Higher Order Finite Difference and Analytical Approach in Drying

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Abstract. Symbolic computation, rule based and functional programming are applied with the generalised integral transform method, GITT, to analytical and numerically solve a Luikov non-linear problem, using automatic manipulation over analytical formulae. A filtering technique split up the general problem to accelerate the convergence and the homogeneous problem is transformed in a initial value problem and solved by Mathematica packages. A Table exhibits excellent numerical results for the dimensionless temperature and moisture distributions that are compared with values obtained by the higher order finite difference method, HOFDM.

Keywords: Symbolic computation, Integral transform, Luikov problem

1 Introduction

New advances in computer science as interactive and intelligent programming techniques and hybrid analytical-numerical methods when combined with symbolic computation are named hybrid computation and they open new possibilities to advanced codes with less programming effort that permit to store some analytical formulae and to analyse physical behaviour. The physical analysis of heat and mass transfer in capillary porous media constitutes a very important problem in thermal sciences. The *classical integral transform technique*, CITT (Cotta et al., 1991) was extended to treat a variety of heat and mass diffusion and convective-diffusion problems as presented in Özisik and Murray (1974) and Mikhailov (1975). Cotta (1986, 1992), Cotta et al. (1986) and Luikov (1966) extended the formalisms of the CITT, generating a new approach named by GITT *generalised integral transform technique* . The GITT formalisms efficiently solved a lot of more general and a priori non-transformable linear and non-linear problems. Additionally formalisms of computer science as symbolic declarative, functional programming and rule based programming (Wolfram, 1998, Gray, 1998) are being used to create intelligent and automatic manipulation of analytical advanced mathematical formalisms. In the present stage, it is possible to present benchmark numerical results to compare with classical solutions obtained by purely numerical methods such as finite differences and finite elements.

2 ANALISYS

Let us consider the non-linear Luikov drying problem version for the case 1D, with boundary condition of first and second kind, with variable termophysical properties that is analytically detailed in (Duarte, 1998). Without loss of generality and for application of GITT formalisms, this dimensionless version is algebraically rearranged and expressed as:

$$a_{q0}\ \rho\ c_q(\Theta_1(X,\tau))\frac{\partial\Theta_1(X,\tau)}{\partial\tau} = \frac{\partial}{\partial X}[(K_q(\Theta_1(X,\tau))+$$

$$+\epsilon(\Theta_2(X,\tau))\ \lambda a_m(\Theta_2(X,\tau))\ \rho\ \delta(\Theta_2(X,\tau)))\frac{\partial\Theta_1(X,\tau)}{\partial X}]+$$

$$-\frac{\partial}{\partial X}[\epsilon(\Theta_2(X,\tau))\ \lambda\ a_m(\Theta_2(X,\tau))(\frac{U_0-U_s}{T_s-T_0})\ \rho\ c_m\frac{\partial\Theta_2(X,\tau)}{\partial X}] \tag{1}$$

$$a_{q0}\ c_m\frac{\partial\Theta_2(X,\tau)}{\partial\tau} = \frac{\partial}{\partial X}[a_m(\Theta_2(X,\tau))\ c_m\frac{\partial\Theta_2(X,\tau)}{\partial X}]+$$

$$-\frac{\partial}{\partial X}[a_m(\Theta_2(X,\tau))\delta(\Theta_2(X,\tau))(\frac{T_s-T_0}{U_0-U_s})\frac{\partial\Theta_1(X,\tau)}{\partial X}] \tag{2}$$

the initial condition take the form:

$$\Theta_1(X,0) = \Theta_2(X,0) = 0\ \ ;\ \ 0 \le X \le 1 \tag{3}$$

and the boundary conditions:

$$\frac{\partial\Theta_1(X,\tau)}{\partial X}\Big|_{X=0} = 0;\quad \tau > 0 \tag{4}$$

$$\frac{\partial\Theta_2(X,\tau)}{\partial X}\Big|_{X=0} = 0;\quad \tau > 0 \tag{5}$$

$$\Theta_1(1,\tau) = 1\ ;\quad \tau > 0 \tag{6}$$

$$\Theta_2(1,\tau) = 1\ ;\quad \tau > 0 \tag{7}$$

Thermophysics parameters having temperature or moisture dependency assume the form

$$c_q\Theta_1(X,\tau) = c_{qz} + c\ \Theta_1(X,\tau) \tag{8}$$

$$K_q \Theta_1(X,\tau) = K_{qz} + K \ \Theta_1(X,\tau) \tag{9}$$

$$a_m \Theta_2(X,\tau) = a_{mz} + a \ \Theta_2(X,\tau) \tag{10}$$

$$\delta \Theta_2(X,\tau) = \delta_z + \delta_v \ \Theta_2(X,\tau) \tag{11}$$

$$\epsilon \Theta_2(X,\tau) = \epsilon_z + \epsilon_v \ \Theta_2(X,\tau) \tag{12}$$

The variable with index z stores the initial values used to non-linearities. Θ_1 and Θ_2 express the dimensionless temperature and moisture content distributions, the other parameters are presented in the literature (Duarte, 1998). To accelerate the convergence rate a powerful analytical filtering technique is applied in this stage (Duarte, 1998, Duarte & Ribeiro, 1997, Cotta and Mikhailov, 1997). So the focused problem is split up in two components, a steady-state, $\Theta_{\iota s}$, and a homogeneous, $\Theta_{\iota h}$, that results:

$$\Theta_\iota(X,\tau) = \Theta_{\iota s}(X) + \Theta_{\iota h}(X,\tau); \qquad \iota = 1,2 \tag{13}$$

The solution achieved for each steady-state equation is easily found:

$$\Theta_{1s}(X) = \Theta_{2s}(X) = 1 \tag{14}$$

The code analytically rearranges the equations and generates the homogeneous problems:

$$\begin{aligned} a_{qz}\ \rho\ c_q(1+\Theta_{1h}(X,\tau))\frac{\partial \Theta_{1h}(X,\tau)}{\partial \tau} &= \frac{\partial}{\partial X}[K_q(1+\Theta_{1h}(X,\tau))+ \\ &+\lambda \rho a_m(1+\Theta_{2h}(X,\tau))\delta(1+\Theta_{2h}(X,\tau))\epsilon(1+\Theta_{2h}(X,\tau))\frac{\partial \Theta_{1h}(X,\tau)}{\partial X}]+ \\ &-\frac{\partial}{\partial X}[C1 c_m \lambda \rho a_m(1+\Theta_{2h}(X,\tau))\epsilon(1+\Theta_{2h}(X,\tau))\frac{\partial \Theta_{2h}(X,\tau)}{\partial X}] \end{aligned} \tag{15}$$

and

$$\begin{aligned} a_{qz}\ c_m\ \frac{\partial \Theta_{2h}(X,\tau)}{\partial \tau} &= \frac{\partial}{\partial X}[c_m\ a_m(1+\Theta_{2h}(X,\tau))\frac{\partial \Theta_{2h}(X,\tau)}{\partial X}]+ \\ &-\frac{\partial}{\partial X}[C2\, a_m(1+\Theta_{2h}(X,\tau))\ \delta(1+\Theta_{2h}(X,\tau))\frac{\partial \Theta_{1h}(X,\tau)}{\partial X}] \end{aligned} \tag{16}$$

with initial conditions:

$$\Theta_{1h}(X,0) = -\Theta_{1s}(X); \qquad 0 \le X \le 1; \tag{17}$$

$$\Theta_{2h}(X,0) = -\Theta_{2s}(X); \qquad 0 \le X \le 1; \tag{18}$$

and homogeneous boundary conditions:

$$\frac{\partial \Theta_{1h}(X,\tau)}{\partial X}\Big|_{X=0} = 0; \quad \tau > 0 \tag{19}$$

$$\frac{\partial \Theta_{2h}(X,\tau)}{\partial X}\Big|_{X=0} = 0; \quad \tau > 0 \tag{20}$$

and:

$$\Theta_{1h}(1,\tau) = 0; \qquad \tau > 0 \tag{21}$$

$$\Theta_{2h}(1,\tau) = 0; \qquad \tau > 0 \tag{22}$$

where

$$C1 = \frac{U_0 - U_s}{T_s - T_0} \tag{23}$$

$$C2 = \frac{T_s - T_0}{U_0 - U_s} \tag{24}$$

According to GITT formalisms (Duarte and Ribeiro, 1997) let us assume the auxiliary associated problem of classical Sturm-Liouville type, necessary to solve the homogeneous problem:

$$\frac{\partial \Psi_i(X)}{\partial X} + \mu_i^2 \Psi_i(X) = 0; \qquad 0 < X < 1 \tag{25}$$

$$\frac{\partial \Psi_i(X)}{\partial X}\Big|_{X=0} = 0 \tag{26}$$

$$\Psi_i(X)\Big|_{X=1} = 0 \tag{27}$$

$$\frac{\partial \Gamma_j(X)}{\partial X} + \lambda_j^2 \Gamma_j(X) = 0; \qquad 0 < X < 1 \tag{28}$$

$$\frac{\partial \Gamma_j(X)}{\partial X}\Big|_{X=0} = 0 \tag{29}$$

$$\Gamma_j(1)\Big|_{X=1} = 0 \tag{30}$$

The next step is to construct the inverse-transform pairs as follows
Transform

$$\overline{\Theta}_{1i}(\tau) = \int_{X=0}^{X=1} \frac{\Psi_i(X)}{N_i^{\frac{1}{2}}} \Theta_{1h}(X,\tau)\, dX \tag{31}$$

Inverse

$$\Theta_{1h}(X,\tau) = \sum_{i=1}^{\infty} \frac{\Psi_i(X)}{N_i^{\frac{1}{2}}} \overline{\Theta}_{1i}(\tau) \tag{32}$$

and,

Transform

$$\overline{\Theta}_{2j}(\tau) = \int_{X=0}^{X=1} \frac{\Gamma_j(X)}{N_j^{*\frac{1}{2}}} \Theta_{2h}(X,\tau)\; dX \tag{33}$$

Inverse

$$\Theta_{2h}(X,\tau) = \sum_{j=1}^{\infty} \frac{\Gamma_j(X)}{N_j^{*\frac{1}{2}}} \overline{\Theta}_{2j}(\tau) \tag{34}$$

and from the normalisation integrals results:

$$N_i = \int_{X=0}^{X=1} \Psi_i^2(X)\; dX \tag{35}$$

$$N_j^* = \int_{X=0}^{X=1} \Gamma_j^2(X)\; dX \tag{36}$$

According the GITT formalisms the solution of the auxiliary problems are:

$$\Psi_i(X) = cos\; \mu_i X \tag{37}$$

$$\Gamma_j(X) = cos\; \sigma_j X \tag{38}$$

the correspondent eigenvalues are easily computed and expressed as:

$$\mu_i = (i - \frac{1}{2})\pi; \qquad i = 1, 2, 3.... \tag{39}$$

$$\sigma_j = (j - \frac{1}{2})\pi; \qquad j = 1, 2, 3.... \tag{40}$$

In this stage functional programming and rule based programming are used to automatically execute some extensive and tedious analytical manipulations. This methodology is more efficient for advanced modelling and permits to reduce months of job requested in the stages of human handling the analytical calculus and algorithm implementation. Without loss of generality, the next step is to reduce the PDE system to an ODE system, eliminating the spatial dependency and minimising the computational effort (Duarte, 1998):

$$\frac{a_{qz}\, c\, \rho}{N_i} \sum_{s=1}^{\infty} \sum_{m=1}^{\infty} (\int_{X=0}^{X=1} \Psi_i(X)\, \Psi_m(X)\, \Psi_s(X)\; dX)\, \overline{\Theta}_{1m}(\tau)\, \frac{\partial \overline{\Theta}_{1s}(\tau)}{\partial \tau} +$$

$$+a_{qz}\, c\, \rho\, \frac{\partial \overline{\Theta}_{1i}(\tau)}{\partial \tau} + a_{qz}\, c_{qz}\, \rho\, \frac{\partial \overline{\Theta}_{1i}(\tau)}{\partial \tau} =$$

$$-\frac{K}{N_i} \sum_{k=1}^{\infty}\sum_{m=1}^{\infty} Int1\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{1m}(\tau) - \frac{3\, a\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) +$$

$$-\frac{2\, a_{mz}\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}\, \overline{\Theta}_{2j}(\tau) - \frac{2\, a\, \delta_z\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) +$$

$$-\frac{a_{mz}\, \delta_z\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) - \frac{2\, a\, \delta_v\, \epsilon_z\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) +$$

$$-\frac{a_{mz}\, \delta_v\, \epsilon_z\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) - \frac{a\, \delta_z\, \epsilon_z\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int2\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) +$$

$$-\frac{3\, a\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int3\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}^2_{2j}(\tau) - \frac{a_{mz}\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int3\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}^2_{2j}(\tau) +$$

$$-\frac{a\, \delta_z\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int3\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}^2_{2j}(\tau) - \frac{a\, \delta_v\, \epsilon_z\, \lambda\, \rho}{N_i\, N_j^{*\frac{1}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int3\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau) +$$

$$-\frac{a\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j^{*\frac{3}{2}}} \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} Int4\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}^3_{2j}(\tau) + \frac{2\, a\, C1\, c_m\, \epsilon_v\, \lambda\, \rho}{N_i^{\frac{1}{2}}\, N_j} \sum_{j=1}^{\infty}\sum_{l=1}^{\infty} Int5\, \overline{\Theta}_{2j}(\tau)\, \overline{\Theta}_{2l}(\tau) +$$

$$+\frac{a_{mz}\, C1\, c_m\, \epsilon_v\, \lambda\, \rho}{N_i^{\frac{1}{2}}\, N_j} \sum_{j=1}^{\infty}\sum_{l=1}^{\infty} Int5\, \overline{\Theta}_{2j}(\tau)\, \overline{\Theta}_{2l}(\tau) +$$

$$+\frac{a\, C1\, c_m\, \epsilon_z\, \lambda\, \rho}{N_i^{\frac{1}{2}}\, N_j} \sum_{j=1}^{\infty}\sum_{l=1}^{\infty} Int5\, \overline{\Theta}_{2j}(\tau)\, \overline{\Theta}_{2l}(\tau) +$$

$$+\frac{a\, C1\, c_m\, \epsilon_v\, \lambda\, \rho}{N_i^{\frac{1}{2}}\, N_j^{*\frac{3}{2}}} \sum_{j=1}^{\infty}\sum_{l=1}^{\infty} Int6\, \overline{\Theta}^2_{2j}(\tau)\, \overline{\Theta}_{2l}(\tau) +$$

$$-\frac{6\, a\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j} \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty} Int7\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau)\, \overline{\Theta}_{2kk}(\tau) +$$

$$-\frac{2\, a_{mz}\, \delta_v\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j} \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty} Int7\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau)\, \overline{\Theta}_{2kk}(\tau) +$$

$$-\frac{2\, a\, \delta_z\, \epsilon_v\, \lambda\, \rho}{N_i\, N_j} \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty} Int7\, \overline{\Theta}_{1k}(\tau)\, \overline{\Theta}_{2j}(\tau)\, \overline{\Theta}_{2kk}(\tau) +$$

$$-\frac{2\,a\,\delta_v\,\epsilon_z\,\lambda\,\rho}{N_i\,N_j}\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty}Int7\,\overline{\Theta}_{1k}(\tau)\,\overline{\Theta}_{2j}(\tau)\,\overline{\Theta}_{2kk}(\tau)+$$

$$-\frac{3\,a\,\delta v\,\epsilon v\,\lambda\,\rho}{N_i\,N_j^{*\frac{3}{2}}}\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty}Int8\,\overline{\Theta}_{1k}(\tau)\,\overline{\Theta}_{2j}^2(\tau)\,\overline{\Theta}_{2kk}(\tau)+$$

$$-\frac{3\,a\,\delta_v\,\epsilon_v\,\lambda\,\rho}{N_i\,N_j^{*\frac{3}{2}}}\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty}Int9\,\overline{\Theta}_{1k}(\tau)\,\overline{\Theta}_{2j}(\tau)\,\overline{\Theta}_{2kk}^2(\tau)+$$

$$+\frac{2\,a\,C1\,c_m\,\epsilon_v\,\lambda\,\rho}{N_i^{\frac{1}{2}}\,N_j^{*\frac{3}{2}}}\sum_{l=1}^{\infty}\sum_{j=1}^{\infty}\sum_{kk=j+1}^{\infty}Int10\,\overline{\Theta}_{2l}(\tau)\,\overline{\Theta}_{2j}(\tau)\,\overline{\Theta}_{2kk}(\tau)+$$

$$-\frac{6a\delta_v\epsilon_v\lambda\rho}{N_iN_j^{*\frac{3}{2}}}\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{j1=j+1}^{\infty}\sum_{j2=j1+1}^{\infty}Int11\,\overline{\Theta}_{1k}(\tau)\,\overline{\Theta}_{2j}(\tau)\,\overline{\Theta}_{2j1}(\tau)\,\overline{\Theta}_{2j1}(\tau)+$$

$$+\frac{a\,C1\,c_m\,\epsilon_v\,\lambda\,\rho}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{j=1}^{\infty}Int12\,\mu_i^2\,\overline{\Theta}_{2j}(\tau)+$$

$$+\frac{a_{mz}\,C1\,c_m\,\epsilon_v\,\lambda\,\rho}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{j=1}^{\infty}Int12\,\mu_i^2\,\overline{\Theta}_{2j}(\tau)+\frac{a\,C1\,c_m\,\epsilon_z\,\lambda\,\rho}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{j=1}^{\infty}Int12\,\mu_i^2\,\overline{\Theta}_{2j}(\tau)+$$

$$+\frac{a_{mz}\,C1\,c_m\,\epsilon_z\,\lambda\,\rho}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{j=1}^{\infty}Int12\,\mu_i^2\,\overline{\Theta}_{2j}(\tau)-K\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)+$$

$$-K_{qz}\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)-a\,\delta_v\,\epsilon_v\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)+$$

$$-a_{mz}\,\delta_v\,\epsilon_v\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)-a\,\delta_z\,\epsilon_v\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)-a_{mz}\,\delta_z\,\epsilon_v\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)+$$

$$-a\,\delta_v\,\epsilon_z\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)-a_{mz}\,\delta_v\,\epsilon_z\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)+$$

$$-a\,\delta_z\,\epsilon_z\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau)-a_{mz}\,\delta_z\,\epsilon_z\,\lambda\,\rho\,\mu_i^2\,\overline{\Theta}_{1i}(\tau) \qquad (41)$$

The second governing equation is systematically generated in the code and has the final aspect:

$$a_{qz}\,c_m\,\frac{\partial\overline{\Theta}_{2j}}{\partial\tau}=\frac{2\,a\,C2\,\delta_v}{N_i^{\frac{1}{2}}\,N_m^*}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}Int13\,\overline{\Theta}_{1k}\,\overline{\Theta}_{2m}+\frac{a_{mz}\,C2\,\delta_v}{N_i^{\frac{1}{2}}\,N_j^*}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}Int13\,\overline{\Theta}_{1k}\,\overline{\Theta}_{2m}+$$

$$+\frac{a\,C2\,\delta_z}{N_i^{\frac{1}{2}}\,N_j^*}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}Int13\,\overline{\Theta}_{1k}\,\overline{\Theta}_{2m}-\frac{a\,c_m}{N_m^{*\frac{3}{2}}}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}Int14\,\overline{\Theta}_{2l}\,\overline{\Theta}_{2m}+$$

$$+\frac{a\,C2\,\delta_v}{N_i^{\frac{1}{2}}\,N_j^{*}}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty} Int15\,\overline{\Theta}_{1k}\,\overline{\Theta}_{2m}^2+\frac{2\,a\,C2\,\delta_v}{N_i^{\frac{1}{2}}\,N_j^{*\frac{3}{2}}}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\sum_{kk=1+m}^{\infty} Int16\,\overline{\Theta}_{1k}\,\overline{\Theta}_{2m}\,\overline{\Theta}_{2kk}^2+$$

$$+\frac{a\,C2\,\delta_v}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{i=1}^{\infty} Int17\,\sigma_j^2\,\overline{\Theta}_{1i}+\frac{a_{mz}\,C2\,\delta_v}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{i=1}^{\infty} Int17\,\sigma_j^2\,\overline{\Theta}_{1i}+\frac{a\,C2\,\delta_z}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{i=1}^{\infty} Int17\,\sigma_j^2\,\overline{\Theta}_{1i}+$$

$$+\frac{a_{mz}\,C2\,\delta_z}{N_i^{\frac{1}{2}}\,N_j^{*\frac{1}{2}}}\sum_{i=1}^{\infty} Int17\,\sigma_j^2\,\overline{\Theta}_{1i}-a\,c_m\,\sigma_j^2\,\overline{\Theta}_{2j}-a_{mz}\,c_m\,\sigma_j^2\,\overline{\Theta}_{2j} \tag{42}$$

The symbols "Int" were defined to express some analytical integrals originated from combinations of equations (37-38).

The transform initial conditions are easily obtained using the initial conditions from the homogeneous problem, see equations (17-18) and without lost of generality:

$$\overline{\Theta}_{1i}(0)=-\frac{sin\,\mu_i}{\sqrt{N_i}\,\mu_i} \tag{43}$$

$$\overline{\Theta}_{2j}(0)=-\frac{\sin\sigma_j}{\sqrt{N_j}\,\sigma_j} \tag{44}$$

At this stage we choose a prescribed error tolerance for this initial value problem and truncate the expansion series at a finite and sufficient order, N.

In the following section Table (1) is presented illustrating the numerical results for dimensionless temperature and moisture profiles obtained by GITT and HOFDM.

3 Results and Discussion

The variant of the non-linear Luikov problem was implemented using a Pentium processor with a 233 MHz of speed and 128Mb of RAM memory, under Windows 95 OS. The code implementation was made using a symbolic programming language, the software Mathematica® 3.01 (Wolfram, 1998), generating solutions with excellent precision and permitting graphical visualisation. The thermophysical parameters were collected from the literature (Duarte, 1998). The initial value transform problem was numerically solved using the NDSolve Mathematica function. The results were compared with the solution obtained by the high order finite difference method, HOFDM, presented in Duarte (1998).

Table (1) presents the excellent results computed for dimensionless temperature and moisture potentials, Θ_1 and Θ_2 using GITT, with increasing truncation order, from $N=3$ to $N=5$ and for the following dimensionless time values, $\tau=20$; $\tau=40$ and $\tau=70$ for temperature potential and $\tau=145$; $\tau=245$ and $\tau=440$ for moisture potential. It was possible to obtain only five terms on the eigenfunction series due to hardware limitations (memory and CPU time). Although the results are not fully numerically converged, in the physical aspects the temperature and mass distributions results agree with those obtained by HOFDM. Comparing both numerical results, it can be observed two to three converged digits.

It is observed an excellent agreement between GITT end HOFDM results, although for a critical comparison more terms in the GITT solution are necessary. In the results obtained for temperature and moisture distributions it is pointed out that according to thermophysical properties assumed, the massical inertial effect is four orders of magnitude higher than the thermal inertial effect and this behaviour requires more control on the numerical convergence that is more critical for short times. Consequently the temperature distribution develop faster than the mass distribution in the porous media, so it is expected that more terms are needed in the expansions series to achieve best results.

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REFERENCES

Campos Silva, J. B., Cotta, R. M. and Aparecido, J. B., **"Analytical Solutions to Simultaneously Developing Flow Inside Parallel-Plates Channel"**, Int. J. Heat & Mass Transfer, V. 35, pp. 887-895, 1992.

Cotta, R. M., **"Diffusion in Media with Prescribed Moving Boundaries: - Application to Metals Oxidation at High Temperatures"** , Anais do II Congresso Latino-Americano de Transferência de Calor e Massa, V. 1, 502-513, São Paulo, 1986.

Cotta, R. M. and Mikhailov, M. D.,**"Heat Conduction - Lumped Analysis, Integral Transforms, Computation"**, John Wiley, 1997.

Cotta, R. M. and Özisik, M. N., **"Laminar Forced Convection in Ducts with Periodic Variations of Inlet Temperature"**, Int. J. Heat & Mass Transfer, V. 29, 1495-1501, 1986.

Duarte, J. B. F., Abordagem Analítico-Numérica em Meios Multidimensionais Capilares Porosos, Ph.D. Thesis, U.F.C., Fortaleza, Brazil, 1998.

Duarte, J. B. F. and Ribeiro, J. W., **"Integral Transform Solution of Three-Dimensional Luikov Problem"**, EUROTHERM 53, Mons, Belgium, October, 1997.

Gray, J. W., **"Mastering Mathematica: Programming Methods and Applications"** , Academic Press, 1998.

Luikov, A. V., **"Heat and Mass Transfer in Capillary Porous Bodies"**, Pergamon Press, Oxford, 1966.

Mikhailov, M. D., **"On the Solution of the Heat Equation with Time Dependent Coefficient"**, Int. J. Heat and Mass Transfer, V. 18, pp. 344-345, 1975.

Özisik, M. N. and Murray, R.L., **"On the Solution of Linear Diffusion Problems with Variable Boundary Conditions Parameters"**, J. Heat Transfer, V. 96, pp. 48-51, 1974.

Wolfram, S., **"The Mathematica Book"**, Third Edition, Cambridge University Press, 1998.

Table 1: Dimensionless temperature and moisture distribution for the variant of the Luikov problem, variable thermophysical properties (Duarte, 1998) using GITT and HOFDM

$\Theta_1(X;\tau)$				
$\tau = 0,20$				
X\N	3	4	5	HOFDM
0,0	0,3058	0,3042	0,3044	0,3025
0,6	0,6144	0,6132	0,6133	0,6097
1,0	1,000	1,000	1,000	1,000
$\tau = 0,40$				
X\N	3	4	5	HOFDM
0,0	0,6342	0,6338	0,6335	0,6297
0,6	0,7901	0,7899	0,7901	0,7862
1,0	1,000	1,000	1,000	1,000
$\tau = 0,70$				
X\N	3	4	5	HOFDM
0,0	0,8574	0,8577	0,8573	0,8530
0,6	0,9149	0,9152	0,9155	0,9120
1,0	1,000	1,000	1,000	1,000
$\Theta_2(X;\tau)$				
$\tau = 145,0$				
X\N	3	4	5	HOFDM
0,0	0,3729	0,3763	0,3744	0,3711
0,6	0,5737	0,5771	0,5784	0,5749
1,0	1,000	1,000	1,000	1,000
$\tau = 245,0$				
X\N	3	4	5	HOFDM
0,0	0,5725	0,5740	0,5732	0,5710
0,6	0,7182	0,7197	0,7202	0,7181
1,0	1,000	1,000	1,000	1,000
$\tau = 440,0$				
X\N	3	4	5	HOFDM
0,0	0,7817	0,7820	0,7817	0,7807
0,6	0,8622	0,8625	0,8626	0,8618
1,0	1,000	1,000	1,000	1,000