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Global optimisation of randomly excited structures with damage constraints.

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Abstract: This work presents a global optimisation strategy for randomly excited linear elastic structures with fatigue life constraints, based on coupling stochastic and deterministic methods involving sensitivity analysis. One of the originalities is the use of a method for evaluating eigen sensitivities and damage sensitivities. In the recent years, efficient new approaches for multiaxial random fatigue life problems have been developed using spectral methods producing reduced computational costs. These frequency formulations are well suited to random vibration problems and give a fast and accurate estimation of the structural fatigue life from the response power spectral densities (PSD) and more precisely the stress PSD. Thus by using spectral analysis properties an original formulation of damage sensitivities is presented according to the chosen frequency formulation of the Crossland's damage criterion. The estimation strategy appears to be computationally efficient because only one modal analysis is needed to obtain the overall damage sensitivities. This approach eliminates one of the essential difficulties in sensitivity analysis considering fatigue life of structures subject to random excitations, which is rarely attempted due to computational cost reasons.

Keywords: multiaxial fatigue, stochastic optimisation, sensitivity analysis, random vibrations

NOMENCLATURE

C = damping matrix ($N\times N$), N.s.m^{-1}
f = generalized excitation vector ($N\times 1$), N
F = random excitation force vector ($N\times 1$), N
H = system frequency response matrix ($n\times n$), m.N^{-1}
K = stiffness matrix ($N\times N$), N.m^{-1}
m_0 = zero order spectral moment
m_2 = second order spectral moment
M = mass matrix ($N\times N$), kg
n = number of considered modes, dimensionless
N = number of degrees of freedom, dimensionless
nel = number of elements, dimensionless
p = hydrostatic pressure, MPa
q = modal co-ordinate vector, dimensionless
u = mode shape matrix normalized with respect to the mass matrix, m
x = displacement vector ($N\times 1$), m
$\dot{\mathbf{x}}$ = velocity vector ($N\times 1$), m.s^{-1}
$\ddot{\mathbf{x}}$ = acceleration vector ($N\times 1$), m.s^{-2}

Greek Symbols

ζ = damping factor, %
λ = square of the natural frequency
Φ_f = power spectral density of the generalized force, N^2.Hz^{-1}
Φ_σ = stress power spectral density, MPa^2.Hz^{-1}
Φ_c = equivalent Von Mises stress power spectral density of the generalized force, MPa^2.Hz^{-1}
Φ_p = power spectral density of the hydrostatic pressure, MPa^2.Hz^{-1}
σ_c = Von Mises stress, MPa
σ_{cm} =mean of the Von Mises stress, MPa
$\boldsymbol{\sigma}$ = modal stress, MPa
ω = circular frequency of the undamped system, rad.s^{-1}

INTRODUCTION

The definition of the using conditions of a structure is an important task in the design stage of this one. Indeed in many cases the structural solicitations are not well defined due to the random nature of the structure environment. For example the structures such as bridges or buildings are subjected to the random kind of the wind or the earthquakes. A definition of these solicitations in the time domain appears to be complicated and on the other hand statistical properties of these loads can be defined. By assuming ergodic excitations, spectral analysis is used to deals with random vibration problems. The excitations are then fully described by their second order statistical properties: their mean value, their autocorrelation functions and also their power spectral density function (PSD). The PSD is a significant mathematical tool in random vibration analysis, giving the information on the signal frequency content as well as the signal variance.

In the light of this, some multiaxial failure criteria initially formulated in the time domain has been transposed in the frequency domain in order to deal with random vibration problems. These formulations have the advantage to be directly applicable after a spectral analysis and have proved to give accurate results and to produce drastic computer saving. An important recent contribution in this area was made by Pitoiset and Preumont (2000). Many multiaxial failure criteria have been proposed through the literature (Weber, 1999), however an optimisation process involving damage evaluations and also damage sensitivity analysis requires a computationally efficient and relatively accurate criterion for damage assessment technique. Among these criteria, a frequency formulation of the Crossland's damage criterion has been chosen as damage assessment technique. The time domain approach of this criterion (Crossland, 1956) based on a global approach has been validated for multiaxial periodic loads and appears to be one of the most widely used in high cycle fatigue.

Design sensitivity analysis of structures deals with the study of change in system response with respect to a design variable variation. Sensitivity analysis is particularly useful in various domains of mechanical engineering such as parameter identification, reanalysis of modified structures, dynamic analysis of large structures, control vibrations and structural optimisation with search direction. In this paper damage sensitivity is studied for structures subjected to random excitations. The frequency formulation of the Crossland's damage criterion requires the computation of the eigen solutions and random dynamic response of the structure. So in order to evaluate the damage sensitivities the determination of the eigen sensitivities and random dynamic response sensitivities are first needed and more precisely the random stress spectral moment sensitivities. Problems with light damping and distinct eigenvalues are considered and by using spectral analysis properties an original formulation of damage sensitivities is presented according to the chosen frequency formulation of the Crossland's damage criterion. The presented formulation assumes that the excitations on all degrees of freedom of the system are ergodic and not correlated with each other, giving computationally efficient results. The presented damage sensitivities will find their application in this paper through an optimisation process.

Indeed this work introduces a global optimisation strategy for randomly excited linear elastic structures with damage constraints, based on coupling stochastic and deterministic methods involving damage sensitivity analysis. We use a general algorithm combining local search, random perturbation and evolutionary iterations: a basic descent method is modified by the adjunction of a suitable random perturbation. Mathematical results of convergence may be found in the literature (Souza de Curzi and Pogu, 1994 or Souza de Curzi and Autrique, 1997). Population based versions leading to evolutionary algorithms and associated convergence results have also been introduced. The numerical experiments illustrate the determination of multiple solutions, the determination of minima lying on the boundary of the search region, the resolution of problems involving a few hundreds of unknowns. It is shown that the numerical performance can be improved by applying an efficient pure local search to the results furnished by the evolutionary algorithm.

In the first part of the paper we set the problem statement and give the framework for stochastic/deterministic hybridisation. Then tools for evaluation of the constraints and constraint gradients of the problem are reviewed: main conclusions of the random dynamic theory for linear system among which the definition of the random stress spectral moment are given and the frequency formulation of the Crossland's damage criterion is detailed. Next we introduce the calculation of the spectral moment sensitivities which will be used for the calculation of the damage sensitivities. Finally the global optimisation process is applied to a mechanical example. The influence of the parameters involved in the optimisation is discussed and conclusions are given.

OPTIMISATION PROCESS

Problem statement

An optimisation problem usually reads as:

$$\begin{cases} \min f(x) \\ \text{subject to} \quad g_j(x) \le 0 \end{cases}, \quad j \in J \tag{1}$$

$f: R^n \to R$ is the *objective function* to be minimised under the *feasible domain* defined by the *constraints* $g_j: R^n \to R$; $x \in R^n$ is a vector of unknowns, referred as *design variables*. Extensions to multiobjective optimisation may be found in the literature but will not be considered here.

Usually, the aim of structural optimisation is to determine the best compromise between the performance and the cost of a structure. In this work, we consider the best compromise between lightness and safety: the objective function is the volume V of the structure formed by a homogeneous material and the constraints correspond to a safe domain associated to the Crossland's failure criterion: they involve a damage evaluation on the whole structure. We assume that the three-dimensional structure is defined by a thickness map $h: \Omega \to R$, where $\Omega \subset R^2$ is a mean surface or the planar bottom of the structure. Such a problem involves infinite dimensional function spaces: in order to obtain a finite dimensional problem corresponding to the formulation (1), we introduce a finite element discretisation of the structure involving a given mesh: the constraints are evaluated at each element and our $n = nel$ design variables are the thicknesses h_e of the elements: we have $x=h=(h_1, \ldots, h_{nel})$. This leads to the following mathematical problem:

$$\min_h V(h) = \sum_{e=1}^{nel} h_e \times s_e \quad \text{subject to:} \quad \max_{e=1,\ldots,nel} D_e \le D_U \tag{2}$$

where h_e, s_e and D_e are respectively the thickness, the surface and the damage, evaluated with the frequency formulation of Crossland's damage criterion, of the element e. D_U is a predefined upper damage value.

Optimisation method

Usually, an optimisation problem is numerically solved by descent iterations, which starts at an initial guess and iteratively modify the actual value h^k of the design variables in order to get a feasible and more performant structure defined by the design variables $h^{k+1} = Q^k(h^k)$ (k is the iteration number and Q^k is the iteration function associated to the method). The constraints are usually treated by using either a dual approach (such as Uzawa's, for instance) or penalty methods. In this work, we shall consider a penalty approach, where the problem defined by the eq. (2) is approached by:

$$\min_h \; V_\lambda(h) = \sum_{e=1}^{nel} h_e \times s_e + \lambda P(h) \;\; ; \quad P(h) = \frac{1}{2}\sum_{e=1}^{nel}\left[\frac{(D_e - D_U) + \mathrm{abs}(D_e - D_U)}{2}\right]^2 \tag{3}$$

Here, $\lambda > 0$ is a parameter destined to be chosen large enough: let h be a solution of the eq. (2) and h_λ be a solution of the eq. (3). We have the inequalities $V(h_\lambda) \le V_\lambda(h_\lambda) \le V_\eta(h_\eta) \le V(h)$, for $\eta \ge \lambda$. Thus, the sequence of optimal volumes is increasing, upper bounded by the optimal value $V(h)$. It follows that, on the one hand, $P(h_\lambda) \to 0$ and, on the other hand, $\liminf V(h_\lambda) \le \limsup V(h_\lambda) \le \lim V_\lambda(h_\lambda) \le V(h)$. Thus, any cluster point of the family h_λ corresponds to a solution of the eq. (2): it is expected that, for $\lambda \to +\infty$, we get a solution of the eq. (2). In practice, λ is simply chosen large enough.

The optimisation problem defined by the eq. (3) involves also numerical difficulties connected to its nonconvexity: under the lack of convexity assumptions, iterative descent methods may converge to local minima and not to a global minimiser h_λ . In order to prevent such an event, we use a hybrid method involving random perturbations of a descent method: the standard descent iterations are modified as $h^{k+1} = Q^k(h^k) + \Pi^k$, where Π^k is a convenient random variable. Convergence results and convenient choices of Π^k may be found in the literature (Souza de Curzi and Autrique, 1997). We consider an evolutionary version of the random perturbations, analogous to the one introduced by Souza de Curzi and Gonçalves (2001). The standard descent method is the gradient method, with a step determined by Wolfe's rule.

The optimisation procedure involves many computations of the constraints but also of the constraint gradients; here the constraints have been defined as the damage of each element of the structure. Classical multiaxial damage assessment techniques usually produce high computer costs especially for randomly excited structure problems. This results in prohibitive computational costs when global optimisation considering damage is carried out. However the frequency formulation of the Crossland's damage criterion proposed by Pitoiset now allows to consider this problem. Moreover considering some assumptions, the evaluations of the damage gradients also appear to be computationally efficient so global optimisation considering damage can now be treated.

DAMAGE EVALUATION FOR RANDOMLY EXCITED STRUCTURES

Stress response PSD of structure subject to random loads

In this section only main conclusions are given but complete demonstration can be found in literature (Meirovitch, 1975). The finite element equation for a discrete vibration problem can be given as:

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{C}\,\dot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) = \mathbf{F}(t) \tag{4}$$

For a linear proportionally damped system (*i.e.* $\mathbf{C}$=a$\mathbf{K}$+b$\mathbf{M}$ with a and b real positive scalar values), the solution of equation (4) is sought as $\mathbf{x}(t) = \mathbf{u}\;\mathbf{q}(t)$ such that the mass normalised eigenvector matrix $\mathbf{u} = [\mathbf{u}_1\,\mathbf{u}_2 \ldots \mathbf{u}_n]$ satisfies:

$$\mathbf{K}\mathbf{u} = \mathbf{M}\,\mathbf{u}\Lambda \;\text{ with }\; \mathbf{u}^T\mathbf{M}\,\mathbf{u} = \mathbf{I}\,, \tag{5}$$

and $\mathbf{q}(t)$ satisfies:

$$\ddot{q}_r(t) + 2\zeta_r\omega_r\dot{q}_r(t) + \omega_r^2 q_r(t) = f_r(t) \qquad r = 1,\ 2, ..., n \tag{6}$$

since

$$\mathbf{u}^T\mathbf{K}\,\mathbf{u} = \Lambda = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_n^2 \end{bmatrix} \text{ and } \mathbf{u}^T\mathbf{C}\,\mathbf{u} = \begin{bmatrix} 2\zeta_1\omega_1 & & \\ & \ddots & \\ & & 2\zeta_n\omega_n \end{bmatrix} \tag{7}$$

where the subscript r is relative to the rth mode and n is the number of evaluated mode.

It is assumed that the structure is subject to white noise excitation of zero mean. A white noise process is characterized by a PSD with a constant value through the frequency range it has been defined, thus the excitation spectral matrix does not depend on ω. The extension to other form of PSD can be numerically treated by discretising

this one into constant parts. Moreover we assume that the white noise excitations on all degrees of freedom are not correlated to each other. Doing so the stress response PSD to a white noise excitation is defined as:

$$\Phi_\sigma(\omega) = \boldsymbol{\sigma}^T \mathbf{H}(\omega)\, \boldsymbol{\Phi}_\mathrm{f}\, \mathbf{H}^*(\omega)\, \boldsymbol{\sigma} \tag{8}$$

where $\mathbf{H}(\omega)$ and $\mathbf{H}^*(\omega)$ stand respectively for the frequency response functions matrix and its transposed complex conjugate and $\boldsymbol{\Phi}_\mathbf{f}$ is the matrix of the power spectral density of the generalized force ($\boldsymbol{\sigma}$ is the modal stress matrix).

A spectral analysis is carried out over a frequency range which usually results in a number n of calculated modes. n must be sufficiently large to capture the dynamic properties of the structure over the considered frequency range. So by using a subscripted notation the eq. (8) gets the form of a summation over n:

$$\Phi_\sigma(\omega) = \sum_r^n \sum_s^n \sigma_r \sigma_s \Phi_\mathrm{f}(r,s)\, H_r(\omega) H_s^*(\omega) \tag{9}$$

$$\text{with } H_r(\omega) = \frac{1}{\omega_r^2 - \omega^2 + 2i\zeta_r \omega \omega_r}$$

The zero and second order spectral moment of Φ_σ are obtained by integrating this expression over ω:

$$m_0(\Phi_\sigma(\omega)) = \sum_r^n \sum_s^n \sigma_r \sigma_s \Phi_\mathrm{f}(r,s) \int_{-\infty}^{\infty} H_r(\omega) H_s^*(\omega)\, d\omega \tag{10a}$$

$$m_2(\Phi_\sigma(\omega)) = \sum_r^n \sum_s^n \sigma_r \sigma_s \Phi_\mathrm{f}(r,s) \int_{-\infty}^{\infty} \omega^2\ H_r(\omega) H_s^*(\omega)\, d\omega \tag{10b}$$

In these two expressions, the cross correlation (when $r \neq s$) can be neglected by considering distinct eigenvalue problems with light damping (Preumont, 1994). We set the frequency range of interest for the sensitivity calculation to $[-\infty, +\infty]$, in order to further evaluate damage sensitivities by an analytical way. Hence the expressions of the spectral moments can be reduced and written as functions of the eigen frequencies, the modal stresses and Φ_f as:

$$m_0(\Phi_\sigma) = \sum_r^n \sigma_r^2 \times \Phi_\mathrm{f}(r,r) \times \int_{-\infty}^{\infty} \left|H_r(\omega)\right|^2 d\omega = \sum_r^n \sigma_r^2 \times \Phi_\mathrm{f}(r,r) \times \pi \Big/ 2\zeta_r \omega_r^3 \tag{11a}$$

$$m_2(\Phi_\sigma) = \sum_r^n \sigma_r^2 \times \Phi_\mathrm{f}(r,r) \times \int_{-\infty}^{\infty} \omega^2 \left|H_r(\omega)\right|^2 d\omega = \sum_r^n \sigma_r^2 \times \Phi_\mathrm{f}(r,r) \times \pi \Big/ 2\zeta_r \omega_r \tag{11b}$$

The frequency range and the considered number of modes act directly on the spectral moments values. However we only want to get a change indicator in the fatigue life due to a parametrical modification and not an exact value. Consequently the chosen frequency range of interest for the damage sensitivity calculation is not significant while the taken frequency range remains the same for each sensitivity calculation.

Frequency formulation of the Crossland's damage criterion

The Crossland's damage criterion was initially formulated in the time domain by Crossland (1956). It is based on the invariants of the stress tensor and on the deviatoric stress. The criterion assumes the structure reliability after a period T if the following inequality is satisfied at every point of the structure:

$$g(\sigma(t),T) = \frac{\sqrt{J_{2,a}} + \alpha \max p(t)}{\beta} \leq 1, \quad t \in [0,T] \tag{12}$$

$\sqrt{J_{2,a}}$ is the maximum amplitude of the second invariant of the stress deviator and this expression is related to the Von Mises stress $\sigma_c(t)$ and its mean value σ_{cm} by:

$$\sqrt{J_{2,a}} = \frac{1}{\sqrt{3}} \max_{0 \leq t \leq T} \left| \sigma_c(t) - \sigma_{cm} \right| \tag{13}$$

$p(t)$ is the hydrostatic pressure defined as a function of the first invariant of the stress tensor, α and β are material parameters function of endurance limits.

The frequency formulation proposed by Pitoiset (2000), partly relies on the peak factor theory and can be applied directly after a spectral analysis as classically performed in random vibration. Its application supposed that the structure is linear and subjected to stationary Gaussian loads of zero mean.

Over an observation period T the peak factor in our case allows to estimate the extreme value reach by a process based on the Von Mises stress $\sigma_c(t)$ and the hydrostatic pressure $p(t)$, thus:

$$\sqrt{J_{2,a}} \approx \frac{1}{\sqrt{3}}\sqrt{m_0(\Phi_c(\omega))}\ \eta_c \tag{14}$$

$$\max p(t) \approx \sqrt{m_0(\Phi_p(\omega))}\ \eta_p \tag{15}$$

$m_0(\Phi_c)$ and $m_0(\Phi_p)$ are respectively the zero order spectral moments of the Von Mises stress PSD and of the hydrostatic pressure PSD. The equivalent Von Mises stress PSD Φ_c and the PSD of the hydrostatic pressure $\Phi_p(\omega)$ can be calculated from the PSD matrix $\mathbf{\Phi}_\sigma(\omega)$ of the stress vector $\sigma(t)$ and the procedure for the evaluation of these terms is detailed by Segalman et al. (2000).

η_c and η_p stand for the peak factors of the PSD $\Phi_c(\omega)$ and $\Phi_p(\omega)$. The mean of the process peak factor could be approximated as a function of the process spectral moments by the following Davenport's expression (Davenport, 1964):

$$F(N_i) = \sqrt{2\ln(N_i)} + 0.5772 \Big/ \sqrt{2\ln(N_i)} \tag{16}$$

where N_i corresponds to the mean number of observed cycles along the process during the period T calculated as:

$$N_i = \frac{1}{2\pi}\sqrt{\frac{m_2(\Phi_i(\omega))}{m_0(\Phi_i(\omega))}}\ T \tag{17}$$

$m_2(\Phi_i(\omega))$ is the second order spectral moment of the process $\Phi_i(\omega)$. The obtained value for the criterion can be considered as the resulting damage after a period T. The relations between damage and expected fatigue life have been widely treated during the last century and the fatigue life theory will not be developed in this paper.

This frequency domain formulation appears to be computationally by far more efficient than the time domain formulation. Moreover this formulation can be expressed as a function of spectral moments which is particularly well suited to the damage sensitivity evaluation.

SENSITIVITY ANALYSIS

Random stress spectral moment sensitivity

In order to evaluate the random stress spectral moment sensitivities we need first to explain the frequency and mode shape sensitivities. The finite element analysis is used to carry out the sensitivity analysis. So a given structure is discretised and following the finite element method, a design variable p of the studied structure is slightly modified.

Deriving the eq. (5), we obtained:

$$\left(\mathbf{K} - \lambda_r \mathbf{M}\right)\frac{\partial \mathbf{u}_r}{\partial p} - \frac{\partial \lambda_r}{\partial p}\mathbf{M}\,\mathbf{u}_r = -\left(\frac{\partial \mathbf{K}}{\partial p} - \lambda_r \frac{\partial \mathbf{M}}{\partial p}\right)\mathbf{u}_r \tag{18}$$

$$\mathbf{u}_r^T \mathbf{M}\frac{\partial \mathbf{u}_r}{\partial p} + \frac{1}{2}\mathbf{u}_r^T \frac{\partial \mathbf{M}}{\partial p}\mathbf{u}_r = 0 \tag{19}$$

Then we can write:

$$\begin{bmatrix} \mathbf{K} - \lambda_r \mathbf{M} & -\mathbf{M}\,\mathbf{u}_r \\ -\mathbf{u}_r^T \mathbf{M} & 0 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \mathbf{u}_r}{\partial p} \\ \dfrac{\partial \lambda_r}{\partial p} \end{Bmatrix} = \begin{Bmatrix} -\left(\dfrac{\partial \mathbf{K}}{\partial p} - \lambda_r \dfrac{\partial \mathbf{M}}{\partial p}\right)\mathbf{u}_r \\ \dfrac{1}{2}\mathbf{u}_r^T \dfrac{\partial \mathbf{M}}{\partial p}\mathbf{u}_r \end{Bmatrix} \tag{20}$$

The eigenpair sensitivities can be found by solving eq. (20). Used for distinct or multiple eigenvalue problem the algorithm of the method is very simple, compact and numerically stable. The evaluation of the eigenpair sensitivities is carried out in a short time and only one modal analysis is necessary.

From eq. (10a), (10b) and (20) the spectral moment sensitivity due to the variation of the design variable p is as follows:

$$\frac{\partial m_0(\Phi_\sigma)}{\partial p} \approx \beta_p \tag{21a}$$

$$\beta_{pi} = \sum_{r=1}^{n} \frac{\pi \sigma_{ir} \Phi_{\mathrm{f}}(r,r)}{\xi_r \lambda_r^3} \times \frac{\partial \sigma_{ir}}{\partial p} - \frac{3\pi \sigma_{ir}^2 \Phi_{\mathrm{f}}(r,r)}{2\xi_r \lambda_r^5} \times \frac{\partial \lambda_r}{\partial p} + \frac{\pi \sigma_{ir}^2}{\xi_r \lambda_r^3} \times \sum_{k=1}^{N} u_{kr} \frac{\partial u_{kr}}{\partial p} f_r \tag{21b}$$

$$\frac{\partial m_2(\Phi_\sigma)}{\partial p} \approx \gamma_p \tag{22a}$$

$$\gamma_{pi} = \sum_{r=1}^{n} \frac{\pi \sigma_{ir} \Phi_{\mathrm{f}}(r,r)}{\xi_r \lambda_r} \times \frac{\partial \sigma_{ir}}{\partial p} - \frac{\pi \sigma_{ir}^2 \Phi_{\mathrm{f}}(r,r)}{2\xi_r \lambda_r^2} \times \frac{\partial \lambda_r}{\partial p} + \frac{\pi \sigma_{ir}^2}{\xi_r \lambda_r} \times \sum_{k=1}^{N} u_{kr} \frac{\partial u_{kr}}{\partial p} u_{kr} f_r \tag{22b}$$

Damage sensitivity

Assume that the material parameters α and β do not change for a design variable. We denote $\partial D/\partial p$ (nel×1) the damage change for the overall elements due to the variation of the eth design variable:

$$\frac{\partial D}{\partial p} = \frac{1}{\beta} \left[\frac{\partial \left(E\left[\sqrt{J_{2,a}} \right] \right)}{\partial p} + \alpha \frac{\partial \left(E[\max\, p(t)] \right)}{\partial p} \right] \approx \frac{1}{\beta\sqrt{3}} \frac{\partial \left(\sqrt{m_0(\Phi_c(\omega))}\, F(2N_c) \right)}{\partial p} + \frac{\alpha}{\beta} \frac{\partial \left(\sqrt{m_0(\Phi_p(\omega))}\, F(N_p) \right)}{\partial p} \tag{23}$$

Only the first part of the equation is provided for sake of brevity but the second part can be treating following the same way. Denoting $m_0(\Phi_c(\omega))=m_0$ and $m_2(\Phi_c(\omega))=m_2$, we can write:

$$\frac{\partial \left(\sqrt{m_0}\, F(N_c) \right)}{\partial p} = \frac{\partial \left(\sqrt{m_0} \right)}{\partial p} F(2N_c) + \sqrt{m_0} \frac{\partial \left(F(2N_c) \right)}{\partial p} \tag{24}$$

$$\frac{\partial \left(\sqrt{m_0} \right)}{\partial p} = \frac{1}{2\sqrt{m_0}} \frac{\partial (m_0)}{\partial p} \tag{25}$$

$$\frac{\partial (F(N_c))}{\partial p} = \frac{\partial (N_c)}{\partial p} \times \left(\frac{2\ln(N_c) - 0.5772}{2\sqrt{2}\, N_c \left(\ln(N_c) \right)^{\frac{3}{2}}} \right) \tag{26}$$

$$\frac{\partial (N_c)}{\partial p} = \frac{T}{4\pi} \frac{m_0 \frac{\partial (m_2)}{\partial p} - m_2 \frac{\partial (m_0)}{\partial p}}{(m_0)^2 \sqrt{m_2/m_0}} \tag{27}$$

The obtained distribution of the derivatives is relatively close to the exact distribution with cross correlations and exact frequency range taking into account. Its application is computationally efficient because only one modal analysis is needed to compute any damage derivatives.

APPLICATION

We apply the optimisation process to a steel rectangular plate with the following material properties:

$$E = 2.1\ 10^{+11} N/m^2;\ \rho = 7800\ kg/m^3;\ \nu = 0.3$$

The dimensions of the plate are 2.5m×3.5m with an initial thickness h uniform set to 5mm. We arbitrarily set D_U to 0.8. The plate is supposed simply fixed on its four edges. The endurance limits for the reversed tension stress and torsion stress f_{-1} and t_{-1} are stated after 2.10^6 cycles as:

$$f_{-1} = 252\ MPa;\quad t_{-1} = 182\ MPa$$

The plate is modelled by 36 eight-node square elements which produce no out-of-plane stress so the dimension d of the problem is equal to 36. The structure is subjected to a random load defined by a band limited white noise process (Figure 1), with Φ_{Fx} the PSD (constant) of the band limited white band process. The frequency range of interest is then set to [0, 1050] (Hz) and only the first mode is computed to capture the dynamic properties of the structure over the frequency range, according to the Modal Participation Factors (Géradin and Rixen,1998).

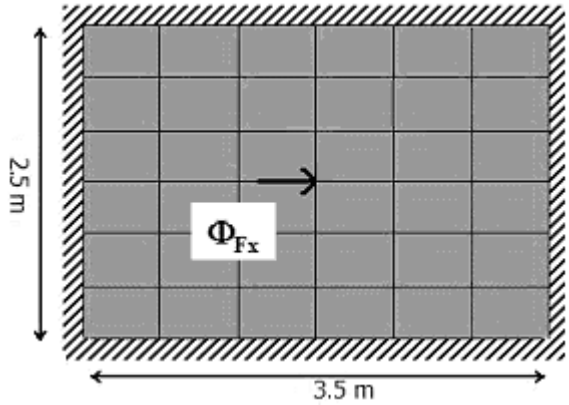


Figure 1 – Initial design of the plate

We denote n the number of iterations, and p, r and t are parameters usually used in evolutionary optimisation, they correspond respectively to the size of the population, the number of recombinations and the number of mutations. The parameter λ is set to 1000. Mathematical results have shown that increasing the values of these parameters will statistically lead to an improved solution. Here numerical experiments are carried out varying these parameters. Due to the random nature of the perturbations involved in the process the obtained solutions slightly vary, so ideally a number of runs would have to be carried out and a statistical study (mean and variance) leaded over the obtained results. Indeed even for this simple application, it is not possible to completely explore the optimisation strategy due to computational costs involved with the mechanical problem. That is why only one run is carried for each numerical experiment and we consider the result close to the expected mean result. We denote n_F and n_Q the number of evaluations of the objective function and the number of evaluations of the descent method. f_{opt} and g_{max} stand for the sum of h and for the corresponding maximum damage of the optimized structure. We illustrate the influence of the number of mutations, recombinations, iterations in the tables 1, 2, 3 and the influence of the size of the population in the table 4.

Table 1 – Results furnished for different numbers of mutation for n = 25, p = 5, r = 10

t	0	10	50	100	500
f_{opt} (mm)	0.063	0.059	0.058	0.059	0.057
g_{max}	0.800	0.800	0.799	0.800	0.800
Cpu (s)	10308	13332	29680	36439	158000
n_F	530	16400	6030	28030	138030
n_Q	1375	1375	1375	1375	1375

Table 2 – Results furnished for different numbers of recombination for n = 25, p = 5, t = 10

R	0	10	20	50	100
f_{opt} (mm)	0.073	0.059	0.059	0.058	0.058
g_{max}	0.800	0.800	0.800	0.799	0.794
Cpu (s)	146	13332	26286	61412	127790
n_F	1155	16400	31380	76380	151400
n_Q	105	1375	2625	6375	12625

Table 3 – Results furnished for different numbers of iteration for p = 5, r = 10, t = 10

n	1	5	10	25	50
f_{opt} (mm)	1.404	0.068	0.065	0.059	0.060
g_{max}	0.436	0.797	0.799	0.800	0.800
Cpu (s)	192	1415	3255	13332	22784
n_F	660	3280	6555	16400	32755
n_Q	55	1375	550	1375	2750

Table 4 – Results furnished for different sizes of population for n = 25, r = 10, t = 10

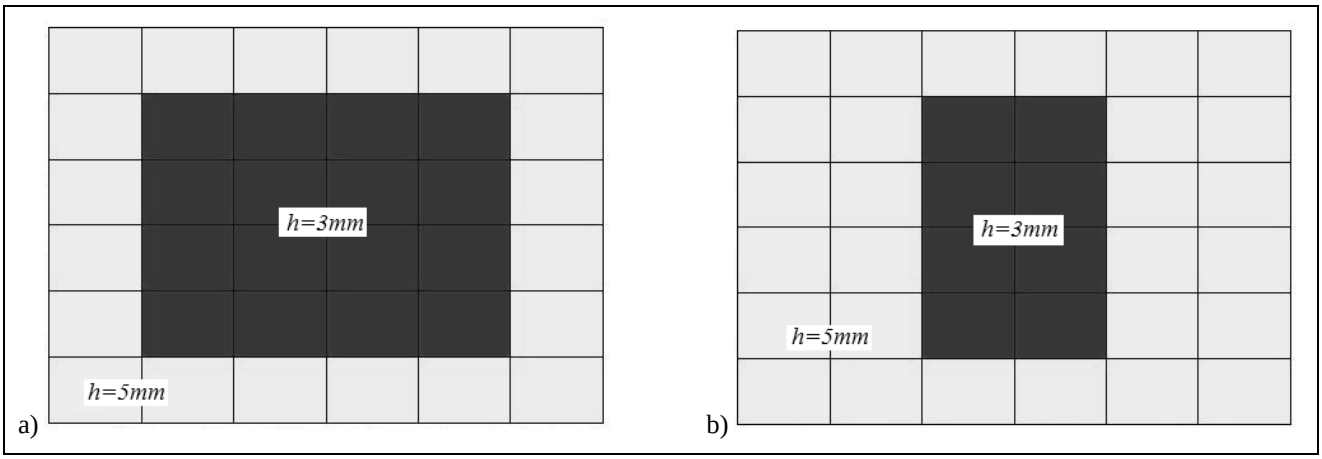
p	1	5	10	20	50
f_{opt} (mm)	0.062	0.059	0.059	0.057	0.059
g_{max}	0.800	0.800	0.799	0.799	0.799
Cpu (s)	3353	13332	26618	54881	128770
n_F	3280	16400	32800	65600	164000
n_Q	275	1375	2750	5500	13750

Compared to optimisation scheme without random perturbation, the chosen optimisation method has successfully improved the results (see table 2 the results without recombination). As expected increasing the parameters values improves the results, but also increases considerably the computational costs. Thus the initial parameters values should be a good compromise between computational cost and result accuracy. However if we want to increase the size of the problem, we will have to increase also the parameters values. So the considered optimisation method applied to problem with damage constraints seems to be only adapted for problem with a low number of design variables.

We now illustrate the influence of the initial thickness when starting the optimisation process in the table 5. We keep the predefined value n = 25, p = 5, r = 10 and t = 10.

Table 5 – Results furnished for different initial thicknesses

Case	a)	b)	c)	d)	e)
f_{opt} (mm)	0.075	0.066	0.061	0.068	0.096
g_{max}	0.799	0.797	0.800	0.800	0.799



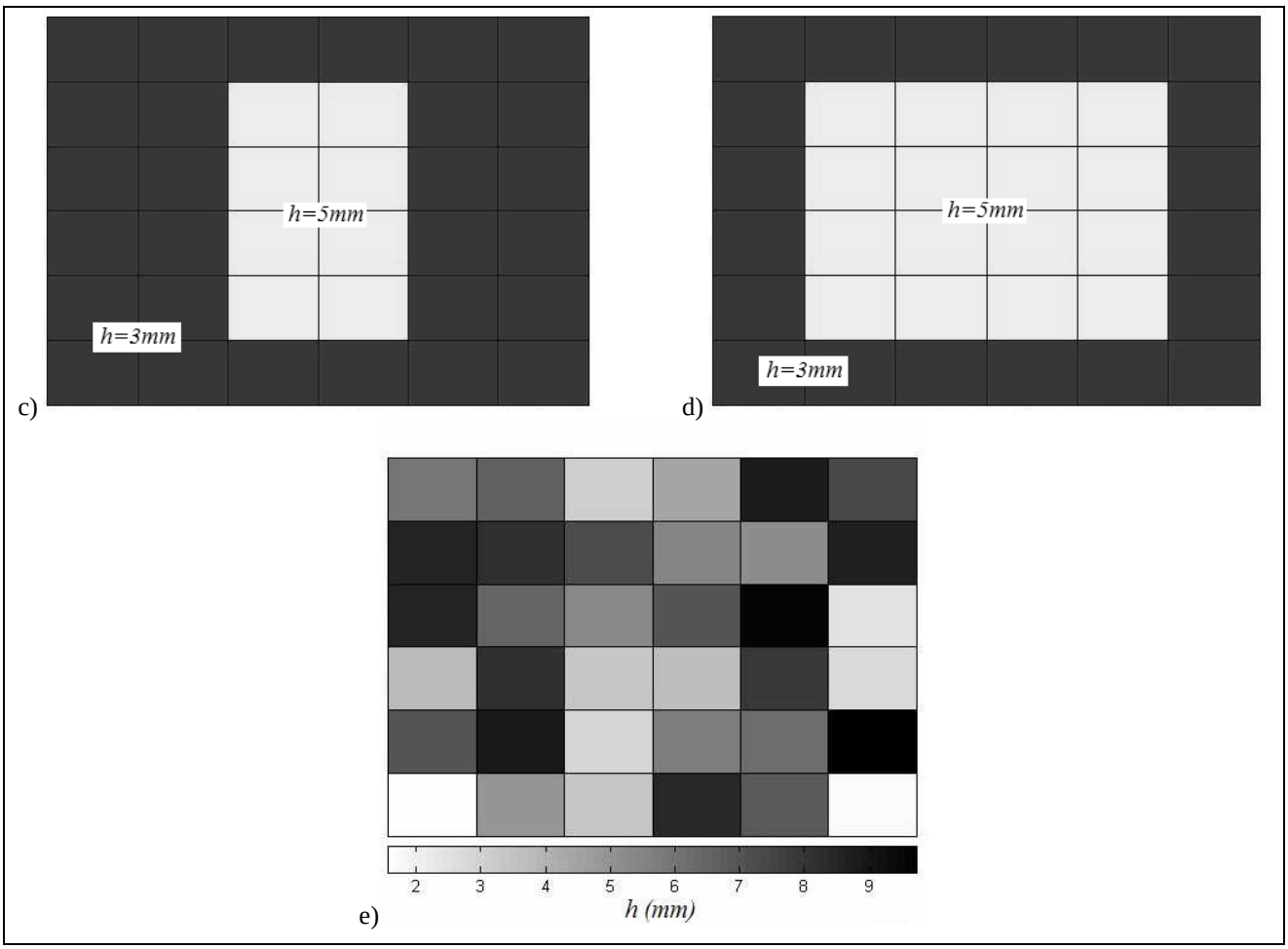


Figure 2 – Initial thickness cases

For uniform initial thickness we have obtained f_{opt} = 0.0587. The obtained results (Table 5) show that the initial thickness has a great influence on the given solution. Thus particular attention should be given to the initial design variables when considering the number of recombinations and mutations and also the size of the population low.

Over the experiments the one with the following parameters n = 25, p = 20, r = 10 and t = 10 has leaded to the best result f_{opt} = 0.057. The figure 3 shows the corresponding obtained design.

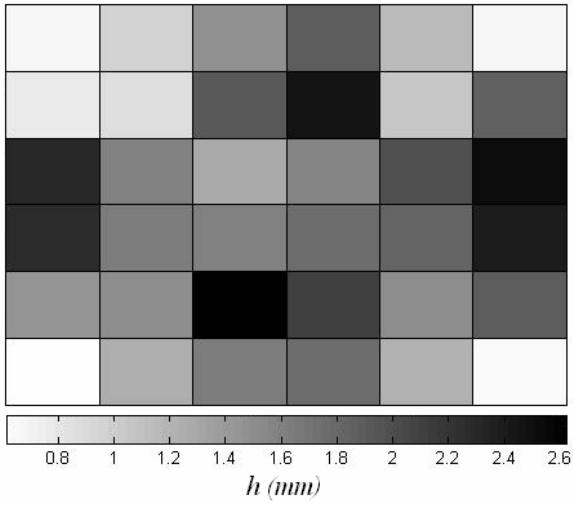


Figure 3 – Best optimal design obtained with for n = 25, p = 20, r = 10, t = 10

CONCLUSION

This paper deals with the optimisation of randomly excited structures. Optimum is stated here as a minimum mass objective for a given, prescribed, damage. An evolutionary based optimisation procedure is chosen in this work for its

global minimum search capacity. An essential feature of this procedure concerns the coupling between stochastic and deterministic strategies involved in the optimisation method: local search, random perturbation and evolutionary concepts are combined adjoining a suitable random perturbation to a basic descent method. Then, the resulting optimisation algorithm, which used sensitivities, is less computationally expensive than purely stochastic ones, allowing to be applied to mechanical problems.

The second feature of this work, essential to the success of the proposed methodology, concerns the procedure defined to evaluate the damage in structures subject to random loads: this procedure is based on a well suited frequency formulation of the Crossland's damage criterion, allowing a highly computational efficient evaluation of the optimisation constraint. Moreover the sensitivities constraints evaluation through the use of the presented method for the evaluation of the damage gradients has made the global optimisation process considering damage feasible with reasonable computational costs. Indeed although damage sensitivity numbers are estimated for each finite element of the problem, the estimation is computationally efficient because only one modal analysis is needed to obtain the overall sensitivity numbers. To demonstrate the viability of such a methodology, an application to a plate is handled on a personal computer. Even for this simple application, it is not possible to completely explore the optimisation strategy due to computational costs involved with the mechanical problem. However, a good compromise between computational cost and result accuracy has been determined, showing that interesting results could be obtained by iterating the algorithm: due to the probability concepts involved in the stochastic search combined to the deterministic strategy used, the optimisation algorithm should always give the minimum obtained with a basic deterministic method (at least in the worst case) and a better solution with some chance or supplementary iterations. This is an advantage from an engineer point of view, ensuring to have at least one good solution, and possibly a better one if computational time is allowed.

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