HIGHER ORDER LUMPED ANALYSIS OF TRANSIENT HEAT TRANSFER IN A NUCLEAR FUEL ROD

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Abstract. *Transient heat transfer in a nuclear fuel rod is analysed by an improved lumped parameter approach. The circunferential symmetry is assumed with the heat transfer through the gap modelled by a heat transfer coefficient. Higher order ($H_{1,1}$) Hermite approximation for integration is used to obtain the average temperatures in the radial direction. A significant improvement over the classical lumped parameter formulation has been achieved. The proposed fuel rod heat conduction model can be used in stability analysis of BWR, simplified model of PWR or real-time simulator of nuclear power plants.*

Keywords *Transient Heat Conduction, Lumped Parameter Analysis, Hermite Approximation, Nuclear Reactor Thermohydraulics*

1. Introduction

The lumped parameter approach has been widely used in the thermohydraulic analysis of nuclear reactors. As in the analysis of other complex thermal systems, this classical approach is extremely useful and sometimes even mandatory when a simplified formulation of the transient heat conduction is seeked. Together with the neutron point kinetics model, the lumped parameter approach for fuel rod heat conduction is essential in the simplified models of pressurized water reactors (PWRs) and in real-time simulators of nuclear power plants (Zanobetti, 1989).

Recently, the dynamics of chaotic instabilites in boiling water reactors has aroused increased interests. In such studies, the lumped parameter approach has been the unique option in the fuel dynamics models. For example, Rao et al. (1995) performed a linear stability analysis in the frequency domain to study the basic mechanism of coupled nuclear-thermal instabilities in a boiling channel, using a one-node lumped parameter

model for the fuel dynamics. Even with the simplest fuel dynamics model, they found that the fuel-time constant was one of the parameters determing the density-wave instablity. Chang and Lahey (1997) used one-dimensional homogeneous equilibrium assumptions for diabatic two-phase flow, a one-node lumped parameter approach for heated wall dynamics, and neutron point kinetics for the consideration of nuclear feedback in a boiling water reactor (BWR) loop. They found that a boiling channel coupled with a riser could experience chaotic oscillations. Lin et al. (1998) found a strip of limit cycle oscillation of a nuclear-coupled boiling channel with a two-node lumped parameter model for the fuel dynamics, where one node was for the fuel and the other for the cladding.

As an inherent limitation of the lumped parameter approach, moderate to low temperature gradients within the region are assumed, which govern the accuracy of such approximate formulations through the associated problem parameters. As a rule of thumb, the classical lumped parameter approach, where uniform temperature is assumed within the region, is in general restricted to problems with Biot number less than 0.1. In most nuclear reactor engineering problems, the Biot number is much higher. In other words, the moderate to low temperature gradient assumption is not reasonable in such applications, thus more accurate approach should be adopted. Cotta and Mikhailov (1997) proposed a systematic formalism to provide improved lumped parameter formulation for steady and transient heat conduction problems based on Hermite approximation for integrals that define averaged temperatures and heat flux integrals. This approach has been shown to be efficient in a great variety of pratical applications (Aparecido and Cotta (1989), Scofano Neto and Cotta (1993), Cheoroto et al. (1997)).

Regis et. al. (2000) have proposed an improved lumped parameter model for the transient heat conduction in a nuclear fuel rod where the one-sided corrected trapezoidal rule ($H_{1,0}$ approximation) was employed in the averaged temperature integral for the fuel, and plain trapezoidal rule ($H_{0,0}$ approximation) was used in the averaged temperature integral for the cladding and the heat fluxes. In this work, we present a higher order lumped parameter analysis that uses the two-sided corrected trapezoidal rule ($H_{1,1}$ approximation) in the averaged temperature integrals for both fuel and cladding, and plain trapezoidal rule ($H_{0,0}$ approximation) is used in the averaged heat fluxes. The proposed lumped parameter model can be used in stability analysis of boiling water systems, in simplified model of pressurized water reactors, or in real-time nuclear power plant simulators.

2. Analysis

We consider the transient heat conduction in a cylindrical nuclear fuel rod such as those that can be found in pressurized water reactors (PWRs), boiling water reactors (BWRs), or liquid metal cooled fast breeder reactors (LMFBRs). In order to illustrate the main idea of improved lumped analysis, we simplify the problem by assuming axisymmetry of temperature distribution and neglecting the axial heat conduction term and the spatial variation of the heat generation across the fuel rod. The thermal conductivities are assumed to be independent of temperature, while this assumption is not essential in the lumped parameter approach as can be seen further in the analysis. The heat generation in the cladding is neglected. With the above assumptions, we have the following governing equations with appropriate boundary and initial conditions for one-dimensional transient heat conduction in a fuel rod,

$$\rho_f c_f \frac{\partial T_f}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\Big(k_f r \frac{\partial T_f}{\partial r}\Big) + g(t), \qquad 0 < r < r_{fo} \tag{1a}$$

$$T_f(r,0) = T_{f0}(r), \tag{1b}$$

$$-k_f \beta_f \frac{\partial T_f}{\partial r}\Big|_{r=r_{fo}} = hg\big(T_f(r_{fo},t) - T_c(r_{ci},t)\big), \tag{1c}$$

$$\rho_c c_c \frac{\partial T_c}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\Big(k_c r \frac{\partial T_c}{\partial r}\Big), \qquad r_{ci} < r < r_{co} \tag{2a}$$

$$T_c(r,0) = T_{c0}(r), \tag{2b}$$

$$-k_c \beta_c \frac{\partial T_c}{\partial r}\Big|_{r=r_{ci}} = hg\big(T_f(r_{fo},t) - T_c(r_{ci},t)\big), \tag{2c}$$

$$-k_c \frac{\partial T_c}{\partial r}\Big|_{r=r_{co}} = h(T_f(r_{co},t) - T_m), \tag{2d}$$

where T_f and T_c are temperatures in fuel and cladding, ρ_f, ρ_c their densities, c_f, c_c the specific heats, k_f, k_c the respective thermal conductivities, g the volumetric heat generation in fuel, h_g the heat transfer coefficient for the gap, and h the heat transfer coefficient between the cladding and the coolant, while r_{fo}, r_{ci}, r_{co}, $\beta_f = r_{fo}/r_{ci}$ and $\beta_c = r_{ci}/r_{fo}$ are geometric parameters of the fuel rod.

Introducing the following dimensionless variables,

$$\theta_f = \frac{T_f - T_m}{T_{ref} - T_m}, \qquad \theta_c = \frac{T_c - T_m}{T_{ref} - T_m},$$

$$R = \frac{r}{r_{co}}, \qquad \tau = \frac{k_f t}{\rho_f c_f r_{co}^2},$$

$$K = \frac{k_f \rho_c c_c}{k_c \rho_f c_f}, \qquad Bi = \frac{h r_{co}}{k_c},$$

$$Bi_{gc} = \frac{h_g r_{co}}{\beta_c k_c}, \qquad Bi_{gf} = \frac{h_g r_{co}}{\beta_f k_f},$$

$$G = \frac{r_{co}^2 g}{k_f (T_{ref} - T_m)},$$

we get the following dimensionless equations,

$$\frac{\partial \theta_f}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\Big(R \frac{\partial \theta_f}{\partial R}\Big) + G(\tau), \qquad 0 < R < R_{fo} \tag{3a}$$

$$\theta_f(R,0) = \theta_{f0}(R), \tag{3b}$$

$$-\frac{\partial \theta_f}{\partial R}|_{R=R_{fo}} = Bi_{gf}\big(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)\big), \tag{3c}$$

$$\frac{\partial \theta_c}{\partial \tau} = \frac{K}{R}\frac{\partial}{\partial R}(R\frac{\partial \theta_c}{\partial R}), \qquad R_{ci} < R < R_{co} \tag{4a}$$

$$\theta_c(r,0) = \theta_{c0}(r), \tag{4b}$$

$$-\frac{\partial \theta_c}{\partial R}|_{R=R_{ci}} = Bi_{gc}(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)), \tag{4c}$$

$$-\frac{\partial\theta_f}{\partial R}|_{R=1} = Bi\theta_c(1,\tau). \tag{4d}$$

The corresponding spatially averaged dimensionless temperatures are defined by

$$\theta_{f,av}(\tau) = \frac{\int_0^{R_{fo}} 2\pi R\theta_f(R,\tau)dR}{\pi R_{fo}^2} = \frac{2}{R_{fo}^2}\int_0^{R_{fo}} R\theta_f(R,\tau)dR, \tag{5a}$$

$$\theta_{c,av}(\tau) = \frac{\int_{R_{ci}}^1 2\pi R\theta_c(R,\tau)dR}{\pi(1-R_{ci}^2)} = \frac{2}{(1-R_{ci}^2)}\int_{R_{ci}}^1 R\theta_c(R,\tau)dR. \tag{5b}$$

Operating Eq.(3a) by $(2/R_{fo}^2)\int_0^{R_{fo}} RdR$, and using the definition of average temperature, Eq.(5a), we get,

$$\frac{d\theta_{f,av}(\tau)}{d\tau} = \frac{2}{R_{fo}}\frac{\partial\theta_f}{\partial R}\Big|_{R=R_{fo}} + G(\tau). \tag{6}$$

Similarly, we operate Eq.(4a) by by $(2/(1-R_{ci}^2))\int_{R_{ci}}^1 RdR$, using the definition of average temperature, Eq.(5b), which yields,

$$\frac{d\theta_{c,av}(\tau)}{d\tau} = -\frac{2K}{(1-R_{ci}^2)}\Big(R_{ci}\frac{\partial\theta_c}{\partial R}\Big|_{R=R_{ci}} - \frac{\partial\theta_c}{\partial R}\Big|_{R=1}\Big). \tag{7}$$

Now, using the boundary conditions Eqs.(3c, 4c, 4d), we get

$$\frac{d\theta_{f,av}(\tau)}{d\tau} = -\frac{2Bi_{gf}}{R_{fo}}\big(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)\big) + G(\tau), \tag{8}$$

$$\frac{d\theta_{c,av}(\tau)}{d\tau} = -\frac{2K}{(1-R_{ci}^2)}\Big[(Bi_{gc}R_{ci}\big(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)\big) - Bi\theta_c(1,\tau)\Big]. \tag{9}$$

The equations (8) and (9) are equivalent integro-differential formulation of the original mathematical model, with no approximation involved. Supposing that the temperature gradients are sufficiently smooth over the whole spatial solution domain, the classical lumped system analysis (CLSA) is based on assuming that the boundary potentials can be reasonably well approximated by the averaged potentials, as

$$\theta_f(R_{fo},\tau) \cong \theta_{f,av}(\tau),$$
$$\theta_c(R_{ci},\tau) \cong \theta_{c,av}(\tau),$$
$$\theta_c(1,\tau) \cong \theta_{c,av}(\tau),$$

which leads to the simplified lumped formulation,

$$\frac{d\theta_{f,av}(\tau)}{d\tau} = -\frac{2Bi_{gf}}{R_{fo}}\big(\theta_{f,av}(\tau) - \theta_{c,av}(\tau)\big) + G(\tau), \tag{10}$$

$$\frac{d\theta_{c,av}(\tau)}{d\tau} = \frac{2K}{(1-R_{ci}^2)}\Big[Bi_{gc}R_{ci}\big(\theta_{f,av}(\tau) - \theta_{c,av}(\tau)\big) - Bi\theta_{c,av}(\tau)\Big], \tag{11}$$

to be solved with the initial conditions for the averaged temperatures,

$$\theta_{f,av}(0) = \theta_{f,av0}, \tag{12a}$$

$$\theta_{c,av}(0) = \theta_{c,av0}. \tag{12b}$$

We now seek improved lumped-differential formulations, in an attempt to offer enhanced characteristic to the approximation path previously proposed. The basic idea is to provide a better relation between the boundary potentials and the averaged potentials, which are to be developed from Hermite-type approximations of the integrals that define the average temperatures and heat fluxes.

In this formulation, the two-sided corrected trapezoidal rule ($H_{1,1}$ approximation) is employed in the averaged temperature integrals for both fuel and cladding, and plain trapezoidal rule ($H_{0,0}$ approximation) is used in the averaged heat fluxes, in the following form:

$$\begin{aligned}\theta_{f,av}(\tau) =& \frac{2}{R_{fo}^2}\Big[\frac{R_{fo}}{2}(R\theta_f)\Big|_{R=0} + \frac{R_{fo}}{2}(R\theta_f)\Big|_{R=R_{fo}} \\ &+ \frac{R_{fo}^2}{12}\frac{\partial(R\theta_f)}{\partial R}\Big|_{R=0} - \frac{R_{fo}^2}{12}\frac{\partial(R\theta_f)}{\partial R}\Big|_{R=R_{fo}}\Big],\end{aligned} \tag{13}$$

$$\int_0^{R_{fo}} \frac{\partial\theta_f(R,\tau)}{\partial R}dR = \theta_f(R_{fo},\tau) - \theta_f(0,\tau) = \frac{1}{2}\Big(\frac{\partial\theta_f}{\partial R}\Big|_{R=0} + \frac{\partial\theta_f}{\partial R}\Big|_{R=R_{fo}}\Big), \tag{14}$$

$$\begin{aligned}\theta_{c,av}(\tau) =& \frac{2}{(1-R_{ci}^2)}\Big[\frac{1}{2}(R\theta_c)\Big|_{R=R_{ci}} + \frac{1}{2}(R\theta_c)\Big|_{R=1} \\ &+ \frac{1}{12}\frac{\partial R\theta_c}{\partial R}\Big|_{R=R_{ci}} - \frac{1}{12}\frac{\partial R\theta_c}{\partial R}\Big|_{R=1}\Big],\end{aligned} \tag{15}$$

$$\int_{R_{ci}}^1 \frac{\partial\theta_f(R,\tau)}{\partial R}dR = \theta_c(1,\tau) - \theta_c(R_{ci},\tau) = \frac{1}{2}\Big(\frac{\partial\theta_c}{\partial R}\Big|_{R=R_{ci}} + \frac{\partial\theta_c}{\partial R}\Big|_{R=1}\Big). \tag{16}$$

Using boundary conditions Eqs.(3c, 4c, 4d), the Eqs.(14, 16) become

$$\theta_{f,av}(\tau) = \frac{5}{6}\theta_f(R_{fo},\tau) + \frac{1}{6}\theta_f(0,\tau) + \frac{1}{6}(R_{fo})Bigf\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau), \tag{17}$$

$$\begin{aligned}\theta_{c,av}(\tau) = \frac{1}{1+R_{ci}}\Big[&R_{ci}\theta_c(R_{ci},\tau) + \theta_c(1,\tau) + \frac{1-R_{ci}}{6}\theta_c(R_{ci},\tau) \\ &-R_{ci}Bigc\big(\theta_f(R_{fo},\tau) - \theta_c(R_{ci},\tau)\big) + Bi\theta_c(1,t) - \theta_c(1,t)\Big].\end{aligned} \tag{18}$$

Eqs.(13,15,17,18) form a system of four linear algebraic equations for four unknowns, $\theta_f(0,\tau)$, $\theta_f(R_{fo},\tau)$, $\theta_c(R_{ci},\tau)$ and $\theta_c(1,\tau)$, that is solved to provide the seeked relations between boundary potentials and averaged potentials, besides an approximate relation for the central temperature, $\theta_f(0,\tau)$. These relations are then used in the equations (8) and (9) to close the two ordinary differential equations for the averaged temperatures, to be solved with the initial conditions (12a, b).

3. Results and Discussion

The proposed higher order lumped parameter model for one-dimensional transient heat conduction is solved numerically with a fourth order Runge-Kutta method, implemented in Fortran PowerStation 4.0. The solution is then compared with a finite difference solution of the original partial differential equations and with the classical lumped parameter model as described in Levy (1999).

Numerical results were obtained for typical parameter values encountered in nuclear reactor enginnering applications (Levy, 1999). The parameters used in the test cases are given in Tables 1 and 2.

Table 1. Fixed Parameters for Selected Test Cases

k_f	(W/m K)	4.1	k_c	(W/m K)	12.8
ρ_f	(kg/m^3)	10980.0	ρ_c	(kg/m^3)	6570.0
c_f	(J/kg^{o}C)	299.0	c_c	(J/kg^{o}C)	330.0
r_{fo}	(m)	0.00445	r_{ci}	(m)	0.00455
r_{co}	(m)	0.00503	h_g	(W/m^2 ^{o}C)	5670.0
g	(W/cm^3)	482.256			

Tabel 2. Heat Transfer Coefficients for Selected Test Cases

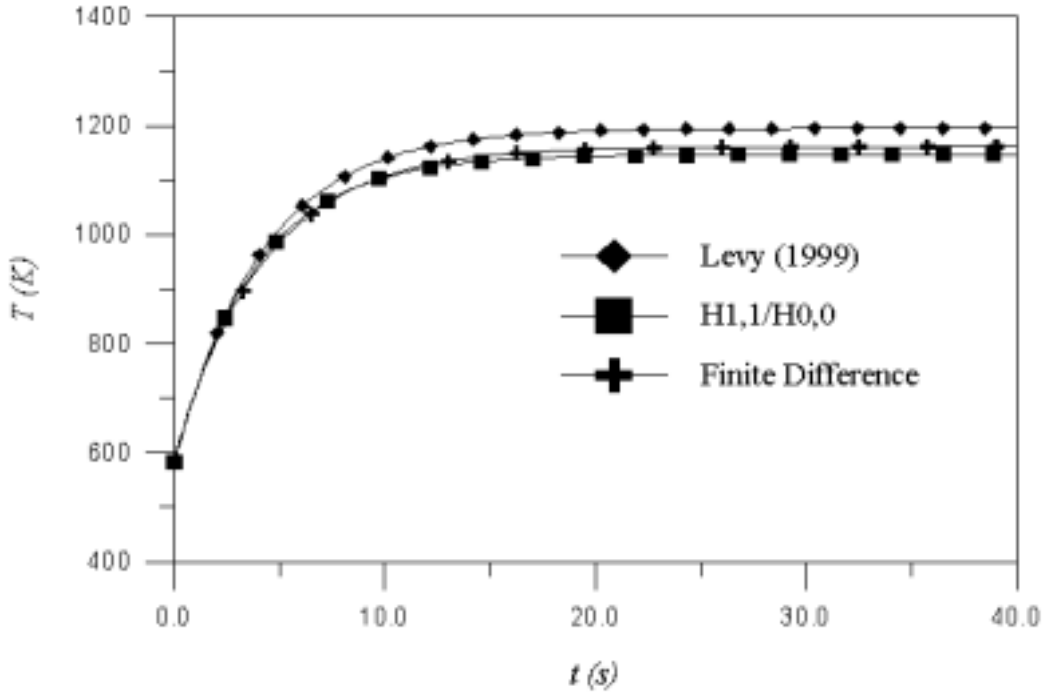
Case	h (W/m^2 ^{o}C)	Bi
1	10658.3	4.18838
2	21316.6	8.37676
3	31974.9	12.5651

In Table 1, we give the parameters used in all the three test cases. In Table 2, the cladding-coolant heat transfer coefficients are given for the three test cases.

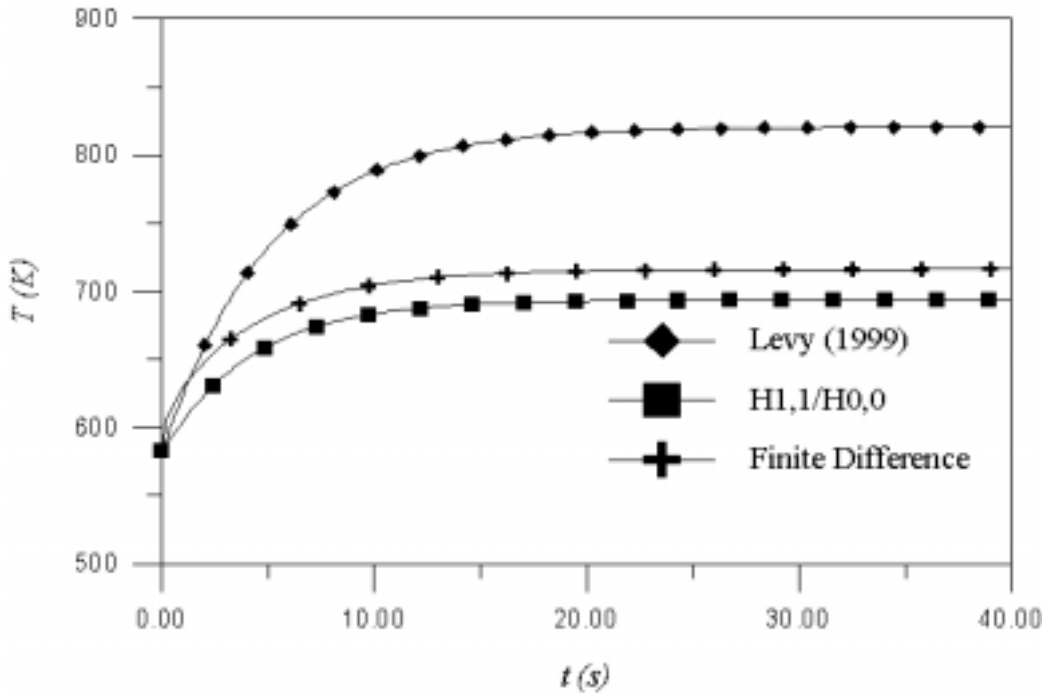
In Figure 1 it is shown the averaged fuel and cladding temperatures for case 1 ($Bi = 4.18838$) obtained by the improved and classical lumped formulations, compared with a finite difference solution with fully implicit scheme and central difference in spatial coordinate. The steady state solution obtained by the finite difference method agrees perfectly with the exact analytical solution. We can observe that the averaged fuel temperature obtained by the improved lumped parameter formulation is in excellent agreement with the finite difference solution, while the agreement for averaged cladding temperature is still quite reasonable. The cladding temperatures are shown in the lowermost set of curves in each graph. There is a significant improvement over the classical lumped parameter formulation also for the averaged cladding temperature. With increased Biot numbers, there is an improvement in the agreement between the improved lumped parameter formulation and the finite difference solution, as can be observed in Figure 2 for $Bi = 8.37676$ and in Figure 3 for $Bi = 12.5651$.

4. Conclusion

A higher order lumped parameter analysis is applied to the transient heat transfer in a nuclear reactor fuel rod to provide a simplified formulation that can be used in stability analysis of BWRs, simplified models of PWRs or real-time simulators of nuclear power plants. With application of $H_{1,1}$ and $H_{0,0}$ Hermite approximations for the averaged

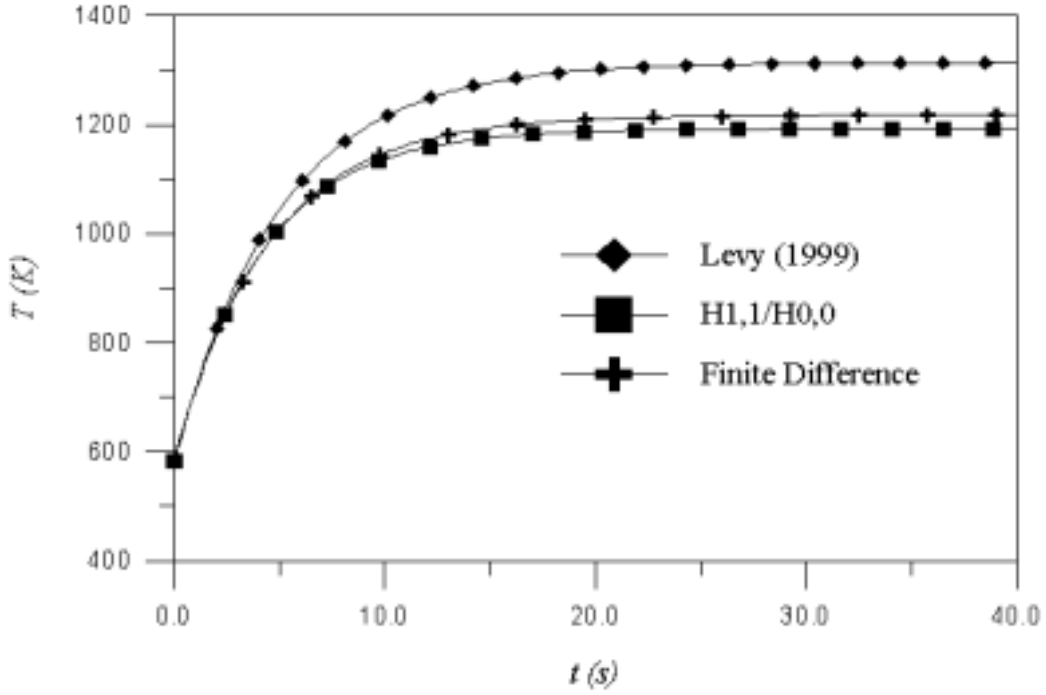


(a) Average fuel temperature

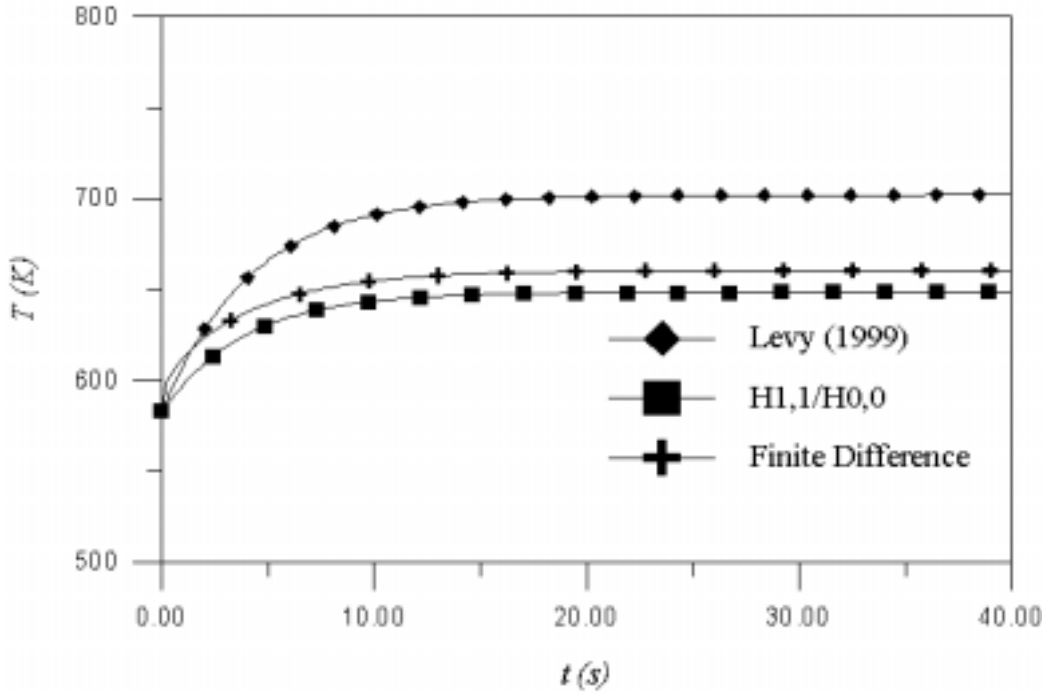


(b) Average cladding temperature

Figure 1 - Comparison of solutions for $Bi = 4.18838$.

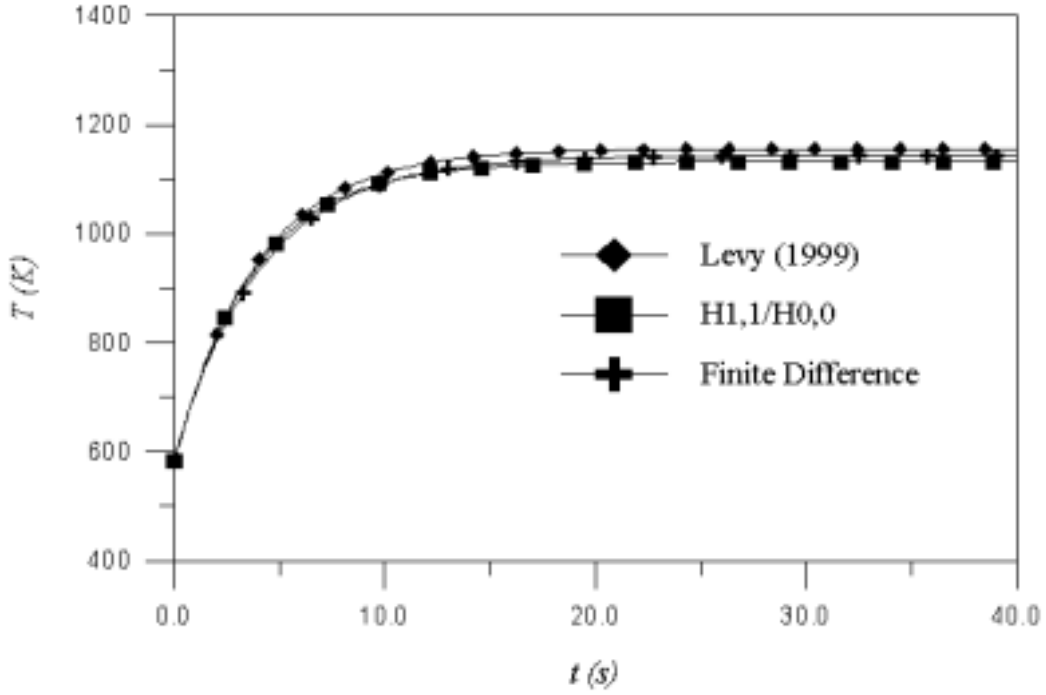


(a) Average fuel temperature

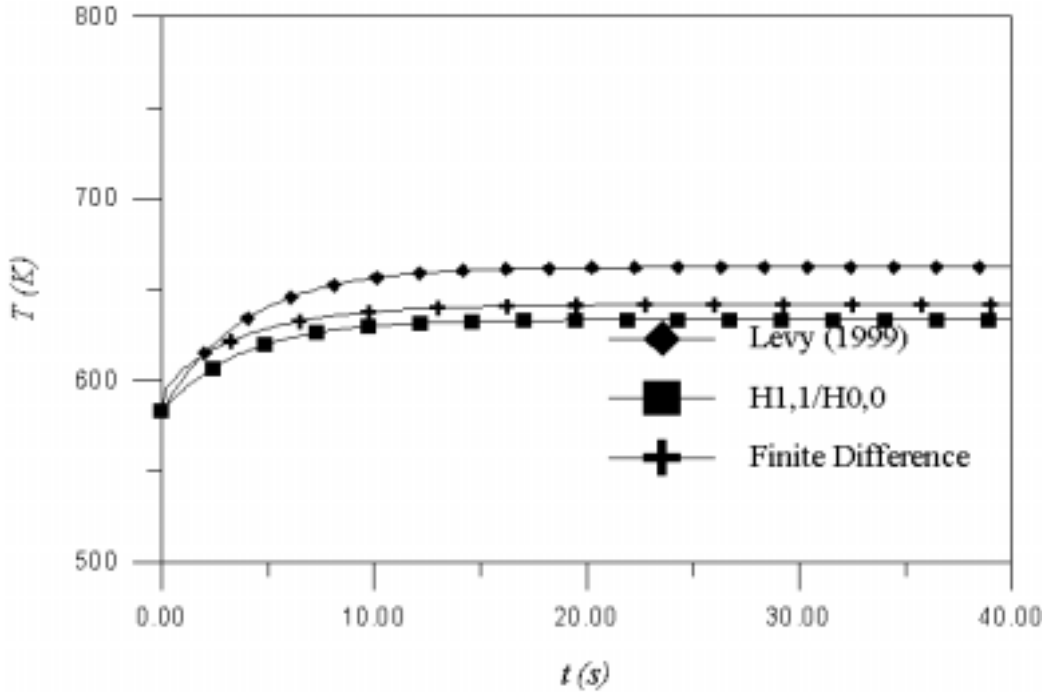


(b) Average cladding temperature

Figure 2 - Comparison of solutions for $Bi = 8.37676$.



(a) Average fuel temperature



(b) Average cladding temperature

Figure 3 - Comparison of solutions for $Bi = 12.5651$.

temperature and heat flux integrals, a significant improvement has been achieved over the classical lumped parameter analysis for the range of parameters typically encountered in nuclear reactor applications. Both averaged fuel and cladding temperature are in excellent agreement with finite difference solution of the original partial differential equation for the range of the Biot number studied.

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