2\right) d\xi d\eta} \int_0^1 \int_1^{\frac{L_2}{L}} M(\xi,\eta)\left(\xi^2 - \eta^2\right) d\xi d\eta \tag{8}$$

In these equations, V is the total volume in the domain considered for the new coordinate system, calculated according to Magnus *et al.* (1966).

2.2 Heat transfer

In the heat transfer process, Φ is the temperature θ, $\Gamma^{\Phi} = k$ is the thermal conductivity and $\psi = \rho c_p$ (density times specific heat). In this manner, the transient heat conduction equation is given by:

$$\frac{\partial \theta}{\partial t} = \left[\frac{1}{L^2\left(\xi^2 - \eta^2\right)}\frac{\partial}{\partial \xi}\left(\left(\xi^2 - 1\right)\alpha\frac{\partial \theta}{\partial \xi}\right)\right] + \left[\frac{1}{L^2\left(\xi^2 - \eta^2\right)}\frac{\partial}{\partial \eta}\left(\left(1 - \eta^2\right)\alpha\frac{\partial \theta}{\partial \eta}\right)\right] \tag{9}$$

where $\alpha = k/(\rho c_p)$ is the thermal diffusity.

The boundary conditions are:

- Free surface. The heat convective flux supplied to the body surface equals to the heat diffusive flux plus the energy necessary to evaporate the liquid water and to heat the vapor produced at the surface of the prolate spheroid from surface temperature to the air drying temperature,

$$-\frac{k}{L}\sqrt{\frac{\left(\xi^2 - 1\right)}{\left(\xi^2 - \eta^2\right)}}\frac{\partial \theta}{\partial \xi}\Bigg|_{\xi=\xi_f} = h_c\left[\theta_e - \theta(\xi,\eta,t)\right]\Big|_{\xi=\xi_f} + \frac{\rho_s V}{S}\frac{\partial \overline{M}}{\partial t}\left[h^*_{fg} + c_v\left(\theta_e - \theta(\xi,\eta,t)\right)\right]\Big|_{\xi=\xi_f} \tag{10}$$

In this equation h^*_{fg} is the evaporating heat of the product, S is surface area, h_c is the convective heat transfer coefficient, ρ_s is the dry solid density and θ_e is the equilibrium temperature of the solid.

- Planes of symmetry. The angular and radial gradients of temperature are equal to zero at the planes of symmetry,

$$\frac{\partial \theta(\xi;1;t)}{\partial \eta} = 0 \qquad \frac{\partial \theta(\xi;0;t)}{\partial \eta} = 0 \qquad \frac{\partial \theta(1;\eta;t)}{\partial \xi} = 0 \tag{11}$$

- Constant initial condition in the interior of the solid

$$\theta(\xi;\ \eta;\ 0) = \theta_o \tag{12}$$

The average temperature $\bar{\theta}$ of the body during diffusion phenomenon is calculated as follows:

$$\bar{\theta} = \frac{1}{V}\int_V \theta dV \tag{13}$$

Various numerical methods have been used to solve the problem of transient diffusion, such as, finite-difference, finite-element, boundary element and finite-volume methods. In particular, in this work, the numerical method used was the finite-volume method. In the simulation of diffusion phenomenon in prolate spheroids a certain smaller domain was utilized, due to the symmetry of the body. A schema of the physical domain considered is illustrated in Fig. 2, where the nodal points are presented. In this figure, Φ'' refers to the flux of Φ per unit of area.

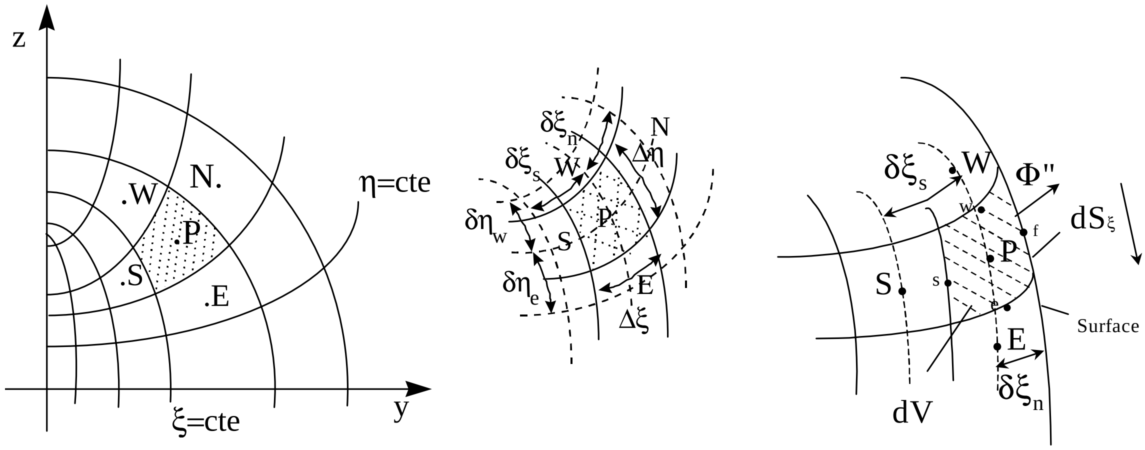

Figure 2- Geometrical configuration of the physical problem.

Considering the following dimensionless parameters,

$$\Phi^* = \frac{\Phi - \Phi_e}{\Phi_o - \Phi_e}\,, \qquad \eta^* = \eta\,, \qquad \xi^* = \xi\,, \qquad t^* = \frac{\Gamma^{\Phi} t}{L^2}\,, \qquad V^* = \frac{V}{L^3} \tag{14}$$

the Eqs. (3) – (13) may be re-written in the dimensionless form.

Assuming fully implicit formulation, all terms are estimated in $t^*+\Delta t^*$, by integrating Eqs. (3) and (9) (on the dimensionless form) in the control volume presented in Fig. 2, that correspond to the internal points of the domain, and also in the dimensionless time (t^*). Eq. (3) and (9) were discretized by a finite-volume method utilizing practice B (nodal points in the center of control-volume) in a grid of uniform size (Patankar, 1980; Maliska, 1995).

The discretization equation is given by:

$$A_P\Phi_P^* = A_E\Phi_E^* + A_W\Phi_W^* + A_N\Phi_N^* + A_S\Phi_S^* + A_P^0\Phi_P^0 + b \tag{15}$$

With known values of potential from the immediately preceding solution of Eq. (15), the set of equations was solved iteratively using Gauss-Seidel method. The calculation starts with the given initial condition and stops when the following convergence criteria were satisfied at each point of the computational domain:

$$\left|\Phi^{*k+1}-\Phi^{*k}\right|\le 10^{-8}, \qquad \frac{\sum\left|A_{nb}\right|}{\left|A_P\right|}\le 1 \quad \text{for all equations} \tag{16}$$

where k represents the k th iteration in each time step and nb represents neighbor points. In addition, we can quote the positivity of all coefficients A_{nb}. A numerical grid with 20x20 points was used. Others details about the numerical procedure may be found in Lima *et al.* (1997), Lima & Nebra (1997), Lima & Nebra (1999a), Lima & Nebra (1999b), Lima (1999) and Lima & Nebra (2000).

3. RESULTS AND DISCUSSIONS

A computational code utilizing the Microsoft Fortran Power Station and called SPREROIDIFF was written to solve the set of equations generated during the numerical formulation. As an application and to validate the numerical model, numerical results were compared with experimental data of moisture content and temperature obtained during the drying of wheat kernels given by Fortes *et al.* (1981). The initial dimensions and physical properties of the wheat and air and the diffusion and mass transfer coefficients are given below.

The dimensions and mean density of the moist wheat are given by Brooker *et al.* (1992). The air drying conditions are: temperature T_a=87,8°C; relative humidity UR_a= 5,6 % and air velocity v_a=1,71 m/s. The initial temperature of the product is θ_P= 26°C. The heat transfer coefficient was obtained considering the grain as a sphere with the same volume of the ellipsoid according with Fortes *et al.* (1981).

For diffusion coefficient it was used an equation proposed by Fioreze (1986) that considered the grain like sphere. In order it is necessary to modify this equation to the present work that consider the particles like ellipsoidal. For successive trials the diffusion and mass transfer coefficients were obtained. The fit to be represented in the equation of diffusion coefficient by the constant 0.0001508.

* **Dimensions and physical properties of the grain and air drying conditions**

L_2= 0.0032760 m; L_1= 0.0015748 m; M_o= 0.2110 (d. b.); M_e= 0.0165 (d. b.)

$$D = 0.0001508\,\overline{M}^{(2.8554.10^{-5}T_a+1.6432)}\, e^{[(0.4113T_a-30.2634)\overline{M}+(-0.022776T_a-9.7271)]}\ \text{m}^2/\text{s}$$

k_p= 0.1170+0,00113[100$\overline{M}$/(1+$\overline{M}$)] W/(m.K)

c_P=1.394+0.0409[100$\overline{M}$/(1+$\overline{M}$)] kJ/(kg.K); c_v=1.919849 kJ/(kgK)

ρ=769.0 kg/m³ ; ρ_s=1265.0 kg/m³

$$h^*_{fg} = 2502.2 - 2.39\theta_f\,[1+2.5348e^{(-23.628\,\overline{M})}]\ \text{kJ/kg}$$

h_c=129.413 W/(m² K); h_m=9.0.10^{-7} m/s;

Figure 3 illustrates the comparison between the average moisture content obtained numerically and the one given by Fortes *et al.* (1981). As may be observed there exists satisfactory agreement between the results.

Figure 4 shows the comparison of the center temperature obtained numerically with the given by Fortes *et al.* (1981). As may be observed there exists almost complete concordance between the results.

The least square error and standard deviation for the average moisture were 0.003980 (d.b.) and 0.000829 (d.b.), respectively. For the center temperature of the grain were obtained 0.035800 and 0.006400, respectively.

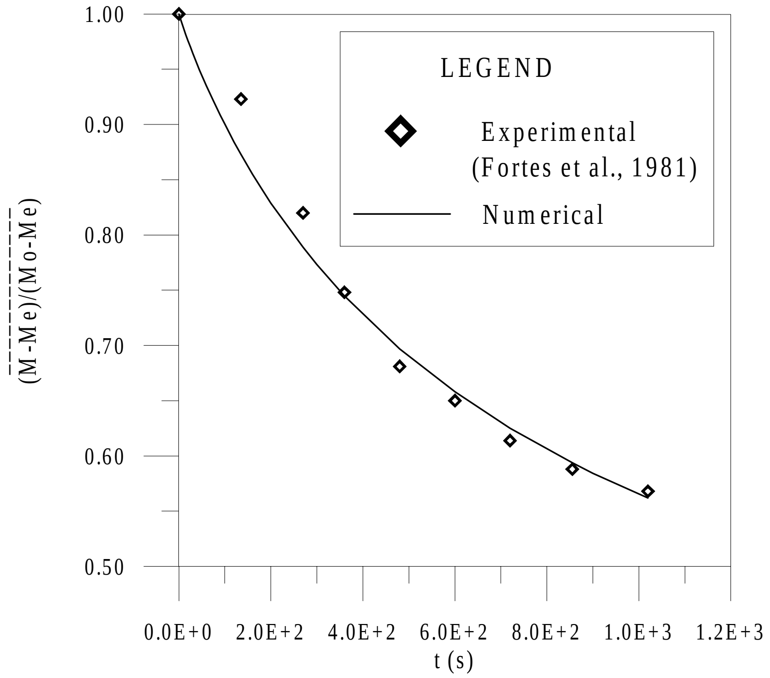

Figure 3 - Comparison between predicted and experimental values of the average moisture content of wheat grain during the drying process

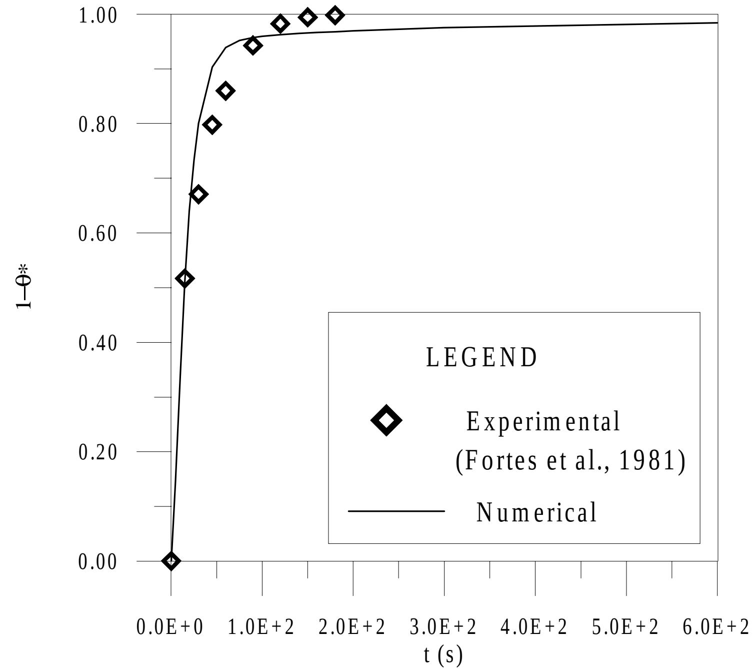

Figure 4 - Comparison between predicted and experimental values of the center temperature of wheat during the drying process, wre $\theta^{*}=(\theta_e-\theta)/(\theta_e-\theta_o)$

Due to the good agreement obtained, we can say that the model is satisfactory to predict the simultaneous heat and mass diffusion phenomenon inside a prolate spheroidal solid with particular reference to the drying process.

4. CONCLUSIONS

A general fully numerical method for the solution of diffusion equation has been developed and applied to the simultaneous heat and mass diffusion phenomenon in prolate spheroidal solids. The method uses a system of prolate spheroidal co-ordinates. With the improved treatment of the diffusion equation, the method quickly achieves convergence in each iteration in the non-steady numerical simulation. The effect of singularity in the symmetry points of the spheroid has been minimized using a regular grid. Satisfactory prediction of average moisture content and center temperature inside the solid was obtained.

This work shows that the methodology may applied for other bodies, change the aspect ratio only. This is a facilities this methodology in comparison with that used for diffusion problem in sphere and cylinder. In these cases, it is necessary different diffusion equations applied to each geometry specified.

REFERENCES

Brooker, D. B., Bakker-Arkema, F.W., Hall, C.W., 1992, Drying and storage of grains and oilseeds. AVI Book, New York.

Cheroto, S.; Silva Guigon, S. M.; Ribeiro, J. W. & Cotta, R. M., 1997, Lumped-differential formulation for drying in capillary porous media, Drying Technology, vol. 15, n. 3-4, pp. 811-835.

Fasina, O. O., Sokhansanj, S. & Tyler, R. T., 1997, Optimizing the drying and cooling of alfalfa pellets. Proceedings of the Inter-American Drying Conference (IADC), July 15-18, Itu, vol. B, pp. 530-537.

Fioreze, Romeu The intermittent drying agricultural crops with particular reference to energy requeriments. Ph. D. Thesis, Granfield Institute of Technology, Silsoe College, 1986.

Fortes; M., 1978, A non-equilibrium thermodynamics approach to transport phenomena in capillary-porous media with esperial reference to drying of grains and foods. Ph.D. Thesis, Faculty of Purdue University, United States of America.

Fortes, M. & Okos, M. R., 1981, Non-equilibrium thermodynamics approach to heat and mass transfer in corn kernels, Transactions of the ASAE, vol. 24, pp. 761-769.

Fortes, M., Okos, M.R. & Barret Jr., J. R., 1981, Heat and mass transfer analysis of intra-kernel wheat drying and rewetting, Journal Agricultural Engineering Research, vol. 26, pp.109-125.

Irudayara, J. & Wu, Y., 1996, Analysis and application of Luikov's heat, mass and pressure transfer model to a capillary porous media, Drying Technology, vol. 14, n. 3-4, pp. 803-824.

Jumah, R. Y. & Mujumdar, A.S., 1996, A mathematical model for constant and intermittent batch drying of grains in a novel rotating jet spouted bed. Drying Technology, vol. 14, n. 3-4, pp. 765-802.

Kameoka, T.; Hosokawa, A. & Morishima, H., 1986, Simulation of heat and mass transfer during through-drying process of rough rice. Drying of solids: recent international developments. pp. 271-278. John Wiley & Sons, New York, Lima, A. G. B., 1999, Diffusion phenomenon in prolate spheroidal solids. Case studied: Drying of banana. Doctor Thesis, State University of Campinas, Campinas, Brazil. (In Portuguese).

Lima, A. G. B. & Nebra, S. A., 1997, Influência de parâmetros geométricos na cinética de secagem de elipsóides sólidos, Anais do XXV Congresso Brasileiro de Sistemas Particulados (XXV ENEMP), Outubro 19-22, São Carlos, Brasil, pp. 195-202.

Lima, A. G. B. & Nebra, S. A., 1999a, The finite-volume approach for the solution of the transient diffusion equation applied to prolate spheroidal solids. Proceedings of the 15th Brazilian Congress of Mechanical Engineering, November 22-26, Águas de Lindóia, Brazil. vol. 1. CD-ROM

Lima, A. G. B. & Nebra, S. A., 1999b, Heat conduction process modeling in prolate spheroids. Proceedings of the XX Iberian Latin-American Congress on Computational Methods in Engineering, November 3-5, São Paulo, Brazil, CD-ROM. pp.34.1-34.14

Lima, A. G. B. & Nebra, S. A., 2000, Theoretical analysis of the diffusion process inside prolate spheroidal solids. Drying Technology, vol. 18, n. 1-2, pp. 21-48.

Lima, A. G. B., Nebra, S. A. & Altemani, C. A. C., 1997, Simulation of the drying kinetics of the silkworm cocoon considering diffusive mechanism in elliptical coordinate, Proceedings of the Inter-American Drying Conference (IADC), July 15-18, Itu, Brazil, vol. B, pp. 317-324.

Magnus, W.; Oberhettinger, F. & Soni, R. P., 1966, Formulas and theorems for the special functions of mathematical physics. Springer-Verlag,

Maliska, C.R., 1995, Transferência de calor e mecânica dos fluidos computacional, LTC, Rio de Janeiro.

Mikhailov, M. D. & Shishedjiev, B. K, 1975, Temperature and moisture distributions during contact drying of a moist porous sheet. International Journal Heat and Mass Transfer, vol. 18, pp. 15-24.

Oliveira, L. S., Franca, A. S. & Haghighi, K., 1995, An adaptive approach to finite element modeling of drying problems. Drying Technology, vol. 13, n. 5-7, pp. 1167-1185.

Parti, M., 1993, Selection of mathematical models for drying grain in thin-layers. Journal of Agricultural Engineering Research, vol. 54, pp. 339-352.

Patankar, S.V., 1980, Numerical heat transfer and fluid flow, Hemisphere Publishing Corporation, New York.

Sokhansanj, S., 1987, Improved heat and mass transfer models to predict grain quality. Drying Technology, vol. 5, n. 4, pp. 511-525.

Young, J. H., 1969, Simultaneous heat and mass transfer in porous higroscopic solid. Transactions of the ASAE, vol. 12, n. 5, pp. 720-725.