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Gravitational Capture in the Bi-Circular Restricted Four-Body Problem.

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Abstract: The objective of the present paper is to study the ballistic gravitational capture in a dynamical model that has the presence of four bodies In particular, the earth-moon-sun-spacecraft system is considered. This phenomenon is explained in terms of the integration of the perturbing forces with respect to time. Numerical simulations are performed to show the differences between the models with three and four bodies in the trajectories of the spacecraft.

Keywords: capture, four-body, bi-circular.

1.0 INTRODUCTION

A gravitational capture occurs when a spacecraft (or any particle with negligible mass) change from a hyperbolic orbit with a small positive energy around a celestial body into an elliptic orbit with a small negative energy without the use of any propulsive system. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of the third and the fourth bodies involved in the dynamics. In this way, those forces are used as a zero cost control, equivalent to a continuous thrust applied in the spacecraft. One of the most important applications of this property is the construction of trajectories to the Moon. The concept of gravitational capture is used together with the basic ideas of the gravity-assisted maneuver and the bi-elliptic transfer orbit to generate a trajectory that requires a fuel consumption smaller than the one required by the Hohmann transfer. This maneuver consists of the following steps: i) the spacecraft is launched from an initial circular orbit with radius r_0 to an elliptic orbit that crosses the Moon's path; ii) a Swing-By with the Moon is used to increase the apoapsis of the elliptic orbit. This step completes the first part of the bi-elliptic transfer, with some savings in ΔV due to the energy gained from the Swing-By; iii) With the spacecraft at the apoapsis, a second very small impulse is applied to rise the periapsis to the Earth-Moon distance. The solar effects can reduce even more the magnitude of this impulse; iv) The transfer is completed with the gravitational capture of the spacecraft by the Moon.

The application of this phenomenon in spacecraft trajectories is recent in the literature. The first demonstration of this was in Belbruno, 1987. Further studies include Belbruno (1990 and 1992); Krish (1991); Krish, Belbruno and Hollister (1992); Miller and Belbruno (1991); Belbruno and Miller (1990 and 1993). They all studied missions in the Earth-Moon system that use this technique to save fuel during the insertion of the spacecraft in its final orbit around the Moon. Another set of papers that made fundamental contributions in this field, also with the main objective of constructing real trajectories in the Earth-Moon system, are those of Yamakawa, Kawaguchi, Ishii and Matsuo (1992 and 1993) and Yamakawa (1992). The first real application of a ballistic capture transfer was made during an emergency in a Japanese spacecraft (Belbruno and Miller, 1990). After that, some studies that consider the time required for this transfer appeared in the literature. Examples of this approach can be found in the papers by Vieira-Neto and Prado (1995 and 1998). An extension of the dynamical model to consider the effects of the eccentricity of the primaries is also available in the literature (Vieira-Neto and Prado, 1996; Vieira-Neto, 1999). A study of this problem, from the perspective of invariant manifolds, was developed by Belbruno (1994). Regarding applications of the bicircular problem, some of the important papers are: Simo, Gomez, Jorba and Masdemont (1995), Gomez, Jorba, Martinez and Simo (2001), Gomez, Llibre, Martinez and Simo (2001) and Castella and Jorba (2000).

The present paper has the goal of developing analytical equations to estimate the effects of the fourth body in the gravitational capture. The main developments are made for the Earth-Moon-Sun-Spacecraft system, but many results are valid for any system of primaries.

2.0 Mathematical models.

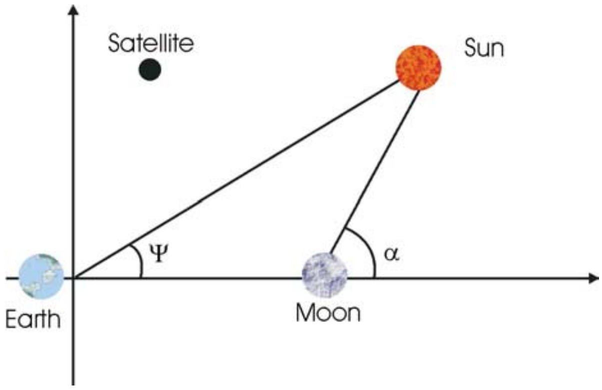

Figure 1 – Bi-circular problem.

In the bi-circular problem (Cronin, Richards and Russel, 1964), we have (Figure 1): $i)$ The Earth and the Moon constitute the two main primaries of the system, both in circular orbits around the common center of mass; $ii)$ The Sun is the third body, assumed to be in a circular orbit around the center of mass of the Earth-Moon system and its orbit is coplanar with the orbit of the Moon around the Earth; $iii)$ The trajectory of a massless fourth body that has its motion governed by the three primaries is studied. The center of mass of the Earth-Moon system is the origin of the reference system. The positions of the bodies are: $(-\mu_2, 0, 0)$ for the Earth and $(\mu_1, 0, 0)$ for the Moon. The values of the masses are given by $\mu_1 = 0.9878493317$ for the Earth, $\mu_2 = 0.0121506683$ for the Moon and $\mu_{Sun} = 328900.48$ for the Sun (canonical units). To obtain the equations of motion, the same procedure used by Yamakawa (1992) is used here. The Lagrangian is written as:

$$L = T + U$$

where T is the kinetic energy given by:

$$T = \frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right) + \left(x\frac{dy}{dt} - \frac{dx}{dt}y\right) + \frac{1}{2}\left(x^2 + y^2\right)$$

and U is the potential energy given by:

$$U = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \mu_{sun}\left[\frac{1}{r_{sun}} - \frac{xx_s + yy_s}{R_{sun}^{\ 3}}\right]$$

r_1, r_2, and r_{Sun} represents the distances between the spacecraft and the Earth, the Moon and the Sun, respectively. $R_{Sun} = 389.1723985$ is the distance between the Sun and the origin of the coordinate system. (x_s, y_s, z_s) represents the position of the Sun with respect to the rotating system. Those variables are given by:

$$r_1^{\ 2} = (x + \mu_2)^2 + y^2 + z^2,$$

$$r_2^{\ 2} = (x - \mu_1)^2 + y^2 + z^2,$$

$$r_{sun}^{\ 2} = (x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2 = (x + R_{sun}\cos\psi)^2 + (y - R_{sun}\sin\psi)^2 + z^2$$

$$\psi = \overline{\alpha} + (\omega_{E-M} - \Omega_{sun})t$$

where $\overline{\alpha}$ is the phase angle of the Sun, given by (anti-Sun direction)-(origin)-(Moon), that is the initial position of the Moon in a system that has the Earth and the Sun in fixed positions; $\omega_{E\text{-}M}$ is the angular velocity of the Earth-Moon system (= 1.00); Ω_{sun} is the angular velocity of the Sun with respect to the system of reference (= 0.07480133).

From the Lagrangian, it is possible to obtain the equations of motion, that are given by:

$$\begin{aligned} &\frac{d^2x}{dt^2} - 2\frac{dy}{dt} - x - \frac{\mu_{sun}}{R_{sun}^2}\cos\psi = -\frac{\partial U_n}{\partial x} \\ &\frac{d^2y}{dt^2} + 2\frac{dx}{dt} - y + \frac{\mu_{sun}}{R_{sun}^2}\sin\psi = -\frac{\partial U_n}{\partial y} \quad (1) \\ &\frac{d^2z}{dt^2} = -\frac{\partial U_n}{\partial z} \end{aligned}$$

where $U_n = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{\mu_{sun}}{r_{sun}}$,

$$\begin{aligned} \frac{\partial U_n}{\partial x} &= \frac{\mu_1}{r_1^3}(x+\mu_2) + \frac{\mu_2}{r_2^3}(x-\mu_1) + \frac{\mu_{sun}}{r_{sun}^3}(x + R_{sun}\cos\psi) \\ \frac{\partial U_n}{\partial y} &= \frac{\mu_1}{r_1^3}(y) + \frac{\mu_2}{r_2^3}(y) + \frac{\mu_{sun}}{r_{sun}^3}(y - R_{sun}\cos\psi) \\ \frac{\partial U_n}{\partial z} &= \frac{\mu_1}{r_1^3}(z) + \frac{\mu_2}{r_2^3}(z) + \frac{\mu_{sun}}{r_{sun}^3}(z) \end{aligned}$$

The standard canonical system of units is used, in which: the unit of distance is the distance between M_1 (the Earth) and M_2 (Moon); the angular velocity (ω_{E-M}) of the motion of M_1 and M_2 is set to unity; the mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1 + m_2)$ (where m_1 and m_2 are the real masses of the Earth and the Moon, respectively) and the mass of M_2 is (1-μ); the unit of time is defined such that the period of the motion of the two primaries is 2π and, as a consequence of those choices, the gravitational constant is unity.

When considering the restricted three-body problem, the well known equations shown below are used, also in the canonical system of units:

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}, \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y},$$

where Ω is the pseudo-potential given by: $\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}$.

3.0 Numerical Simulations.

To show better the differences between the dynamical systems including three and four bodies, we made several simulations. The numerical integration performed here and in all phases of the present research used the Runge-Kutta 7-8 method.

In Fig. 2, we have parabolic orbits, case $C_3 = 0$. In Fig. 3 we have elliptical orbits with $C_3 = -0.10$. In both cases: red is for $\alpha = 0^\circ$, green is for $\alpha = 45^\circ$, light blue is for $\alpha = 90^\circ$, pink is for $\alpha = 135$, dark blue is for $\alpha = 180$, brown is for $\alpha = 225$, dark grenn is for $\alpha = 270$. In all plots $\psi = 0$.

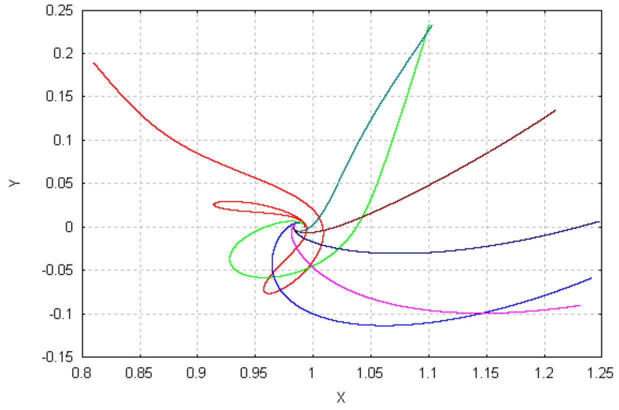

Figure 2 – Parabolic orbits.

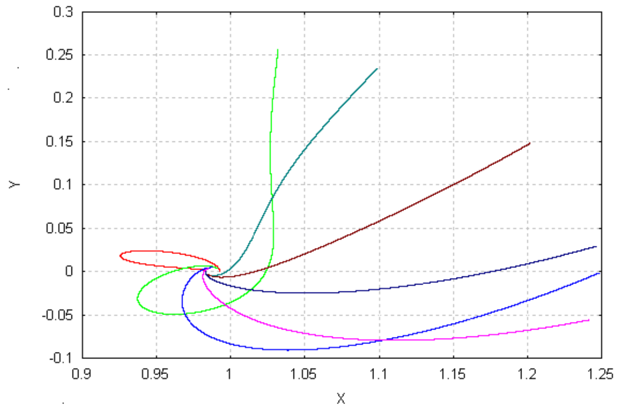

Figure 3 – Elliptical orbits.

In Fig. 4, we have direct orbit and in Fig. 5 we have a retrograde orbit. In the figs below we have: $\alpha = 180^{\circ}$ e $\psi = 0^{\circ}$. Com $C_3 = -0.6$.

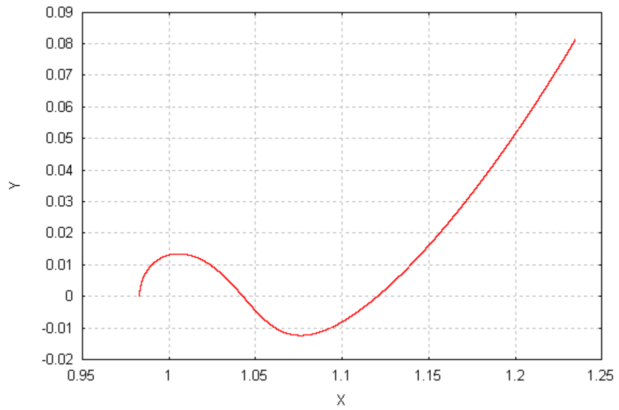

Figure 4 – Direct orbit.

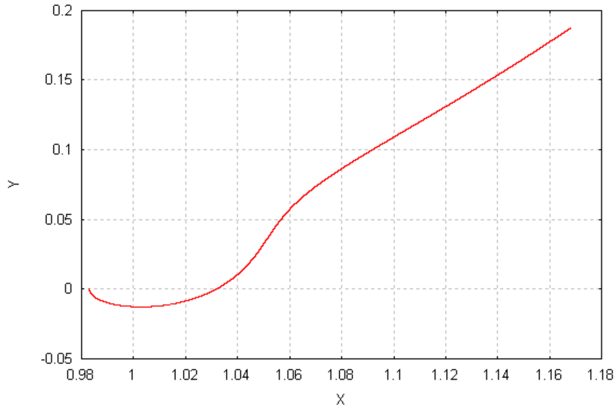

Figure 5 – Retrograde orbit.

Then we study C_3. We have $\alpha = 180^{\circ}$ e $\psi = 0^{\circ}$. For the pink plot we have $C_3 = -0.6$, for the blue one we have $C_3 = -0.4$, for the green $C_3 = -0.2$, for the red $C_3 = 0.0$. In fig. 6 we have direct orbits and in Fig. 7 retrograde orbits.

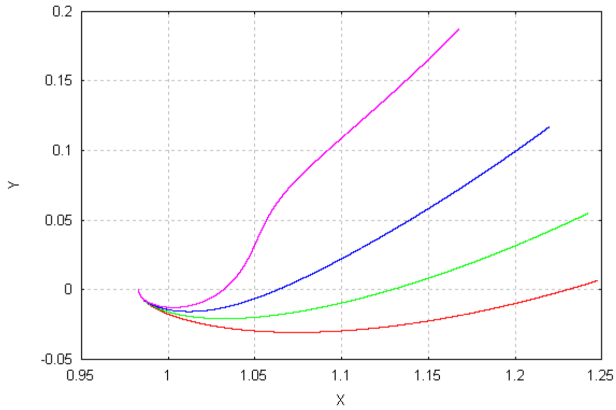

Figure 6 – Directs orbits.

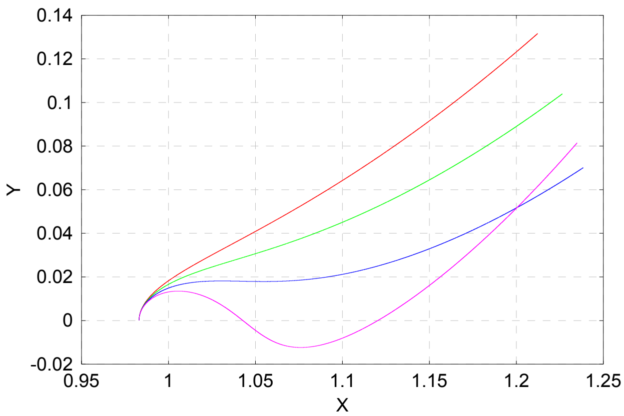

Figure 7 – Retrograde orbit.

Now we make $C_3 = -0.15$ e $\alpha = 120^{\circ}$ and vary ψ. In red we have $\psi = 0^{\circ}$, in green $\psi = 90^{\circ}$, in blue $\psi = 180^{\circ}$, in pink $\psi = 270^{\circ}$. In fig. 8 we have direct orbits and in Fig. 9 retrograde orbits.

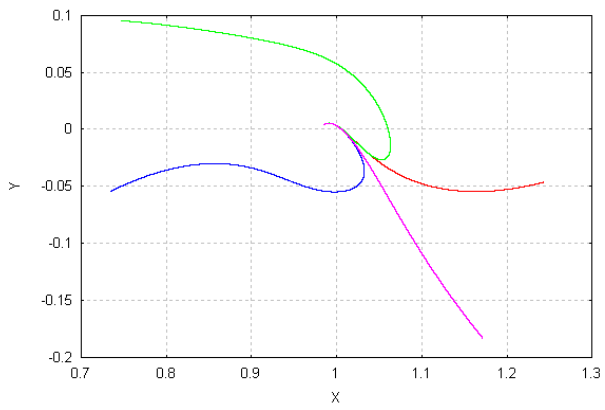

Figure 8 – Directs orbits.

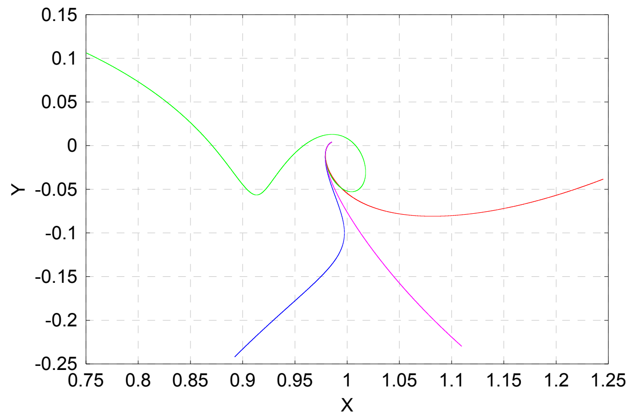

Figure 9 – Directs orbits.

4.0 Conclusions.

This paper had the main goal of studying the ballistic gravitational capture problem under the model given by the bicircular problem. The numerical results showed that large savings can be obtained under the bi-circular four body problem when compared to the restricted three body problem. In order to obtain those savings, it is necessary to find a proper geometry to start the maneuver, such that the force of the Sun acts to reduce de velocity of the spacecraft that is approaching the Moon. If this is not done, the savings can be reduced or even desapear.

The next topics to be covered in this research will be: analysis of the time of gravitational capture, study of the minimum consumption of fuel and some numerical results of temporary gravitational capture for other mathematical models, those will be studied in the plan and in the space.

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