A NEW EXPRESSION FOR THE HEIGHT OF MAXIMUM SPEED-UP ON THE ATMOSPHERIC BOUNDARY LAYER OVER LOW HILLS

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Abstract. *The atmospheric flow over low isolated hills has attracted considerable attention in the past 25 years. The ability to predict the height where there is a maximum speed-up, often called l, has been of particular interest to meteorologists, engineers and wind energy researchers. In this paper, we propose a new expression for calculating l under neutral stability conditions, based on a modified logarithmic law for the vertical wind distribution, recently proposed. The results are compared with the Askervein hill experimental data and good agreement is observed. We also tentatively study how other expressions for l can be obtained from other pre-existing vertical distributions of wind velocity.*

Key words: Maximum speed-up, Inner-layer depth, Low hills, Atmospheric boundary layer.

1. INTRODUCTION

There is a considerable technological interest in knowing the height of maximum wind speed-up in the atmospheric boundary layer over low hills. An important application is the siting of wind turbines in regions of enhanced flow. Because the power generated by the turbine is proportional to the cube of the wind velocity, a small wind speed-up corresponds to a large power increase. The correct calculation of the height of maximum wind speed-up is also important in itself, since it allows for the adequate prediction of the value of maximum wind speed-up by simple substitution in the appropriate expression for the vertical distribution of the wind speed.

A number of expressions for estimating the height of maximum speed-up, often denoted by l, are available in the literature. A comparative study of the relative merits of the mostly well known expressions for l can be found in Walmsley and Taylor (1996). Pellegrini and Bodstein (2000a) (hereafter referred as PBa) also present a short review of these expressions and propose a new one, which reads

$$l^+ \ln^2(l^+) = 2.4\kappa^2 L_h{}^+ , \tag{1}$$

with $l^+ = l/z_0$ and $L^+{}_h = L_h/z_0$, where z_0 is the roughness length and k the von Karman's constant, adopted here as 0.39, as suggested by a recent result from Frenzen and Voguel (1995). L_h is the half-length of the hill, defined following Jackson and Hunt (1975) as 'the distance from the hilltop to the upstream point where the elevation is half its maximum' (see fig. 1 ahead).

In the present work, we use the equation for the vertical profile of the wind speed-up recently obtained by Pellegrini and Bodstein (2000b), hereafter referred to as PBb, to propose a new expression for *l*. It is obtained through a formal analysis of the behaviour of the speed-up function. We compare this expression with all available results from the Askervein field experiment (Taylor and Teunissen, 1983, 1985) and find fairly good agreement. We also compare it with Eq. (1) and, again, good results are found. Finally, we use the vertical distribution of the wind velocity proposed by Taylor and Lee (1984) to obtain another expression for *l*, and we show this expression to be formally identical to the one early proposed by Jackson and Hunt (1975).

2. DEFINITION OF THE PROBLEM

Because the present analysis is built upon the results established in a previous work (PBb), the restrictions for the case under study are the same as for that case, which we state below. Consider an isolated 2D hill in the middle of an otherwise flat terrain, of constant roughness and under a neutrally stratified atmosphere. For our purposes, we consider a hill to be a topographical variation with characteristic length of the order of 5 Km and height less than 500m. A hill is called low when its slope never exceeds 20^o. The vertical co-ordinate *z* is defined as the height above the local terrain. For the cases of very large roughness elements, *z* is considered to be the displaced height above the local terrain. Fig. 1 illustrates the main features of a typical low hill.

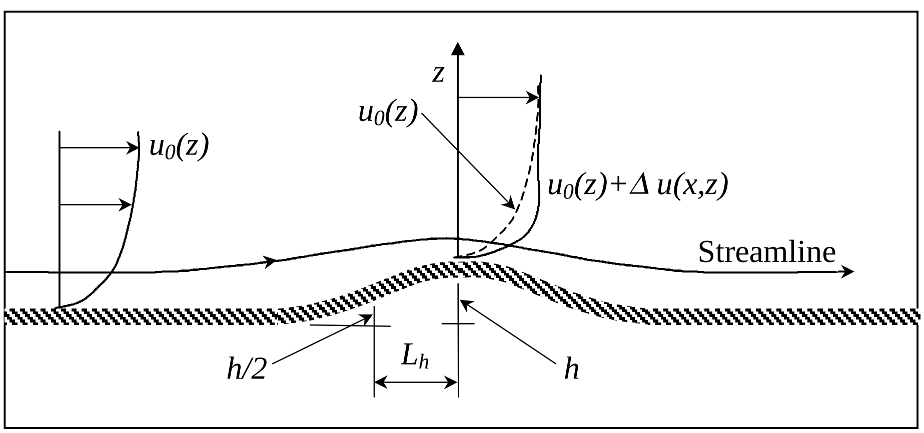

Fig.1. Definitions of h, L_h, Δu, u_0 and z.

We assume that the vertical profile of the mean horizontal wind velocity is essentially logarithmic far from the hill and refer to it, hereafter, as $\bar{u}_0(z)$. Further, we refer to the location upwind of the hilltop (HT) where $\bar{u}_0(z)$ is observed as the reference site (RS). In the atmospheric flow over low hills, the RS mean profile is influenced by the hill in such a way that it is modified by a speed-up quantity $\Delta\bar{u}(x,z)$ and becomes $\bar{u}(x,z)$ at a given point over the hill. Thus, we can write $\bar{u}(x,z) \equiv \bar{u}_0(z) + \Delta\bar{u}(x,z)$, where $\Delta\bar{u}$ is positive at HT, because the flow is accelerated to satisfy the continuity equation. If we divide the speed-up by the RS velocity, we have the relative speed-up, defined as

$$\Delta S(x,z) \equiv \overline{u}(x,z)/\,\overline{u}_0(x,z) - 1\,. \tag{2}$$

Following the work of PBb, we adopt the streamline co-ordinate system here. In this system, z is now the height above the local terrain measured normally to the local streamline, x is the distance along the local streamline, $\overline{u}$ and $\overline{u}_0$ are the mean velocities in the x direction and the other former definitions are kept unchanged.

3. ANALYSIS

A direct way to obtain an expression for the height of maximum speed-up in flows over hills is to calculate $\partial\Delta\overline{u}/\partial z \equiv 0$, for $z=l$, as long as $\Delta\overline{u}(x,z)$ is known. In spite of that, this method was, however, seldom used in the literature. If some expressions for $\Delta\overline{u}(x,z)$ are in fact not suitable to this end, others present no real difficulty. Jackson and Hunt's (1975) and Finnigan's (1992) solutions, for example, cannot be not easily differentiated, the former because it depends on a generic function and the latter because it is implicit in $\overline{u}$. On the other hand, the result proposed by Taylor and Lee (1984), which is essentially an empirical exponentially damping function of the maximum speed-up with height, is easily differentiated, as we show below. In what follows, we calculate $\partial\Delta\overline{u}/\partial z \equiv 0$ to determine l from the expression for $\Delta\overline{u}(x,z)$ obtained by PBb and Taylor and Lee (1984) and analyse the results.

3.1. The height of maximum speed-up based on PBb's vertical profile of the wind

We start by writing down the expression for $\Delta\overline{u}(x,z)$ recently proposed by PBb:

$$\Delta\overline{u} \approx \frac{u_* - u_{*0}}{k}\ln\frac{z}{z_0} + \frac{u_*}{k}\sum_{n=1}^{\infty}\frac{\left(z/R_h\right)^n - \left(z_0/R_h\right)^n}{n\cdot n!}\,. \tag{3}$$

In the equation above, u_* is the friction velocity at the site, defined as $u_*=(\tau_s/\rho)^{1/2}$, where τ_s is the surface stress, and u_{*0} is the friction velocity at the RS. The parameter R_h, called *radius length*, appeared due to the presence of R, the radius of curvature of the streamlines, in the momentum equations written in streamline coordinates (Finnigan, 1983), which was used to obtain equation (3). As proposed by PBb, R_h is neither identical to R nor to R_{h0}, the radius of curvature of the hill. It is related to R_{h0} in the same way as the roughness length, z_0, is related to the real height of the roughness elements. In other words, R_{h0} is a theoretical lenght used to integrate the equation of motion in the streamwise direction. In their work, PBb also shows that R_h seems to be proportional to R_{h0}, which, in turn, is dependent on the geometry of the hill.

The approximation sign in expression (3) is due to the assumption that $z_0 << R_h$, made during the calculation of one of the integration constants.

To begin the analysis, we recall from PBb that the power series in Eq. (3) can be shown to converge for all z. Therefore, the function $\Delta\overline{u}(x,z)$ is analytical and (consequently) continuous for all z. We also recall that Eq. (3) is defined in the closed interval $[z_0, z^*]$, being z^* the upper limit where the approximations made during the derivation of Eq. (3) still holds.

Therefore, we conclude that $\Delta\bar{u}(x,z)$ must have absolute maximum and minimum in the interval [z_0 ,z*].

An important step in obtaining the height of maximum $\Delta\bar{u}(x,z)$ is searching for the zeros of $\Delta\bar{u}(x,z)$, which enables us to distinguish between points of maximum and minimum. Simple inspection shows that $z=z_0$ is one zero of $\Delta\bar{u}(x,z)$. Other zeros are not trivially obtained. Another important step is to calculate the critical points of the function. For that, we need the expression for the derivative of the profile, which can also be obtained from the original work of PBb:

$$\left(kz\frac{\partial\bar{u}}{\partial z}\right)^2 = C_1(x)e^{2z/R_h} . \tag{4}$$

In their work, the authors show that $C_1(x) \approx u_*{}^2$ and thus $\partial\bar{u}/\partial z \approx (u_*/kz)e^{z/R_h}$. Recalling that $\bar{u}(x,z) \equiv \bar{u}_0(z) + \Delta\bar{u}(x,z),$ we thus have

$$\frac{\partial\Delta\bar{u}}{\partial z} \approx \frac{u_{*0}}{kz}\left(\frac{u_*}{u_{*0}}e^{z/R_h} - 1\right). \tag{5}$$

The critical points, if any, can be obtained by solving

$$0 \approx \frac{u_{*0}}{kz}\left(\frac{u_*}{u_{*0}}e^{z/R_h} - 1\right), \tag{6}$$

for $z = z_{crit}$. Considering that $u_{*0} > 0$ and $z > z_0 > 0$, we have $(u_*/u_{*0})e^{z_{crit}/R_h} \approx 1$, which has the unique solution

$$z_{crit} \approx R_h \ln\left[\frac{u_{0*}}{u_*}\right]. \tag{7}$$

So far, we have shown that expression (3) has just one critical point, given by Eq. (7). However, since the *x*-dependent functions u_* and R_h of Eq. (7) can have positive as well as negative values, this relation must be interpreted with some caution. Indeed, $u_* > 0$ in general, but we may observe $u_* < 0$ in regions of reversed flow. Furthermore, we have $R_h > 0$ if the centre of curvature of the surface lies in the direction of increasing *z* whereas $R_h < 0$ otherwise (see PBb). Theoretically, four possibilities exist: $u_* > 0$ and $R_h > 0$, $u_* < 0$ and $R_h < 0$, $u_* < 0$ and $R_h > 0$ and $u_* > 0$ and $R_h < 0$. However, the function $\exp(z_{crit}/R_h)$ in Eq. (5) is always positive, implying that expression (7) does not hold for $u_* < 0$. This is probably due to the fact that the approximations made on the derivation of Eq. (3) do not hold for reversed flow either. In addition, eq (7) means that if $u_* > u_{*0}$, then $R_h < 0$ and $u_* < u_{*0}$, then $R_h > 0$ (remember that $z_{crit} > z_0 > 0$). This is exactly what is generally observed in the real case. Over the HT, where $R_h < 0$, the speed-up is always positive and, consequently, $u_* > u_{*0}$. Over the upwind slope, where we must have $R_h > 0$ at some point, a deceleration of the flow is often observed, implying that $u_* < u_{*0}$. Thus, in fact, only two possibilities exist, both for $u_* > 0$: $R_h > 0$ (for $u_* < u_{*0}$) and $R_h < 0$ (for $u_* > u_{*0}$).

We must now determine in which case z_{crit}, defined by Eq. (7), is a point of maximum or minimum. Let us return to Eq. (4) to calculate the intervals where the function increases or decreases. We know that if $\partial\Delta\bar{u}/\partial z > 0$, then $\Delta\bar{u}$ increases and, conversely, if $\partial\Delta\bar{u}/\partial z < 0$, then $\Delta\bar{u}$ decreases. Recalling that $k > 0$, $z_0 > 0$ and $u_{*0} > 0$, these conditions allows us to write:

$$u_* e^{z/R_h} > u_{*0} \Rightarrow \Delta\bar{u} \text{ increasing,} \tag{8}$$
$$u_* e^{z/R_h} < u_{*0} \Rightarrow \Delta\bar{u} \text{ decreasing.} \tag{9}$$

Expressions (8) and (9) are easier to analyse if we rewrite them in terms of z_{crit}. From Eq. (6) we have $(u_*/u_{*0})e^{z_{crit}/R_h} \approx 1$, which means $u_{*0} \approx u_* e^{z_{crit}/R_h}$. Substitution of this into eqns. (8) and (9), recalling that $u_* > 0$ always, yields

$$e^{z/R_h} > e^{z_{crit}/R_h} \Rightarrow \Delta\bar{u} \text{ increasing,} \tag{10}$$
$$e^{z/R_h} < e^{z_{crit}/R_h} \Rightarrow \Delta\bar{u} \text{ decreasing.} \tag{11}$$

Now, we have the two possibilities sketched before:

Case 1. $R_h > 0$, for $u_* < u_{*0}$.
In this case, solution to eqns. (10) and (11) is simply

$$z > z_{crit} \Rightarrow \Delta\bar{u} \text{ increasing,} \tag{12}$$
$$z < z_{crit} \Rightarrow \Delta\bar{u} \text{ decreasing,} \tag{13}$$

and as z_{crit} is the only critical point of the function, it is an *absolute minimum.*

Case 2. $R_h < 0$ for $u_* > u_{*0}$.

$$z < z_{crit} \Rightarrow \Delta\bar{u} \text{ increasing,} \tag{14}$$
$$z > z_{crit} \Rightarrow \Delta\bar{u} \text{ decreasing,} \tag{15}$$

and as z_{crit} is the only critical point of the function, it is an *absolute maximum.*

In the far more common case, where we are interested in the value of l over the top of the hill, we have $R_h < 0$. Thus, it is a case of absolute maximum and l can be calculated from Eq. (7). At the upwind slope, in cases where there is no reversed flow, we always have $R_h > 0$ somewhere. In this region, an absolute minimum occurs and the corresponding height is again calculated from Eq. (7). Because Eq. (6) is the expression for the critical value of z, we adopt the symbol l for both the cases of minimum and maximum and write

$$l \approx R_h \ln\left[\frac{u_{0*}}{u_*}\right]. \tag{16}$$

This is the new expression for the critical height of the speed-up on neutral flows over low hills. For practical purposes, careful distinction must be made between the maximum and minimum speed-up cases. A thorough analysis of this expression is presented at section 4.

3.2. The height of maximum speed-up based on Taylor's and Lee (1984) vertical profile

This derivation is included here to show that the method is valid for other expressions for $\Delta\overline{u}$ found in the literature. A complete analysis of all the expressions currently available is not intended.

We consider the work of Taylor and Lee (1984), which proposes that

$$\Delta S(z) = \Delta S_{\max} e^{-Az/L_h}, \tag{17}$$

where A=4 for 3-D hills and A=3 for 2-D hills. For 3-D hills that are elongated in shape, the authors suggest A=3.5. The definition of ΔS implies that

$$\Delta\overline{u}(z) = \overline{u}_0(z)\Delta S_{\max} e^{-Az/L_h}. \tag{18}$$

Differentiating Eq. (17) with respect to z and setting it equal to zero at the critical points, we obtain $0 = \Delta S_{\max}[\partial\overline{u}_0/\partial z\, e^{-Az/L_h} + \overline{u}_0(-A/L_h)e^{-Az/L_h}]$ at $z=z_{crit}$. Recalling that $\overline{u}_0 = (u_*/\kappa)\ln(z/z_0)$, it follows that $0 = \Delta S_{\max} e^{-Az/L_h}[(u_*/z\kappa) + (u_*/\kappa)\ln(z/z_0)(-A/L_h)]$, at the critical points and we thus have,

$$\frac{1}{z_{crit}} = \frac{A}{L_h}\ln\frac{z_{crit}}{z_0}. \tag{19}$$

Substituting z_{crit} for l, multiplying and dividing by κ^2 and rearranging, we finally obtain

$$\frac{l}{L_h}\ln\frac{l}{z_0} = \left(\frac{1}{A\kappa^2}\right)\kappa^2. \tag{20}$$

We recognise this expression to be almost identical to the one found in Jackson and Hunt (1975), which reads $(l/L_h)\ln(l/z_0) = 2\kappa^2$. The only difference is the value of the constant multiplying κ^2. Considering a value of $\kappa = 0.39$, as stated earlier, we have $(1/A\kappa^2) = 1.6$ for 3-D hills, $(1/A\kappa^2) = 2.2$ for 2-D hills and $(1/A\kappa^2) = 1.9$, for 3-D elongated hills. The values for the 2-D and elongated 3-D hills are very close to Jackson and Hunt's value of 2. In all cases, however, the conclusion is that *Taylor and Lee's (1984) wind profile implies Jackson and Hunt's (1975) law for the height of maximum speed-up*. This conclusion, is spite of being very easy to draw, has apparently passed unnoticed through the years.

4. COMPARISON WITH FIELD DATA AND DISCUSSION

The results of expression (16) were compared with all available results of the Askervein hill experiment, conducted in 1982 and 1983 (Taylor and Teunissen, 1983, 1985). The Askervein results were chosen because they are believed to be 'the most complete field experiment to date' (Kaimal and Finnigan, 1994) and to 'still represent a benchmark for such studies', according to a rather recent evaluation by Walmsley and Taylor (1996). The original data were directly obtained from the authors in the form of tables for the vertical velocity profiles. The processing of the data is described below and the results are summarised at Table 1.

The first column in Table 1 is the designation of the run. Following Mickle *et al.* (1988), the letters TU mean 'turbulence' and refer to the data obtained during the year of 1983 and the letters MF denote 'mean flow' and refer to 1982 data. The numbers refer to the day the run was made. When more than one run was made on the same day, letters 'A' for the first run, 'B' for the second and so on were added. All runs obtained on October 5th, 1983, were not included because they were considered suspicious by the authors. In fact, we verified that no coherent results could be derived from those runs.

Table1. Height of maximum speed-up.

Run Number	ϕ (^{0})	z_0 (mm)	L_h (m)	R_h (m)	l Eq.(16) (m)	l Eq.(1) (m)	Exp. l (m)	% diff. Eq.(16)	% diff. Eq.(1)
TU25	210	12	200	5,3	4,58	2,56	4,5	1,9	-43,6
TU30A	135	41	700	5,3	4,01	8,90	5,0	-19,8	77,6
TU30B	130	59	700	3,9	1,89	9,84	2,0	-5,7	390,9
TU01A	170	23	315	6,0	4,66	4,25	5,0	-6,8	-12,2
TU01B	180	15	280	7,6	5,33	3,48	5,0	6,5	-30,2
TU01C	185	22	260	5,6	4,86	3,66	5,0	-2,9	-26,7
TU01D	200	54	210	3,0	2,96	4,12	4,0	-26,0	2,3
TU02	165	28	380	5,9	4,29	5,15	5,0	-14,2	2,9
TU03A	210	18	200	4,3	3,84	2,86	4,0	-4,0	-28,4
TU03B	210	27	200	3,9	3,65	3,22	5,0	-26,9	-35,9
TU06A	215	31	200	3,2	3,00	3,36	5,0	-40,0	-32,7
TU06B	230	42	220	2,9	2,76	3,93	5,0	-44,8	-21,5
TO07A	240	31	230	4,1	3,16	3,70	5,0	-36,8	-25,6
TU07B	260	23	300	5,2	3,42	4,11	5,0	-31,5	-17,6
MF25	120	30	650	4,1	2,19	7,75	1,5	46,0	417,0
MF28	175	32	300	3,5	2,95	4,51	3,5	-15,7	29,0
MF29A	225	39	210	1,99	1,70	3,72	1,8	-11,2	107,0
MF29B	235	29	220	2,45	2,16	3,52	2,0	8,1	76,2
MF01A	155	24	530	12,4	6,02	6,29	6,0	0,4	4,3
MF02	200	35	210	2,5	2,42	3,60	2,0	20,8	79,7
MF03	155	17	520	6,3	3,59	5,67	5,1	-29,6	11,6
Average absolute % difference								20,0	67,0
Average % difference								-10,0	44,0
Average absolute % difference, ϕ=120^o—135^o excluded								18,2	32,6
Average % difference, ϕ=120^o—135^o excluded								-12,8	2,1

The direction of the wind is presented in the second column of Table 1. The estimates for z_0 and R_h are obtained by fitting the modified log law to the field data, as explained in PBb. The values of L_h are obtained from a curve fit to the values presented by Taylor and Lee (1984), for a limited range of directions, which brings a certain amount of arbitrariness to the interpolation process. The columns designated by 'l Eq.(16)', 'l Eq.(1)' and 'Exp. l' represent l calculated from Eq. (16), Eq. (1) and the experimental value of l, respectively. The last two columns are the relative difference between Eq. (16) and Eq. (1) with respect to the experimental value of l, respectively. The last four lines of Table 1 are four different kinds of average values, calculated with those differences. Before those values are analised, a word of caution is necessary about the experimental value of *l*, however.

Experimental values of *l* were estimated directly from the raw data received from the authors. These estimates were made (whenever a plausible one could be made) by computing the difference between the wind velocities at HT and RS, i.e. calculating $\Delta\bar{u}$, and searching for its maximum. However, the limited number of measurement points and the fact that there was often just one point between the supposed maximum and the ground, made it very difficult to establish the real point of maximum $\Delta\bar{u}$. The strategy adopted to overcome this difficulty was to draw a best-fit line for $\Delta\bar{u}$ and to estimate the maximum directly from it. We recognise that this approach is limited to the amount of data points available (and to the curve fit itself) but we believe that our conclusions still hold. In all but a few cases, the value estimated for *l* coincided with the largest experimental value of $\Delta\bar{u}$. In a small number of exceptions, we thought a better estimate could be made. Those were the cases where two neighbouring values were very close to each other, suggesting that the maximum might be somewhere between them or cases where the difference was still growing at the measurement point closest to the ground.

As an additional possible source of scattering for the estimations of *l*, the MF observations (1982's measurements) were not taken at the same levels at the RS and HT towers. Therefore, to calculate $\Delta\bar{u}$ we needed to interpolate the values of the wind velocity at RS logarithmically at the measuring heights used at HT.

The values of the last two columns of Table 1 shows that Eq. (16) provides a better description of the field data than Eq. (1). The two values in the first of the last four lines, 20.0% and 67.0%, are the average of the absolute values of the differences, which tend to confirm this conclusion. The averages of the differences (sign included) are –10.0%, for Eq. (16), and 44.0%, for Eq. (1), and are shown in the second of the last four lines. They suggest that Eq. (1) tends to overestimate *l* whereas Eq. (16) tends to underestimate it slightly. The last two columns also suggest that Eq. (1) does not predict *l* correctly when the wind is blowing from the $120^{o}-135^{o}$ direction range, probably because the estimate of L_h is difficult to perform in this case. In fact, if such cases are excluded, the average of the absolute difference for Eq. (1) goes down to 32.6% (third line), whereas the average difference (sign included) goes down to 2.1% (fourth line), indicating only a very slight tendency to overestimate it. No recognisable dependence of the results obtained from Eq. (16) with direction was detected. The scatter of the results appear to be a simple consequence of the scatter in the determination of u_* and u_{*0} and of the measurement limitations mentioned above. In all cases, Eq. (16) seems to furnish the best available description of the phenomena. Indeed, in a previous work, PBa have shown Eq. (1) to describe experimental data better than any other expression, including Eq. (20) or its similar, proposed by Jackson and Hunt (1975). As we have shown that Eq. (16) describes field data better than Eq. (1), we conclude that Eq. (16) is the best available at the moment, as far as the Askervein data is concerned. However, expressions of the kind of Eq. (1) are easier to use than Eq. (16), because the former is purely geometrical while the latter is essentially dynamic. Therefore, no use can be made of Eq. (16) unless z_0, u_* and R_h are calculated. However, one should consider that:

- most equations of the kind of Eq. (1) also depend on the dynamics of the flow because they include $\bar{u}_0(z)$ implicitly in them. This is usually where the term $\ln(l/z_0)$ comes from;
- because most equations of the kind of Eq. (1) come from order of magnitude analysis, they generally include constants to be determined against experimental data. Eq. (16) includes none, which is a clear advantage;

- l is indeed to be expected to depend on the dynamics of the flow and the *detailed* geometry of the hill and not only on the ratio $L^{+}_{h} \equiv L/z_0$.
- the need to calculate z_0, and u_* is typical of boundary layer solutions using flux-profile relationships, like Eq. (2). Our description is, therefore, coherent with the results of PBb, upon which it is based.

As we have shown in section 3.2, different expressions for l can be obtained using different author's expressions for $\Delta\overline{u}$. One was used as an example and we suggest others to be tried. We have shown that Taylor and Lee's (1984) expression for $\Delta\overline{u}$ implies Jackson and Hunt's (1975) expression for l. Consequently, as Eq. (16) give better results than Jackson and Hunt's (1975), it is suggested that our expression for $\Delta\overline{u}$ is also better than Taylor and Lee's (1984). This conclusion has also been confirmed by direct comparison made in PBb previously.

Finally, we stress that careful distinction must be made between the minimum and maximum cases when using Eq. (16). If we recall the fact that z_0 is a zero of the speed-up function, we see that, in the minimum case, the value of speed-up at l is negative. Misuse of Eq. (16) in that sense can lead, for example, to a wind turbine being sited at the height of minimum (and negative) speed-up in a region where $R_h > 0$. This can be very important in siting arrays of wind turbines on the upwind slope at the top of a hill. Implicit in this discussion is the fact that we expect Eq. (16) to hold on the slopes of the hill as well as on the HT. This is due to the fact that Eq. (3) is expected to also hold in those regions, according to PBb. At the moment, however, this claim cannot be proved nor refuted due to the lack of complete vertical wind profiles on the slopes.

5. CONCLUSIONS

We have obtained a new expression for the height of maximum speed-up in flows over low isolated hills under neutral atmosphere. The task was accomplished by analysing the behaviour of the speed-up function proposed by PBb. Although we have shown the resulting expression to describe the Askervein field data better than any other expression available in the literature, we point out that it is essentially dynamic in nature and, therefore, demands the atmospheric boundary layer to be fully resolved. We tentatively obtained another expression for l by the same method, analysing Taylor and Lee's (1984) expression for the speed-up. We also suggest that other available expressions for $\Delta\overline{u}$ should be analysed to yield new expressions for l.

Our results also pointed out the need of a greater density of velocity measurements at the HT, obtained at the same measuring levels of the RS, so that the position of l can be more firmly established.

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