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Modeling and qualitative analysis of tire wear for steady state cornering maneuvers

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Abstract: Nowadays, modeling tools are available for the engineer during the design or improvement of a product. Considering the study of vehicle dynamics, there are commercial dedicated softwares which aid the designer during the conceptual and detailed design phase. Therefore, some metrics are promptly available after some simulations, such as yaw velocity, lateral velocity, etc., for standard maneuvers. However, it is still a challenge to connect these metrics with some empirical quantities that usually depend on several external parameters and design specifications. This is the case of tire wear, which depends on the friction work developed by the tire-road contact. Using this statement, an approach is derived to estimate the tire-road friction during longitudinal and cornering drive using the wheel slip, which is the relative difference in wheel velocities. In this way, a qualitative formula for tire wear is obtained and some sensitivity analysis considering cornering maneuvers can be performed using a simple model for lateral vehicle dynamics such as the single-track model. The influence of design parameters such as cornering stiffness, the distance between the axes, and the steering relation between the axes has been evaluated. This approach is also extended for vehicle with two steering axes, where an extension of the single-track model is used. Some results are shown, and some statements found in the literature are highlighted which allows the designer to infer about the vehicle behavior considering tire wear using simple models and standard statements.

Keywords: tire wear, qualitative analysis, lateral vehicle dynamics, simple models

NOMENCLATURE

A,B = state space matrices
a = distance between the centre of gravity and the first axle
b = distance between the centre of gravity and the second axle
c = distance between the centre of gravity and the point between the third and the fourth axes
Ab = abrasion coefficient, 1/N
C = cornering stiffness, N/rad
F = force, N
I = inertia, kg.m^2
L = work, Nm
M = mass, kg
R = wheel radius, m
S = distance, m
T = time, s
V = vehicle velocity, m/s
W = tire wear, m
dl = infinitesimal distance
dt = infinitesimal time
dx = infinitesimal distance in the longitudinal direction
dy = infinitesimal distance in the lateral direction

Greek Symbols

Δ = ½ distance between the third and the forth axes
α = side slip angle, rad
β = slip ratio, dimensionless
δ = steer angle, rad
ω = angular velocity, rad/s

Subscripts

a relative to friction
t relative to tangential direction
x relative to longitudinal direction
y relative to lateral direction
w relative to wheel
z relative to the vertical direction
N relative to normal
0 relative to the reference
1,2,3,4 relative to the first, second, third and fourth axes respectively

INTRODUCTION

In other to face the design problems, nowadays, modeling tools are available for the engineer. Considering the study of vehicle dynamics, there are commercial dedicated softwares which aid the designer during the conceptual and detailed design phase. Therefore, some metrics are promptly available after simulations, such as yaw velocity, lateral velocity, etc., for standard maneuvers. However, it is still a challenge to connect these metrics with empirical quantities that usually depend on several external parameters and design specifications.

This is the case of tire wear, which depends on the friction work developed by the tire-road contact. Using this statement, an approach is derived to estimate the tire-road friction work during longitudinal and cornering drive using the wheel slip, which is the relative difference in wheel velocities. Similar approach is also used to include tread wear in advanced tire models available in commercial dedicated multibody softwares, such as FTire (Gipser, 2003). Some properties of FTire model, such as compression stiffness and damping, shear stiffness and damping, heat capacity and tread mass distribution are updated depending on the tread depth. FTire models are based on the friction power and a functional relationship between friction power and tire wear rate (Gipser, 2003).

Tire wear is unavoidable, but it can be delayed. Therefore, the designer should be aware of this problem during the design phase and try to minimize it. This may became a huge issue for vehicles with two steering axes. For this kind of vehicle, an improper geometry can cause a rapid deterioration of the tire condition.

The relationship between the wheel slip ratio and the friction coefficient is well-know and can be described by the 'magic formula' (Bakker *et. al.*, 1989). This relationship depends on the road condition (dry, wet, etc.), road type (asphalt, concrete, etc.), tire type (carcass or radial), tread type and tread depth. Several works were done in this framework, for instance, real time estimation of the friction coefficient based on slip estimation (Lee *et. al.*, 2004).

The methodology proposed in this paper is a qualitative analysis of the tire wear considering some vehicle dynamic responses. The tire wear can be correlated with the friction work by the abrasion coefficient. Besides, the friction work can be described in terms of dynamic responses for steady state longitudinal and lateral dynamics. This approach is described in the next section. Since a qualitative formula is obtained, some sensitivity analysis considering cornering maneuvers can be performed using a simple model for lateral vehicle dynamics such as the single-track model. A short review of the single track model considering two steering axes is shown in the section Simple Models for Lateral Dynamics. Using the qualitative formula and the single-track model, the evaluation of parameters such as cornering stiffness, the distance between the axes and the steering relation between the axes has been performed and are shown in the section Qualitative Analyses. Some results are addressed, and some statements found in the literature are highlighted, which allows the designer to infer about the vehicle behavior considering tire wear using simple models and standard statements.

QUALITATIVE FORMULATION FOR TIRE WEAR

A qualitative formulation for tire wear is derived for steady state longitudinal and lateral dynamic responses in the next sections. Firstly, the friction work is written depending on vehicle dynamic responses. Afterward, the friction work is used to describe the tire wear using the abrasion coefficient.

Friction work definition

The tire wear is proportional to the friction work that is defined by Eq. (1) (Argemiro, 2000).

$$L_a = \int_0^S F_t \cdot dl \tag{1}$$

where L_a represents the friction work, F_t the tangential force and S the distance. For both, steady state longitudinal and lateral dynamics the Eq. (1) can be rewritten.

Considering steady state longitudinal dynamics, the tangential force F_t, from Eq. (1), can be substituted by the longitudinal force F_x and the relative displacement developed in the longitudinal direction, *dx*. Therefore, Eq. (1) can be rewritten, obtaining:

$$L_a = \int_0^S F_x \cdot dx = \int_0^T F_x \cdot V_x \cdot dt = \int_0^T F_x \cdot (\omega_w R - V) \cdot dt \tag{2}$$

where V_x represents the vehicle velocity in the longitudinal direction, ω_w the wheel angular velocity, R the wheel radius and V the vehicle velocity. The slip ratio, β, can be defined by Eq. (3) (Winkler, 1998).

$$\beta = \frac{(\omega_w R - V)}{V} \tag{3}$$

(3)

Using the definition in Eq. (3) in Eq. (2), yields:

$$L_a = \int_0^T F_x \cdot \beta \cdot V \cdot dt \tag{4}$$

Considering longitudinal steady state maneuvers, the longitudinal force and the slip ratio are constants. Therefore, the friction work can be described by the Eq. (5).

$$L_a = F_x \cdot \beta \cdot \int_0^T V \cdot dt = F_x \cdot \beta \cdot S \qquad (5)$$

Consequently, considering longitudinal steady state maneuvers, the friction work is proportional to the longitudinal force, the slip ratio and the distance.

In a similar way, considering steady state lateral dynamics, the tangential force F_t can be substituted by the lateral force F_y and the relative displacement developed in the lateral direction. Therefore, for steady state lateral dynamics, the friction work reads:

$$L_a = \int_0^S F_y \cdot dy = \int_0^T F_y \cdot V_y \cdot dt \qquad (6)$$

where F_y represents the lateral force and V_y the lateral velocity. The side slip angle, α, can be defined by Eq. (7), (Gillespie, 1992).

$$\alpha = \arctan\left(\frac{V_y}{V_x}\right) \qquad (7)$$

where V_x represents the longitudinal velocity of the vehicle. For small side slip angles, the longitudinal velocity can be approximated by the velocity of the vehicle Eq. (8).

$$\alpha \approx \frac{V_y}{V_x} \approx \frac{V_y}{V} \qquad (8)$$

Therefore, the friction work for steady state lateral dynamics can be rewritten as in Eq. (9), since the lateral force and the side slip angles are constants.

$$L_a = \int_0^T F_y \cdot \alpha \cdot V \cdot dt = F_y \cdot \alpha \cdot S \qquad (9)$$

Consequently, considering lateral steady state maneuvers, the friction work is proportional to the lateral force, the side slip angle and the distance.

Qualitative formulation for tire wear during lateral steady state maneuvers

Tire wear, W, and the friction work, L_a, are proportional by the term Ab, abrasion coefficient, Eq. (10), (Argemiro, 2000). Abrasion coefficient depends on some tire characteristics: hardness, molecular structure, elongation at break, wearing resistance, vulcanization degree, quantity of carbon black, etc (Argemiro, 2000).

$$W = Ab \cdot L_a \qquad (10)$$

At this point, it is necessary to highlight that the normal weight also plays an important role in the tire wear phenomena. In this work a linear relation between the tire wear and the normal weight is considered. Therefore, the adopted qualitative formula for the tire wear, R_w, considering lateral steady state maneuvers is described by Eq. (11).

$$W \propto Ab \cdot F_y \cdot \alpha \cdot S \cdot \left(\frac{F_N}{F_{N0}}\right) = Ab \cdot C_\alpha \cdot \alpha^2 \cdot S \cdot \left(\frac{F_N}{F_{N0}}\right) \qquad (11)$$

where F_N represents the normal force and F_{N0} the reference for the normal force. The lateral force F_y is proportional to the cornering stiffness C_α and the side slip angle α.

In order to minimize the tire wear, the product of lateral force and the side slip angle should be minimized. These parameters can be extracted from simplified models such as single-track model of a vehicle.

SIMPLIFIED VEHICLE MODEL FOR LATERAL DYNAMICS

The procedure described at the previous section can be used for qualitative analysis during the design phase of a vehicle. For steady-state cornering maneuvers, the simplest model is known as single track or bicycle model (Gillespie, 1992). This model was used to predict some responses (necessary to the qualitative analysis) for two vehicles with one and two steering axes. The single track model for vehicles with two axes (one steering axis) is widely described in the literature (Gillespie, 1992).

An extension of this model for vehicles with four axes (two steering axes) is derived in the same way. A state-space model can be derived from the summation of forces and moments in the bicycle model shown at Fig. (1). The lateral transfer load is considered negligible, therefore, the model is valid for low lateral accelerations. Further details can be found at (Winkler, 1998).

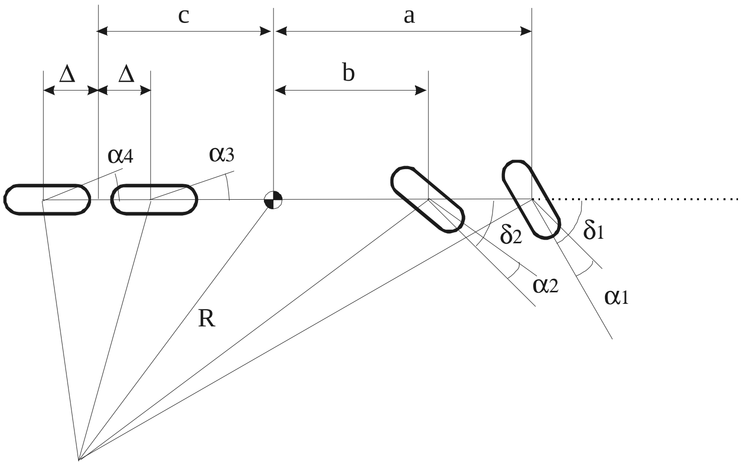


Figure 1 – Single Track Model for four axes vehicle – two steering axes

The state-space model is shown at Eq. (12), where the matrices **A** and **B** are defined by Eqs. (13) and (14), respectively. They depend on the mass M , the inertia I , the cornering stiffness C_α of the different axes, and the distances between the axes a , b , c and Δ , see Fig. 1. The inputs of the system are the steer angles δ_1 and δ_2 , and the state variables are the angular velocity ω_z and the slip ratio β .

$$\begin{bmatrix} \dot{\beta} \\ \dot{\omega}_z \end{bmatrix} = \mathbf{A}\begin{bmatrix} \beta \\ \omega_z \end{bmatrix} + \mathbf{B}\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \tag{12}$$

$$\mathbf{A} = \begin{bmatrix} -\dfrac{\left(C_{\alpha 1} + C_{\alpha 2} + C_{\alpha 3} + C_{\alpha 4}\right)}{MV} & -1 - \dfrac{\left(aC_{\alpha 1} + bC_{\alpha 2} - (c-\Delta)C_{\alpha 3} - (c+\Delta)C_{\alpha 4}\right)}{MV^2} \\ -\dfrac{\left(aC_{\alpha 1} + bC_{\alpha 2} - (c-\Delta)C_{\alpha 3} - (c+\Delta)C_{\alpha 4}\right)}{I} & -\dfrac{\left(a^2C_{\alpha 1} + b^2C_{\alpha 2} + (c-\Delta)^2C_{\alpha 3} + (c+\Delta)^2C_{\alpha 4}\right)}{IV} \end{bmatrix} \tag{13}$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{C_{\alpha 1}}{MV} & \dfrac{C_{\alpha 2}}{MV} \\ \dfrac{aC_{\alpha 1}}{I} & \dfrac{bC_{\alpha 2}}{I} \end{bmatrix} \tag{14}$$

From this formulation, the angular velocity ω_z and the slip ratio β can be calculated. The side slip angles, that are required for the tire wear estimation during cornering maneuvers, are given by the Eqs. (15).

$$\alpha_1 = \beta + \frac{a\omega_z}{V} - \delta_1 ,\ \alpha_2 = \beta + \frac{b\omega_z}{V} - \delta_2 ,\ \alpha_3 = \beta - \frac{(c-\Delta)\omega_z}{V} \text{ and } \alpha_4 = \beta - \frac{(c+\Delta)\omega_z}{V} \tag{15}$$

QUALITATIVE ANALYSES

The qualitative formulation for tire wear, which is based on lateral steady state dynamic responses, derived at the previous sections can be used to perform sensitivity analyses. Some sensitivity analyses were performed for two vehicles: with two and four axes. Typical values of mass, inertia, mass distribution, distance between the axes, etc. are the inputs for the simplified models that supplies some dynamic responses requested by the sensitivity analyses. The analyses of the vehicle with two axes, i.e. one steering axis, were made using typical parameters of light commercial vehicles. For a vehicle with two steering axes, typical parameters of heavy passenger vehicle were used. Before presenting the results for these typical vehicles, some statements about Ackerman geometry will be addressed.

Ackerman geometry

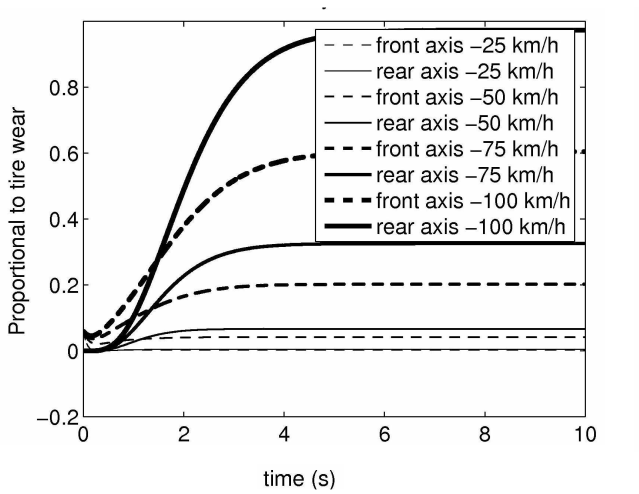
According to Maxwell (2006), deviations from Ackerman angles for the right or left steer angles can significantly affect tire wear. However, deviations do not significantly affect directional response for low speed turns. Consequently, the proposed methodology is not useful to evaluate tire wear caused by deviations from Ackerman angles. Since, the proposed approach is based on the lateral dynamics responses from a simplified model; it is only valid for low lateral accelerations. Therefore, these responses are only valid for low speed turns.

In spite of this approach does not fulfill the evaluation of the Ackerman angles, it is important to highlight that a good specification of the steer angles can prevent early tire wear phenomena. Since, it avoids high values of friction forces during cornering maneuvers.

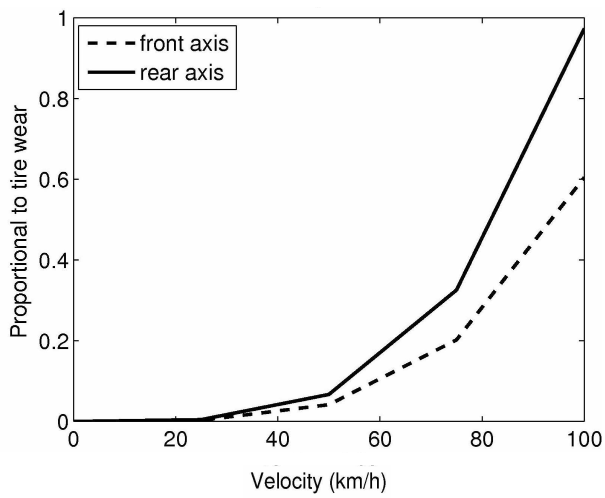
One steering axis vehicle

Since the proposed methodology is formulated for steady state maneuvers, a step steer input was chosen to simulate and evaluate the tire wear for several conditions and different parameters. The output responses are considered when the system is in steady state condition and are proportional to the tire wear according to Eq. (11).

A step steer input of 1º was applied to the vehicle at different velocities. The responses of a bicycle model of a light commercial vehicle for several velocities were calculated. The results which are proportional to the tire wear are shown in Fig. 2a. The steady state values for different velocities are shown in Fig. 2b. The tire wear of the rear axis is more severe than that of the front axis. A linear proportion is kept for larger step steer input.



(a)



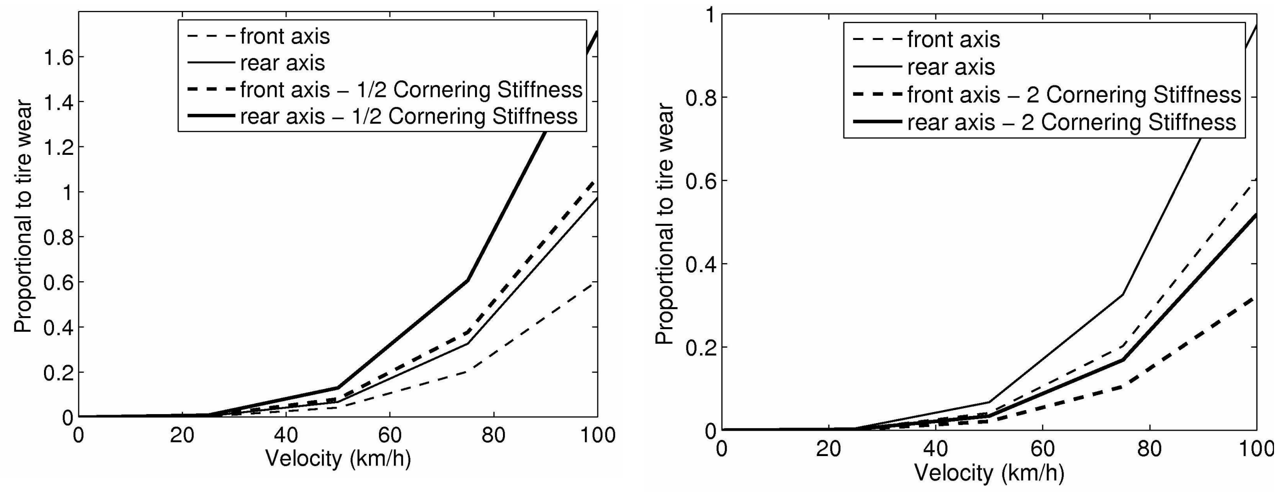
(b)

Figure 2 – (a) Qualitative tire wear values for a step steer input at different velocities (b) Steady-state qualitative tire wear values for different velocities for a vehicle with one steering axis

For the sensitivity analyses, some changes were made and the impact of these changes was evaluated. For the evaluation of a single steering axis vehicle, the sensitivity analyses were made changing the cornering stiffness coefficient and the location of the axes. The results are shown in the next sections.

Sensitivity analyses - cornering stiffness coefficient

Two situations were evaluated: (a) a reduction of 50% at the initial cornering stiffness coefficient and (b) an increase of 100% at the initial cornering stiffness coefficient. The comparison between the configurations is shown in Fig. 3. In both cases, the rear axis is still suffering the most severe tire wear. The lateral force is proportional to the cornering stiffness coefficient; consequently, the proportionality appears at the analysis.

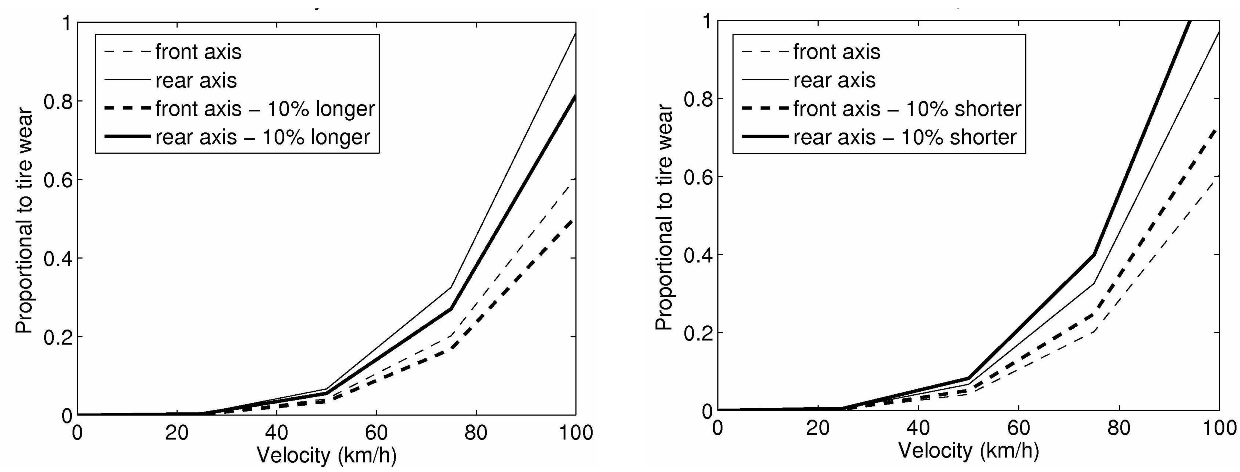


(a) **(b)**

Figure 3 – Comparison between the original configuration and the modified ones (a) half and (b) twice the cornering stiffness coefficient

Sensitivity analyses - location of the axes

Two situations were evaluated: (a) a vehicle 10% longer (b) a vehicle 10% shorter than the original. The comparisons between the configurations are shown in Fig. 4. These modifications were also not able to change the rear axis condition. The 10% longer configuration presents better results than the original configuration. This improvement was about 15%.

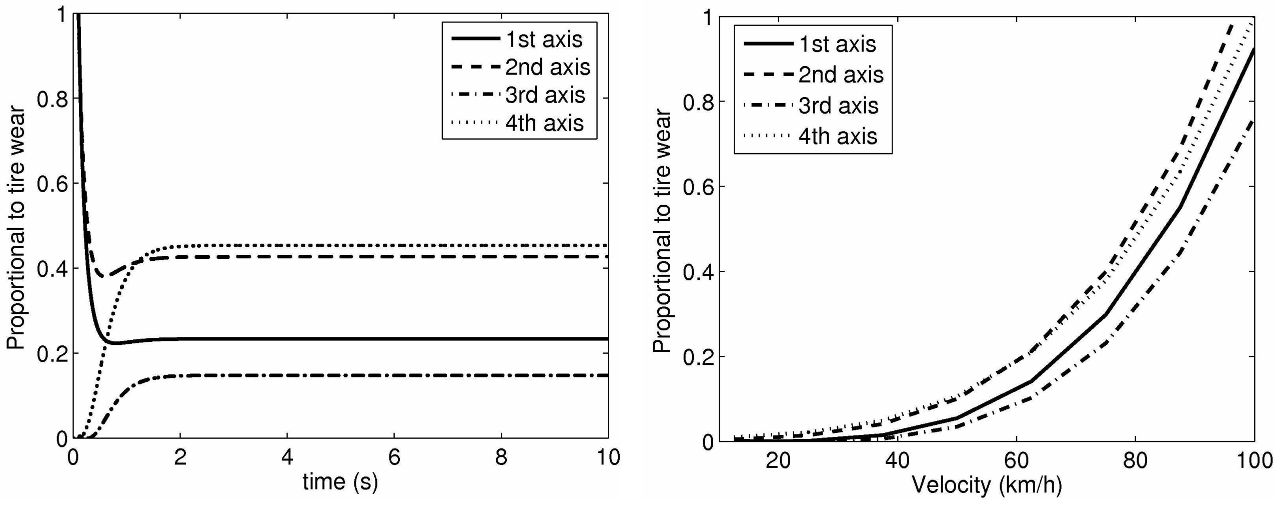


(a) **(b)**

Figure 4 – Comparison between the original configuration and the modified ones (a) half and (b) twice the cornering stiffness coefficient

Vehicles with two steering axes

The same approach was done to analyze the vehicle with two steering axes. A step steer input of 1º was applied to the vehicle at different velocities. The results for 75 km/h are shown at Fig. 5a. The steady state values for different velocities are shown at Fig. 5b. In this case, the tire wear of the second axis is more severe than the others. Some modifications were studied to change this condition. For the evaluation of the two steering axes vehicle, the steering relation between the axes and the location of the axes were the parameters evaluated.

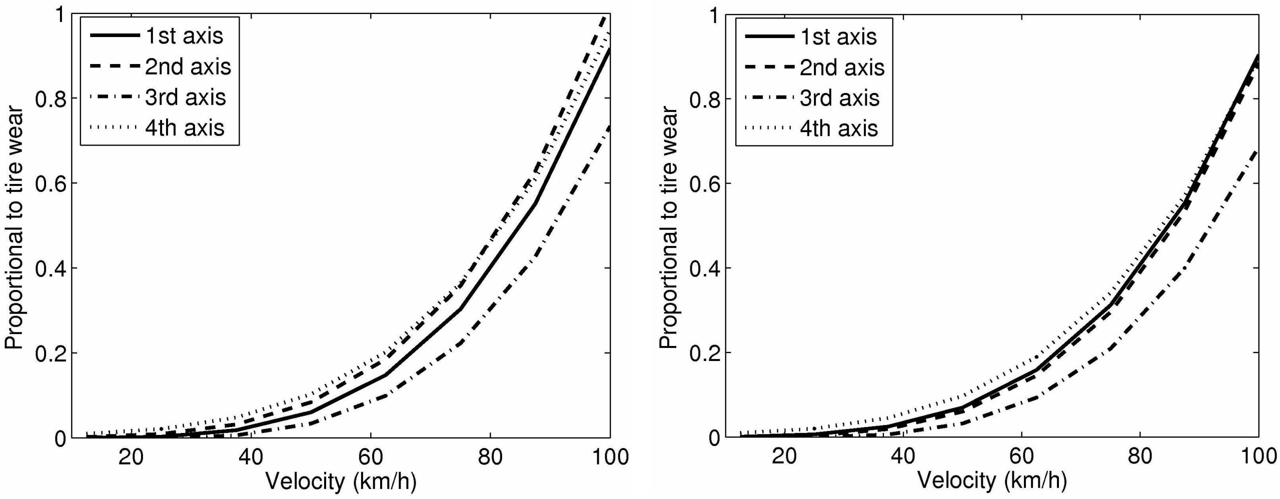


(a) **(b)**

Figure 5 – (a) Qualitative tire wear values for a step steer input at 75 km/h (b) Steady-state qualitative tire wear values for different velocities for a vehicle with two steering axes

Sensitivity analyses - steering relation between the axes

Two situations were evaluated: (a) a steering relation 5% smaller (b) a steering relation 14% smaller than the original. The comparisons between the configurations are shown in Fig. 6. Both modifications were able to reduce the tire wear of the second, third and the fourth axes. In fact, the second configuration was the optimal case resulted from an extensive search for the best steering relation between the axes.



(a) **(b)**

Figure 6 – Steady-state qualitative tire wear values for modified configurations: a steering relation (a) 5% smaller and (b) 14% smaller

Sensitivity analyses - location of the axes

Considering the scheme in Fig. 7 for the location of the axes, the Table 1 can be interpreted as the four studied configuration: the distance between the first and the second axes, Ld, (a) 20% shorter and (b) 20% longer, and the distance between the third and the fourth axes, Lt, (c) 20% shorter and (d) 20% longer. The comparisons between the configurations are shown in Fig. 8. These configurations were evaluated with the optimal steering relation between the axes. No significant improvement was made by changing the location of axis on this vehicle.

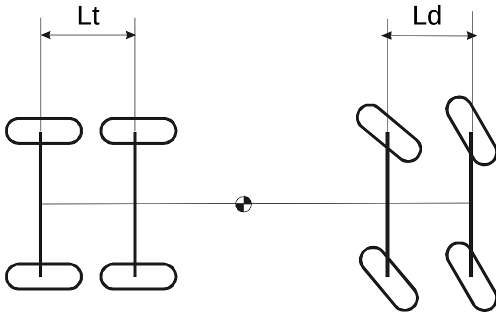
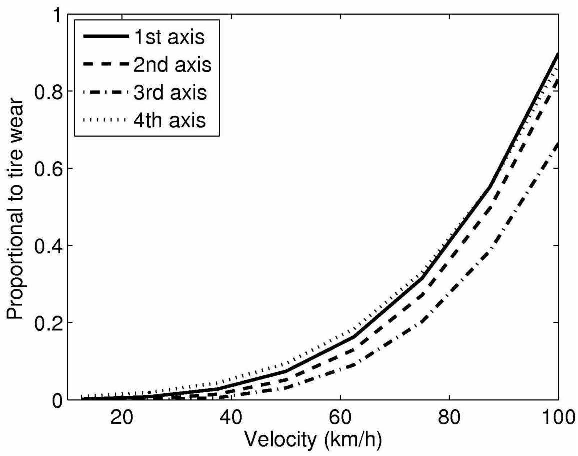


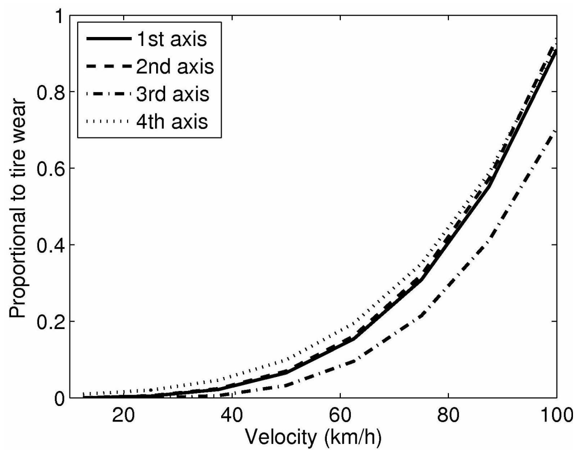
Figure 7 – Distance between the front and the rear axis

Table 1 – Study case description

Case	Ld	Lt	Results
a	0.8 Ld	Lt	slightly better
b	1.2 Ld	Lt	worst
c	Ld	0.8 Lt	no significant change
d	Ld	1.2 Lt	worst



(a)



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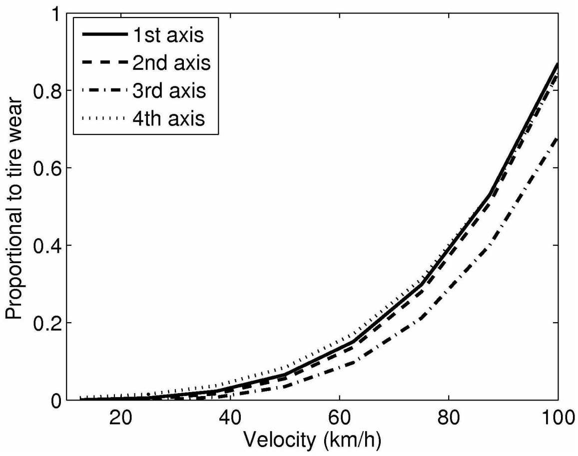
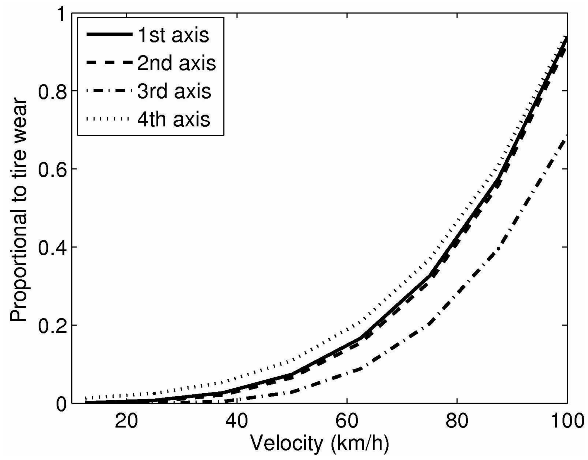


(c)



(d)

Figure 8 – Steady-state qualitative tire wear values for modified configurations: Ld (a) 20% shorter and (b) 20% longer, and Lt (c) 20% shorter and (d) 20% longer

CONCLUSIONS

An empirical metric was derived for tire wear evaluation during steady state maneuvers. As tire wear depends on the friction work developed by the tire-road contact, an approach can be derived to estimate it during steady state longitudinal and cornering drive using the wheel slip, which is the relative difference in wheel velocities. Since a qualitative formula was obtained, some sensitivity analysis considering cornering maneuvers could be performed using a simple model for lateral vehicle dynamics such as the single-track model. Using this criteria and the single-track model, some new configurations were evaluated. This approach was also extended for vehicle with two steering axis, where an extension of the single-track model was used. Some results were shown, and some statements found in the literature were highlighted which allows the designer to infer about the vehicle behavior considering tire wear using simple models and standard statements.

Conclusions can just be drawn for specific vehicles. However, two clear conclusions can be observed from the sensitivity analyses. First, there is an optimal configuration for the location of the axes for vehicles with two axes. Second, for vehicles with two steering axes, there is an optimal steering relation between the axes.

In spite of this approach does not fulfill the evaluation of the Ackerman angles, it is important to highlight that a good specification of the steer angles can prevent early tire wear phenomena, since it avoids high values of friction forces during cornering maneuvers.

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