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Dynamics and Control of Robots with Parallel Kinematical Structures

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Abstract: A new approach for the identification of friction and rigid-body dynamics of complex parallel kinematical structures is presented in this paper. The approach is based on optimal excitation trajectories with the linear estimation theory as the mathematical background. The trajectories are bounded in such a way that they are easy to be fit into the small and hard constrained workspace of PKM.

Keywords: robotics, closed kinematical chains, model-based control, computed control

INTRODUCTION

Due to their precision, stiffness and dynamics parallel kinematical machines (PKM) are becoming more and more interesting in the field of machine tools and robots. Model-based control algorithms are necessary to take advantage of the possibilities offered by such structures. To utilize multi-body models for control, the dynamic model must be efficiently formulated in order to meet real-time requirements. Furthermore, the model parameters like masses and moments of inertia as well as friction parameters must be known. If they are not given from design data, they have to be experimentally identified. Till now, this experimental determination is restricted to simple models and adaptive control algorithms (e. g. Honegger *et al.* (2000), Sirouspour and Salcudean (2001)).

Therefore, a new approach for the identification of friction and rigid-body dynamics of complex parallel kinematical structures is presented in this paper. The approach is based on optimal excitation trajectories. The trajectory are bounded, such they are easy to be fit into the small and hard constrained workspace of PKM.

Two Examples for Parallel Kinematical Machines

Hydraulic Hexapod (MSI)

The hexapod MSI is a test stand to simulate vibrations and movement appearing in vehicles. The actuators are hydraulic cylinders with a maximum force of 15 kN each. To reduce forces acting in the joints the platform is a lightweight construction. Therefore, the maximal acceleration of HSP is larger than 50 g. The valves are able to realise frequencies of more than 50 Hz. The maximal speed is limited to 1 m/s by the size of the valves.

Fig.1 shows the test stand both as a MBS model and as a physical realisation. The table left to the platform is used to fix the test samples. The platform is shown in centred position. Some necessary hydraulic components like pipes and accumulators are not included in the MKS model.

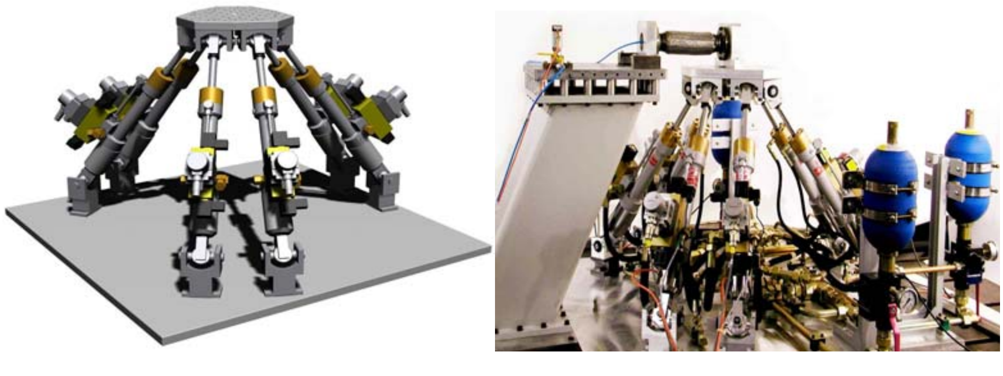

Figure 1 – Hydraulic Stewart-Gough-platform of the MZH: MBS model and test bed.

Contrary to most hydraulic test beds the workspace is not limited by the hydraulic actuators. Therefore, each TCP-platform position must be checked with respect to the workspace. That means, the control unit includes some safety features, like path tracing errors and other safety functions.
As already mentioned, the platform is used in a standard regime as a test bed for frequencies of about 5 Hz and amplitudes of about 25 mm. In applications with only small forces compared to the power of the actuators the dynamic forces have only limited influence due to the resulting low accelerations. It follows that the accuracy of the platform is mainly appointed by the accuracy of one single actuator. In order to reduce the path error the model of the hydraulic actuator has to be improved. Therefore, the hydraulic actuator model is more important than the dynamic ones of the parallel mechanism. That means, the control of the HSP can be reduced to a non-linear decoupled single axis controller. The actuator speed is used for a feedforward control.

Hexapod with linear direct drives (PaLiDA)

The hexapod PaLiDA uses electrical direct linear drives. They have several advantages compared to conventional ball screw drives: reduced mechanical components, no backlash and low inertia with a minimized number of wear parts. Furthermore, higher control bandwidth and extremely high accelerations can be achieved.

Fig. 2 shows both the MBS-model and the realised hexapod.

Figure 2 – Hexapod PaLiDa: MBS model and test bed.

For the application of direct linear drives in PKMs a compact design is essential when a linear motor is to be used for a strut variable in length. This is an argument for the cylindrical version of a linear motor, which on the one hand enlarges the magnetically active area to the whole magnet surface, and on the other hand compensates the permanent attraction force between the primary and the secondary part of the motor. This force is a considerable load for the guides in the bench-type construction.

A commercial electromagnetic linear motor, originally designed for fast lifting movements, is the basic element for the developed actuator. To use this motor as a PKM actuator, several modifications were necessary. It had to be enhanced guidance of the slider with minimized radial backlash.

The electronic control hardware is modified to realise a direct force control (current control) of the motor, which is a necessary condition for the implementation of model-based feedforward control. Except the current control, the drive control is implemented on a DSP card including the calculation of the actuator position from the position signal of the Hall-sensors. Using a simple PID-controller and the developed compensation strategies, the positioning accuracy of the actuator could be improved by factor 10. However, it has to be mentioned that the amount of information of the Hall-sensor is limited. An accuracy of approx. 20 µm can be achieved for the repeating accuracy for PTP (point to point) motions. Nevertheless, an external position measuring system has still to be chosen for applications that require absolute accuracy of less than 100 µm. The maximum force of a single actuator is 200 N.
The first prototype was designed after an evaluation of different construction alternatives (Toenshoff et al., 2000). The favoured solution is a cardan joint, which encloses the motor. Its platform has to be lightweight to reduce dynamic forces.

MODELING

Newton-Euler-equations in combination with d'Alembert's or Jourdain's principle, which are equivalent, are known to be highly efficient for the solution of the inverse dynamics of parallel robots (e. g. Zhang and Song (1993)). Therefore, an approach is utilized which combines Jourdain's principle in parameter-linear form with analytical rules for the determination of the minimal parameter set (Grotjahn *et al.* (2002)). The approach leads to very efficient C-code.

It yields a formulation of the rigid-body dynamics, which is linear with respect to the base parameter vector $\boldsymbol{p}_{\mathrm{rb,min}}$ that consists of the unknown and minimal inertial parameters of the mechanism:

$$\boldsymbol{Q}_{\mathrm{a,rb}} = \boldsymbol{A}_{\mathrm{rb}}\left(\boldsymbol{x},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}\right)\boldsymbol{p}_{\mathrm{rb,min}}, \tag{1}$$

where $\boldsymbol{Q}_{\mathrm{a,rb}}$ is the vector of the actuator forces caused by rigid-body dynamics. The matrix $\boldsymbol{A}\left(\boldsymbol{x},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}\right)$ comprises all kinematical quantities and is a function of the generalized coordinates, velocities and accelerations. These are chosen as position and orientation of the TCP-platform $\boldsymbol{x}$ and its velocities, respectively: $\dot{\boldsymbol{\theta}} = \left[{}_{(0)}\boldsymbol{v}_{\mathrm{E}}^{\mathrm{T}} \quad {}_{(0)}\boldsymbol{\omega}_{\mathrm{E}}^{\mathrm{T}}\right]^{\mathrm{T}}$.

Besides rigid-body dynamics, the losses in actuators, gears and bearings are taken into account by friction models. The losses are normally modeled as a force characteristic which is only a function of the joint velocity $\dot{q}_i$, e. g. by a sum of viscous damping and dry friction:

$$Q_{\mathrm{f}_i} = r_{1,i}\dot{q}_i + r_{2,i}\operatorname{sign}\left(\dot{q}_i\right). \tag{2}$$

These single joint friction models are not restricted to actuated joints. Friction models of passive joints can simply be taken into account by using Jourdain's principle. With the variables of all considered joints $\boldsymbol{q}$ and the corresponding friction forces $\boldsymbol{Q}_{\mathrm{f}}$, the effective actuator friction forces $\boldsymbol{Q}_{\mathrm{a,f}}$ result in

$$\boldsymbol{Q}_{\mathrm{a,f}} = \left(\frac{\partial\dot{\boldsymbol{q}}}{\partial\dot{\boldsymbol{q}}_{\mathrm{a}}}\right)^{\mathrm{T}}\boldsymbol{Q}_{\mathrm{f}} = \boldsymbol{J}^{\boldsymbol{T}}\left(\frac{\partial\dot{\boldsymbol{q}}}{\partial\dot{\boldsymbol{\theta}}}\right)^{\mathrm{T}}\boldsymbol{Q}_{\mathrm{f}}, \tag{3}$$

where $\dot{\boldsymbol{q}}_{\mathrm{a}}$ are the velocities of the actuated joints. Thus, the effective actuator friction forces depend directly on the actual configuration $\boldsymbol{x}$ since the Jacobian $\left(\partial\dot{\boldsymbol{q}}\ /\ \partial\dot{\boldsymbol{q}}_{\mathrm{a}}\right)^{\mathrm{T}}$ is a function of $\boldsymbol{x}$. By taking previous equations into account, it is easy to formulate the effective friction forces in a parameter-linear form:

$$\boldsymbol{Q}_{\mathrm{a,f}} = \underbrace{\boldsymbol{J}^{\boldsymbol{T}}\left(\frac{\partial\dot{\boldsymbol{q}}}{\partial\dot{\boldsymbol{\theta}}}\right)^{\mathrm{T}}\left[\dot{\boldsymbol{q}} \quad \operatorname{sign}(\dot{\boldsymbol{q}})\right]}_{A_f}\underbrace{\begin{bmatrix}\boldsymbol{r}_1\\ \boldsymbol{r}_2\end{bmatrix}}_{p_{f,\min}} = \boldsymbol{A}_{\mathrm{f}}\left(x,\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}\right)\boldsymbol{p}_{\mathrm{f,min}}, \tag{4}$$

Combining equations (1) and (4) yields the final form of the integral dynamics

$$\boldsymbol{Q}_{\mathrm{a}} = \boldsymbol{Q}_{\mathrm{a,rb}} + \boldsymbol{Q}_{\mathrm{a.f}} = \boldsymbol{A}_{\mathrm{rb}}\left(\boldsymbol{x},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}\right)\boldsymbol{p}_{\mathrm{rb,min}} + \boldsymbol{A}_{\mathrm{f}}\left(\boldsymbol{x},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}\right)\boldsymbol{p}_{\mathrm{f,min}} = \boldsymbol{A}\left(\boldsymbol{x},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}\right)\boldsymbol{p}. \tag{5}$$

IDENTIFICATION APPROACH OF DYNAMICAL PARAMETERS

The main advantage of applying linear estimators is their computational efficiency. This explains their wide reputation in the identification of the dynamics for serial robotic manipulators (Grotjahn *et al.* (2001)). For parallel structures, linear estimators have also proven their efficiency in recursive Honegger *et al.* (2000) as well as in the general form (Grotjahn *et al.* (2004)).

Linear Parameter Estimation

The formulation of the estimation problem can be derived from (5) as

$$\underbrace{\begin{bmatrix}\boldsymbol{Q}_{\mathrm{a}}^{(1)}\\ \vdots\\ \boldsymbol{Q}_{\mathrm{a}}^{(N)}\end{bmatrix}}_{\boldsymbol{\Gamma}} = \underbrace{\begin{bmatrix}\boldsymbol{A}\left(\boldsymbol{x}^{(1)},\dot{\boldsymbol{\theta}}^{(1)},\ddot{\boldsymbol{\theta}}^{(1)}\right)\\ \vdots\\ \boldsymbol{A}\left(\boldsymbol{x}^{(N)},\dot{\boldsymbol{\theta}}^{(N)},\ddot{\boldsymbol{\theta}}^{(N)}\right)\end{bmatrix}}_{\boldsymbol{\Psi}}\boldsymbol{p} + \underbrace{\begin{bmatrix}\boldsymbol{e}^{(1)}\\ \vdots\\ \boldsymbol{e}^{(N)}\end{bmatrix}}_{\boldsymbol{\eta}}, \tag{6}$$

with the measurement vector $\boldsymbol{\Gamma}$, the information or regression matrix $\boldsymbol{\Psi}$ and the error vector $\boldsymbol{\eta}$ that accounts for disturbances. The formulation of PKM's dynamics in a linear form reduces the system complexity to a simple LP (*linear in its parameter*) model structure. The solution of the over-determined equations system (6) in a Least-Squares sense yields the estimation $\hat{\boldsymbol{p}}$ of the parameter vector

$$\hat{\boldsymbol{p}} = \min\left(\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta}\right) \quad \Rightarrow \quad \hat{\boldsymbol{p}} = \left(\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Gamma}. \tag{7}$$

This corresponds to an upper bound for estimation uncertainty (Grotjahn *et al.* (2004))

$$\frac{\left\| \boldsymbol{p}_{\min} - \hat{\boldsymbol{p}}_{\min} \right\|}{\left\| \boldsymbol{p}_{\min} \right\|} \le \mathrm{cond}(\boldsymbol{\Psi}) \frac{\left\| \boldsymbol{\eta} \right\|}{\left\| \boldsymbol{\Gamma} \right\|}, \tag{8}$$

where $\kappa = \mathrm{cond}(\boldsymbol{\Psi}) = \frac{\sigma_{\max}(\boldsymbol{\Psi})}{\sigma_{\min}(\boldsymbol{\Psi})}$ and $\sigma_{\max}(\boldsymbol{\Psi})$, $\sigma_{\min}(\boldsymbol{\Psi})$ are respectively the largest and smallest singular value of the information matrix. The condition number κ can be used as a criterion for parameter excitation (Swevers *et al.* (1996), GK92 Grotjahn *et al.* (2004)), since it is the quotient of the largest and smallest singular values of $\boldsymbol{\Psi}$. Its minimization yields uniform excitation of all elements of the parameter vector $\boldsymbol{p}$ and minimizes therefore the upper bound of the estimation uncertainty (8). Hence, the experiment design is the task to find a *N*-dimensional set of configurations (see (6)), such that the corresponding information matrix has a minimal condition number. In the case of an indirect identification approach, this set is composed of single discrete *N* configurations that resulted from *N* different and simple trajectories (Grotjahn *et al.* (2004)). In the direct estimation the *N* elements of the information matrix correspond to one continuous trajectory, which has to be designed to fulfill the optimality criterion of minimal condition number. It is also very important to notice, that this criterion is sensible in the deterministic framework, where the disturbances of the information matrix $\boldsymbol{\Psi}$ is assumed to be negligible in comparison to the measurement noise $\boldsymbol{\eta}$.

Input-Trajectory Optimization and Design of Experiments

Generally, excitation trajectories are obtained by means of nonlinear optimization with motion constraints (e.g. joint limitation, workspace, etc.). The mathematical description of the motion is crucial for success and computational efficiency of the optimization, because the trajectory parameters are the degrees of freedom of the optimization problem. By considering $\boldsymbol{\xi}$ as the vector regrouping such parameters, a trajectory in the space of minimal coordinates can be describes as following:

$$\Im(\boldsymbol{\xi}) = \left\{ \boldsymbol{x} \in R^6, \boldsymbol{f_t}(\boldsymbol{x}, t, \boldsymbol{\xi}) = \boldsymbol{0}, \forall t \right\}. \tag{9}$$

The optimal excitation trajectory has to minimize the resulted condition number and is associated with the optimal parameter set $\boldsymbol{\xi}_0$

$$\boldsymbol{\xi}_0 = \arg \min_{\boldsymbol{\xi}} \left(\mathrm{cond}(\boldsymbol{\psi}(\Im(\boldsymbol{\xi}))) \right), \tag{10}$$

with the respect to the constraints of workspace, as well as the kinematical and actuation constraints. For serial manipulators several approaches have been presented, which use different trajectory parametrization (Swevers *et al.* (1996)), which is equivalent to the functional expression of the holonomic constraint $\boldsymbol{f_t}(\boldsymbol{x}, t, \boldsymbol{\xi})$. The optimization (10) in the case of PKM is in general very challenging (Grotjahn *et al.* (2004), Abdellatif *et al.* (2004)), which is mainly due to the strong workspace constraints. Minimal slopes and tilting of the tool platform yield important reduction of the workspace. Using polynomial approach for optimal input design, such it is known from serial manipulators leads to huge computational effort and do not guarantee convergence or satisfying minimization of the condition number κ However, the approach presented in (Abdellatif *et al.* (2004)) has been successfully adapted for the dynamics of parallel kinematics. The excitation trajectories consist of a finite sum of harmonic sine and cosine functions in a form of a finite Fourier series (Swevers *et al.* (1996)). They are expressed in cartesian frame, since they present the minimal or general coordinates

$$f_{t,i}\left(x_i, t, \boldsymbol{\xi}^i\right) = x_i(t) - x_0^i - \sum_{k=1}^{n} \left(\frac{\mu_k^i}{k\omega_f} \sin(k\omega_f t) - \frac{\nu_k^i}{k\omega_f} \cos(k\omega_f t) \right). \tag{11}$$

Each general coordinate x_i corresponds to an appropriate trajectory parameter vector:

$$\boldsymbol{\xi}^i = \left[x_0^i, \ \mu_1^i, \ \cdots \ \mu_n^i, \ \nu_1^i, \ \cdots \ \nu_n^i \right]^{\mathrm{T}}. \tag{12}$$

The fundamental pulsation ω_f of the Fourier series is the same for all degrees of freedom. Thus the trajectory is periodic with a period $T_f = 2\pi / \omega_f$. The vector of all trajectory parameters $\boldsymbol{\xi}$ groups $\boldsymbol{\xi}^i$ and the fundamental pulsation ω_f Its dimension is equal to $6 \times (2 \times n + 1) + 1 = 12\,n + 7$ and depends only on n, which can be chosen arbitrarily. Fig. 3 depicts exemplarily one period of an optimized trajectory.

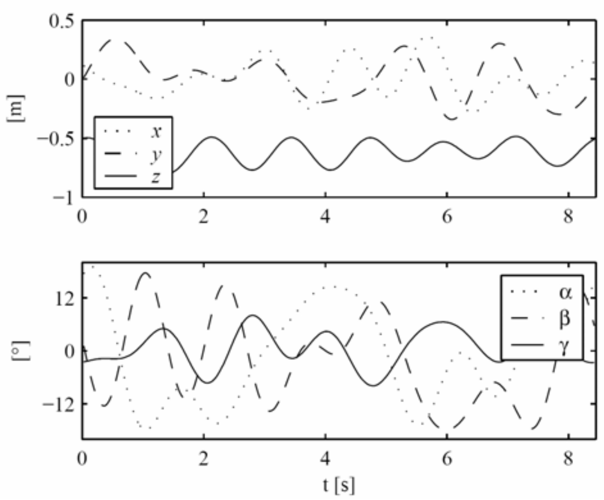

Figure 3 - Example of an optimized excitation trajectory in cartesian displacements (above) and tilting angles (below).

Indications on the Kinematical Computation

A further challenging point about the dynamic identification of complex parallel manipulators is the computation of minimal kinematical variables $\boldsymbol{x}$, $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$. The minimal coordinates are generally obtained by solving numerically the direct kinematics (Merlet (2000)). Conventionally, the computation of velocities and accelerations is done by numerical differentiation (Swevers *et al.* (1996), Grotjahn *et al.* (2004)). This yields in most of cases to important accumulation and amplification of numerical and measurement errors (Abdellatif *et al.* (2004)). The assumption of noise poor $\boldsymbol{\Psi}$ is not maintainable any more. Thus it is recommended to apply the methods suggested in (Swevers *et al.* (1996)) and in (Abdellatif *et al.* (2004)). Since the trajectory is available analytically (11), the time derivatives can be determined. By means of the forward kinematics, the obtained data $\boldsymbol{x}$ is approximated by a finite Fourier series with the same form as the desired motion given by (11). The parameters of the measured trajectory $\hat{\boldsymbol{\xi}}$ are estimated by applying LS-estimation and inserted in the time derivatives of $\boldsymbol{f_t}(\boldsymbol{x}, t, \boldsymbol{\xi})$ in order to obtain $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$. The efficiency and precision of the approach were demonstrated on experimental data in (Abdellatif *et al.* (2004)).

APPLICATION TO THE HEXAPOD PALIDA

The presented algorithms are applied to the Hexapod PaLiDA, an innovative Stewart-Gough- platform with linear direct drives as extensible struts, which was developed by the Institute of Production Engineering and Machine Tools at the University of Hannover (e. g. Tönshoff *et al.* (2002). The identified models are validated by a lot of different trajectories. The close agreement of measured and modeled actuator forces proves the capacity of the presented approach. The application to model-based feedforward control yields significant reductions of tracking errors. Figure 4 shows the tracking errors of two actuators during a circular trajectory with a diameter of 0.3 m and endeffector path velocity of 1 m/s. The largest tracking errors are already reduced by the rigid-body model without friction. These deviations occur at the beginning and the end during acceleration and deceleration phases. However, the deviations during constant velocity phase still remain. These errors are reduced after incorporation of the friction model.

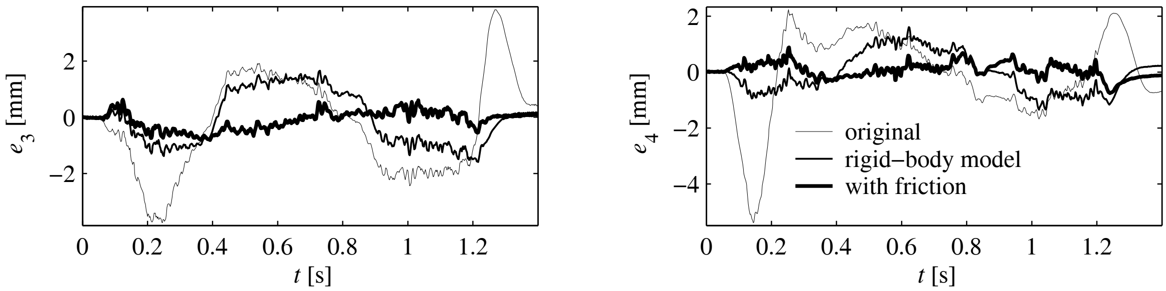

Figure 4 - Reduction of control errors by model-based feedforward control for a circular trajectory.

For the same trajectory, Cartesian path errors are also investigated by computing numerically the direct kinematics. The results are shown in Fig. 5, where the pose deviation Δx and the orientation error in respect of the second cardan-angle Δβ are depicted. In analogy to the actuator errors, the implementation of the identified model yields excellent reductions of path errors.

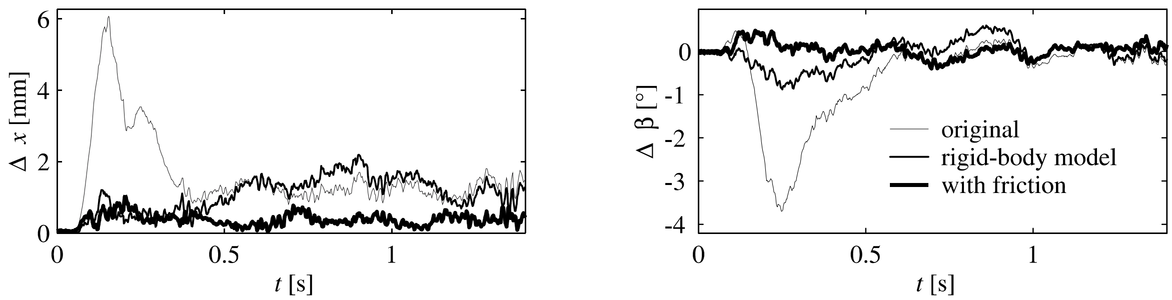

Figure 5 - Improvement of positioning accuracy by model-based feedforward control for a circular trajectory.

CONCLUSION

In this paper a new approach is presented that makes the identification of the dynamics of complex parallel mechanisms possible. The presented identification methodology of the dynamic parameters for parallel manipulators is based on a direct approach. Hereby, only single optimised trajectories are needed for well conditioned measurements and therefore good estimation results. The paper focused also on the differences to serial manipulators to give an overview about the main challenges, which have to be considered and solved for parallel manipulators. The success of the method, in terms of trajectory design, as well as the evaluation and parameter identification was proven by experimental results, achieved on a hexapod platform. The developed methodology and the obtained good results should contribute to advances in the identification of parallel mechanisms, towards closing the gap in this research area. The utilization of the identified model for model-based control yields significant reductions of tracking errors and, consequently, improvements of path accuracy. Accurate models of rigid-body dynamics are crucial for the accuracy improvement while motion with high dynamics. Meanwhile friction has to be considered to reduce errors in phases with constant velocity.

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