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ON THE PARAMETER ESTIMATION OF AN INTERNAL VARIABLE BASED VISCOELASTIC MODEL

D.A. Castello [1], F.A. Rochinha [2], N. Roitman [3] and C. Magluta [3]

[1] Theoretical and Applied Mechanics Laboratory, UFF, Niterói, Brasil, castello@vm.uff.br

[2] Solid Mechanics Laboratory COPPE/UFRJ, Rio de Janeiro, Brasil,rochinha@adc.coppe.ufrj.br

[2] Structures Laboratory, COPPE/UFRJ, Rio de Janeiro, Brasil, roitman@labest.coc.ufrj.br and magluta@labest.coc.ufrj.br

Abstract: The present work is aimed at modelling and characterizing viscoelastic materials. A constitutive equation for viscoelastic materials, in time domain, is proposed based on the concepts of internal variables and the thermodynamics of irreversible processes. The proposed constitutive equation is capable of dealing with common viscoelastic behavior such as creep and relaxation phenomena. Once one has chosen the parsimony of the model, a finite element model of the system, which is parameterized by a set of constitutive parameters, is built. The constitutive parameters required to describe the dynamic behavior of the viscoelastic material are estimated by means of the solution of the associated inverse problem which was formulated in frequency domain. The inverse problem has been solved by means of a modified Levenberg-Marquardt technique. The effectiveness of the proposed approach has been evaluated through experimental data obtained out of a viscoelastic sandwich beam. The structure was instrumented with an electromechanical shaker and three piezoelectric accelerometers. The frequency response function of the first accelerometer was used for the estimation process. The second and third accelerometer data were used in the validation step.

Keywords: Viscoelasticity, Constitutive Equation, Levenberg-Marquardt

1 INTRODUCTION

Nowadays, modeling plays a crucial role in controlling and optimizing industrial process by providing means of better understanding the involved phenomena and of improving the capacity of predicting future behavior, probably its key feature. The use of such models is mainly based on computational simulations, giving rise to two shortcomings concerning the reliability of the results: numerical pitfalls inherent to approximation methods, and uncertainties associated to non modeled dynamics and to parameters values. This last drawback can be alleviated by System Identification, which consists of the process of improving a mathematical model for a real system by combining physical principles with experimental or field data. The main idea is to identify a set of parameters such that, over a desired range of operating conditions, the model outputs are close, in some well-defined sense, to the system outputs, when both are submitted to the same inputs. Due to the incompleteness of available information and unavoidable measurement errors, system identification only achieves an approximation of the actual system.

The present work is motivated by the need of improving the capability of predicting the mechanical response of damping materials when applied to control the level of vibrations in structures or mechanical components. Usually those materials present a viscoelastic behavior, which is the main focus of the present work.

The dissipation mechanisms inherent to those materials are tied to chemical micro-structure and, therefore, a viscoelastic constitutive equation could be derived from a multiscale perspective. The main drawback of this approach would be the computational effort that a multiscale computation takes, which can lead to prohibitive costs for analyzing real applications. Taking this in consideration several phenomenological models have been proposed in order describe the viscoelastic behavior in terms of macro variables [6], [10] and [11] . Those models introduce a set of parameters that must be identified, which represents the core of the present paper.

A constitutive equation for viscoelastic materials, in time domain, is proposed based on the concepts of internal variables and the thermodynamics of irreversible processes. The proposed constitutive equation is capable of dealing with common viscoelastic behavior such as creep and relaxation phenomena [6], [2] and [10].

The constitutive parameters required to describe the dynamic behavior of the viscoelastic material are estimated by means of the solution of the associated inverse problem which was formulated in frequency domain. The inverse problem has been solved by means of the Levenberg-Marquardt technique [4].

The effectiveness of the proposed approach has been evaluated through experimental data obtained out of a viscoelastic sandwich beam. The structures were instrumented with an electromechanical shaker and three piezoelectric accelerometers. The frequency response function of the first accelerometer was used for the estimation process. The second and third accelerometer data were used in the validation step.

The article is organized as follows. The second section presents the proposed constitutive equation. The third section presents the Levenberg-Marquardt parameter estimation technique. The fourth section presents an example. Lastly, the fifth section presents the concluding remarks and future works.

CONSTITUTIVE EQUATION

Aiming at proposing a constitutive equation for viscoelastic materials the Thermodynamics of Irreversible Processes have been considered along with the concept of internal variables [2] and [6]. Considering a small strain thermomechanical process, the free energy function ψ and the pseudo-potential of dissipation φ were chosen as follows

$$\psi(\varepsilon(\mathbf{x},t),\xi_1(\mathbf{x},t),\ldots,\xi_I(\mathbf{x},t)) = \frac{1}{2\rho}\left[E\,\varepsilon(\mathbf{x},t)\cdot\varepsilon(\mathbf{x},t) + \sum_{r=1}^{I} E_r\,(\varepsilon(\mathbf{x},t)-\xi_r(\mathbf{x},t))\cdot(\varepsilon(\mathbf{x},t)-\xi_r(\mathbf{x},t))\right] \tag{1}$$

$$\varphi(\dot{\varepsilon}(\mathbf{x},t),\dot{\xi}^1(\mathbf{x},t),\ldots,\dot{\xi}^I(\mathbf{x},t)) = \frac{1}{2\rho}\left[\eta\,\dot{\varepsilon}(\mathbf{x},t)\cdot\dot{\varepsilon}(\mathbf{x},t) + \sum_{r=1}^{I}\eta_r\,\dot{\xi}^r(\mathbf{x},t)\cdot\dot{\xi}^r(\mathbf{x},t)\right] \tag{2}$$

where ρ is the specific mass, E,E_1,...,E_I and η,η_1,...,η_I are constitutive material parameters, ε is the total strain tensor and ξ_1,...,ξ_I are the internal variables tensors. Once the free energy function ψ and the pseudo-potential of dissipation φ had been chosen one can obtain the constitutive equation by means of the fulfillment of the Clausius-Duhem Inequality. Therefore, the constitutive equation renders as follows

$$\sigma(\mathbf{x},t) = E\,\varepsilon(\mathbf{x},t) + \sum_{r=1}^{I} E_r\,(\varepsilon(\mathbf{x},t)-\xi_r(\mathbf{x},t)) + \eta\,\dot{\varepsilon}(\mathbf{x},t) \tag{3}$$

$$\dot{\xi}_r(\mathbf{x},t) = b_r\,(\varepsilon(\mathbf{x},t)-\xi_r(\mathbf{x},t)), \quad r = 1,\ldots,I \tag{4}$$

where σ is the stress tensor and the parameter b_r is defined as the inverse of the relaxation time and it is defined as follows

$$b_r = \frac{E_r}{\eta_r} \tag{5}$$

It should be emphasized that the constitutive equations (3) and (4) should be able to reproduce some common dynamic behaviour of viscoelastic materials such as creep and relaxation. The ability of these constitutive equations to reproduce such phenomena can be shown through some mathematical manipulations with equations (3) and (4) [2]. The physical meaning of the constitutive parameters E, E_r and b_r, $r = 1,\ldots,I$, can be easily understood by means of the the stress relaxation response of a one-dimensional system whose constitutive equation is given by equations (3) and (4). Such a stress relaxation response is shown in equation (6)

$$\sigma(t) = E\,\varepsilon_0\left[1 + \sum_{r=1}^{I}\Delta_r\,e^{-b_r t}\right] \tag{6}$$

where Δ_r is defined as the ratio of E_r and E. From equation equation (6) one can conclude that Δ_r and b_r are associated to the magnitude of relaxation and to the inverse of the relaxation time of the r-th internal variable. Another aspect that should be highlighted is the fact that such constitutive equations are able to reproduce the behaviour of a viscoelastic material whose loss factor is approximately uniform over a certain frequency range.

2 PARAMETER ESTIMATION

In order to fully characterize a mechanical system it is required to estimate a set of unknown parameters which is representative to its dynamics. Therefore, for the sake of simplicity, it is defined a vector $\mathbf{p}$, which contains information concerning all the unknown parameters of a system, as follows

$$\mathbf{p} = \{p_1, p_2, \ldots, p_{N_p}\}^T \tag{7}$$

where N_p corresponds to the number of unknown parameters. In the inverse problem formulation one considers that the set of parameters $\mathbf{p}$ is unknown and that there is available a set of experimental data concerning the response of the system $\mathbf{y}^E(t)$ to a certain excitation/stimulus within the period of time $[0,t_f]$. The basic idea behind the inverse problem formulation is to find the set of parameters $\mathbf{p}$ that best correlates the response $\mathbf{y}(t)$, which is obtained from the mathematical model of the system under study, with the experimental response $\mathbf{y}^E(t)$, when they are subject to the same excitaion/stimulus. Therefore, it is required to define a function S to measure the difference between these two responses $\mathbf{y}^E(t)$ and $\mathbf{y}(t)$. If one assumes the hypothesis that the measurement errors have zero mean, constant variance, Gaussian distribution and that they are additive and non-correlated, the error function S that provides the minimum variance estimates is the ordinary least squares norm defined as follows [1] and [4]

$$S(\mathbf{p}) = [\bar{\mathbf{Y}} - \mathbf{Y}]^T [\bar{\mathbf{Y}} - \mathbf{Y}] \tag{8}$$

where $(\bullet)^T$ indicates the transpose of $(\bullet)$ and $\bar{\mathbf{Y}}$ and $\mathbf{Y}$ contain information about the experimental and the estimated responses of the system respectively and are defined as follows

$$\bar{\mathbf{Y}}^T = \{\bar{\mathbf{Y}}_{1^T}, \ldots, \bar{\mathbf{Y}}_{N_t}^T\} \tag{9}$$

$$\mathbf{Y}^T = \{\mathbf{Y}_1^T, \ldots, \mathbf{Y}_{N_t}^T\} \tag{10}$$

where N_t corresponds to the total number of measured instants of time. The column vectors $\bar{\mathbf{Y}}_j$ and $\mathbf{Y}_j$ contains experimental and estimated information respectively and they are organized such that $[\bar{\mathbf{Y}}_j]_s$ and $[\mathbf{Y}_j]_s$ represent measurements of the s-th sensor taken at the j-th instant of time. It should be emphasized that the error function S can be defined in different ways [5] and a broader definition could have been adopted, for example, as follows

$$S(\mathbf{p}) = \int_0^{t_f} \mathbf{Z}(\mathbf{y}^E(t), \mathbf{y}(t), t)\, dt \tag{11}$$

where the function $\mathbf{Z}$ could be simply the difference between the experimental and the estimated response or a more complex function . A representation such as the one shown in (11) considers that the measured data can be approximated as being continuous [4]. In the present article the ordinary least squares norm (8) will be adopted as the error function.

Therefore, once the error function had been properly defined, the inverse problem consists in determining the set of parameters which minimizes such a function, viz.

$$\min_{\mathbf{p}} S(\mathbf{p}) \qquad \mathbf{p} \in \mathscr{P} \tag{12}$$

where every constraint associated to the inverse problem is represented by the solution set $\mathscr{P}$.

The inverse problem defined in (12) will be solved, in the present article, by means of the Levenberg-Marquardt method, which corresponds to a powerful iterative method for solving nonlinear least squares problems of parameter estimation [1] and [4]. This technique was first derived by Levenberg [7] in 1944. Later, in 1963, Marquardt [8] derived the same technique by a different approach. The Levenberg-Marquardt technique tends to the Steepest Descent Method at the neighborhood of the initial guess used for the iterative procedure and tends to the Gauss Method at the neighborhood of the minimum of the ordinary least squares norm. AIming at minimizing the functional S in equation (8) one has to obtain the derivative of $S(\mathbf{p})$ with respect to the set of unknown parameters $\mathbf{p}$, and then equals such derivative to zero, i.e., the optimality condition is given as follows

$$\frac{\partial S(\mathbf{p})}{\partial p_j} = 0, \qquad j = 1, \ldots, N_p \tag{13}$$

The optimality condition in (13) can be rearranged in matrix notation as follows

$$\nabla S(\mathbf{p}) = -2\mathbf{J}[\bar{\mathbf{Y}} - \mathbf{Y}(\mathbf{p})] = \mathbf{0} \tag{14}$$

where the matrix $\mathbf{J}$ is the Sensitivity Matrix whose components, named as sensitivity coefficients, are defined as follows

$$J_{ij} = \frac{\partial Y_i}{\partial p_j}, \qquad i = 1, \ldots, N_s \times N_t \quad \text{and} \quad j = 1, \ldots, N_p \tag{15}$$

The iterative procedure of the Levenberg-Marquardt method is given by

$$\mathbf{p}^{k+1} = \mathbf{p}^k + [\mathbf{J}^{k^T}\mathbf{J}^k + \mu^k\Gamma^k]^{-1}\mathbf{J}^{k^T}[\bar{\mathbf{Y}} - \mathbf{Y}(\mathbf{p}^k)] \tag{16}$$

where $\mathbf{J}$ is the Sensitivity Matrix, μ is a stabilization parameter, Γ is a diagonal matrix and the superscript $(\bullet)^k$ denotes the iteration number. The purpose of the term $\mu^k\Gamma^k$ in equation (16) is to reduce the oscillations or instabilities due to the ill-conditioning associated to the problem. The decrease of these instabilities or oscillations can be achieved by adopting a matrix $\mu^k\Gamma^k$ whose components are relatively large as compared to the components of the matrix $\mathbf{J}^T\mathbf{J}$ [4]. At the beginning of the iterative process a large parameter μ is chosen and the Levenberg-Marquardt Method tends to the Steepest Descent Method. The parameter μ is gradually reduced as the iterative process approaches the solution of the problem and then the Levenberg-Marquardt Method tends to the Gauss Method. The parameter μ^k is chosen such that $S(\mathbf{p}^{k+1}) < S(\mathbf{p}^k)$ remains valid at every iteration. The stopping criteria adopted for the iteration process are the ones suggested by Dennis and Schnabel [3] as follows

$$S(\mathbf{p}^{k+1}) < \varepsilon_1 \tag{17}$$
$$||\,\mathbf{J}(\mathbf{p}^{k+1})^T\,[\bar{\mathbf{Y}} - \mathbf{Y}(\mathbf{p})]\,|| < \varepsilon_2 \tag{18}$$
$$||\,\mathbf{p}^{k+1} - \mathbf{p}^k\,|| < \varepsilon_3 \tag{19}$$

where ε_1,ε_2 and ε_3 are user-prescribed and $||\bullet||$ corresponds to the Euclidean norm. Different versions of Levenberg-Marquardt method can be found in the literature, depending on the choice the diagonal matrix Γ and on the form chosen for the variation of the parameter μ [4]. For the present work it has been chosen the matrix Γ as follows

$$\Gamma^k = \mathrm{d}iag[\mathbf{J}^{k^T}\,\mathbf{J}^k] \tag{20}$$

Assuming that a set of experimental data $\bar{\mathbf{Y}}$ is available, the Levenberg-Marquardt algorithm used in the present work is detailed bellow [4]

1. $k = 0$ and $\mu^0 = 0.001$.
2. Solve the direct problem for the initial parameter vector $\mathbf{p}^k$.
3. Compute $S(\mathbf{p}^k)$.
4. Compute the sensitivity matrix $\mathbf{J}^k$ and Γ^k using the current parameter vector $\mathbf{p}^k$.
5. Solve the system of equations

$$[\mathbf{J}^{k^T}\,\mathbf{J}^k + \mu^k\,\Gamma^k]\Delta\mathbf{p}^{k+1} = \mathbf{J}^{k^T}\,[\mathbf{y}^E - \mathbf{y}(\mathbf{p}^k)] \tag{21}$$

6. Compute the new estimate $\mathbf{p}^{k+1} = \mathbf{p}^k + \Delta\mathbf{p}^k$
7. Solve the direct problem using the new estimate $\mathbf{p}^{(k+1)}$ in order find $\mathbf{y}(\mathbf{p}^{k+1})$. Compute $J(\mathbf{p}^{k+1})$.
8. If $S(\mathbf{p}^{k+1}) \geq S(\mathbf{p}^k)$, replace μ^k by $10\mu^k$ and return to step (4).
9. If $S(\mathbf{p}^{k+1}) < S(\mathbf{p}^k)$, adopt the new estimate $\mathbf{p}^{k+1}$, replace μ^k by $0.1\mu^k$ and return to item 4.
10. Check the stopping criteria (17), (18) e (19). The iterative process stops if any of the them is satisfied, otherwise, replace k by $k+1$ and return to item 3.

3 ILLUSTRATIVE EXAMPLES

3.1 Sandwich Beam

The system under analysis is a sandwich beam whose core is made of a viscoelastic material and whose sketch is shown in Figure (1). The base layer and the constraining layer are made of aluminium and the core of the sandwich is a viscoelastic tape produced by $3M$. The specification of the tape is 4950.

The length of the beams is $1.46m$. The system is instrumented with four piezoelectric accelerometers ($PCBSN$ 13575) placed at 1/4, 2/5, 1/2 and 3/4 of the beam length and with an electromechanical shaker collocated with accelerometer number one. The first layer, called base layer, is the only one which is connected to the support as shown in Figure(1). The base layer is hinged at both ends.

As a mathematical model of the system shown in Figure(1) is required to be used in the estimation process, a finite element model of this system was built [2]. This finite element model takes the constitutive equations (3) and (4) into account and the kinematics that was adopted for this model is shown in figure (2).

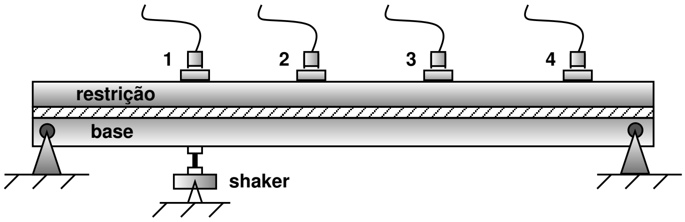

Figure 1 – Sketch of the viscoelastic sandwich beam.

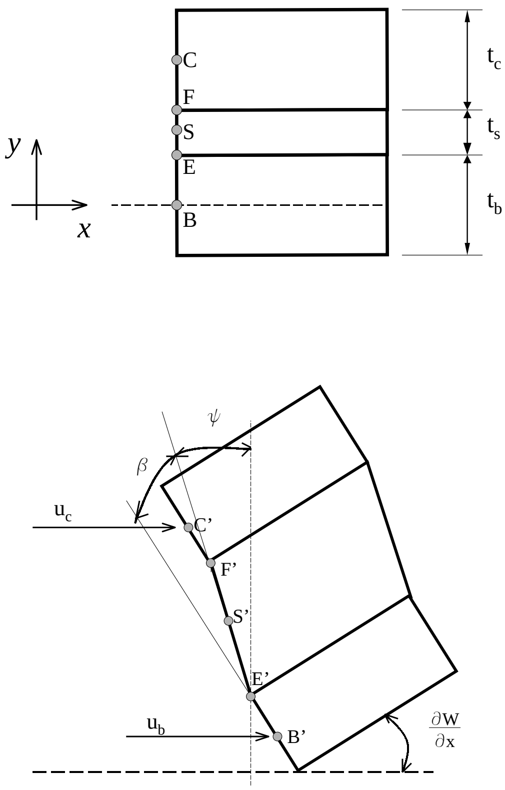

Figure 2 – Finite element model.

The estimation process considers the FRF of the first and second accelerometers within the band $(0-100)Hz$, containing 200 points each, and it was measured at a laboratory at 25 ^0C. The first analysis considers the viscoelastic model containing one internal variable and the parameters have been denoted as follows:

$$G = p_1 \times 10^6 \quad (22)$$

$$G_1 = p_2 \times 10^6 \quad (23)$$

$$b_1 = p_3 \quad (24)$$

As no test has been done previously in order to obtain initial estimates for the parameters it was considered a simple test to determine the order of magnitude of parameter G. The test that has been performed considered a sandwich beam similar to the under analysis but with an elastic core whose first three natural frequencies were evaluated for a set of values of G. It was concluded that these three natural frequencies are close to the information contained in the FRFs for values of G within (3.5,5.5) MPa. Such information was important to determine the initial guesses for G and G_r. Unfortunately the authors did not have a specific pretest for determining a suitable range of initial guesses for b_1. Three different initial guesses were tested, namely: $\mathbf{p}^{(0)} = \{5,1,1\}^T$, $\mathbf{p}^{(0)} = \{5,1,10\}^T$ and $\mathbf{p}^{(0)} = \{3,3,1\}^T$. All of the converged to the estimated vector $\hat{\mathbf{p}} = \{1.58, 10.82, 652.6\}^T$. Figure (3) graphs the experimental and estimated frequency response functions of the accelerometers 1 and 2 respectively.

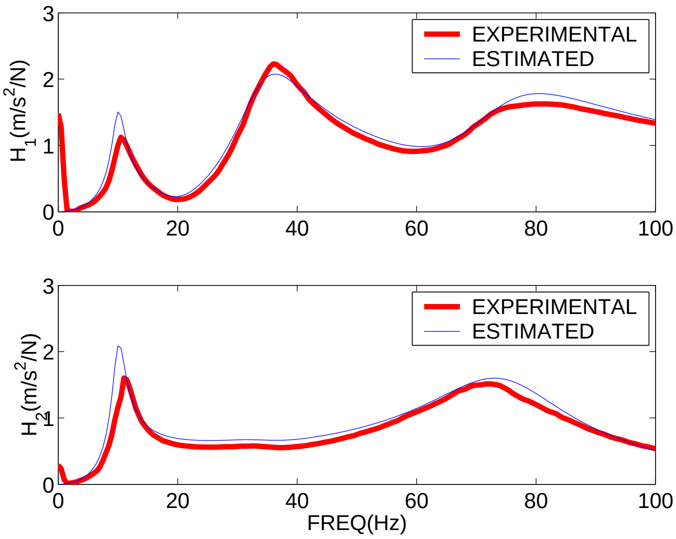

Figure 3 – Experimental an estimated FRFs for accelerometers 1 and 2.

From figure (3) one may conclude that the estimated and the experimental FRFs are in agreement. The L_2 norm of the difference of the experimental and estimated FRFs of the accelerometers 1 and 2 are 12.45 and 8.88, respectively.

In order to evaluate the effect of the inclusion a new internal variable it is considered a new model for the viscoelastic core whose dynamics is assumed to be described by two internal variables. The vector on unknown parameters may be defined as follows

$$G = p_1 \times 10^6 \tag{25}$$

$$G_1 = p_2 \times 10^6 \tag{26}$$

$$G_2 = p_3 \times 10^6 \tag{27}$$

$$b_1 = p_4 \tag{28}$$

$$b_2 = p_5 \tag{29}$$

It was used the experimental FRFs of the accelerometers 1 and 2 and the initial guess was chosen as follows $\mathbf{p}^{(0)} = \{2,2,2,1,1\}^T$. The estimated parameter vector is $\mathbf{p}^{(0)} = \{0.469, 2.14, 12.6, 59.95, 1090\}^T$. Figure (4) show the experimental and estimated FRFs of acceleromters 1 and 2, respectively. One can clearly see from figure (4) that the level of agreement between the estimated FRFs and the experimental ones is higher than the one shown in figure (3). The L_2 norm of the difference of the experimental and estimated FRFs of the accelerometers 1 and 2 for this 2 internal variable model are 11.88 and 7.14, respectively.

authors decided to con

Aiming at validating the provided results it is considered a new set of experimental data. The first validation considers time response of the first and third acceleromters when the system is excited with a sine-chirp sweeping the band (0,100) Hz. The validation for accelerometer number one is shown in figure (5) and for acclerometer number three in shown in figure (6). The responses graphed in figures (5) and (6) are in favor of the estimated parameters. Although the estimated responses provided by the one internal variable model and by the two internal variable model seems to be quite similar in figures (5) and (6) the model which best describes the system is the one which contains two internal variables. Such a conclusion can be obtained out of the comparison between figures (3) and (4) and by the H_2 norm of the differences between the experimental and estimated FRFs for these two models.

4 CONCLUDING REMARKS

The present work proposed a internal variable based constitutive equation to describe the dynamical behavior of viscoelastic materials. This constitutive equation is linear and it seems that it is able to describe common viscoelastic behavior such as creep and relaxation. The parameters that characterize the constitutive equation have been estimated by means of the classical Levenberg-Marquardt technique.

The suitability of the proposed constitutive equation has been assessed on a set of experimental data out of a sandwich beam whose core is made of viscoelastic material. The inverse problem has been formulated in

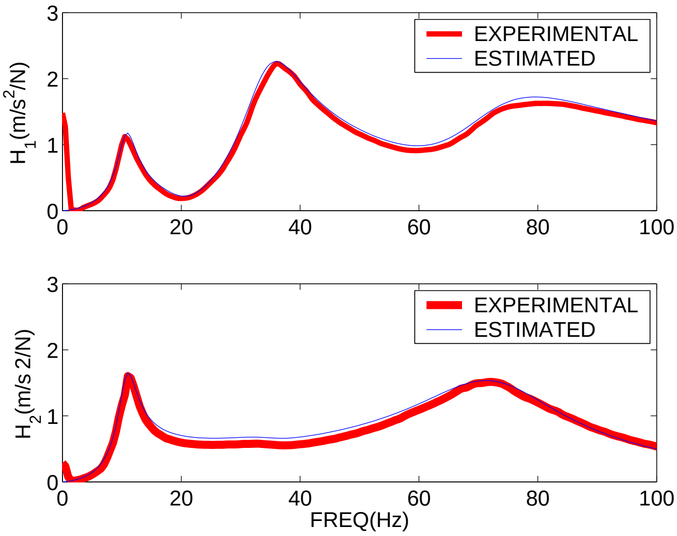

Figure 4 – Experimental an estimated FRFs for accelerometers 1 and 2.

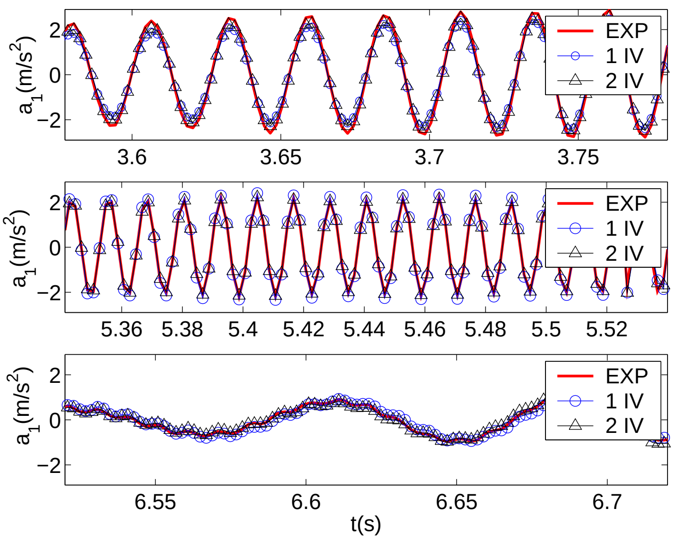

Figure 5 – Experimental and estimated time responses measured by accelerometer number one for a sine chirp excitation.

frequency domain and it has been used the frequency response function of two of the accelerometers within the band (0,100) Hz. As a means of validating the estimative obtained for the parameters, it has been been used the time domain response of two acceleroters due to a sine chirp excitation. The validation step showed agreement between the experimental and the predicted response.

The contribution of this work is to provide a constitutive equation for linear viscoelastic materials in time domain. As this constitutive equation is defined in time domain it is straightforward to build a time domain mathematical model of the system after the estimation of the constitutive parameters. Such a time domain model can be used to simulate the dynamical behavior of the system under different environmental conditions.

For future works the authors will: (i) perform estimation within wider frequency bands, (ii) consider time domain data for the estimation process and (iii) analyze a systematic way of determining the number of internal variables required to describe the dynamical behaviour of the system.

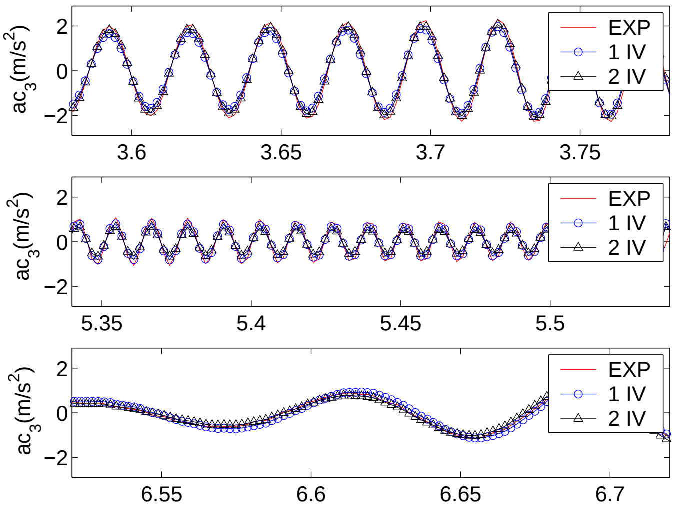

Figure 6 – Experimental and estimated time responses measured by accelerometer number three for a sine chirp excitation.

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