Proceedings of the XII International Symposium on Dynamic Problems of Mechanics (DINAME 2007),
Varoto, P. S. and Trindade, M. A. (Editors), ABCM, Ilhabela, SP, Brazil, February 26 - March 2, 2007

A Hybrid Control Approach for Vibration Reduction of a Tower with a Pendulum Absorber

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Abstract: The oscillations of tall slender towers under environmental loads are usually of large amplitude. In order to protect these structures, suitable absorbers can be installed to control or limit the amplitude of oscillation. In this paper, passive and hybrid control strategies are used to improve the dynamic response of a tall tower. A pendulum attached to the top of the tower is used as a vibration absorber. The tower is modeled as a bar with variable cross-section and concentrated masses. First, the vibration modes and frequencies of the tower are obtained analytically. The primary structure and absorber together constitute a coupled system which is discretized as a two degrees of freedom nonlinear system, using the normalized eigenfunctions and the Galerkin method. These equations are either solved numerically, using the Runge-Kutta method, or analytically by the use of the Galerkin-Urabe method. A detailed numerical analysis illustrates the influence of the system parameters on the non-linear response of the tower and the effectiveness of pendulum-type passive absorbers to control the oscillations of the system. The pendulum-absorber is particularly effective when the tower is subjected to periodic excitation. In order to improve the effectiveness of the control during the transient response and under non-periodic forces, a hybrid control system is suggested. The added control force is implemented as a non-linear variable stiffness device based on position and velocity feedback. A marked decrease in vibration amplitudes is obtained. The same is observed for velocities and accelerations. The influence of time delay, an unavoidable fact in practical problem, is also investigated. The obtained results show that this strategy of nonlinear control is attractive, has a good potential and can be used to minimize the response of slender structures under various types of excitation.

Keywords: tall tower, vibration absorber, pendulum absorber, passive and hybrid control, time delay.

NOMENCLATURE

f = magnitude of the control force – non-dimensional.
w = transversal displacement of the column, m.

Greek Symbols

β = non-dimensional control parameter of the control force.
ζ = non-dimensional parameter of displacement of the column.
ζ_s = amplitude of the excitation force, non-dimensional.
θ = angular displacement of the pendulum absorber.
μ = relation between the mass of the pendulum and the mass of the column.
ξ_c = damping ratio of the column
ξ_p = damping ratio of the pendulum absorber.
τ = non-dimensional time parameter.
ω_c = natural frequency of the column, rad/s.
ω_p = natural frequency of the pendulum, rad/s.
ω_e = excitation frequency, rad/s.

INTRODUCTION

Constant advances in the areas of structural materials and construction techniques, allied to the development of new design methods and increasing technological necessities, structures such as telecommunication towers are constantly increasing in height and, consequently, becoming more flexible. Figure 1 shows some examples of tall telecommunication towers.

(a) CN Tower, Toronto, 553m.

(b) TV Tower, Brasília, 224m.

Figure 1– Tall telecommunication towers

These towers, due to its height and slenderness, are vulnerable to the occurrence of extreme vibrations caused by dynamic loads, such as, for example, wind and earthquakes. The high vibration levels induced by these loading conditions can cause discomfort and, worse, compromise the structure safety and integrity. The action of wind is of utmost importance in towers, since it generates flexural vibrations, causing large displacements and rotations at the top of the tower. These vibrations in towers usually are caused by the vortex shedding, generating a force perpendicular to the incidence of the wind. This type of vibration occurs around the wind speed where the periodic vortex-shedding frequency coincides with one of the natural frequencies of the structure. In towers, the worst case occurs when the vortex-shedding frequency coincides with the frequency of the first vibration mode of the tower. This lateral force is practically harmonic (Korenev & Reznikov, 1993; Kitagawa *et al.,* 1997). The protection of civil engineering structures against undesirable vibrations, including their material contents and human occupants, is without doubt a worldwide priority. An alternative to minimize these vibrations, widely studied in the last decades, is the application of control strategies One can change the structural properties by installing external devices or applying external forces to the structure. There are basically three types of structural control: active, passive and hybrid. Basic concepts, experiments and practical applications of these devices are found in Korenev & Reznikov (1993) and Soong & Dargush (1997), amongst others. Avila (2002) presents a detailed literature revision of structural control in civil engineering.

In the passive control, the magnitude of the control forces depends only on the physical properties of mass, stiffness and damping of the auxiliary system. The basis of passive absorption is energy pumping from the main elastic subsystem into the absorber. According to Marques (2000), the choice of absorber parameters is based on the tuning of its natural frequency to the frequency of the harmonic excitation whose value is admitted as fixed. The main advantage of the passive absorber is that it does not need any external energy source and the control is always stable. However there are some limitations in the use of this technology, since the passive devices are designed to work efficiently within a small frequency range. One passive control device proposed in literature is the pendulum absorber. They have already been used in practice to reduce the vibration level of steel vent tubes and smoke stacks, supports of communication lines, support towers of solar telescopes, high buildings, etc. Pendulum absorbers were studied by Mustafa & Ertas (1995), Ertas *et al.* (2000), Cuvalci (2000) and Yaman & Sen (2004), among others. They studied the influence of a pendulum attached to a column or spring-mass system. All these authors confirmed the efficiency of this control device in the reduction of the vibrations of the main structure. Its ability to control rather than dampen is the key to its success. Adaptive absorbers are those whose physical parameters of mass, stiffness and damping can be adjusted, conferring to the devices the capacity of tuning in a larger frequency range (Marques, 2000). In this context, the recent technological advances in the production of intelligent materials can offer ample possibilities for the proposal of new adaptive configurations (Williams *et al.,* 2002).

The hybrid control approaches combine active controllers with passive devices. The active portion of a hybrid system requires much less power than a similar active system, while providing better structural response than the passive system alone (Avila, 2002; Oueini *et al.* 1999). Here a hybrid control approach is proposed based on the simultaneous use of a pendulum absorber with an external force applied at the pendulum-tower connection. The active force is based on the basic ideas of the switched stiffness approach (Wu et al., 2005; Winthrop et. al, 2005, Ramaratnam and Jalili, 2005). The control law is implemented by measuring the position and velocity of the tower.

Many cantilevered tall towers can be treated as cantilever bars with variable cross-section for the analysis of their free and forced vibrations (Qiusheng et al., 1994). Here, the tower is modeled as a bar with variable cross-section with concentrated masses. First, the vibration modes and frequencies of the tower are obtained analytically. Using these vibration modes as interpolating functions, the natural frequencies and modes of the column-pendulum system are obtained by the Rayleigh-Ritz method. For a pendulum tuned to the lowest frequency of the tower, only the two first vibration modes and frequencies of the tower-pendulum system are important, since the subsequent frequencies of the cantilevered tower are much higher than the first ones. So, using these two vibration modes, two degrees of freedom nonlinear system is obtained. These equations are either solved numerically, using the Runge-Kutta method, or analytically by the use of the Galerkin-Urabe method. Detailed numerical analysis illustrates the influence of the system parameters on the non-linear response of the tower and the effectiveness of pendulum-type passive absorbers on the control of oscillations of the system. Results show that the non-linearity of the tower is not relevant, but the non-linearity of the pendulum leads to complex responses in the vicinity of the main resonance region. The pendulum-absorber is particularly effective when the tower is subjected to harmonic excitation, due to, for example, vortex shedding. The importance of non-linearities in control problems was addressed recently by Pinto & Gonçalves (2002). Marked decrease in vibration amplitude is obtained. The same is observed with velocities and accelerations. The obtained results show that this strategy of nonlinear control is attractive, has good potential and can be used to minimize the response of slender structures under various types of excitation.

FORMULATION OF THE PROBLEM

The tower is modeled as a clamped-free column with variable cross-section (Qiusheng et al., 1994). Platforms, antennas and equipments are modeled as discrete masses along the tower. Examples of this class of structures are shown in Figure 1. The pendulum is considered as a discrete element along the tower. In terms of efficiency, the best position of the pendulum is the top of the tower. However, in some cases, for practical reasons, the pendulum must be placed in a less convenient position.

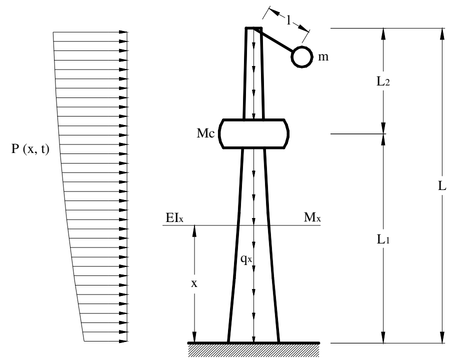


Figure 2: Column-absorber system.

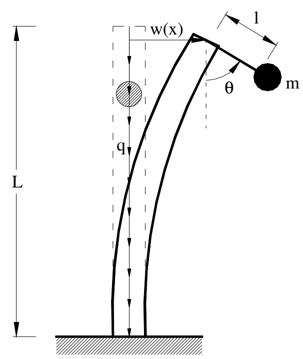


Figure 3: Column of constant cross-section.

The behavior of the column-pendulum system shown in Fig. 2 is described by the following set of partial differential equations of motion (Orlando, 2006):

$$\frac{d^2}{dx^2}\left[EI_o(1+\eta x)^{n+2}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]+\frac{d}{dx}\left[N_o(1+\eta x)^{n+1}\left(\frac{\partial w}{\partial x}\right)\right]+M_o(1+\eta x)^n\left(\frac{\partial^2 w}{\partial t^2}\right)+M_c\delta(x-L_1)\left(\frac{\partial^2 w}{\partial t^2}\right)+$$
$$m\delta(x-L)\left(\frac{\partial^2 w}{\partial t^2}\right)+ml\delta(x-L)\left(\frac{d^2\theta}{dt^2}\right)\cos(\theta)-ml\delta(x-L)\left(\frac{d\theta}{dt}\right)^2\sin(\theta)=0 \tag{1a}$$

$$ml^2\left(\frac{d^2\theta}{dt^2}\right)+K_p\theta+mgl\sin(\theta)+ml\delta(x-L)\left(\frac{\partial^2 w}{\partial t^2}\right)\cos(\theta)=0 \tag{1b}$$

where EI_o is the flexural stiffness at the base of the column, N_0, the normal force at the base due to the tower self weight, q_x, M_o, the mass per unit length at the base, M_c, a concentrated mass at a distance L_1 from the base, L, the length of the column, m, the mass of the pendulum, K_p, the stiffness of the pendulum and l, the pendulum length. Finally η and n are parameters that define the variation of the column cross-section, δ is the Dirac delta function and g is the acceleration of the gravity.

State space equations

First the analytical free vibration modes and frequencies of the column are obtained using symbolic algebra. Then, these modes are used together with the Rayleigh-Ritz method to obtain the free vibration modes and frequencies of the column-pendulum system. Finally, a two degrees of freedom model, capable of describing with precision the behavior of the system in the neighborhood of the basic frequency of the column, is derived, from which the following set of nonlinear equations of motion, in the non-dimensional form, is obtained (Orlando, 2006):

$$\begin{cases} (1+\mu)\zeta,_{\tau\tau} + 2\xi_c \dfrac{\omega_c}{\omega_e}\zeta,_{\tau} + \left(\dfrac{\omega_c}{\omega_e}\right)^2 \zeta + \mu\theta,_{\tau\tau}\cos(\theta) - \mu\theta,_{\tau}{}^2\sin(\theta) = \zeta_s\left(\dfrac{\omega_c}{\omega_e}\right)^2\sin(\tau) & \text{(2a)} \\ \mu\theta,_{\tau\tau} + 2\mu\xi_p \dfrac{\omega_p}{\omega_e}\theta,_{\tau} + \mu\zeta,_{\tau\tau}\cos(\theta) + \mu\left(\dfrac{\omega_p}{\omega_e}\right)^2\sin(\theta) = 0 & \text{(2b)} \end{cases}$$

where $\zeta = x/L$, $\tau = \omega_e t$, ω_c is the natural frequency of the column; ξ_c is the damping ratio of the column; ω_p is the natural frequency of the pendulum absorber; ξ_p is damping ratio of the pendulum absorber; μ is the mass ratio ; ζ_s is the amplitude of the excitation force and ω_e is the excitation frequency.

The external control force is applied directly to the main structure, in the opposite direction of the excitation force. It is given, in its non-dimensional form, as:

$$Fc = f\tanh(\beta\zeta\zeta,_{\tau})\zeta \tag{3}$$

This force is evaluated at all instant as a function of the displacement and velocity measured at the top of the column. For this it is necessary to place sensors at the top of the tower to measure the displacements and its velocities and send the data to the control system. The control force is a function of two parameters: f and β. The influence of these parameters is investigated in detail in the present work.

Considering that the external control force is applied at the connection between the column and the pendulum absorber, the state equations are:

$$\begin{cases} \dot{y}_1 = y_2 & \text{(4a)} \\ \dot{y}_2 = \left[\zeta_s\left(\dfrac{\omega_c}{\omega_e}\right)^2\sin(\tau) - f\tanh(\beta y_1 y_2)y_1 - 2\xi_c\dfrac{\omega_c}{\omega_e}y_2 - \left(\dfrac{\omega_c}{\omega_e}\right)^2 y_1 - \mu\dot{y}_4\cos(y_3) + \mu y_4{}^2\sin(y_3)\right]/(1+\mu) & \text{(4b)} \\ \dot{y}_3 = y_4 & \text{(4c)} \\ \dot{y}_4 = -2\xi_p\dfrac{\omega_p}{\omega_e}y_4 - \dot{y}_2\cos(y_3) - \left(\dfrac{\omega_p}{\omega_e}\right)^2\sin(y_3) & \text{(4d)} \end{cases}$$

where y_1 is the displacement, y_2, the velocity and $\dot{y}_2$, the acceleration of the column, and y_3 is the displacement, y_4, the velocity and $\dot{y}_4$, the acceleration of the pendulum absorber.

Algebraic non-linear equations

The nonlinear equations of motion can not be solved analytically. So, the solution can be either obtained by numerical integration or approximately by the use of perturbation techniques or harmonic expansions. Here an approximate solution is obtained by the use of the Galerkin-Urabe method, which transforms the system (2) into a system of nonlinear algebraic equations (Orlando, 2006).

First consider that the solution of the system, subjected to a harmonic excitation of frequency ω_e, is of the form

$$w = \overline{w}\cos(\omega_e t) \tag{5a}$$

$$\theta = \overline{\theta}\cos(\omega_e t + \varphi) \tag{5b}$$

where φ it is the phase between the response of the column and that of the pendulum

Consider now that there is a phase angle ψ of the force relative to the response of the tower. Thus the force can be written in the form:

$$F_o\sin(\omega_e t + \psi) = F_c\cos(\omega_e t) + F_s\sin(\omega_e \mathrm{t}) \tag{6}$$

Substituting expressions (5) and (6) into the system of equations of motion (2), multiplying these equations by the weight functions $\phi_1 = \cos(\omega_e t)$ and $\phi_2 = \sin(\omega_e t)$, respectively, and integrating each one of the four resultant equations from 0 to $2\pi/\omega_e$, the following nonlinear system of nonlinear algebraic equations is obtained

$$\begin{cases} \zeta\left(-1-\mu+\left(\dfrac{\omega_c}{\omega_e}\right)^2\right)-\mu\bar{\theta}\cos(\varphi)\left(J_0(\bar{\theta})-J_2(\bar{\theta})+\dfrac{\bar{\theta}}{2}J_1(\bar{\theta})\right)=\zeta_s\left(\dfrac{\omega_c}{\omega_e}\right)^2\cos(\psi) & \text{(7a)} \\ -2\zeta\xi_c\dfrac{\omega_c}{\omega_e}+\mu\bar{\theta}\sin(\varphi)\left(J_0(\bar{\theta})-J_2(\bar{\theta})+\dfrac{\bar{\theta}}{2}J_1(\bar{\theta})\right)=\zeta_s\left(\dfrac{\omega_c}{\omega_e}\right)^2\sin(\psi) & \text{(7b)} \\ -\mu\left(\bar{\theta}\cos(\varphi)-\zeta\left(\cos(2\varphi)J_2(\bar{\theta})-J_0(\bar{\theta})\right)\right)-2\mu\xi_p\dfrac{\omega_p}{\omega_e}\bar{\theta}\sin(\varphi)+2\mu\left(\dfrac{\omega_p}{\omega_e}\right)^2\cos(\varphi)J_1(\bar{\theta})=0 & \text{(7c)} \\ \sin(\varphi)\mu\left(2\left(\dfrac{\omega_p}{\omega_e}\right)^2J_1(\bar{\theta})-\bar{\theta}\right)+\mu\zeta J_2(\bar{\theta})\sin(2\varphi)+2\mu\dfrac{\omega_p}{\omega_e}\bar{\theta}\cos(\varphi)=0 & \text{(7d)} \end{cases}$$

where $J_0(\bar{\theta})$, $J_1(\bar{\theta})$ and $J_2(\bar{\theta})$ are Bessel functions.

This system of nonlinear equations is already in its non-dimensional form. The nonlinear algebraic equations possess as variables the frequency of excitation ω_e, the amplitudes ζ and $\bar{\theta}$, and the phase angles φ and ψ.

RESULTS

First the behavior of the column-absorber system with the passive control is investigated. Then the behavior of the system with the proposed hybrid control is analyzed. A column of constant section with length *L*, an axial load uniformly distributed throughout its length (self weight) *q* and a transversal displacement *w*, and a pendulum absorber with mass *m*, length of connecting rod *l* and an angular displacement represented by θ, as illustrated in Figure 3, is considered. The main parameters of the system used in this investigation are: $\omega_c = 1.255438$ rad/s, $\xi_c = 0.7\%$, $\xi_p = 0.0\%$, $\mu = 0.04$ (4,0% of the modal mass of the first mode) and $\zeta_s = 0.007$.

Behavior of the system with passive control

Here the displacements, velocities and accelerations of the column and of the pendulum absorber are obtained by numerical integration of equation (2), through the fourth-order Runge-Kutta method. Fixing the frequency of the column (ω_c) and varying the pendulum (ω_p) and excitation (ω_e) frequencies so that all possible frequency relations of (ω_e / ω_c) occur, one can verify when the pendulum is effective in reducing the response of the original column.

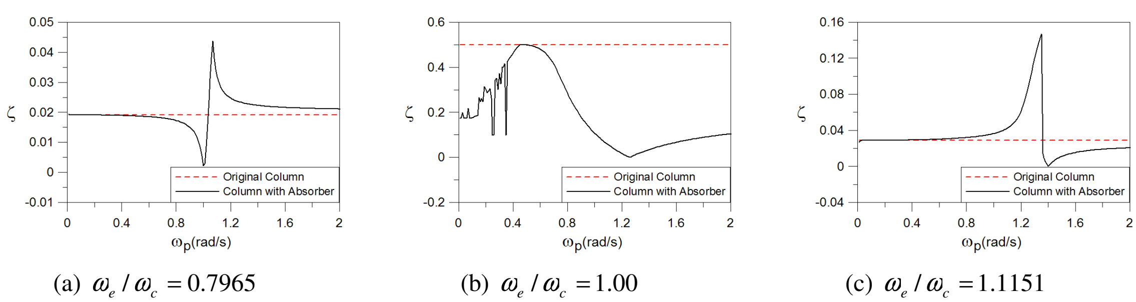
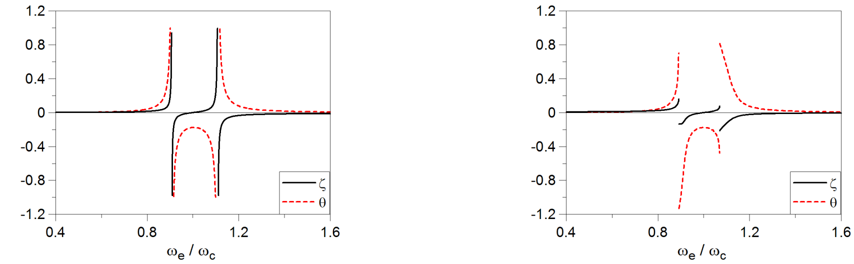


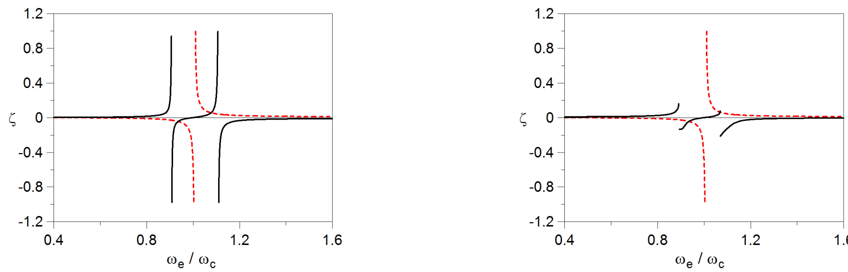
Figure 4 –Maximum amplitude of the displacement of the column with and without absorber in the permanent state as a function of ω_p.

In Figure 4 the maximum amplitude of displacement of the column in the permanent state for different magnitudes of the excitation frequency, is compared with the maximum displacement of the original column (dotted line). Analyzing these results, one concludes that when $\omega_e < \omega_c$, if $\omega_p < \omega_e$ there is a reduction in the magnitude of the oscillations, but if $\omega_p > \omega_e$, there is an increase in the amplitude of the column. For $\omega_e > \omega_c$, the inverse is observed: if $\omega_p < \omega_e$ the maximum displacement of the column increases, while for $\omega_p > \omega_e$, it decreases. Finally it is observed that, when $\omega_e = \omega_c$ (Figure 4(b)) the maximum response always decrease.



(a) Linear behavior (b) Non-linear behavior

Figure 5 - Magnification factor versus ω_e / ω_c frequency ratio for $\omega_p / \omega_c = 1.00$, considering in the modeling the pendulum as (a) linear and (b) non-linear.



(a) Linear behavior (b) Non-linear behavior

Figure 6 – Comparison of the steady-state response of the column with and without the pendulum absorber for $\omega_p / \omega_c = 1.00$.

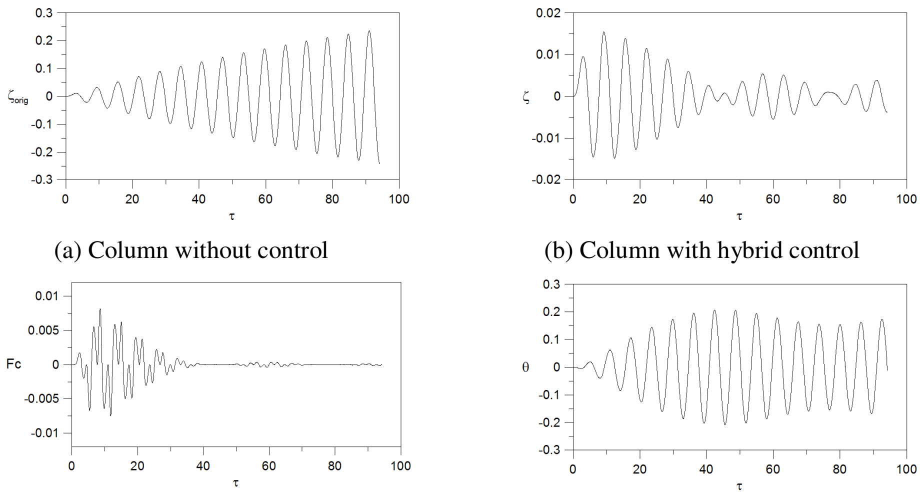
Figure 5 shows the variation of the column magnification factor as a function of the frequency of excitation for $\omega_p = \omega_c$. The response is obtained by the solution of the system of algebraic equations (7) by the Newton-Raphson method. Figure 5(a) shows the response considering a linear formulation for the pendulum, an usual simplification in literature. Figure 5(b) shows the response of the column-pendulum system considering full non-linearity. The results show that the nonlinearity of the pendulum cannot be neglected in this class of problem and that the not-linearity affects positively the vibrations of the tower, decreasing markedly the vibration amplitudes in the resonance region.

Figure 6 shows a comparison of the resonance curve of the original column without absorber with the response of the column-pendulum system, corroborating the beneficial influence of the tuned pendulum in the main resonance region.

Behavior of the system with hybrid control

Passive vibration absorbers are efficient in the neighborhood of the reference frequency for a harmonic excitation around the natural frequency of the column for which it was calibrated. For slender towers the absorber is more efficient when $\omega_c \approx \omega_p \approx \omega_e$, allowing some variations in the value of the frequency of excitation in the neighborhood of this point. Also, the absorber is not efficient under loads with short duration and during the initial transient response.

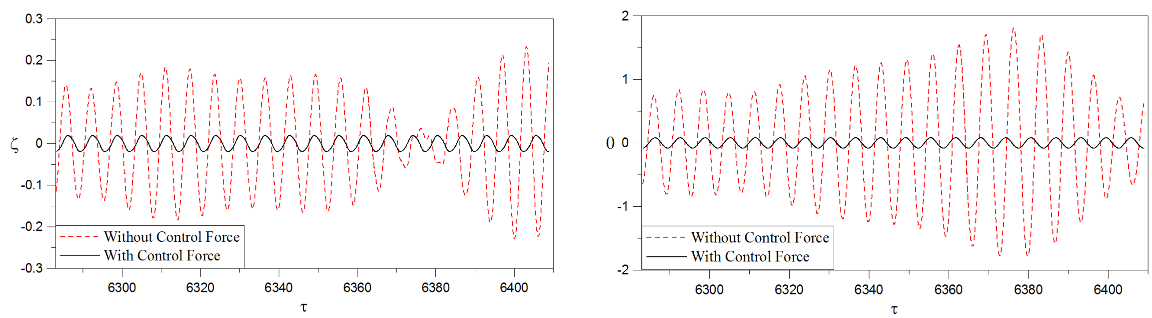
In order to improve the effectiveness of the control during the transient response, a semi-active control system is suggested. The added control force is implemented as a non-linear variable stiffness device based on position and velocity feedback. Figure 7 clarifies the influence of the present control strategy on the behavior of the tower. Figures 7(a) and 7(b) show, respectively, the uncontrolled and controlled response of the tower (maximum normalized displacement vs. time). A marked decrease in vibration amplitudes is obtained. The same is observed for velocities and accelerations. Figure 7(c) shows the evolution of the control force, while Figure 7(d) shows the response of the pendulum. First, while the response of the pendulum increases steadily from rest, the added control force acts to control the response of the tower but soon tends to zero as the pendulum reaches its full potential, absorbing most of the vibration energy. In this analysis it was adopted $\omega_c = \omega_p = \omega_e$. In this example, the control force is computed considering $f = 1.00$ and $\beta = 6000$.



(a) Column without control (b) Column with hybrid control

(c) Control force (d) Pendulum absorber

Figure 7 - Behavior of the system with hybrid control. $f = 1.00$ and $\beta = 6000$.



(a) Displacement of the column (b) Displacement of the pendulum

Figure 8 - Comparison of the amplitude of displacement with and without the control force.

To evaluate the efficiency of the hybrid control, results were obtained for $\omega_e / \omega_c = 0.8991$ resulting in $\omega_e = 1.12876$ rad/s. This point coincides with the point where the absorber-tower system reaches the maximum amplitude (first resonance). Figure 8 shows a comparison of the tower and pendulum steady-state state response with and without the active force. The results show that the hybrid control system practically eliminates the vibrations in the resonance region

Influence of the force control parameters

The control force depends on two parameter: f and β. The parameter f is the magnitude of the force while β measures the velocity of the change of the control force from $+f$ to $-f$ and vice-versa. In other works dealing with a switched stiffness approach, the function $sign(\zeta\zeta_{,\tau})$ is used to model the force (Winthrop *et al,* 2005). As will be shown in this paper, this leads to instability of the system in the presence of time delay. Also, in practical situations an instantaneous change of the sign of the force from $+f$ to $-f$ may not be possible. So, the description used in the present work seems to be more feasible and will lead to a more stable control system.

Initially, considering β =60, the parameter f is varies from zero to two. Table 1 shows the influence of the parameter f on the behavior of the maximum displacement, velocity and acceleration of the tower. It is observed that, as parameter f increases, the hybrid control becomes more efficient. However this increase is very small, which is an attractive aspect in a mechanism of active control, since a small force can be adopted leading to less energy consume.

The maximum amplitude of the column in the steady-state regime is not affected by the variation of f. This is explained, as shown previously, by the fact that the pendulum is the responsible for the control of the vibrations in this phase, being the control force practically zero.

Table 1 – Influence of the parameter f on the maximum amplitudes of the column.

f	Maximum displacement	Maximum velocity	Maximum acceleration
0.00	0.033235	0.032756	0.033245
0.40	0.032609	0.032249	0.032623
0.80	0.032024	0.031770	0.032047
1.20	0.031476	0.031315	0.031510
1.60	0.030961	0.030882	0.031036
2.00	0.030480	0.0 3047 1	0.0 30585

Table 2 – Influence of the parameter β on the maximum amplitudes of the column.

β	Maximum displacement	Maximum velocity	Maximum acceleration
0.00	0.033235	0.032756	0.033245
6.00	0.033074	0.032627	0.033075
60.0	0.031746	0.031539	0.031771
600.0	0.025490	0.025113	0.026036
6000.0	0.015469	0.016007	0.018705
60000.0	0.009689	0.010928	0.016703

Now, the parameter β varies while the parameter f is taken equal to 1.00. Table 2 shows the behavior of the maximum displacement, velocity and acceleration of the column. As β increases the function $\tanh(\beta\zeta\zeta_{,\tau})$ tends asymptotically to the behavior of the function $sign(\zeta\zeta_{,\tau})$ and the efficiency of the control force increases markedly decreasing the maximum values that occur during the transient response. The amplitudes of the steady-state response are not altered by the value of β.

Influence of time delay

In active, hybrid or adaptive control systems where feedback strategies are used, a certain amount of time is necessary to obtain and process the signal and, after that, evaluate and apply the control force. This time delay may cause a deterioration of the control system and can even cause instability. So, the influence of time delay is an essential step in the design of a given control system. Here the time delay, T_d, is given as a percentage o the period of the tower response, T. In this analysis the following values are adopted: $f = 1.00$ and $\beta = 6000$. Table 3 show the influence of time delay on the column response. The results show that the system becomes unstable when the time delay is higher than 20.0% of the tower period. Table 4 show the results for $f = 1.00$ and $\beta = 60$. For these values the response is always stable and the time delay has a negligible influence on the maximum amplitudes.

Based on these observations, a parametric analysis was conducted to evaluate the critical values of f and β as a function of the time delay. The results are presented in Figures 9 and 10, where the variation of the critical values is shown as a function of the time delay. The worst case occurs when $T_d/T = 0.5$. From the results, one can conclude that reasonable values of f and β can be used without instability problems due to time delay. However if higher values of f and β are required, several compensation methods including modifications of phase shift of the measured state variables in the modal domain and methods of updating the measured quantities can be found in literature (Soong, 1988).

Table 3 – Influence of the time delay on the maximum amplitude of the column, for $f = 1.00$ e $\beta = 6000$.

Time delay T_d/T	Maximum displacement	Maximum velocity	Maximum acceleration
0.00	0.015469	0.016007	0.018705
0.10	0.017010	0.015386	0.024899
0.20	0.022849	0.0321433	0.033540
0.23	18.320434	28.965204	37.726468

Table 4 – Influence of the time delay on the maximum amplitude of the column, for $f = 1.00$ e $\beta = 60$.

Time delay T_d/T	Maximum displacement	Maximum velocity	Maximum acceleration
0.00	0.03 1746	0.03 1539	0.03 1771
0.20	0.032688	0.032573	0.032852
0.40	0.034166	0.033685	0.034957
0.60	0.033921	0.033686	0.033245
0.80	0.033076	0.032413	0.033016
1.00	0.032791	0.032544	0.032876
2.00	0.033220	0.032755	0.033232

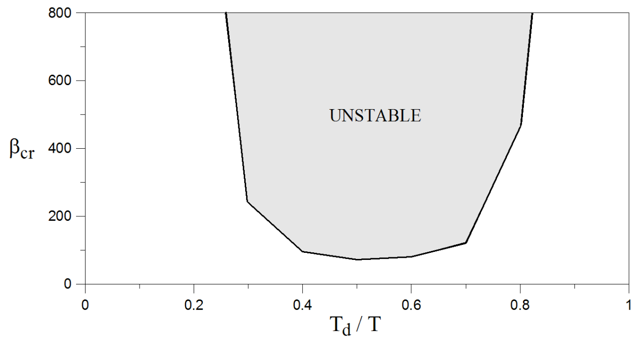


Figure 9 - Variation of the critical value of β as a function of time delay.

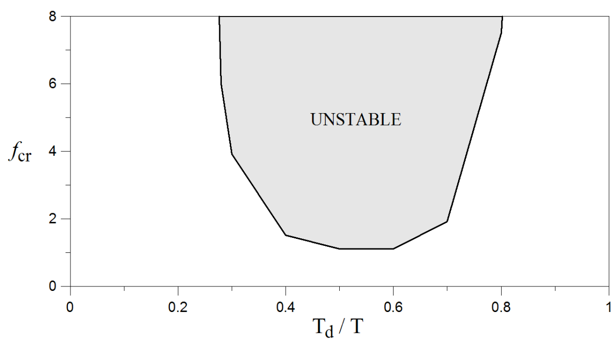


Figure 10 - Variation of the critical value of f as a function of time delay.

CONCLUSIONS

The parametric study of the system with passive control, in the time and frequency domains, clarified the influence of the physical and geometric parameters of the system on the capacity of the pendulum absorber in reducing the vibration amplitudes of a tall and slender tower. The results show that a properly tuned pendulum with a small mass can effectively reduce displacements, velocities and accelerations of the tower in the main resonance region. It is particularly helpful during the steady-state regime. The results also demonstrate that geometric and inertial non-linearities are important in the modeling of this problem and that the non-linearities have a beneficial effect decreasing the vibration amplitudes in the resonance region. However, if the pendulum is not properly tuned, it can amplify the response of the column.

In analysis of the hybrid control system it was demonstrated that the control force acts when the pendulum absorber starts to move, during the transient regime. After the absorber reaches the amplitude necessary to control the oscillations of the column, the amplitude of the control force diminishes significantly. The results show that small force magnitudes can achieve good results. This is important in practical applications, leading to small energy expense. The real-time implementation of switched stiffness devices can be found, for example, in Ramaratnam and Jalili (2006). It was also observed that this control can practically eliminate the oscillations of the system in the main resonance region of the coupled tower-pendulum system. The hybrid control, as shown by Orlando (2006), is also efficient for towers subjected to random loads and loads of short duration, when the passive control alone cannot reduce efficiently the tower vibration amplitudes.

The parametric analysis of the system show that time delay, an unavoidable problem in control, may lead to dangerous instabilities. However, the proper choice of the parameters of the active force can eliminate this loss of stability leading to a robust control system.

So, one can conclude that this strategy of nonlinear control is attractive, has a good potential and can be used as a base for the design of control systems in towers. The hybrid control is much more efficient than the passive one, without a great expense of energy, demonstrating, a priori, to be a good mechanism of vibration control of slender towers. However additional studies are necessary, including the implementation of this control mechanism in real towers, so that the efficiency of the hybrid control can be properly evaluated.

ACKNOWLEDGMENTS

The authors acknowledge the financial support of the Brazilian research agencies CAPES and CNPq.

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