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Multiobjective Optimization of Viscoelastic Structures Combining Robust Condensation and Adaptive Response Surface

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Abstract: In the context of passive control of noise and vibration the use of viscoelastic materials is an interesting means of achieving effective mitigation in various types of industrial applications, where these materials can be applied either as discrete devices or surface treatments at a relatively low cost. The effective design of viscoelastic dampers as applied to real-world complex structures can be conveniently carried out by using modern multi-objective numerical optimization techniques which are capable of dealing with various objective functions, some of which can be conflicting. The large number of evaluations of the cost functions, combined with the typically high dimensions of finite element models of industrial structures, makes multi-objective optimization very costly, some times impossible. Those difficulties motivate the study reported in this paper, in which a strategy is proposed consisting in the use of multi-objective evolutionary algorithms combined with robust condensation and metamodels. After the discussion of various theoretical aspects involved in the study, a numerical application is presented to illustrate the proposed multiobjective optimization methodology for the optimal design of viscoelastic surface treatment applied to a stiffened panel.

Keywords: multiobjective optimization, robust condensation, viscoelastic damping, adaptive response surface

INTRODUCTION

The use of viscoelastic materials has been regarded as a convenient strategy in many types of industrial applications, where these materials can be applied either as discrete devices, such as translational and/or rotational mounts, or surface treatments (free or constrained layers) at a low cost of application (Nashif et al., 1985; Samali and Kwok, 1995; Rao, 2001). Another interesting strategy consists in incorporating viscoelastic materials as a means of adding damping to vibration neutralizers (Espíndola and Silva, 1992; Espíndola and Bavastri, 1997). In the last decades, much effort has been devoted to the development of finite element models capable of accounting for the typical dependence of the viscoelastic behavior with respect to frequency and temperature (Balmès and Germès, 2002). As a result, it is currently possible to perform finite element modeling of complex real-world engineering structures such as automobiles, airplanes, communication satellites, tall buildings and space structures.

A natural extension of modeling capability is the optimization of the viscoelastic devices aiming at the reduction of cost and/or maximization of performance. In the quest for optimization, the engineers are frequently faced with conflicting objectives. Such situations are conveniently dealt with by the so-called multi-objective or multicriteria optimization approach (Eschenauer et al., 1990). However, multi-objective optimization generally requires a large number of evaluations of the cost functions involved. For large finite element models of viscoelastic systems typically composed by many thousands of degrees-of-freedom, if such evaluations are made based on response computations performed on the full matrices, computation times can be prohibitive. The work reported herein intends to propose a general strategy for the reduction of the computational burden of multi-objective optimization of structures containing viscoelastic materials by combining multi-objective evolutionary algorithms (MOEAs), robust condensation and response surface methodology (RSM).

Because of possible contradictions observed between the objective functions in multi-objective optimization, it is not possible to find a single solution that would optimize all the objectives simultaneously. In this case, the optimization result is formed by a set of compromise solutions between several objective functions. These solutions are known as the Pareto optimal solutions (Srinivas and Deb, 1993). Evolutionary algorithms have been widely used to treat multi-objective optimization problems in different domains, notably in the field of structural dynamics and vibrations. Several applications of evolutionary algorithms are reported in the literature, (Srinivas and Deb, 1993; Soteris, 2004).

The motivation for the use of robust condensation is that the computation of the complex response based on the complete FE-model by direct inversion of the complex dynamic stiffness matrix is not feasible. It is suggested herein to use an adapted version of the approach of Masson et al. (2003) and Balmès and Germès (2002), based on the extension of the modal basis of reduction to viscoelastic damped systems. This robust basis is constructed in such a way to take into account the structural modifications introduced by the inclusion of the viscoelastic treatments, in the lower frequency domain, thus avoiding the updating of the basis of reduction by exact re-analysis. This leads to a drastic reduction of the time required to compute the complex frequency response functions.

The RSM consists in approximating the response of the system by low order polynomials with the aim of reducing the time required for evaluation of cost functions in model updating and/or optimizations (Ghanmi et al., 2005). Most of the response surface construction methods use a single quadratic polynomial to represent the entire design space of variables and are classified as global methods (Myers, 2002). The main problem in using the global methods is that they introduce large errors in the response estimation. In order to overcome such difficulties, an *Adaptive Response Surface Methodology* (ARSM) is used. In the following sections, the theoretical foundations of the optimization strategy are first summarized, followed by the description of a numerical example that demonstrates its effectiveness when applied to the optimal design of viscoelastic constrained layer treatments.

THE COMPLEX MODULUS APPROACH

According to the linear theory of viscoelasticity (Christensen, 1982), the one-dimensional stress-strain relation can be expressed, in Laplace domain, as follows:

$$\sigma(s) = \left(G_r + H(s)\right)\varepsilon(s) \tag{1}$$

In the equation above, G_r is the *static modulus*, representing the elastic behavior and $H(s)$ is the *relaxation function*, associated to the dissipation effects. When evaluated along the imaginary axis of the s-plane ($s = i\omega$), Eq. (1) leads to the complex modulus, which can be expressed under the following form:

$$G(\omega) = G'(\omega) + iG''(\omega) = G'(\omega)\left(1 + i\,\eta(\omega)\right) \tag{2}$$

where $G'(\omega)$, $G''(\omega)$ and $\eta(\omega) = G''(\omega)/G'(\omega)$ are the *storage* and *loss moduli* and *loss factor*, respectively. The complex modulus provides a straightforward model the viscoelastic behavior in the frequency domain.

INFLUENCE OF TEMPERATURE ON THE VISCOELASTIC BEHAVIOR

Temperature is usually considered to be single most important environmental factor which exerts influence upon the properties of viscoelastic materials. Thus, it becomes important to account for temperature variations in the modeling of structural systems containing viscoelastic elements. This can be done by making use of the so-called *Frequency-Temperature Superposition Principle – FTSP*, which establishes a relation between the effects of the excitation frequency and temperature on the properties of the thermorheologically simple viscoelastic materials (Nashif et al., 1985). This implies that the viscoelastic characteristics at different temperatures can be related to each other by changes (or shifts) in the actual values of the excitation frequency. This leads to the concepts of *shift factor* and *reduced frequency*. Symbolically, the *FTSP* can be expressed under the following forms:

$$G(\omega,T) = G'(\omega_r, T_0) = G(\alpha_T\omega, T_0), \quad \eta(\omega_r, T_0) = \eta(\alpha_T\omega, T_0) \tag{3}$$

where $\omega_r = \alpha_T(T)\omega$ is the reduced frequency, ω is the excitation frequency, α_T is the *shift* factor, and T_0 is a reference value of temperature. Figure 1 illustrates the *FTSP*, showing that having the modulus and loss factor of a given viscoelastic material for different temperature values, T_{-1}, T_0, T_1, if horizontal shifts along the frequency axis are applied to each of these curves, all of them can be combined into a single one, called master curves. The horizontal shift α_T depends on the temperature.

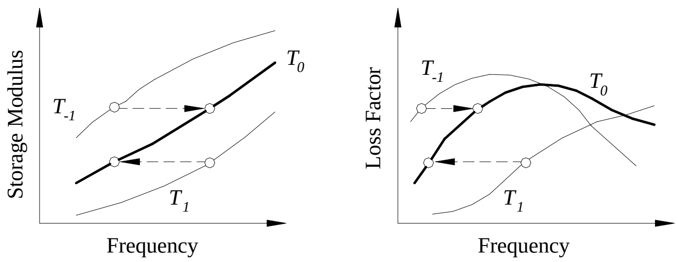

Figure 1 – Illustration of the *Frequency-Temperature Superposition Principle (FTSP)*

The functions $G(\omega_r)$ and $\alpha_T(T)$ can be obtained from experimental tests for specific viscoelastic materials (Nashif et al., 1985; Espíndola et al., 2005). As the result of a comprehensive experimental work, Drake and Soovere (1984) suggest analytical expressions for the complex modulus and shift factor for various commercial viscoelastic materials.

The following equations give these functions for the 3M ISD112™ viscoelastic material, which is considered in the numerical application that follows, as provided by those authors:

$$G(\omega_r) = B_1 + B_2 / (1 + iB_5\omega_r / B_3)^{-B_6} + (i\omega_r / B_3)^{-B_4} \tag{4a}$$

$$log(\alpha_T) = a(1/T - 1/T_0) + 2.303(2a/T_0 - b)log(T/T_0) + (b/T_0 - a/T_0^2 - S_{AZ})(T - T_0) \tag{4b}$$

where :

$B_1 = 0.4307MPa; B_2 = 1200MPa; B_3 = 0.1543MPa; B_4 = 0.6847; B_5 = 3.241; B_6 = 0.18; T_r = 290K; T_L = 210K; T_H = 360K;$
$S_{AZ} = 0.05956K^{-1}; S_{AL} = 0.1474K^{-1}; S_{AH} = 0.009725K^{-1}; C_A = (1/T_L - 1/T_r)^2; C_B = (1/T_L - 1/T_r); C_C = (S_{AL} - S_{AZ}); D_A = (1/T_H - 1/T_r)^2$
$D_B = (1/T_H - 1/T_r); D_C = (S_{AH} - S_{AZ}); a = ((D_B C_C - C_B D_C)/D_E); b = ((D_C C_A - C_C D_A)/D_E)$

Figure 2 depicts the standardized curves representing the variations of the storage modulus, loss modulus and loss factor as functions of the reduced frequency, and a plot of the shift factor as a function of temperature for 3M ISD112™ .

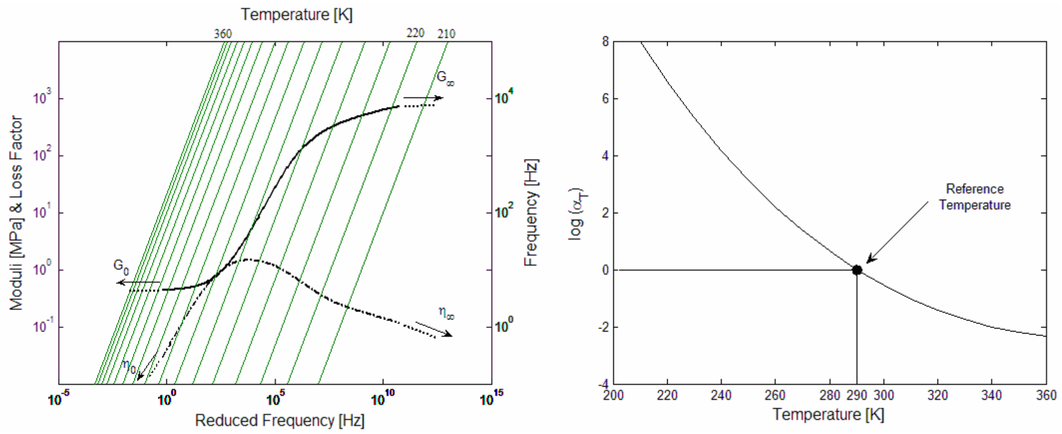

Figure 2 – Master (left) and shift factor curves (right) for the 3M ISD112™ viscoelastic material

VISCOELASTIC BEHAVIOR INCORPORATED INTO FINITE ELEMENT MODELS

Consider the following equations of motion of a viscoelastic structure containing N degrees-of-freedom:

$$\begin{gathered}(\boldsymbol{K}(\omega,T) - \omega^2\boldsymbol{M})\boldsymbol{q}(\omega) = \boldsymbol{Z}(\omega,T)\boldsymbol{q}(\omega) = \boldsymbol{bu}(\omega)\\ \boldsymbol{y}(\omega) = \boldsymbol{cq}(\omega)\end{gathered} \tag{5}$$

where $\boldsymbol{M}, \boldsymbol{K}(\omega,T) \in R^{N\times N}$ are the mass (symmetric, positive-definite) and stiffness (symmetric, nonnegative-definite) matrices. $\boldsymbol{q}(\omega) \in R^N$ and $\boldsymbol{f}(\omega) = \boldsymbol{bu}(\omega) \in R^N$ are the vectors of displacement and external loads, and $\boldsymbol{y}$ is the vector of response. Matrices $\boldsymbol{b}$ and $\boldsymbol{c}$ are boolean matrices which enable to select, among the finite element degrees of freedom, those in which the excitation forces are applied and responses are computed, respectively.

It is assumed that the structure contains both elastic and viscoelastic elements, so that the stiffness matrix can be decomposed as follows:

$$\boldsymbol{K}(\omega,T) = \boldsymbol{K_e} + \boldsymbol{K_v}(\omega,T) \tag{6}$$

where $\boldsymbol{K_e}$ is the stiffness matrix corresponding to the purely elastic substructure and $\boldsymbol{K_v}(\omega,T)$ is the stiffness matrix associated with the viscoelastic substructure. The inclusion of the frequency-dependent behavior of the viscoelastic material can be made by using the so-called *elastic-viscoelastic correspondence principle* (Christensen, 1982), according to which, for a given temperature, $\boldsymbol{K_v}(\omega,T)$ can be first generated for specific types of finite elements (rods, beam, plates, etc.) assuming that the longitudinal modulus $E(\omega,T)$ and/or the shear modulus $G(\omega,T)$ (according to the stress-state) are frequency-independent. Then, after the finite element matrix is constructed, the frequency-dependence of such moduli is introduced according to a particular viscoelastic model adopted. By assuming the widely accepted hypothesis of a constant Poisson ratio for the viscoelastic material, $E(\omega,T)$ becomes proportional to $G(\omega,T)$ through the relation $G(\omega,T) = E(\omega,T)/2(1+\nu)$. Then, one of the two moduli can be factored-out of the stiffness matrix of the viscoelastic sub-structures, $\boldsymbol{K_v}(\omega,T) = G(\omega,T)\overline{\boldsymbol{K}}_{\boldsymbol{v}}$ (were $\overline{\boldsymbol{K}}_{\boldsymbol{v}}$ is a constant matrix), which is combined with Eqs. (5) and (6) to produce the following complex dynamic stiffness matrix:

$$\boldsymbol{Z}(\omega,T) = \boldsymbol{K_e} + G(\omega,T)\overline{\boldsymbol{K}}_{\boldsymbol{v}} - \omega^2 \boldsymbol{M} \tag{7}$$

The complex FRF matrix can be expressed as follows:

$$\boldsymbol{H}(\omega,T) = \boldsymbol{c}\boldsymbol{Z}(\omega,T)^{\boldsymbol{-1}}\boldsymbol{b} \tag{8}$$

ROBUST CONDENSATION OF THE VISCOELASTIC MODEL

The exact responses can be approached by solutions in the reduced space of the following form:

$$\boldsymbol{q}(\omega) = \boldsymbol{T}\boldsymbol{q_r}(\omega) \tag{9}$$

where $\boldsymbol{T} \in R^{N \times NR}$ denotes a reduction basis (or Ritz basis), $\boldsymbol{q_r}(\omega) \in R^{NR}$ were $NR << N$ the number of retained modes.

By considering Eqs. (5) and (9), the transfer function (8) can be rewritten under the following form:

$$\boldsymbol{H_c}(\omega,T) = \boldsymbol{c}\boldsymbol{Z_c}(\omega,T)^{\boldsymbol{-1}}\boldsymbol{b} \tag{10}$$

where $\boldsymbol{Z_c}(\omega,T) = \boldsymbol{T^T}\boldsymbol{K_e}\boldsymbol{T} + G(\omega,T)\boldsymbol{T^T}\overline{\boldsymbol{K}}_{\boldsymbol{v}}\boldsymbol{T} - \omega^2\boldsymbol{T^T}\boldsymbol{M}\boldsymbol{T}$ is the reduced dynamic stiffness matrix.

The reduced dynamic stiffness matrix is then computed frequency by frequency, in a direct way. For viscoelastic systems, the selection of the basis of reduction poses a problem. Owing to the dependence of the stiffness matrix with respect to frequency and temperature, this basis should be also dependent on frequency and temperature. Three solutions are possible: a) one can neglect this dependence by considering the stiffness matrix as independent from frequency and temperature; b) one can use a basis of reduction obtained by the resolution of the nonlinear eigenvalue problem; c) one can use an iterative method, which allows the re-actualization of the basis of reduction according to the frequency (Balmès and Germès, 2002). This later alternative is adopted herein. One fixes the basis of reduction (which is assumed to be independent from frequency and temperature) and one determines the response by using the standard approximation of Ritz-Galerkin. For that, the tangent elastic matrix is a convenient starting point, representing the static behavior of the viscoelastic material. As shown in Fig. 2, in the low frequency, one prolongs the curves by asymptotes, and the extrapolation curves give the real values G_0 and $\eta_0 = 0$. For high frequencies, the extrapolation gives the complex values G_∞ and η_∞. The tangent stiffness matrix can thus be calculated as follows (Balmès and Germès, 2002):

$$\boldsymbol{K_0} = \boldsymbol{K_e} + G_0\overline{\boldsymbol{K}}_{\boldsymbol{v}} \tag{11}$$

The nominal basis of reduction containing the first retained modes of the damped system can thus be obtained by the resolution of the following eigenvalue problem:

$$\begin{aligned} &(\boldsymbol{K_0} - \lambda_i \boldsymbol{M})\phi_i = 0 \qquad i = 1,\ldots,N \\ &\boldsymbol{\phi_0} = [\phi_1 \quad \phi_2 \quad \ldots \quad \phi_{NR}], \quad \Lambda_0 = diag(\lambda_1,\ldots,\lambda_{NR}) \end{aligned} \tag{12}$$

The basis of reduction $\boldsymbol{\phi_0}$ contains the free modes of the system by taking into account the tangent elastic matrix. To enrich this basis, it was proposed by Balmès and Germès (2002) to introduce the residues based on the static displacements associated with imposed loadings, $\boldsymbol{R} = \boldsymbol{K_0}^{\boldsymbol{-1}}\boldsymbol{b}$, called static correction of first order, which are completed by the imaginary part of complex dynamic stiffness matrix, to take into account the viscoelastic forces (damping effects). Thus, the enriched basis is represented as:

$$\boldsymbol{T_0} = \left[\boldsymbol{\phi_0} \quad \boldsymbol{R} \quad \boldsymbol{R_\Delta^0}\right] \tag{13}$$

where $\boldsymbol{R_\Delta^0}$ is the residue associated with the viscoelastic effects of the nominal system.

The basis of reduction represented by the Eq. (13) can be used to reduce the viscoelastic damped systems with accuracy, but it is not “robust” enough to take into account the parametric modifications entailed by the viscoelastic treatment. With the aim of reducing the computing time of response calculation of a new basis of reduction for a parametric modification, for example, during an optimization procedure and/or model updating, one can use the same basis evaluated for the initial model (expression (13)), enriched by residual vectors depending on the modifications (Masson et al. 2003) introduced by the viscoelastic treatment, thus increasing the quality of representation of the dynamic behavior of the modified damped system.

For the viscoelastic surface treatments, the initial model is not perturbed in the entire structure, but only in the zones treated with viscoelastic materials. Thus, one can write the modified dynamic system under the following form:

$$\left(\Delta\boldsymbol{Z}(\omega,T)+\boldsymbol{Z}(\omega,T)\right)\boldsymbol{q}(\omega)=\boldsymbol{f}(\omega) \tag{14}$$

where $\Delta\boldsymbol{Z}(\omega,T)=\Delta\boldsymbol{K}_v(\omega,T)-\omega^2\Delta\boldsymbol{M}_v$ is the dynamic stiffness matrix associated with the parametric modifications related with a viscoelastic treatment. By introducing the concept of forces associated to these modifications, Eq. (14) can be interpreted as the dynamic equilibrium equation of the initial model, subjected to the forces associated to the viscoelastic modifications, $\boldsymbol{f}_\Delta(\omega,T)$:

$$\boldsymbol{Z}(\omega,T)\boldsymbol{q}(\omega)=\boldsymbol{f}(\omega)+\boldsymbol{f}_\Delta(\omega,T) \tag{15a}$$

$$\boldsymbol{f}_\Delta(\omega,T)=-\Delta\boldsymbol{Z}(\omega,T)\boldsymbol{q}(\omega) \tag{15b}$$

The perturbed matrices associated with a modifiable viscoelastic zone can be calculated under the following forms:

$$\Delta\boldsymbol{M_v}=\sum_{i=1}^{mp}\boldsymbol{M_v}(\Delta m_i),\quad \Delta\overline{\boldsymbol{K}}_{\boldsymbol{v}}=\sum_{i=1}^{mp}\overline{\boldsymbol{K}}_{\boldsymbol{v}}(\Delta k_i) \tag{16}$$

where $\boldsymbol{M_v}=\sum_{i=1}^{ve}\boldsymbol{M}_{v_i}^{(e)}$ and $\overline{\boldsymbol{K}}_{\boldsymbol{v}}=\sum_{i=1}^{ve}\overline{\boldsymbol{K}}_{v_i}^{(e)}$ are the matrices associated with the viscoelastic zones composed by ve elements, and $\boldsymbol{M}_{v_i}^{(e)}$ and $\overline{\boldsymbol{K}}_{v_i}^{(e)}$, are the elementary viscoelastic matrices, having mp modifiable parameters. The matrices $\Delta\boldsymbol{M_v}$ and $\Delta\overline{\boldsymbol{K}}_{\boldsymbol{v}}$ are the matrices to be reduced, which are in general nonlinear functions of the design parameters, and Δm_i and Δk_i are the mass and stiffness variations, respectively.

- **Basis of forces associated to the modifications**

The vector of efforts associated with the viscoelastic modifications depend on the response of the modified system $\boldsymbol{q}(\omega)$. By assuming that this response is unknown, the forces associated to the modifications cannot be computed exactly. The essential concept of the robust condensation is expressed in the following steps (Masson et al., 2003): first, one uses relation (15b) to generate a vector of forces which, even though it does not contain the exact forces associated to the modifications, will at least represent a subspace containing these vectors. This is accomplished by introducing the response of the nominal system into (15b); next, the resulting vector of forces is used to generate static responses, once again on the basis of the nominal model; the first two steps are repeated for each design parameter subjected to modifications.

In practice, many types of responses may be introduced into the expression (15b), including the normal modes of the structure and the sensitivity vectors. If the vector basis is composed by a truncated basis of normal modes, one can rewrite equation (15b) as follows:

$$\boldsymbol{f}_\Delta(\omega,T)\approx-\Delta\boldsymbol{Z}(\omega,T)\boldsymbol{\phi_0}\boldsymbol{q_r}(\omega) \tag{17}$$

To generate the basis of forces $\boldsymbol{F}_\Delta$ representing the modifications into a viscoelastic zone, starting from expression (17), for each parameter, p, one must calculate a particular basis. For example, for a parameter intervening linearly in the viscoelastic mass and stiffness matrices, the basis of forces can be calculated by the following form:

$$\boldsymbol{F}_{\Delta_p}=\left[\boldsymbol{f}_{\Delta_p}^{\boldsymbol{M_v}}\quad \boldsymbol{f}_{\Delta_p}^{\boldsymbol{K_v}}\right] \tag{18}$$

where $\boldsymbol{f}_{\Delta_p}^{\boldsymbol{M_v}}=\boldsymbol{M_v}(\Delta p)\boldsymbol{\phi_0}\Lambda_0$ and $\boldsymbol{f}_{\Delta_p}^{\boldsymbol{M_v}}=G_0\overline{\boldsymbol{K}}_{\boldsymbol{v}}(\Delta p)\boldsymbol{\phi_0}$ are the bases of forces associated to the viscoelastic mass and stiffness modifications, respectively.

- **Basis of displacements associated to the modifications**

After obtaining the basis of forces, one can calculate a series of static responses of the system based on the tangent stiffness matrix, which will be used to complete the nominal base, $\boldsymbol{T_0}$, according to the relation:

$$\boldsymbol{T}=\left[\boldsymbol{T_0}\quad \boldsymbol{R}_\Delta\right]=\left[\boldsymbol{T_0}\quad \boldsymbol{K_0^{-1}}\boldsymbol{F}_\Delta\right] \tag{19}$$

The basis $\boldsymbol{R}_\Delta$ is not necessarily of maximum rank and with the aim of obtaining a limited number of residues, it is appropriate to select the dominating directions of this basis. That can be done by separating the larger singular values of the rectangular matrix $\boldsymbol{R}_\Delta$ by using the Singular Value Decomposition technique.

Figure 3 illustrates a cycle of approximate analysis and/or optimization process by using the robust basis, where the standard Ritz basis is increased by the static residues associated to the external forces and the forces associated to the viscoelastic modifications. This procedure is used to approximate the behavior of the modified system without the re-actualization of the basis of the initial model, leading to a drastic reduction of the time required for response computation.

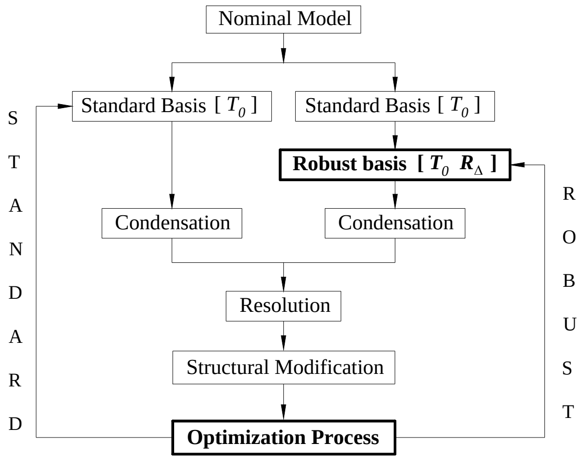

Figure 3 – Cycle of optimization using robust condensation

A THREE-LAYER SANDWICH PLATE FINITE ELEMENT

The formulation of a three-layer sandwich plate FE is summarized based on the original development made by Khatua and Cheung (1973) and implemented by Lima et al. (2005). Figure 4 depicts a rectangular plate FE formed by an elastic base-plate (1), a viscoelastic core (2) and an elastic constraining layer (3), whose dimensions in directions x and y are denoted by a and b, respectively. Space discretization is made by considering 4 nodes and 7 degrees-of-freedom per node, representing the nodal longitudinal displacements of the base-plate middle plane in directions x and y (denoted by u_1 and v_1), the nodal longitudinal displacements of the constraining layer middle plane in directions x and y (denoted by u_3 and v_3), the transverse displacement, w, and the cross-section rotations of the layers (1) and (3) about axes x and y, denoted by θ_x and θ_y.

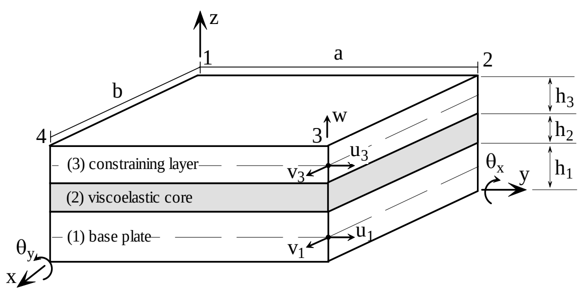

Figure 4 – Three-layer sandwich plate finite element

In the development of the theory, the following assumptions are adopted: normal stresses/strains in direction z are neglected for all the three layers; the elastic layers (1) and (3) are modeled according to Kirchhoff's theory; for the viscoelastic core, Mindlin's theory is adopted (transverse shear); the transverse displacement w is the same for all the three layers and cross-section rotations θ_x and θ_y are assumed to be the same for (1) and (3).

Based on the hypotheses of the stress-states assumed for each layer, the stress-strain relations can be obtained, and the strain and kinetic energies of the sandwich plate finite element are formulated. Lagrange equations are used, considering the nodal displacements and rotations as generalized coordinates, to obtain the element stiffness and mass matrices. After assembling, the stiffness matrix related to the elastic substructure and the viscoelastic substructure, respectively, can be expressed as follows:

$$\boldsymbol{K_e} = \boldsymbol{K}^{(1)} + \boldsymbol{K}^{(3)}, \quad \boldsymbol{K_v}(\omega,T) = G(\omega,T)\overline{\boldsymbol{K}}^{(2)} \quad (\overline{\boldsymbol{K}}^{(2)}: \text{constant matrix}) \tag{20}$$

Details of the derivations are given by Khatua and Cheung (1973) and Lima et al. (2005).

ADAPTIVE RESPONSE SURFACE METHODOLOGY (ARSM)

The adaptive response surface belongs to a family of numerical methods called *Meshless Methods*, in which the goal is to create a discrete approximation domain from the continuous design variables (Carpinteri et al., 2001), as illustrated in Fig. 5. The resulting discretization space forms a group of points called *nodes*, and the values of state variables are called *nodal values*. Each node represents the center of a element, whose form is generally given by a sphere of radius, R which represents its zone of influence.

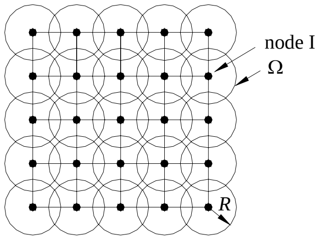

Figure 5 – Discretized domain of continuous variables

The ARSM consists in making an approximation $\hat{y}(x)$ of the real response $y(x)$ in the discretized field of design variables Ω , defined by a linear combination of a polynomial basis, expressed under the following forms:

$$\hat{y}(\boldsymbol{x}) = \sum_{i=1}^{m} p_{\boldsymbol{i}}(\boldsymbol{x}) a_i(\boldsymbol{x}) \text{ or } \hat{\boldsymbol{y}}(\boldsymbol{x}) = \boldsymbol{P}(\boldsymbol{x})^{\mathrm{T}} \boldsymbol{A}(\boldsymbol{x}) \tag{21}$$

where $\boldsymbol{x} = \{x_1 \;\; x_2 \;\; x_3 \ldots x_n\}$ is the vector of design parameters, $\boldsymbol{P}(\boldsymbol{x}) \in \mathrm{R}^{1\times m}$ is the polynomial basis matrix, $\boldsymbol{A}(\boldsymbol{x}) \in R^{m\times 1}$ is the vector of spatial coordinates, and m is the number of sampling points.

The coefficients a_i can be determined by the minimization of the quadratic error function, $J(\boldsymbol{x})$, in which for m samples, the error function can be expressed under the following form:

$$J(\boldsymbol{x}) = \frac{1}{2} \sum_{I=1}^{n} w_{\boldsymbol{I}}(\boldsymbol{x} - x_I) \left[\sum_{i=1}^{m} p_i(\boldsymbol{x}) a_i(\boldsymbol{x}) - y_I(\boldsymbol{x}) \right]^2 \tag{22}$$

where $w_{\boldsymbol{I}}(\boldsymbol{x} - x_I)$ is the weight function associated to the node $I(x_I, y_I)$, presenting non-zero values only into the influence domain, x_I are the coordinates in the discretized domain, and y_I is the real value of the function $y(x)$.

Equation (22) can be rewritten in matrix notation as follows:

$$\boldsymbol{J}(\boldsymbol{A}(\boldsymbol{x})) = \frac{1}{2} (\boldsymbol{P}(x)\boldsymbol{A}(x) - \boldsymbol{Y}(x))^{\boldsymbol{T}} \boldsymbol{W}(x) (\boldsymbol{P}(x)\boldsymbol{A}(x) - \boldsymbol{Y}(x)) \tag{23}$$

where the matrices $\boldsymbol{P}(x)$ and $\boldsymbol{W}(x) \in R^{n\times n}$ are defined, respectively, as:

$$\boldsymbol{P}(\boldsymbol{x}) = \begin{bmatrix} p_1(x_1) & p_2(x_1) & \ldots & p_m(x_1) \\ p_1(x_2) & p_2(x_2) & \ldots & p_m(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ p_1(x_n) & p_2(x_n) & \ldots & p_m(x_n) \end{bmatrix}, \quad \boldsymbol{W}(\boldsymbol{x}) = \begin{bmatrix} w(x - x_1) & 0 & \ldots & 0 \\ 0 & w(x - x_2) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & w(x - x_n) \end{bmatrix} \tag{24}$$

In order to determine the matrix of coefficients, the optimum of the expression (23) can be obtained as follows:

$$\frac{\partial \boldsymbol{J}(\boldsymbol{A}(\boldsymbol{x}))}{\partial \boldsymbol{A}(\boldsymbol{x})} = \overline{\boldsymbol{A}}(\boldsymbol{x})\boldsymbol{A}(\boldsymbol{x}) - \boldsymbol{B}(\boldsymbol{x})\boldsymbol{Y}(\boldsymbol{x}) = \boldsymbol{0} \tag{25}$$

where the matrices $\overline{\boldsymbol{A}}(\boldsymbol{x}) \in R^{m\times m}$ and $\boldsymbol{B}(\boldsymbol{x}) \in R^{m\times n}$ are defined as follows:

$$\overline{\boldsymbol{A}}(\boldsymbol{x}) = \boldsymbol{P}(\boldsymbol{x})^T \boldsymbol{W}(\boldsymbol{x}) \boldsymbol{P}(\boldsymbol{x}), \quad \boldsymbol{B}(\boldsymbol{x}) = \boldsymbol{P}(\boldsymbol{x})^T \boldsymbol{W}(\boldsymbol{x}) \tag{26}$$

and the coefficients of the polynomials can be determined by solving the following relation:

$$\boldsymbol{A}(\boldsymbol{x}) = \overline{\boldsymbol{A}}(\boldsymbol{x})^{-1}\boldsymbol{B}(\boldsymbol{x})\boldsymbol{Y}(\boldsymbol{x}) \tag{27}$$

The approximation represented by Eq. (27) is well defined only when the matrix $\overline{\boldsymbol{A}}(\boldsymbol{x})$ in non-singular. This is true only if there are at least n sampling points in the influence domain of node I such that $n > m$. As already mentioned, the weighting function is non-zero only in the influence domain of the nodes, and equal to zero outside this region. The selection is made such that its value is 1 at the center, and 0 outside of the influence domain. With this aim, the influence domain should be assumed to be a spherical form, defined as following (Ghanmi et al., 2005):

$$w_I(x - x_I) = \begin{cases} \left(1 - \left(\|x - x_I\|/R\right)^2\right)^2 & if \quad \|x - x_I\| \leq R \\ 0 & if \quad \|x - x_I\| > R \end{cases} \tag{28}$$

The zone of influence for one node I corresponds to the area of the domain where the node intervenes directly in the approximation function. This zone, defined by the weighting function, is limited in order to give a local character to the function approximation, having a directly influence on the aspect of the response surface (Ghanmi et al., 2005).

The principal steps of this strategy, required to obtain the adequate metamodel, are summarized as follows: approximation field generation by using the Latin hyper cube (LHC) method, in which the size of the field is equal to the number of design variables; coding of the variables, where the design variables are assumed to have the values 0 or 1, representing the minimum and maximum, respectively; discretization in regular intervals defined by the number of desired points of approximation; choice of the polynomial basis, depending on the number of design parameters, and the choice of the weighting functions.

EVOLUTIONARY ALGORITHMS COUPLED WITH METAMODELS

Evolutionary algorithms (EAs) are widely used due to their general numerical setting and robust optimal solutions (Keys and Rees, 2002; Soteris, 2004; Srivinas and Deb, 1993). Many engineering problems involve simultaneous optimization of multiples objectives, where the goal is to find the best design solutions, which correspond to the minimum or maximum value of the objective functions. In a multi-objective optimization with conflicting objectives, there is no single optimal solution, and the interaction among different objectives gives rise to a set of compromise solutions, classically known as the Pareto optimal solutions (Srivinas and Deb, 1993). Since EAs are stochastic iterative methods which require several evaluations of the exact solutions, the cost of calculation are of primary importance. It is thus necessary to combine the EAs with approximation methods in order to reduce the computing time of the cost functions during the optimization process. In the literature one can find several works dedicated to the coupling process of evolutionary algorithms with metamodels, such as response surface methodology (RSM), or neural networks (ANN) (Keys and Rees, 2002).

A multiobjective problem includes a set of k parameters (decision variables) and a set of n objective functions ($n \geq 2$), that can be summarized as follows:

$$\begin{cases} minimize \quad F(x) = \left(f_1(x), f_2(x), \ldots, f_n(x)\right) \\ \qquad\qquad\qquad x \in C \end{cases} \tag{29}$$

where $x = (x_1, x_2, \ldots, x_k)$ is the vector containing k design variables, $C \subset R^k$ is the design space. For a practical design problem, $F(x)$ is non-linear, multi-modal, and not necessary analytical.

Although simplistic from a biologist's viewpoint, evolutionary algorithms are sufficiently complex to provide robust and powerful adaptive search mechanisms. The algorithm used to solve the multi-objective optimization problem is the non-dominated sorting genetic algorithm (NSGA). The NSGA differs from classical multi-objective genetic algorithm in the way that the selection operator is used, in such a way that after the computation of the Pareto ranking representing the whole of population, a same dummy fitness value is provisionally assigned to the members of the first front of Pareto. This dummy fitness defines an equal reproductive pressure for these individual and, with the aim in maintaining the diversity of the population, a *sharing technique* is then applied and the dummy fitness is recomputed accordingly (Srivinas and Deb, 1993).

Figure 6 illustrates the coupling process NSGA-ARSM, where the mechanism of search for the optimal solutions by this strategy can be briefly described into two steps: first, the response surfaces are built with exact evaluations of the condensed damped model, using the robust condensation strategy previously described. In this step, Monte Carlo simulation is used to generate the variations of the design parameters and compute the amplitudes of frequency response functions associated with these parametric modifications to obtain the polynomials; next, the multi-objective optimization is used to explore the good solutions in the decision space produced by the approximation responses (response surfaces) of the damped system, instead of the full model.

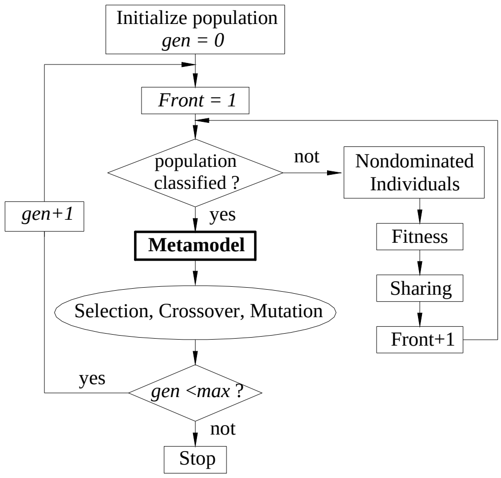

Figure 6 – NSGA coupled with ARSM

NUMERICAL APPLICATION

The following numerical application is presented to illustrate the proposed method by evaluating the damping performance (optimum design) of a stiffened panel treated with viscoelastic constrained layer. Figure 7a depicts the test structures consisting of a free stiffened panel containing 4 stringers, where the geometric dimensions, in millimeters are: internal radius: 938, length: 720, arc length: 680, thicknesses of the panel and the stringers: 1.5 and 0.75, respectively, and the height of stringers: 30. The material properties for both panel and stringers are: Young modulus $E=2.1\times10^{11}$ N/m^2, mass density $\rho=7800$ Kg/m^3 and Poisson's ratio, $\upsilon=0.3$.

The FEM without viscoelastic treatment is composed by 928 shell finite elements having 6840 d.o.f, and the viscoelastic treatment is composed by 15 viscoelastic patches, each one composed by 16 three-layer sandwich plate FE developed accordingly to the theory previously presented. Material properties for the base-plate and the constraining layer are the same of the stiffened panel, and for the viscoelastic core, one uses the modulus function of the ISD112 3M ($\rho=950Kg/m^3$), as shown in Fig. 2.

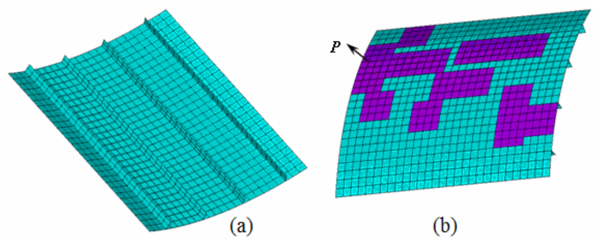

Figure 7 – FE model of the stiffened panel without (left) and with treatment (viscoelastic paths) (right)

The design parameters and their admissible variations are: x_1 - thickness of the constraining layer (h_3=0.5 mm; ±50%); x_2 - temperature of the viscoelastic material (T=25 ºC; ±15%), and x_3 - thickness of the viscoelastic core (h_2=0.254 mm; ±70%). These ranges were chosen arbitrarily in such a way to avoid large variations from the nominal values. Only the range of continuous variables defined previously is taken as constraints.. The cost functions are the amplitudes of the FRFs of the damped system at the frequencies corresponding to the natural frequencies of modes 10 (M10) and 11 (M11), respectively, for an excitation force applied at point P, and responses computed at the same point, as indicated in Fig. 6b. The aim is to minimize the amplitudes of the resonance peaks of these modes. The parameters of NSGA are defined as: population (30 individuals), probability of selection (25 %), probability of crossover (25 %), probability of mutation (25 %), and sharing coefficient, σ=0.2. To evaluate the NSGA-ARSM, the objective functions (amplitudes of FRFs of damped system), are approximated by following second order polynomials, obtained by the ARSM methodology previously described, based on the reduced viscoelastic model, in which the nominal basis of reduction is composed by 121 vectors (60 eigenvectors, 1 vector related to the static residue, and 60 vectors are related to the viscoelastic forces).

$$f_{10}(x_1,x_2,x_3)=6.1816+0.10023x_1+0.26882x_2+0.91513x_3+0.01152x_1x_2+0.0553x_1x_3+\ldots \\ 0.13364x_2x_3+0.0057604x_1^2+0.009217x_2^2+0.25922x_3^2 \tag{30}$$

$$f_{11}(x_1,x_2,x_3)=7.2741-0.90968x_1-0.26966x_2-1.0006x_3+0.0097466x_1x_2+0.053606x_1x_3+\ldots \\ 0.18713x_2x_3+0.020793x_1^2+0.0071475x_2^2+0.47693x_3^2 \tag{31}$$

where the variables, x_1, x_2 and x_3 are expressed under the following decoding forms (Myers and Montgomery, 2002):

$$x_1 = \left(h_3 - h_3^{mean}\right)/\left(h_3^{max} - h_3^{min}\right); \quad x_2 = \left(T - T^{mean}\right)/\left(T^{max} - T^{min}\right); \quad x_3 = \left(h_2 - h_2^{mean}\right)/\left(h_2^{max} - h_2^{min}\right) \tag{32}$$

where h_3, h_2 and T are the current values of constraining and viscoelastic thicknesses and temperature, respectively; the superscripts *mean*, *max* and *min* are related to the nominal, the maximum and the minimum values of variables, calculated according to the perturbations. Figure 8 shows the solutions obtained by using the NSGA (used for as reference for comparison), and by using the coupling procedure NSGA-ARSM. By comparing the first front of Pareto obtained by the two optimization procedures, one can conclude that the second order polynomials f_{10} and f_{11} represent quite well the damped response of the system, as demonstrated by the similarity of the two clouds of optimal solutions.

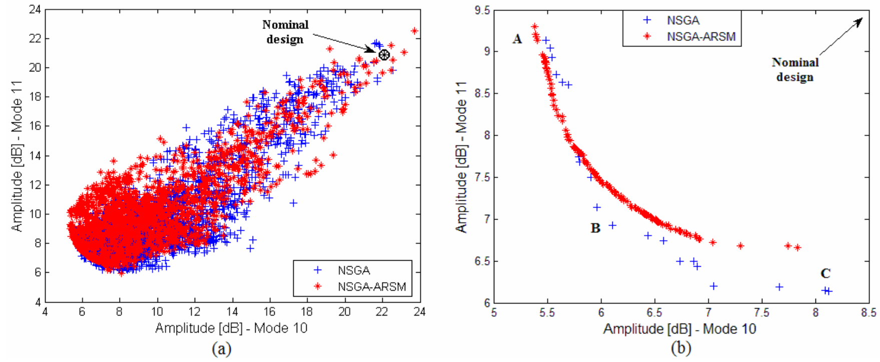

Figure 8 – NSGA and NSGA-ARSM solution evolution (a) and first Front of Pareto (b)

The total number of generations to find the optimal solutions was limited to 100, meaning that the maximum number of evaluations of cost functions is equal to 3000. The CPU time for the two optimization approaches were: NSGA (2016.2 min); NSGA-ARSM (285 min). One can conclude that the NSGA-ARSM procedure allows a significant reduction of the computation time during the optimization process, with a reduction ratio of 86 %. From Figs. 8 and 9 one can conclude that for point *A*, the optimal solution leads to a better damping performance for M10 as compared to M11. In the opposite, for point *C*, the optimal solution leads to better damping performance for M11 than for M10. For point *B*, the solution corresponds to similar damping performance for the modes M10 and M11.

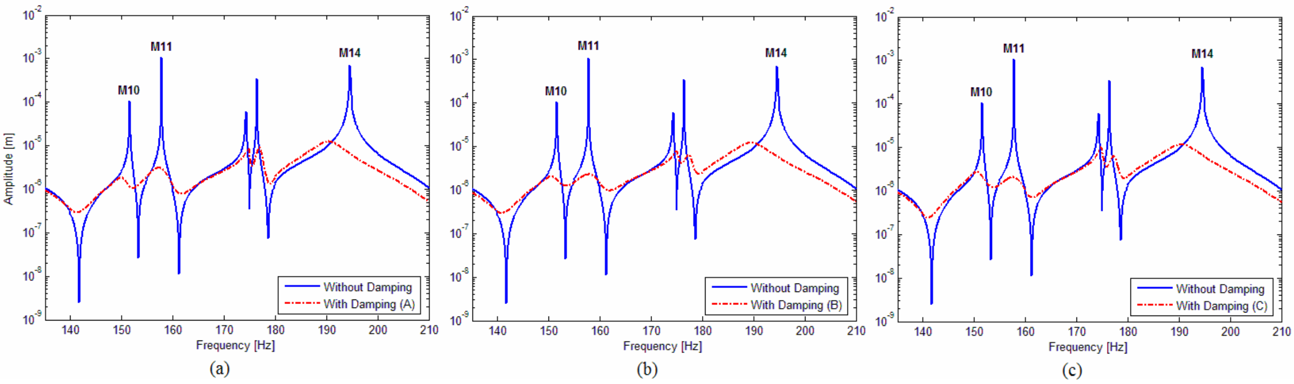

Figure 9 – Amplitudes of FRFs related to the optimal design solutions *A*, *B*, and *C*

For illustration, Tab. 1 shows the optimal values of the thicknesses of the viscoelastic and constraining layers for the optimal points ***A***, ***B*** and ***C***, and the corresponding optimal temperatures of the viscoelastic material.

Table 1 – Optimal values of viscoelastic and constraining layers thicknesses (optimal points *A*, *B* and *C*)

Variables	Point ***A***	Point ***B***	Point ***C***
h_2 [m]	0.07x10^{-3}	0.035x10^{-3}	0.035x10^{-3}
h_3 [m]	1.00x10^{-3}	0.95x10^{-3}	1.00x10^{-3}
T [ºC]	22.50	22.52	22.50

CONCLUDING REMARKS

The multiobjective optimization process combining evolutionary algorithm, robust condensation and response surface methodology for large systems containing viscoelastic damping were investigated. The originality of the methodology suggested consists in introducing an adaptive response surface to be used in the optimization process in order to approximate the exact responses of the large viscoelastic damped structures. In the numerical applications, an optimization problem involving three design variables and three objective functions with the interest oriented towards a relatively complex structure was investigated. The results demonstrate the effectiveness of the optimization strategy in terms of reduction of computation time, which makes it well adapted to be applied to industrial structures.

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