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On the Unsteady Dynamic Response of Telecommunication Submarine Cables during Deployment

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Abstract: In recent years, the advent of fiber optics has made communication systems based on this technology either a viable alternative or a complement to celestial communication systems. As a result, there has been an expansion in the activities of laying cables in the ocean. The laying operation consists of a cable ship capable of storing the whole length of cable to install in a reel. As the ship advances the cable is unreeled at a given pay out rate so that the amount of cable deployed is enough to cover the corresponding distance in the seabed without suspensions. The tension at the seabed is zero, so that the equilibrium of the cable for steady state conditions, i.e., constant forward vessel speed, constant pay out rate, and horizontal seabed, can only be achieved if the normal component of the cable submerged weight equals the drag force on the cable. As a result, the cable configuration may be approximated by straight line. However, there may be time dependent effects during cable laying for which the cable configuration departs from the straight line, i.e., change in course, slow down, speed up, break seizure, among others. This paper presents results for the transient three-dimensional dynamic response of the cable for ship maneuvers and wave motions. This response is obtained by a non-linear time domain finite element model, which has been validated in an earlier work. Results for dynamic tension, displacement, velocity and configuration of the cable clearly show the non-linear behavior of the cable as well as the strong potential for instabilities under certain circumstances.

Keywords: marine cables, low tension cables, dynamics of cables.

NOMENCLATURE

A =cross-sectional area of the cable
$\mathbf{a}_c$ =absolute acceleration vector
C_D =normal drag coefficient
C_M =added mass coefficient
C_T =tangential drag coefficient
d =cable's diameter
EA =cable's axial stiffness
EI =cable's flexural rigidity
$\mathbf{H}$ =angular momentum
$\mathbf{I}, \mathbf{J}, \mathbf{K}$ =unit vectors for the inertial frame of reference
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ =unit vectors for the vessel frame of reference
$\mathbf{L}$ =linear momentum per unit length
$\mathbf{M}$ =bending moment vector
$\mathbf{M_T}$ =twisting moment vector
m =physical mass per unit length
p =stretched arc length
$\mathbf{Q}$ =shear force vector
s =unstretched arc length
$\mathbf{T}$ =tension vector
T =tesnion magnitude
$\mathbf{t}, \mathbf{n}, \mathbf{b}$ =unit vectors for the local frame
t =time
$\mathbf{U}$ =fluid velocity
$\mathbf{u}$ = unknown vector

INTRODUCTION

In recent years, the advent of fiber optics has made communication systems based on this technology either a viable alternative or a complement to celestial communication systems. As a result, there has been an expansion in the activities of laying cables in the ocean. The cable laying operation consists of a cable ship capable of storing the whole length of cable to install in a reel. As the ship advances the cable is unreeled at a given pay out rate so that the amount of cable deployed is enough to cover the corresponding distance in the seabed without suspensions. The tension at the seabed is zero, so that the equilibrium of the cable for steady state conditions, i.e., constant forward vessel speed, constant pay out rate, and horizontal seabed, can only be achieved if the normal component of the cable submerged weight equals the drag force on the cable. As a result, the cable configuration may be approximated by straight line. However, there may be time dependent effects during cable laying for which the cable configuration departs from the straight line, i.e., change in course, slow down, speed up, break seizure, among others.

One important feature of these operations is that the cable operator must share the seabed with other users such as oil and gas related industry and fishing industry. As result, a telecommunication cable must deployed inside a strip of the seabed whose breadth tends to became narrower. The problem with this is that it is very difficult to predict de touchdown point of the cable on the seabed because of the uncertainties involved in the installation process. The problem can be stated as follows: there is a cable ship unreeling a say 50 mm diameter cable from the ocean surface and the cable touchdown point needs to be determined. The ship and the cable are subject to environmental loads due to ocean waves and ocean currents which may induce large three-dimensional displacements. In addition, the changes in speed and the change in the course of the lay vessel induce transient loading on the cable. Departure of a steady state condition is unavoidable because of seabed irregularities. If is being laying on a descending seabed, the coverage of the ground can be achieved by increasing the payout rate. If the cable is being laying in ascending seabed there are two situations to consider depending on the seabed slope. If the slope angle is less than the angle of the straight line configuration of the cable then a reduction on the payout rate would be enough to assure the coverage of the seabed without suspension. However, if the slope of the ascending seabed is greater than the cable angle, the only way of avoiding suspension is to reduce the ship forward speed. Figure 1 shows these three situations. More details on cable laying kinematics can be found in Zajac(1957), Roden (1974) and Pinto(1995).

From the structural viewpoint, telecommunication cables can be classified as low tension cables. Low tension cables can experience very large displacements even for very small loadings. Usually, they are used for signal, control and power transmission between the top facility and the underwater facility. Examples of applications of low tension cables, in addition to telecommunication cables, are umbilicals used in the oil and gas offshore industry as well as remote operated vehicles (ROVs) umbilicals. As these are very flexible structures, virtually all capacity to bear with transverse loading comes from the geometric stiffness of the cable, which is a function of the level of the tension. If the tension is low so is the geometric stiffness and, as a result, the global stiffness of the cable is low. As a consequence, low tension cable tend to have axial torsional stability problems mainly in function of some residual twisting on the cable caused by the manufacturing and storage process. If the cable have built in twist, it may well throw a loop which may cause irreversible damage to structural integrity of the cable. Such instability problems are beyond the scope of this paper, which considers only the global dynamics of the cable.

This paper presents results for the transient three-dimensional dynamic response of the cable for ship maneuvers and wave motions. This response is obtained by a non-linear time domain finite element model, which has been validated in an earlier work. Results for dynamic tension, displacement, velocity and configuration of the cable clearly show the non-linear behavior of the cable as well as the strong potential for instabilities under certain circumstances

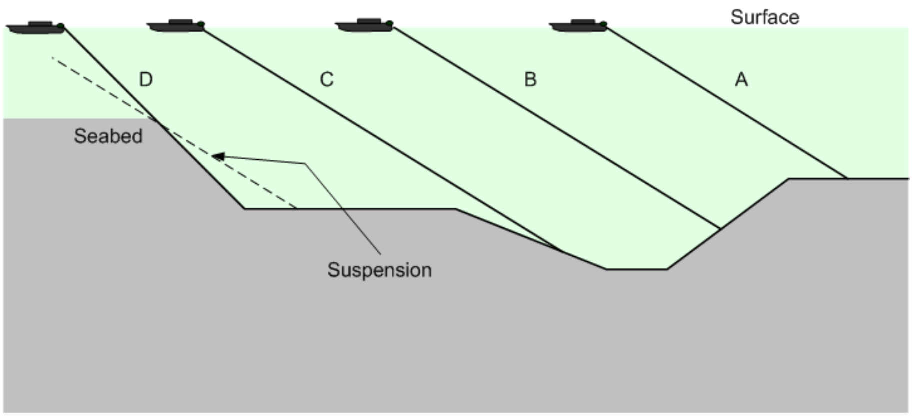

Figure 1 – Sketch for laying in a irregular seabed: A- horizontal seabed; B- Descending Seabed; C – Ascending Seabed with slope lower than the cable inclination; D – Descending Seabed with slope grater than the cable inclination

FORMULATION

A good model for the global analysis of the dynamics of low tension marine cables should be able to simulate large displacements and, more importantly, it should be able to cope with singularities in the geometric stiffness matrix. These singularities in the geometric stiffness matrix occur whenever the tension becomes zero. Another important factor in the modelling of marine cables is the definition of the local co-ordinate system. An arbitrary choice of the local frame of reference usually leads to a situation where the number of unknowns become larger than the number of independent differential equations available. Because of this, an extra assumption has to be made. This extra assumption may introduce further singularities in the analysis. This work follows the cable model proposed by Pinto (1995). This model is capable of dealing with the problems described above. The formulation assumes that the marine cable may undergo large displacements and it takes into consideration the flexural rigidity of the cable. The main reason for this is to overcome singularities in the geometric stiffness matrix, since it is not expected that the flexural rigidity can significantly affect the cable global configuration. However, this assumption makes the analysis much more complex because de cable is now considered as beam. In addition, the model may be applied to both towed cables and cable laying analyses. The main distinction between these analyses lies on the fact that, in general, towed cables have a known suspended length as opposed to a cable being laid whose suspended length may vary since the constraint is that the vertical coordinate equals the water depth.

Assumptions

The model in question makes a number of assumptions which are summarised here. The first assumption considers the cable is a long homogeneous linear elastic circular cylinder with small diameter compared with its length which means that the coupled axial-torsional behaviour of armoured cables is not considered here. This is reasonable for the global analysis of cables which are designed with contra-helical armour layers so that there is torsional balance under tensile load. For unbalanced cables the torque generated by the tension are relatively low owing to the small lay angles normally associated with the armour layers. In addition, the axial torsional coupling is expected to play an important role only in the local stability analysis. The second assumption considers that the configuration of the cable may be represented by a unit speed space curve, so that it can be parameterised by its stretched arc length while the third, as mentioned before, assumes that the cable flexural rigidity is taken into consideration in order to introduce a more rigorous model to overcome singularities in the geometric stiffness. The fourth assumption considers that the cable may undergo large displacements but only small strains so that the cable is free of plastic deformation. The fifth assumption considers that the kinetic energy due to both axial strains and rotatory inertia are small if compared with the kinetic energy of translation motion. This is consistent with the assumptions of small strain and long beam theory. The sixth assumption establishes that there are no twisting moments action on the cable. In other words, there is no residual twist within the cable. The remaining assumptions are concerned with the loading. The forces acting on the cable are due to self-weigh, buoyancy, lift and drag forces, added mass and D'Alembert force. The buoyancy force is considered by applying the Archimedes principle to an element of cable which leads to the concept of submerged weight and effective tension. These are very important concepts in submarine cable analyses. Their derivation is too lengthy do be shown here and it is beyond the scope of this work. Detailed explanation of these concepts can be found in Patel (1989) and Sarpkaya and Isaacson (1981). Finally, it is assumed that wave effects on the cable are small so that the Froude-Krylov forces are negligible. This is consistent with the fact that the cable has a small diameter and the fluid forces are drag dominated. In addition, the cable is very long and usually the critical load occurs in ultra-deep waters where wave effects are restrict to a very narrow strip close the water surface.

Kinematics

The first step to establish the analysis of the geometry of motion is to set a proper reference system. Next section, deals with the equations of motion, which must be written with reference to an inertial frame. However, it is convenient to place a frame of reference at the lay vessel because it is there that almost all observations are made during the lay operation. The problem is that such a frame is not inertial. A third frame of reference is also needed to describe the local properties of the cable. The local frame is also non-inertial and depends on the arc length. Figure 2, shows the three frames of reference used in this paper where the frame defined by the vectors (**I,J,K**) fixed in space and time, therefore inertial, the vessel frame defined by (**i,j,k**) and the local frame of reference which is the intrinsic frame of the curve (**t,n,b**), where **t** is the tangent, **n** is the normal and **b** is the binormal unit vectors.

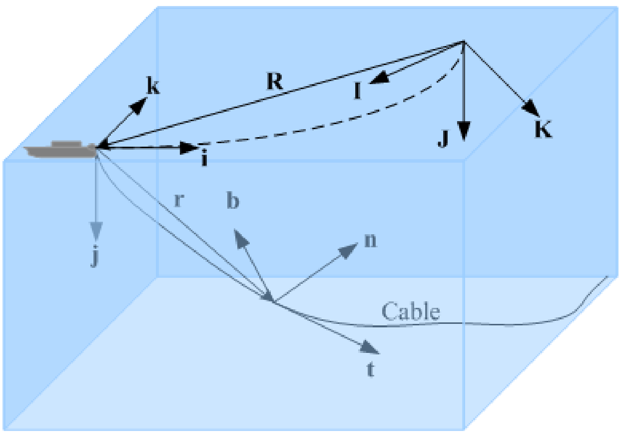

Figure 2 - Reference System: Inertial - (**I,J,K**); Vessel (**i,j,k**); Local (**t,n,b**)

The local frame is defined from differential geometry (Enseinhart, 1947, Do Carmo, 1976). The model assume that the neutral axis of the cable can be modeled by a parametric unit speed curve. Therefore, such a curve can use the stretched arc length, p, as a parameter. Furthermore, the derivative of the vector position with respect to the arc length gives a unit vector that is tangent to the neutral axis of the cable.

$$\mathbf{t} = \frac{\partial \mathbf{r}}{\partial \mathrm{p}} \tag{1}$$

A second unit vector can be obtained from the second derivative of the position vector with respect to the arc length p divided by its magnitude. This vector is normal to the unit tangent vector, since the derivative of a vector with reference to the stretched arc length is always normal to the original vector.

$$\mathbf{n} = \frac{\partial^2 \mathbf{r}}{\partial \mathrm{p}^2} \tag{2}$$

This unit vector is called the principal normal and its magnitude gives a measure of the change in the tangent direction, or the curvature of the cable. The third unit vector called binormal is obtained via the cross product of the unit tangent and the unit normal vector. That is:

$$\mathbf{b} = \mathbf{t} \times \mathbf{n} \tag{3}$$

Once the local frame of reference has been established, one needs to establish the relationship between the triad (**t,n,b**) and their derivatives with respect to the stretched arc length p. Following the references on differential geometry, such a relationship can be written in the following matrix form:

$$\frac{\partial}{\partial \mathrm{p}} \begin{Bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{Bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{Bmatrix} \tag{4}$$

In Equation 4, κ is the scalar curvature defined as the magnitude of the derivative of the unit tangent vector with respect to the arc length p, and τ is the torsion of the curve which gives a measure of the change the binormal vector, or the bending in the normal direction. Therefore, the torsion is the defined as the magnitude of the derivative of the unit binormal vector with respect to the stretched arc length.

The second task is to establish the relationship between the local and the vessel frame of reference. The unit vectors of the vessel frame of reference do not depend on the arc length. As a result the following relationship may be written (see Pinto, 1995):

$$\begin{Bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial p} & \dfrac{\partial y}{\partial p} & \dfrac{\partial z}{\partial p} \\ \dfrac{1}{\kappa}\dfrac{\partial^2 x}{\partial p^2} & \dfrac{1}{\kappa}\dfrac{\partial^2 y}{\partial p^2} & \dfrac{1}{\kappa}\dfrac{\partial^2 z}{\partial p^2} \\ \dfrac{1}{\kappa}\left(\dfrac{\partial y}{\partial p}\dfrac{\partial^2 z}{\partial p^2} - \dfrac{\partial^2 y}{\partial p^2}\dfrac{\partial z}{\partial p}\right) & \dfrac{1}{\kappa}\left(\dfrac{\partial^2 x}{\partial p^2}\dfrac{\partial z}{\partial p} - \dfrac{\partial x}{\partial p}\dfrac{\partial^2 z}{\partial p^2}\right) & \dfrac{1}{\kappa}\left(\dfrac{\partial x}{\partial p}\dfrac{\partial^2 y}{\partial p^2} - \dfrac{\partial^2 x}{\partial p^2}\dfrac{\partial y}{\partial p}\right) \end{bmatrix} \begin{Bmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{Bmatrix} \tag{5}$$

The next step is to establish the position of an element of cable with respect to the inertial frame of reference. Based on Figure 2, such a position is given by:

$$\mathbf{r_c}(\mathrm{p},\mathrm{t}) = \mathbf{R} + \mathbf{r}(\mathrm{p},\mathrm{t}) \tag{6}$$

where t is the time. The relative velocity between the cable and the fluid can be obtained by the sum of the time derivative of Equation (6) and the pay out rate, which is tangent to the cable, minus the fluid velocity. Therefore,

$$\mathbf{V_c}(\mathrm{p},\mathrm{t}) = \frac{\partial \mathbf{r_c}}{\partial \mathrm{t}} + \mathrm{V_{po}}\mathbf{t} - \mathbf{U} \tag{7}$$

Finally, the relative acceleration can be obtained from the time derivative of Equation (7), that is:

$$\mathbf{a_c}(\mathrm{p},\mathrm{t}) = \frac{\partial^2 \mathbf{r_c}}{\partial \mathrm{t}^2} + \frac{\partial \mathrm{V_{po}}}{\partial \mathrm{t}}\mathbf{t} + \mathrm{V_{po}}\frac{\partial \mathbf{t}}{\partial \mathrm{t}} - \frac{\mathrm{D}\mathbf{U}}{\mathrm{Dt}} \tag{8}$$

where D denotes the total derivative, which is necessary to take into account the convective acceleration of the fluid.

Kinetics

The differential equations of motion are obtained from the balance of forces and moments acting on an element of cable with infinitesimal length δp, as shown in Figure 3, where δ is a differential operator defined as:

$$\delta = \frac{\delta p}{2}\frac{\partial}{\partial p} \tag{9}$$

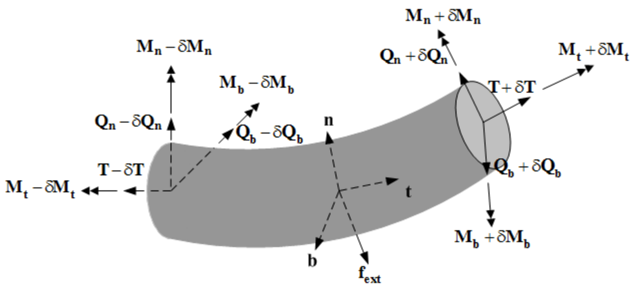

Figure 3 – Forces and Moments in a Element of Cable with Length δp

The application of the D'Alembert principle together with the differential operator the following equations of motion are obtained:

$$\left(\frac{\partial T}{\partial p} - \kappa Q_n\right)\mathbf{t} + \left(\kappa T + \frac{\partial Q_n}{\partial p} - \tau Q_b\right)\mathbf{n} + \left(\frac{\partial Q_b}{\partial p} + \tau Q_n\right)\mathbf{b} + \mathbf{f_{ext}} - \frac{\partial \mathbf{L}}{\partial t} = \mathbf{0} \tag{10}$$

and

$$\left(\frac{\partial M_t}{\partial p} - \kappa M_n\right)\mathbf{t} + \left(\kappa M_t + \frac{\partial M_n}{\partial p} - \tau M_b - Q_b\right)\mathbf{n} + \left(\frac{\partial M_b}{\partial p} + \tau M_n + Q_n\right)\mathbf{b} - \frac{\partial \mathbf{H}}{\partial t} = \mathbf{0} \tag{11}$$

In these equations, T is the magnitude of the effective tension, Q_n and Q_b are the components of the shear force in the normal and binormal directions, M_t is the twisting moment, M_n and M_b are the components of the bending moment in the normal and binormal directions, $\mathbf{f_{ext}}$ is the vector containing all the external forces including lift and drag, $\mathbf{L}$ is the linear momentum and $\mathbf{H}$ is the angular momentum. Following the assumptions made earlier in this section, the angular momentum, the binormal component of the shear force, the normal component of the bending moment, the twisting moment and the angular momentum can be neglected. In addition, the binormal component of the bending moment is given by the product of the cables flexural rigidity (EI) and the curvature. Moreover, the normal component of the shear force can be approximated by the derivative of the bending moment. As a result of these simplifications, the differential equations of motion assume the following expressions:

$$\kappa T - \frac{EI}{(1+\varepsilon)^2}\frac{\partial^2 \kappa}{\partial s^2} + \frac{w}{\kappa(1+\varepsilon)^2}\frac{\partial^2 y}{\partial s^2} - \frac{1}{2}\rho d C_D V_{rn}\sqrt{V_{rn}^2 + V_{rb}^2} - \rho A a_{rn} - m a_n = 0 \tag{12}$$

$$EI\tau\kappa + \frac{w}{\kappa(1+\varepsilon)^3}\left(\frac{\partial^2 x}{\partial s^2}\frac{\partial z}{\partial s} - \frac{\partial x}{\partial s}\frac{\partial^2 z}{\partial s^2}\right) - \frac{1}{2}\rho d C_D V_{rb}\sqrt{V_{rn}^2 + V_{rb}^2} - \rho A a_{rb} - m a_b = 0 \tag{13}$$

$$\frac{1}{1+\varepsilon}\frac{\partial T}{\partial s} + \frac{EI}{1+\varepsilon}\kappa\frac{\partial \kappa}{\partial s} + \frac{w}{1+\varepsilon}\frac{\partial y}{\partial s} - \frac{1}{2}\pi\rho d C_T \left|V_{rt}\right| V_{rt} - m a_t = 0 \tag{14}$$

The above equations introduce the unstretched arc length s and the axial strain ε. The unstretched arc length is related with the stretched one by:

$$\frac{\partial p}{\partial s} = \frac{1}{1+\varepsilon} \tag{15}$$

The axial strain can be obtained as a function of the effective tension and the pressure field. Assuming that the vessel frame of reference is placed at sea surface with the vertical coordinate, y, pointing downwards, the axial strain may be written as:

$$\varepsilon = \frac{T - \rho A y}{EA} \tag{16}$$

where ρ is the fluid density and EA is the cable axial stiffness.

Equations of motion 12 to 14 take into account the submerged weight, w, the drag and lift forces, which are function of the normal drag coefficient C_D and the tangential friction coefficient C_T. These forces are resolved into the tangential, normal and binormal directions. In addition, the model considers the added mass force which is given by the product of the fluid density by the area of the transverse section of the cable. It is assumed implicitly that there is no interference of walls or other bodies. The added mass in the tangential direction was neglected because the capacity of the tangential acceleration of the cable to induce acceleration in the fluid is very limited. The next section deals with the solution of the equations of motion.

NUMERICAL IMPLEMENTATION

The equations of motion are highly non linear. Because of this, the best way of solving them is to use the time domain approach. Under these circumstances, this work uses a finite element approach to perform the space integration and a direct integration for the time integration. The differential equations of motion are of fourth order in space and second order in time. Therefore, one needs four boundary conditions and two initial conditions. The latter depend on the type of analysis. Usually a transient analysis start either from rest or from a steady state condition. For steady state harmonic loading the initial condition is obtained by starting from rest and gradually increasing the amplitude of the excitation. As for the boundary conditions, the usual assumptions for beams apply. Either the displacement and slope or the forces or moments are known at both ends of the cable.

In the case of cable laying analyses, the suspended length of the cable is not known because the actual position of the touchdown point is unknown. The suspended length of the cable can be determined by an additional boundary condition which can be the slope at the seabed.

The finite element model developed here, consider that the coordinates of the vessel frame of reference can be expressed as known functions of the arch length and the nodal coordinates, just like a usual finite element formulation. There are two basic differences in the formulation applied here. Firstly, this model approximates the coordinates themselves rather than the displacements. This approach make it easier to take into account rigid body motions. Secondly, the space integration, which is done by the classic Galerkin weighted residual method, is a line integral along the deformed length of the cable, instead of the local x-coordinate as done in the classical space frame analysis. This allows the model to deal very easily with large displacements with just a few elements. Another advantage of this approach is that it is easier to detect potential instabilities and, moreover, to perform post-instability analyses. However, this issue is beyond the scope of this work.

The coordinates x, y and z are approximated by cubic polynomials so that a given point within the finite element is obtained by the multiplication of the shape or interpolation functions by the unknown nodal values of the coordinates. This is the standard finite element procedure and may be found in any finite element textbooks such as Zienkwicz and Taylor (1989) and Bathe (1982). The result of the space integration is a semi-discrete system of differential equations in time. The time integration of this system is performed by a direct integration technique which uses Newmark beta algorithm. The procedure for solving the equations is as follows: Lets denote the unknown dependent variables (coordinates) by a generic vector **u**. The question posed now is to estimate this vector at the end of the time step, once both the vector and its derivative with respect to time are known at the beginning of the time step (initial condition). The Newmark algorithm uses the following expressions to estimate the velocity vector:

$$^{t+\Delta t}\frac{\partial \mathbf{u}}{\partial t} = {}^{t}\frac{\partial \mathbf{u}}{\partial t} + \left[{}^{t+\Delta t}\frac{\partial^2 \mathbf{u}}{\partial t^2}\delta + {}^{t}\frac{\partial^2 \mathbf{u}}{\partial t^2}(1-\delta) \right]\Delta t \tag{17}$$

while for the position vector the expression is:

$$^{t+\Delta t}\mathbf{u} = {}^{t}\mathbf{u} + {}^{t}\frac{\partial \mathbf{u}}{\partial t} + \Delta t \left[{}^{t+\Delta t}\frac{\partial^2 \mathbf{u}}{\partial t^2}\alpha + {}^{t}\frac{\partial^2 \mathbf{u}}{\partial t^2}\left(\frac{1}{2}-\alpha\right) \right]\Delta t^2 \tag{18}$$

The application of the Newmark method results in a system of non-linear algebraic equations of the type:

$$\mathbf{F}(\mathbf{u}) = \mathbf{0} \tag{19}$$

where **u** is the vector containing the n unknown nodal coordinates and **F** is a set of n non-linear equations.

The solution of the non-linear system of algebraic equations may be obtained by an unconstrained optimization technique which consists in minimize the error of the substitution, in Equation 19, of an approximated value of the unknown vector, $\hat{\mathbf{u}}$ **,** by the least squared method. That is, a scalar function can be constructed according to the expression:

$$\phi(\hat{\mathbf{u}}) = \sum_{i=1}^{n} F_i^2 \tag{20}$$

The problem now is reduced to the unconstraint minimization of a scalar function of several variables. There are a number of algorithms that can be used to perform such a minimization. This work uses the BFGS and the Newton-Raphson algorithms. Notice that this approach is possible because the minimum of the scalar function given by Equation (20) corresponds to the solution of the system of algebraic Equations 19, (See Buchnan and Turner, 1992).

APPLICATIONS

This chapter is concerned with the application of the model to three cases. The first case consists of the analyses of towed cables which have been subject to sea trials. The trial measurements were published by Hopland (1993). The analysis were made for the transient response of the a towed cable when the cable vessel speeds up and slows down. The cables characteristics are shown in Table 1. Notice that the terms normal and tangential drag coefficient are used to compute the normal and tangential forces acting on the cable. The normal drag results from the unsymmetrical distribution of pressure due to the normal component of the relative velocity between the cable and the fluid. The tangential drag is an axial force resulting from the viscous friction between the cable and the fluid. Table 1 also uses the term hydrodynamic constant of the cable. This quantity is obtained by finding the inclination of the cable for which the normal drag equals the normal component of the submerged weight of the cable. The hydrodynamic constant relates to the normal drag coefficient by the expression:

$$H = \sqrt{\frac{2w}{C_D \rho d}} \tag{21}$$

where w is the submerged weight per unit length of the cable, C_D is the normal drag coefficient, ρ is the fluid density and d is the cable diameter.

Table 1 – Properties of the heavy armored (HA) and light armored (LA) cables

Marine Cable	HA	LA
Diameter (mm)	33.2	26.4
Physical mass (kg/m)	2.70	1.64
Hydrodynamic constant (rad*m/s)	0.7974	0.6173
Bending stiffness (N*m^2)	1000	500
Weight in water (N/m)	17.80	10.96
Normal drag coefficient	1.64	2.12
Tangential drag coefficient	0.01	0.01

Figure 4 shows the comparison between the results obtained by the present model and the Hopland’s measurements for the case. Model results agree fairly with the observed data.

The second application is concerned to the response of a cable laying in wave motions. According to the assumptions made by the model wave induced loading directly on the cable is quite small because of the small diameter and the large length of the cable. However, the waves may induce large motions in the lay vessel where the cable is attached. As a result, the main effect of waves on the cable via vessel motions. Figure 5 presents the results obtained by the model for the effective tension for a vessel in a wave with 3 metres of amplitude and 10 s encounter period. The vessel RAO are 0.708 for surge, and 1.619 for heave while the surge and heave phases are –45.9 degrees and 0, respectively.

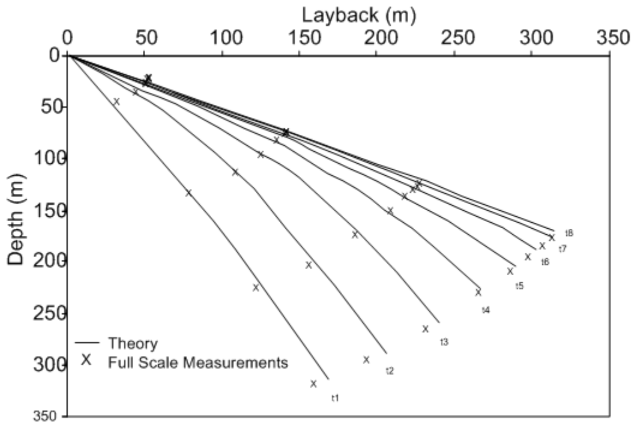

Figure 4 – Configuration for the speeding up simulation of a towed light cable

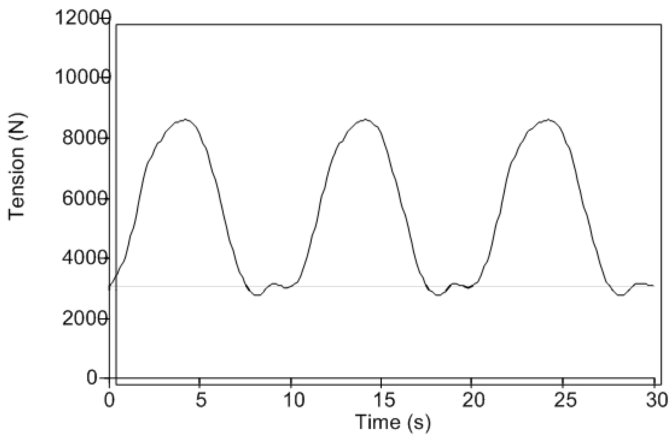

Figure 5 – Top tension for a wave simulation of a light armoured cable

The third application is concerned with the response of the cable in shear current. The analyses are made for a current profile of 1 m/s at the surface and 0.1 m/s at the sea bed for a 1000 m water depth. The current direction was considered for head, bow, quartering beam, stern quartering and following currents. An interesting results is that the suspended length of the cable is 30% greater than the straight line configuration for head sea current and 22% lower for the following sea. Figure 6 shows the foot print diagram which gives the estimate position of the touchdown point for each case.

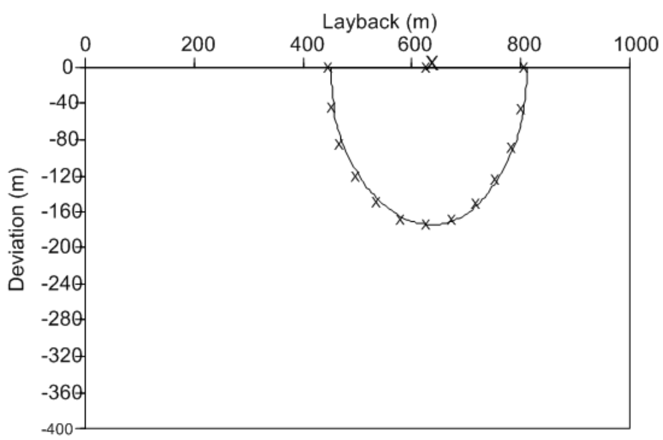

Figure 6 – Foot print diagram for three dimensional shear current simulation

CONCLUSIONS

This work has presented a numerical model for the simulation of low tension marine cable installation with application in telecommunication fibre optics cable. The results of the model were compared with field measurements obtained by Hopland's sea trials. Then the ability of the model in dealing with the transient response as well as three dimensional case due shear currents was demonstrated. The model was able to capture the non-linearity in the top tension response to regular wave excitation.

The performance of the model is encouraging to extend it to incorporate the detection of instabilities such as looping formation as well as hydrodynamic instabilities such as flutter and other flow induced vibration effects.

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