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Bifurcation Control under High-Frequency Excitation and its Application for Motion Control of an Underactuated Manipulator

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Abstract: Some control methods by using nonlinear effects are introduced in this paper. The 1/3-third order subharmonic resonance is stabilized independent of magnitude of disturbance by using nonlinear coupling to an absorber. Nonlinear characteristics of the hunting motion of railway vehicle wheelset are controlled and the system is globally stabilized by the nonlinear feedback. The third topic is on a positive utilization of nonlinear phenomena. Motion control of an underactuated manipulator is carried out under high-frequency excitation without state feedback. The above control strategies are based on control of slowly-varying dynamics. The averaged equations governing the slowly-varying dynamics, which are obtained by the method of multiple scales, center manifold theory, and so on, are modified and the resulting change of nonlinear characteristics realize the stabilization control and motion control.

Keywords: nonlinear dynamics, nonlinear control, bifurcation, high-frequency excitation

NOMENCLATURE

a = amplitude of the main system b = amplitude of the absorber P_1, P_2, P_3 = constant coefficient determined by dimension
ε = dimensionless excitation amplitude
ν = dimensionless excitation frequency
μ_z = damping coefficient of main system
μ_θ = damping coefficient of pendulum
σ = detuning parameter of main system
ρ = detuning parameter of absorber
γ = initial hapse of main system
ϕ = initial phase of absorber
ω_θ = natural frequency of absorber
y = lateral motion of wheelset
ψ = yawing motion
r = representative amplitude on cener manifold
α_1 = linear coefficientet of normal form
α_3 = nonlinear coefficient of normal form

θ_1 = angle of the first joint
θ_2 = relative angle of the second link
θ_{1a} = excitation amplitude of the first link
θ_{1off} = offset of the first link
σ = inverse number of squared excittion frequency of the first link

INTRODUCTION

By using averaging method, the method of multiple time scales, center manifold theory, and so on (Nayfeh and Mook, 1979, Carr, 1981), we obtain averaged equations expressing slowly-varying dynamics which govern essential feature of nonlinear phenomena. Therefore, if we modify the averaged equations, it is possible to stabilize nonlinear resonance and to realize dynamical systems with desired nonlinear features. In this paper, we show some stabilization control methods based on modifications of slowly-varying dynamics and consider the positive utilization of nonlinear phenomena to the motion control for a underactuated manipulator.

In the next section, we consider stabilization of 1/3-order subharmonic resonance. By using a pendulum-type vibration absorber, the main system which is excited with 1/3-order subharmonic resonance is stabilized. The appropriate tuning of the natural frequency of the absorber modifies original averaged equations in the case without absorber. Then, the slowly-varying dynamics is controlled and the resonance is stabilized. Furthermore, we consider the hunting motion of railway vehicle wheelset. The phenomenon is self-excited oscillation which is produced in the case of higher running speed than a critical speed. Finally, we deal with an application of high-frequency excitation to the motion control of an underactuated manipulator. By the modification of slowly-varying dynamics, the various bifurcation phenomena are emerged and the equilibrium states are changed approximately to realize the motion control.

STABILIZATION OF 1/3-ORDER SUBHARMONIC RESONANCE BY AN AUTO-PARAMETRIC VIBRATION ABSORBER

Analytical model and the basic equation

We consider a magnetically levitated body as shown in Fig. 1. The body is freely moved in the vertical direction. The magnets on the body and the base repulse with same magnetic pole. The base magnet is moved sinusoidally in the vertical direction. By taking into account nonlinearly of the restoring force until cubic term, the dimensionless equation of motion of the magnetically levitated body is expressed as (Yabuno et al, 1999):

$$\ddot{z} + \mu_z \dot{z} + (1 + \varepsilon \cos \nu t) z + \alpha_{zz} z^2 + \alpha_{zzz} z^3 = \varepsilon \cos \nu t, \tag{1}$$

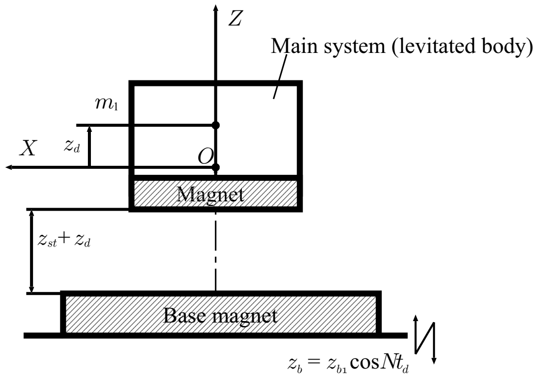


Figure 1 – Analytical model of a magnetically levitated body.

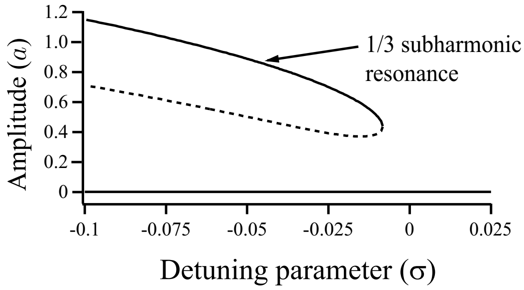


Figure 2 – Frequency response curve of the main system.

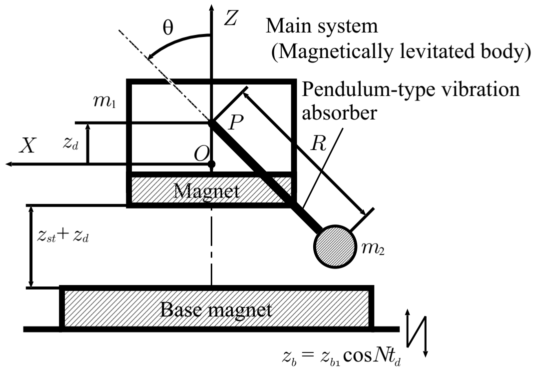


Figure 3 – Magnetically levitated body with an auto-parametric vibration absorber.

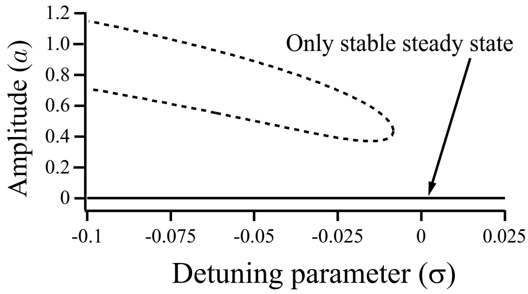


Figure 4 – Frequency response curve of the main system (absorber in action).

where the natural frequency is normalized as 1.

Theoretical analysis

We consider the 1/3-order subharmonic resonance in the case when the excitation frequency is in the neighborhood with triple the linear natural fernery ($\nu = 3 + \sigma$). We seek an approximate solution as

$$z = \varepsilon z_1 + \varepsilon^2 z_2 + \varepsilon^3 z_3 + \cdots. \tag{2}$$

By using the method of multiple scales, we obtain the following approximate solution:

$$z = a(t)\cos\left\{\frac{\nu}{3}t + \gamma(t)\right\} + 2\Lambda\cos\nu t + O(\varepsilon), \tag{3}$$

where the time variations of a and γ are governed with averaged equations:

$$\frac{da}{dt} = -\frac{1}{2}\mu_z a - \frac{P_2}{4}a^2\sin 3\gamma \tag{4}$$

$$a\frac{d\gamma}{dt} = -(\frac{P_1}{2} + \frac{\sigma}{3})a - \frac{P_2}{4}a^2\cos 3\gamma - \frac{P_3}{8}a^3. \tag{5}$$

Letting $da/dt = d\gamma/dt = 0$ yields amplitude a and initial phase γ in the steady date. The frequency response curve is described as Fig. 3, where the solid and dashed lines stand for stable and unstable steady state amplitude. It is noticed that the 1/3-order subharmonic resonance can be produced depending on the magnitude of disturbance. The control objective is here to avoid the occurrence of $1/3$-order subharmonic resonance in dependent of the magnitude of initial conditions and disturbance.

Control strategy

The dependency of the occurrence on the initial condition and the disturbance is found by the averaged equation, which is the condition not to produce the secular term in z_3. By attaching a pendulum-type vibration absorber and changing the averaged equations, we try to stabilize the 1/3-order subharmonic resonance. The effect of the absorber on the main system needs to modify the averaged equation of main system Eqs. (4) and (5). To this end, the natural frequency of the absorber is tuned to be a half the natural frequency of the main system. Then, the averaged equations for the main system and the absorber are

$$\begin{aligned} \frac{da}{dt} &= -\frac{1}{2}\mu_z a - \frac{P_2}{4}a^2\sin 3\gamma - \frac{mr\omega_\theta{}^2\rho}{4}b^2\sin(\gamma-2\phi) \\ &- \frac{mr\omega_\theta^2}{2}\sin(\gamma-2\phi)b^2 \end{aligned} \tag{6}$$

$$\begin{aligned} a\frac{d\gamma}{dt} &= -\left(\frac{P_1}{2}+\frac{\sigma}{3}\right)a - \frac{P_2}{4}a^2\cos 3\gamma - \frac{P_3}{8}a^3 + P_{12}ab^2 - \frac{mr\omega_\theta{}^2\rho}{4}b^2\cos(\gamma-2\phi) \\ &- \frac{mr\omega_\theta^2}{2}b^2\cos(\gamma-2\phi) \end{aligned} \tag{7}$$

The frequency response curve is shown in Fig. 4. Compare with that in the case without absorber, the non-trivial steady state is changed to unstable one. As a result, because only one stable steady state is trivial, the 1/3-order subharmonic resonance cannot be produced independent of disturbance and initial condition. Experimental confirmation of the proposed control method is carried out by using a simple apparatus (Yabuno et al, 1999). In the above control strategy, the averaged equations expressing slowly-varying dynamics is focused and they are modified to avoid the occurrence of the 1/3-order subharmonic resonance.

NONLINEAR CONTROL FOR THE HUNTING MOTION OF A RAILWAY VEHICLE WHEELSET

Railway vehicle wheelset experiences the problem of hunting above a critical running speed (Whiclens, 1965) due to contact force between the wheels and the rails (Kalker, 1990). The resonance is flutter-type destabilization and Hopf bifurcation occurs at the critical speed. The nonlinearity of the bifurcation is subcritical (Molle and FGash, 1992). Even below the critical speed which is obtained by linear analysis, the hunting motion can be produced depending on the magnitude of disturbance. Here, control objective is to avoid the occurrence of the hunting motion below the critical speed independent of the magnitude of disturbance. Also in this section, we focus on slowly-varying dynamics, which is derived through center manifold theory.

Nonlinear analysis and control of hunting motion

We consider the behavior of a single suspended railway wheelset at a constant running speed v on a straight track as shown in Fig. 5. The wheelset can be freely moved in the lateral and yawing directions, y and ψ. The dimensionless

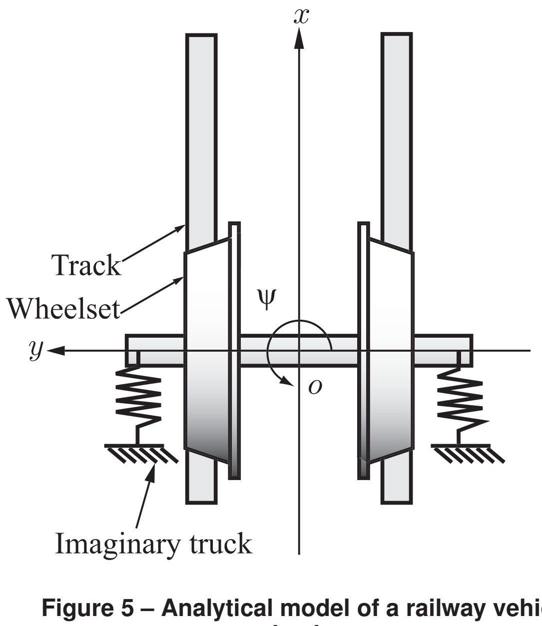


Figure 5 – Analytical model of a railway vehicle wheelset.

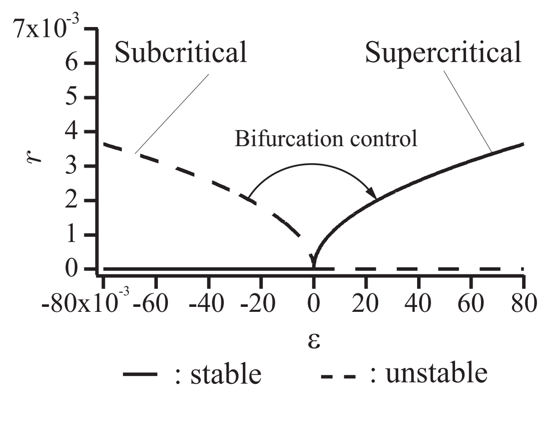


Figure 6 – Bifurcation control on center manifold.

lateral motion y^* and yawing motion ψ are expressed as follows (Yabuno et al, 2002)

$$\dot{x} = Ax + \begin{bmatrix} 0 \\ N_2(x) \\ 0 \\ N_4(x) \end{bmatrix}, x = \begin{bmatrix} y^* & \dot{y^*} & \psi & \dot{\psi} \end{bmatrix}^T \tag{8}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -c_1 & -c_2 & c_3 & 0 \\ 0 & 0 & 0 & 1 \\ -c_5 & 0 & -1 & -c_4 \end{bmatrix}$$
$$N_2(x) = c_1\varepsilon\dot{y}^* + c_6 y^{*3} + c_7 y^{*2}\psi + c_8 y^*\psi^2 + c_9\psi^3$$
$$N_4(x) = c_4\varepsilon\dot{y}^* + c_{10} y^{*3} + c_{11} y^{*2}\psi + c_{12} y^*\psi^2 + c_{13}\psi^3,$$

where dimensionless parameters $c_1 \cdots c_{13}$ are assumed in some studies (Lorant and Stepan, 1996, Molle and Gasch, 1992). The anti-symmetry stiffness matrix can produce self-excited oscillation through Hopf bifurcation. The dimensionless running speed v^* is assumed as follows:

$$v^* = v_c^*(1+\varepsilon)(|\varepsilon| << 1), \tag{9}$$

where v_c^* is the dimensionless linear critical speed which is obtained by using root loci.

Based on center manifold theory, the fast and slow dynamics are separated. Because the slow dynamics determines the stability, dynamics reduced on the center manifold is controlled to suppress the hunting motion below the critical speed. Furthermore, the normal form on the center manifold is expressed as

$$\dot{r} = a_1\varepsilon r + a_3 r^3, \quad \dot{\beta} = \omega^* + b_0\varepsilon + b_2 r^2. \tag{10}$$

Because it has been indicated in some studies (Molle and Gasch, 1992, Lorant and Stepahn, 1996) that the cubic coefficient a_3 is negative, it concludes that the bifurcation at the critical speed is a subcritical Hopf bifurcation and the hunting motion can occur for large disturbance even under the critical speed. In order to suppress the hunting motion independent of the magnitude of disturbance below the critical speed, We design the nonlinear feedback gain on the center manifold to bend the bifurcation branch to the right as shown in Fig. 6 (Yabuno, 2005). By applying cubic nonlinear feedback, $-\alpha r^3$, the above normal form is changed as follows:

$$\dot{r} = a_1\varepsilon r + (a_3-\alpha) r^3, \quad \dot{\beta} = \omega^* + b_0\varepsilon + b_2 r^2, \tag{11}$$

where nonlinear feedback gain α is selected so that the coefficient of the cubic term is negative, i.e., $a_3 - \alpha < 0$. Then the nonlinear characteristic of the bifurcation point is supercritical as shown in Fig. 6, and there is no unstable limit cycle below the critical speed. The validity of the proposed control method is experimentally confirmed by using a simple apparatus (Yabuno et al, 2005). Also in this method, we focus on slowly-varying dynamics and carries out the stabilization of the amplitude

MOTION CONTROL OF AN UNDERACTUATED MANIPULATOR BY BIFURCATION CONTROL UNDER HIGH=FREQUENCY EXCITATION

Manipulators with free joints (links) are called underactuated manipulators and in these systems, the number of generalized coordinates is greater than the number of actuators. There have been many studies on the control of underactuated manipulators (a comprehensive list of references can be found in (Arai et al, 1998)). While previous control methods are based on feedback control with respect to the free link, the method presented in this study does not require any information on the motion of the free link (Yabuno et al, 2004). We focused on slowly-varying dynamics as the topics in the previous sections. However, the dynamics of the controlled state is not slow in cntrast with these topics. We apply high-frequency excitation so that the state dynamics can be regarded as slow dynamics.

Two-link underactuated manipulator

The control objective is for the two-link underactuated manipulator (Fig. 7 shows an analytical model) to swing up the second (free) link, which hangs down in the direction of gravity in the initial state, to the state where the second (free) link points in the direction opposite to that of the gravity and also to stabilize the free link. The control is based on the actuation of the perturbation of bifurcations emerged under high-frequency excitation. The gravity effect is in the positive direction of the x' axis.

Here we set the position of the first link according to

$$\theta_1 = \theta_{1a}\cos\omega t + \theta_{1off}, \tag{12}$$

where the first term is for application of the high-frequency excitation to the second link. The second term expresses the configuration of the first link with respect to the direction of the gravity effect. The essental strategy of the proposed control is utilization of the dynamics stabilization phenomena which is seen in inverted pendulum. Inverted pendulum is stabilized without feedback control under the high-frequency vertical excitation as shown in Fig. 8. The pendulum is known as Kapitza pendulum (Kapitza, 1965). Under the excitation of Eq. (12), the equation governing the motion of the

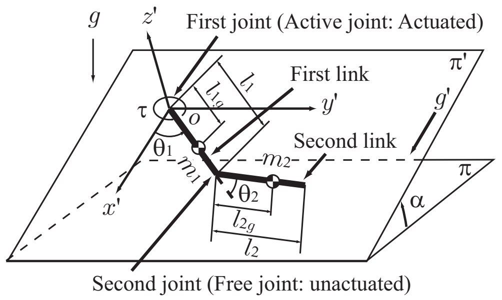


Figure 7 – Analytical model of an underactuated manipulator.

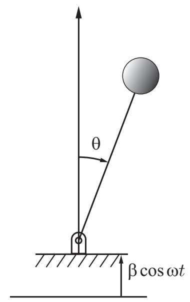


Figure 8 – Inverted pendulum under vertically highi-frequency excitation

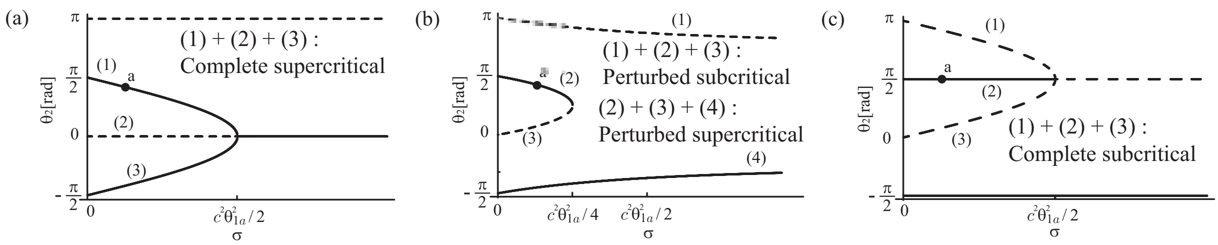


Figure 9 – Strategy of motion control for the underactuated manipulator: swing-up and stabilization by actuating the perturbation of pitchfork bifurcations (solid line: stable, dashed line: unstable) ((a) $\theta_{1off} = 0$**, (b)** $\theta_{1off} = \pi/4$**, (c)** $\theta_{1off} = \pi/2$**) .**

second link (free link) is expressed in dimensionless form as

$$\ddot{\theta}_2 + \mu\dot{\theta}_2 - \theta_{1a}(1 + c\cos\theta_2)\cos t^* \\ + c\theta_{1a}^2 \sin\theta_2 \sin^2 t^* + \sigma\sin(\theta_{1a}\cos t^* + \theta_{1off} + \theta_2) = 0, \quad (13)$$

where $(\dot{\ })$ denotes the derivative with respect to dimensionless time, $t^* = t \cdot \omega$, and the dimensionless parameter values corresponding to the subsequent experiment are $c = 1.49, \sigma = 1.43 \times 10^2/\omega^2$, and $\mu = 0.850/\omega$, where the excitation frequency of the first link ω is variable (Yabuno et al, 2005).

Proposition of the motion control method

By applying the method of multiple scales, Eq. (13), which is a nonautonomous system, can be transformed into the autonomous differential equation for the case of $0 < \sigma << 1$:

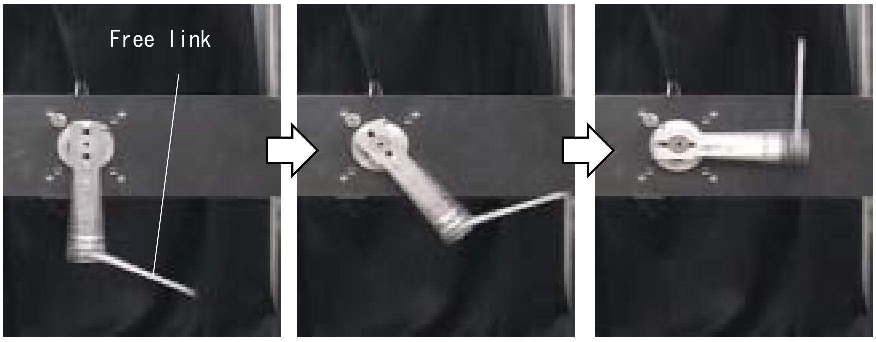
$$\ddot{\theta}_2 + \mu\dot{\theta}_2 + \sigma\sin(\theta_{1off} + \theta_2) - \frac{c^2\theta_{1a}^2}{2}\sin\theta_2\cos\theta_2 = 0. \quad (14)$$

The bifurcation diagram obtained from Eq. (14) is perturbed by the magnitude of the offset of the excitation θ_{1off}, as shown in Fig. 9 (a), (b), and (c). The three branches of (1) in Figs. 9(a) and (2) in Fig. 9(b) and (c), which are stable steady states, are smoothly connected by a continuous change of the value of θ_{1off} from 0 to $\pi/2$. Therefore, the high-frequency excitation indicated by point *a* in Fig. 9, i.e., $\sigma < c^2\theta_{1a}^2/4$, causes the swing-up of the second link to the upright position and stabilize the free link at the upright position.

We experimentally confirm the validity of the proposed method. The swing-up and the stabilization of the free link are carried out as shown in Fig. 10, by applying the high-frequency excitation and by the actuation of the perturbation, i.e., by the change of the offset of the excitation θ_{1off}.

SUMMARY

We proposed stabilization control methods and motion control method. In these methods, the slowly-varying dynamics are derived from the method of multiple scales and center manifold theory for the problems of magnetically levitated body and the railway vehicle wheelset, respectively. In the study of underactuated manipulator, in order to regard the motion of the free link as slowly-varying dynamics, we apply high-frequency excitation and actuate the perturbation of the resulting bifurcation to emerge the various stable steady state. In this paper, we introduce stabilization control methods by using nonlinear effects and motion control by producing the bifurcation phenomena and actuating their perturbation. As the systems to be controlled become more complex, it becomes more important to positively utilize inherently existing nonlinearlity in mechanical systems.



(a) $\theta_{1off} = 0$ (supercritical PF) (b) $\theta_{1off} = \frac{\pi}{4}$ (perturbed PF) (c) $\theta_{1off} = \frac{\pi}{2}$ (subcritical PF)

Figure 10 – Swing-up and stabilization of the underactuated manipulator ($\omega/(2\pi) = 45$Hz).

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