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Experimental Investigation of Non-smooth System Dynamics

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Abstract: Nature is full of non-smooth nonlinearities that are usually related to either the friction phenomenon or the discontinuous characteristics as intermittent contacts. In general, non-smooth characteristics are the source of difficulties for the modeling and simulation of natural systems. This article develops an experimental investigation concerning non-smooth systems with discontinuous support. An experimental apparatus is developed in order to analyze the nonlinear dynamics of a single degree of freedom system with discontinuous support. The apparatus is composed by an oscillator constructed by a car, free to move over a rail, connected to an excitation system. The discontinuous support is constructed considering a mass-spring systems separated by a gap to the car position. This apparatus is instrumented to obtain all the system state variables. System dynamical behavior shows a rich response, presenting dynamical jumps, bifurcations and chaos. Different configurations of the experimental set up are treated in order to evaluate the influence of the internal impact within the car and also support characteristics in the system dynamics.

Keywords: Nonlinear dynamics, non-smooth systems, bifurcation.

INTRODUCTION

Non-smooth nonlinearity is abundant in nature being usually related to either the friction phenomenon or the discontinuous characteristics as intermittent contacts of some system components. Non-smooth systems appear in many kinds of engineering systems and also in everyday life. Examples may be mentioned by the stick-slip oscillations of a violin string or grating brakes (Hinrichs *et al.*, 1998). Some related phenomena as chatter and squeal causes serious problems in many industrial applications and, in general, these forms of vibrations are undesirable because of their detrimental effects on the operation and performance of mechanical systems (Andreaus & Casini, 2001).

The mathematical modeling and numerical simulations of non-smooth systems present many difficulties, which turns their description unusual complex. Moreover, the dynamical behavior of these systems is complex, presenting a rich response. Literature presents many reports dealing with non-smooth systems (Divenyi *et al.*, 2006; Savi *et al.*, 2006).

Since non-smooth systems present an unusual complex behavior and their description involves many mathematical and numerical difficulties, experimental studies are of great importance. Some of the cited references use experimental approaches to verify the proposed numerical methods. Other references discuss just the experimental point of view. Wiercigroch *et al.* (1998) and Wiercigroch & Sin (1998) present an experimental analysis of a base excited symmetrically bilinear oscillator. Virgin and co-workers also develops interesting experimental analyses related to non-smooth systems (Todd & Virgin, 1996, 1997; Begley & Virgin, 1998; Slade *et al.*, 1997; Piiroinen *et al.*, 2004).

This article deals with the nonlinear analysis of dynamical systems with discontinuities. An experimental investigation of a single-degree of freedom system with discontinuous support is considered and, despite the deceiving simplicity of this problem, its nonlinear dynamics is very rich. The apparatus developed to analyze the system dynamical behavior is composed by an oscillator constructed by a car, free to move over a rail, connected to an excitation system. The discontinuous support is constructed considering a mass-spring system separated by a gap related to the car position. This apparatus is instrumented to obtain all system state variables. Different aspects related to the system dynamics are carried out considering the influence of internal impacts within the car and also support characteristics.

EXPERIMENTAL APPARATUS

The dynamical response of discontinuous systems is analyzed from a single-degree of freedom oscillator with discontinuous support, shown in Figure 1. The oscillator is composed by a mass m and part of this mass, m_i, is free to move through a guide within the car, impacting at its ends. The mass is connected to linear springs with stiffness k. The dissipation process can be modeled by linear viscous damping with coefficient c. The support is modeled as a linear oscillator composed by a mass m_s, a spring with stiffness k_s and, again, a dissipation process represented by a linear damping with coefficient c_s. The mass displacement is denoted by x, relative to the equilibrium position and the original distance between the mass and the support is defined by a gap g. Since it is possible to consider different properties and also different gaps to each support, it is employed the superscripts L and R to respectively identify the left and right side support. This system has two possible modes, represented by a situation where the mass presents contact with the support and another situation when there is no contact. Moreover, it is assumed that the system is subjected to a harmonic excitation $F(t) = \rho \sin(\omega t)$. There are many simplifications that can be done to this general system, depending on the system characteristics. When the support relaxation time is much smaller than the time between two contact events, the support dynamics can be neglected and therefore, it may be assumed massless. Another simplification occurs when the impact occurs just in one side. Concerning the internal impact, the impact mass may be fixed to the car and, therefore, the system may not present this kind of behavior.

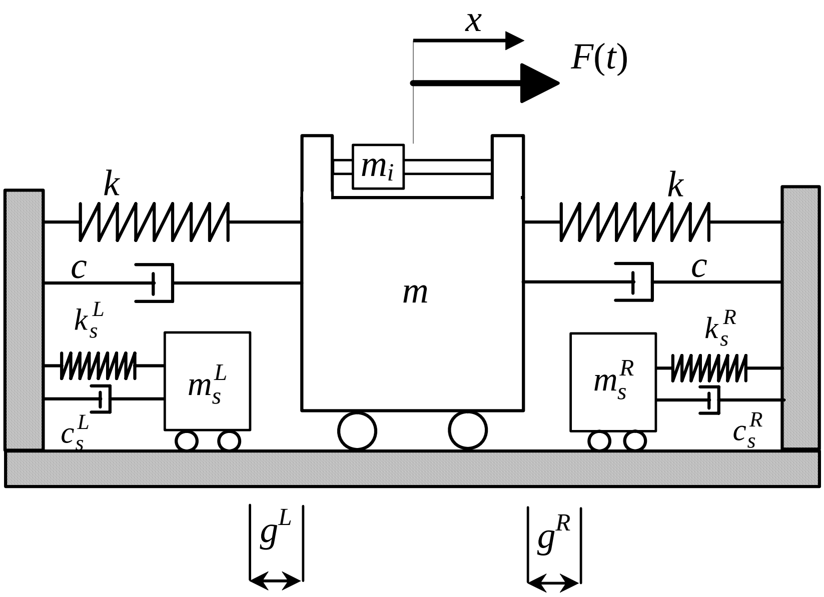

Figure 1. Non-smooth system with discontinuous support.

An experimental apparatus of this system is developed considering an oscillator composed by a car (4), free to move over a rail (2), connected to an excitation system composed by springs (3), strings and a DC motor (1) (PASCO ME-8750 with 0-12 V e 0-0.3 A). Dissipation characteristics may be adjusted by a magnetic damping device (6). The car has an internal mass which can move through a guide (7). The discontinuous support is constructed, at the left side, by considering a spring separated by a gap related to the car position (5). On the other hand, on the right side there is a mass connected to the support spring and fixed by a string in order to always present compressive behavior (8). Actually, different support may be used depending on the analysis. The movement is measured with the aid of a rotary sensor (9), PASCO CI-6538, which has a precision of ±0.25 degrees, maximum velocity of 30 rev/s and maximum sampling frequency of 1000 Hz. The apparatus is shown in Figure 2, together with detailed pictures of some parts.

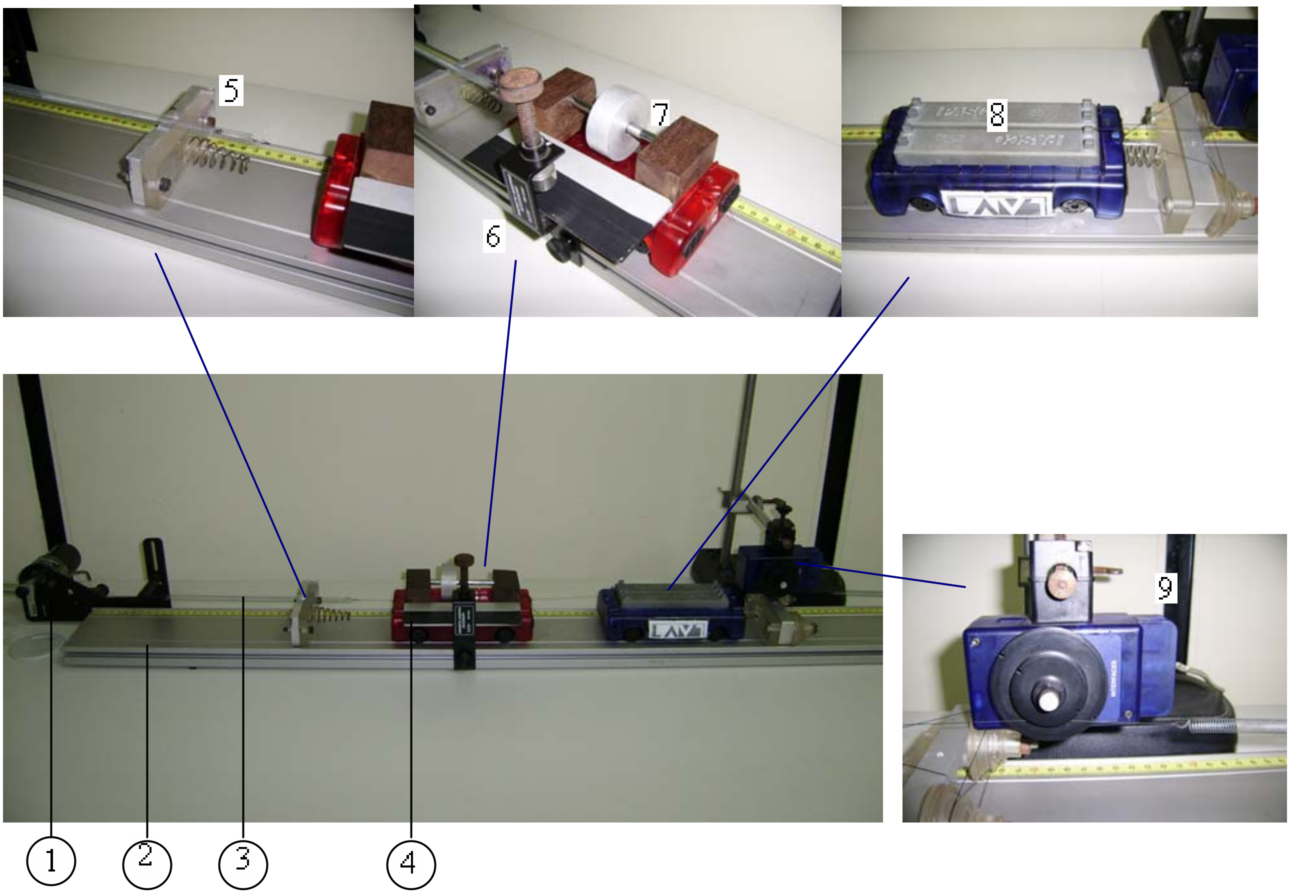

Figure 2. Experimental apparatus.

Parameters identification may be done by different procedures. Savi *et al.* (2006) develop numerical and experimental investigations dealing with an oscillator without internal impact, where the discontinuity is represented by a single support, massless. This reference employs a weight scale to measure the system mass while the spring stiffness is evaluated through the slope of a force-displacement curve, plotted with the aid of two sensors: the rotary sensor shown in Figure 2 and a force transducer. Concerning the dissipation characteristics, it is assumed a linear viscous damping in the considered range and the determination of the oscillator dissipation parameter is done analyzing the system frequency response. The support dissipation parameter is identified assuming the logarithmic decrement procedure, which is defined verifying the ratio between two consecutive displacement amplitudes. Moreover, forcing characteristics, namely the amplitude and frequency forcing, are respectively related to the motor length of rotating crank and motor voltage.

In this article, the system dynamical response is analyzed evaluating the influence of different characteristics related to either the car characteristics or the support. With respect to the car characteristics, it is considered a further impact system within the car. Concerning the support characteristics, the inclusion of two supports and also different properties as stiffness and damping are considered. Moreover, it is investigated the inclusion of inertia aspects, connecting a mass to the support.

INFLUENCE OF THE CAR CHARACTERISTICS

The influence of the car characteristics is now investigated considering a single support. Under this condition, there is only a single gap, and it is assumed $g = g^R$. The goal is to consider an internal impact mass, which may be free to move through a guide within the car, impacting at its ends. Since this impact mass may be fixed to the car, it is possible to establish a comparison between the dynamics with or without internal impacts. The car has a total mass m = 0.471 kg and different configurations are carried out changing the initial position of the impact mass and also the lubrication of the guide.

A comparison of the system dynamics considering the car with and without the internal impact is presented in Figure 3 assuming g = 22.65 mm, ρ = 0.76 N and ω = 1.51 Hz. The situation without impact is generated restricting the internal mass movement. The result with impacts presents smaller amplitudes than those without impacts where the car is in the imminence to jump out of the rail. Internal impacts tend to dissipate system energy and, therefore, small amplitude responses are expected.

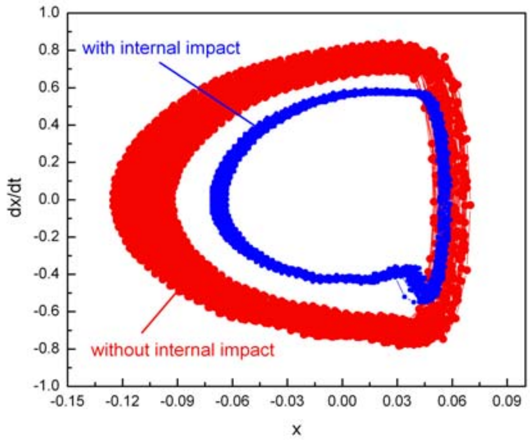

Figure 3. Effect of internal impact. *g* = 22.65 mm, *ρ* = 0.76 N, *ω* = 1.51 Hz.

At this point, it is analyzed the impact mass initial position influence. Two different situations are considered (Figure 4). In the first one, the impact mass starts at the end near the support and therefore, it is associated with a smaller first impact. The second configuration, on the other hand, is associated with the impact mass starting the movement in the opposite end of the support being related to a greater first impact.

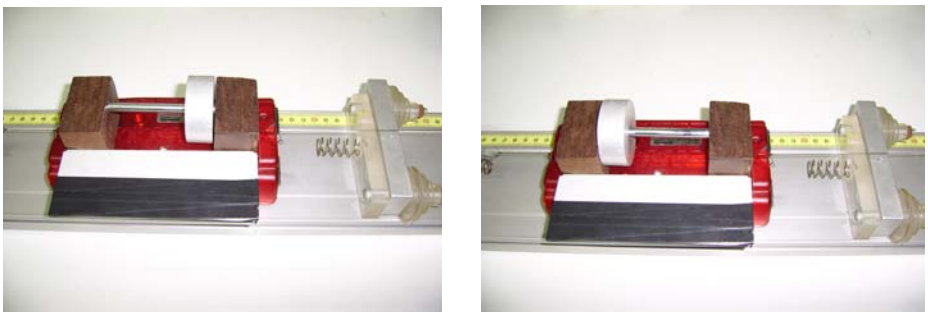

Figure 4. Different configurations related to the initial condition of the impact mass.

A comparison between the responses of both configurations for g = 62.1 mm, ρ = 0.76 N, ω = 1.03 Hz is presented in Figure 5. When the system has a smaller first internal impact, the system response is related to a period-2 orbit. By considering the other situation, the first internal impact changes the orbit and the system response is related to a period-1 orbit.

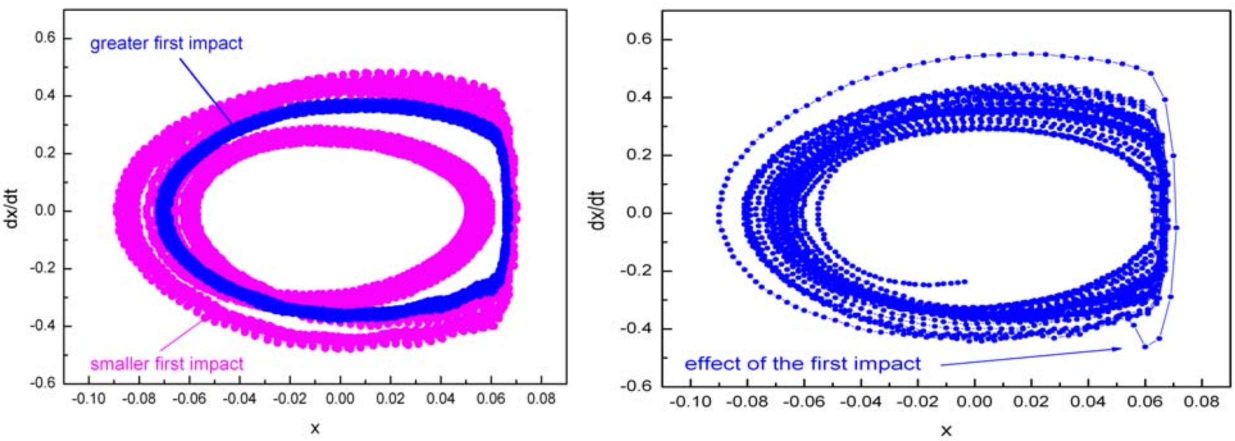

Figure 5. Effect of internal impact: initial position of the internal mass.

***g* = 62.1 mm, *ρ* = 0.76 N, *ω* = 1.03 Hz**

Now, the mass guide is lubricated causing differences in the system response. A comparison between the system with and without a special lubrication assuming g = 50.4 mm, ρ = 0.76 N, ω = 1.41 Hz is presented in Figure 6. Notice that the internal impacts cause perturbations in the system response, and the original orbit presents oscillations during a cycle. These oscillations are probably due to the force increase during internal impacts. Other situations may also cause the increase of the number of impacts for the same energy level.

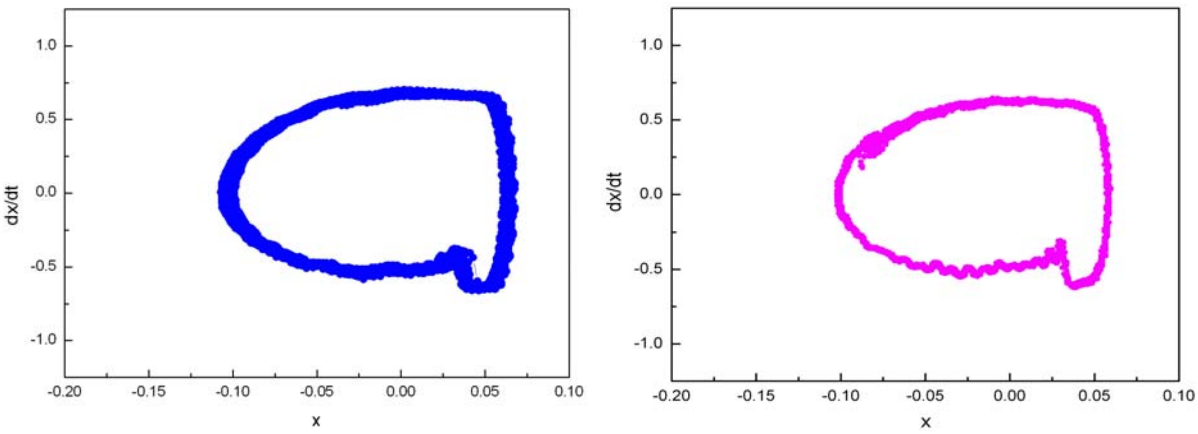

Figure 6. Effect of internal impact: lubricated guide. g= 50.4 mm, ρ = 0.76 N, ω = 1.41 Hz

INFLUENCE OF THE SUPPPORT CHARACTERISTICS

In order to analyze the influence of the support characteristics in the system dynamics, the experimental set up is altered. Once again, a single support is considered ($g = g^R$) and the first consideration is just the change of the support elastic element to a spring with higher stiffness and also to a rigid rubber. The impact mass is now restrict to move through the guide and, therefore, there are no internal impacts. Figure 7 shows different supports employed in the analysis representing three situations: low stiffness spring, high stiffness spring and rubber.

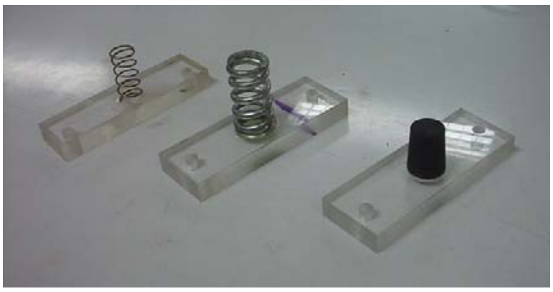

Figure 7. Different supports.

Figure 8 shows the response for each support considering an oscillator with a mass m = 0.838 kg, excitation parameters ρ = 0.75 N, ω = 0.80 Hz and three different gaps: 2.5, 11.0 and 16.3 mm. By increasing the support stiffness causes a reduction on the phase space since the mass is not capable to reach certain positions in the contact region. For the small gap, g = 2.5 mm it is noticeable that the low stiffness spring impacts just one time in a cycle. The other two supports, high stiffness spring and rubber, on the other hand, impacts twice. Therefore, low stiffness spring presents smaller amplitudes than those obtained by the other supports. By changing the gap, the system presents similar qualitative results for all supports.

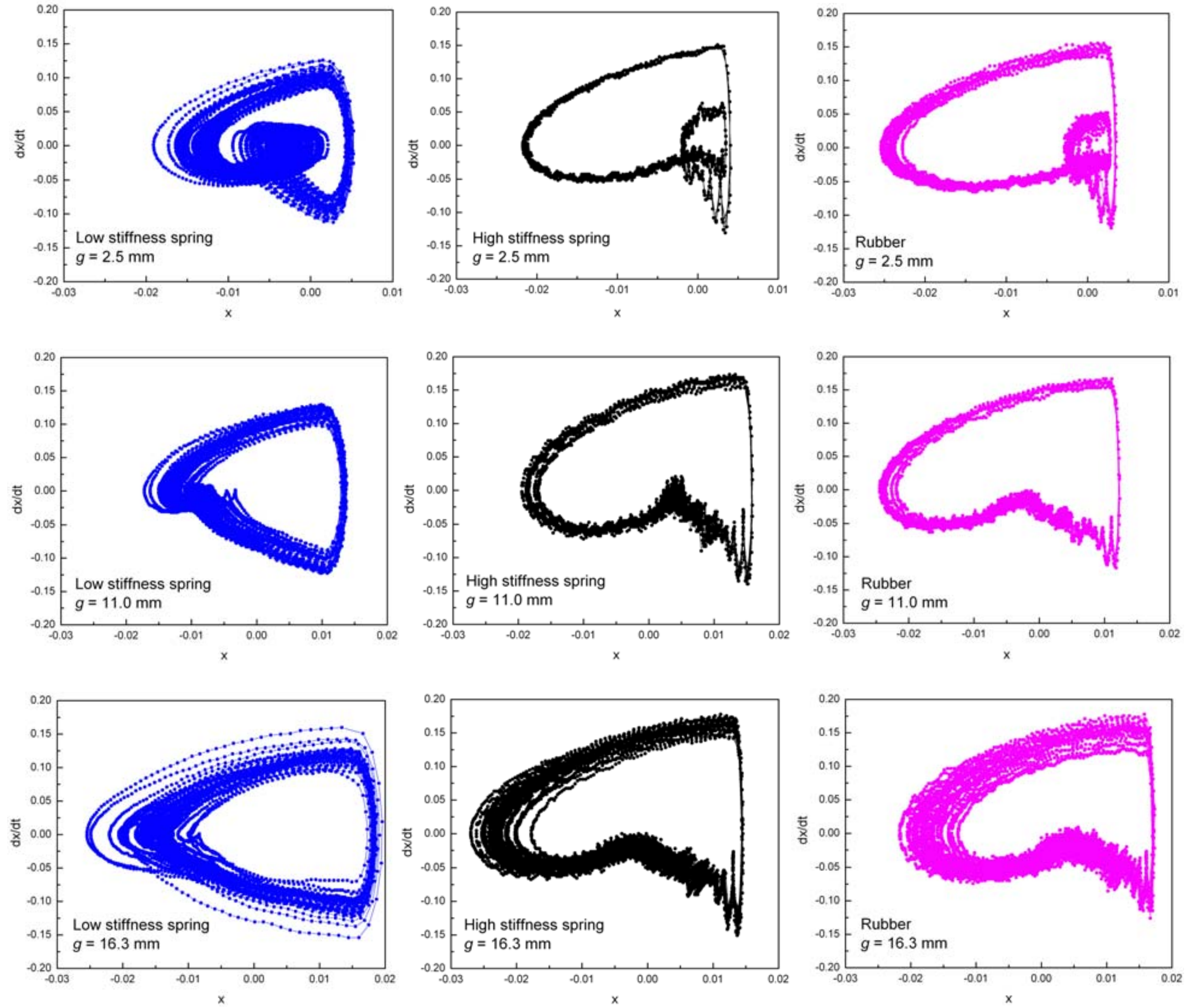

Figure 8. Influence of different support characteristics.

The support inertia is now focused on. This investigation is done considering a system with a single support where a new car is connected to the support (Figure 2). Different forcing frequencies and support masses are analyzed considering an oscillator with mass m = 0.471 kg. Different forcing frequencies are analyzed in order to evaluate the effect of support inertia. The first noticeable effect associated with the support mass increase is the velocity decrease when the car looses the contact. This behavior may be understood just thinking in terms of momentum conservation. It is also related to this effect the characteristics of the non-contact response, when it is increasing the velocity for the next contact with the support. By observing the state space orbit, it is perceptible the orbit inclination as the mass is increased. Besides, by increasing the support mass, the position where the car looses contact also presents smaller values. The forthcoming results are obtained assuming g = 13.35 mm and ρ = 0.75 N. A comparison between the system with and without support inertia for ω = 0.75 Hz is shown in Figure 9. The same comparison is presented in Figure 10 and 11 for ω = 0.97 Hz and ω = 1.29 Hz, respectively.

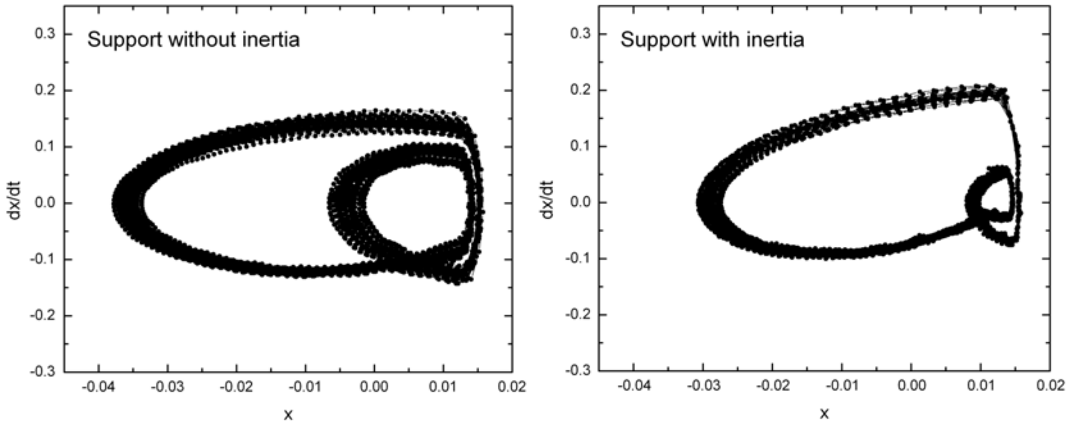

Figure 9. Effect of support inertia for $\omega = 0.75$ Hz.

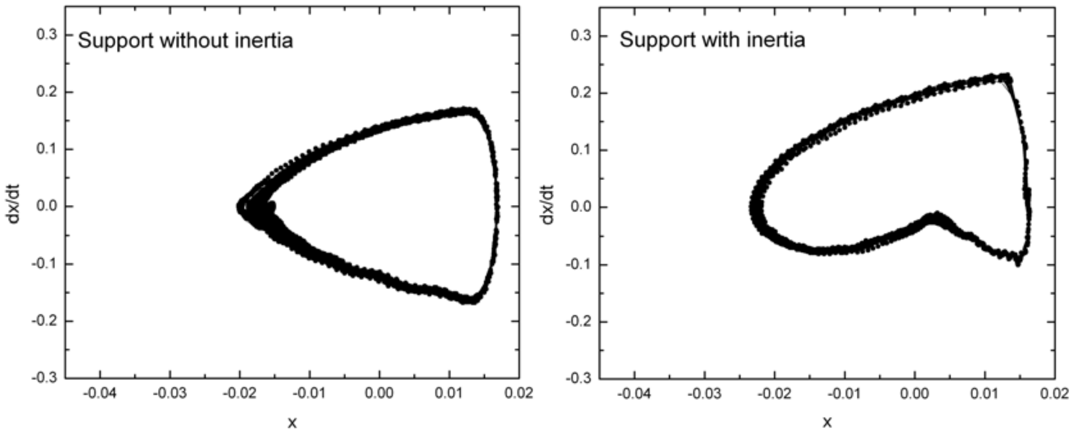

Figure 10. Effect of support inertia for $\omega = 0.97$ Hz.

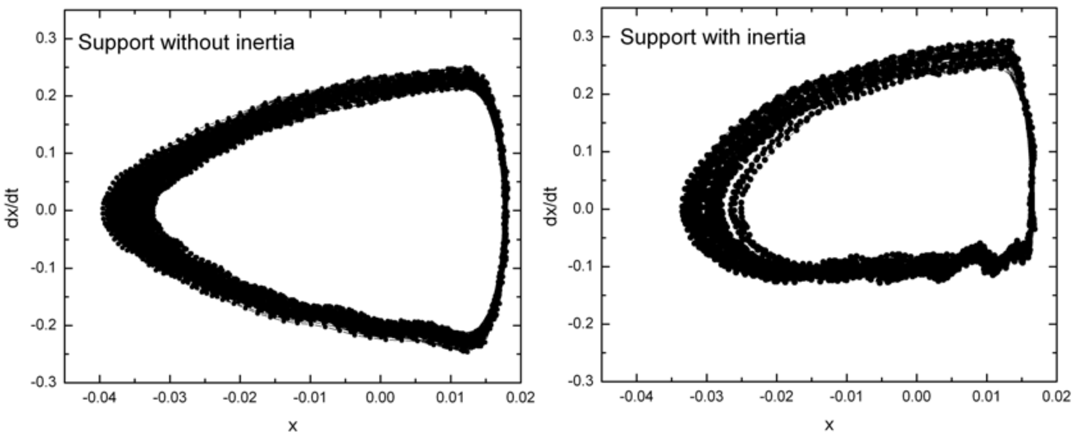

Figure 11. Effect of support inertia for $\omega = 1.29$ Hz

The next results show the influence of the support mass increase, considering three m_s values: 261 g, 511 g, 761 g. This analysis is done assuming g = 5.55mm, ρ = 0.75 N and ω = 0.88 Hz. Figure 12 shows that together with the inclination of the orbit, similar to those of the previous results, there is a decrease of the second oscillation amplitude (internal loop).

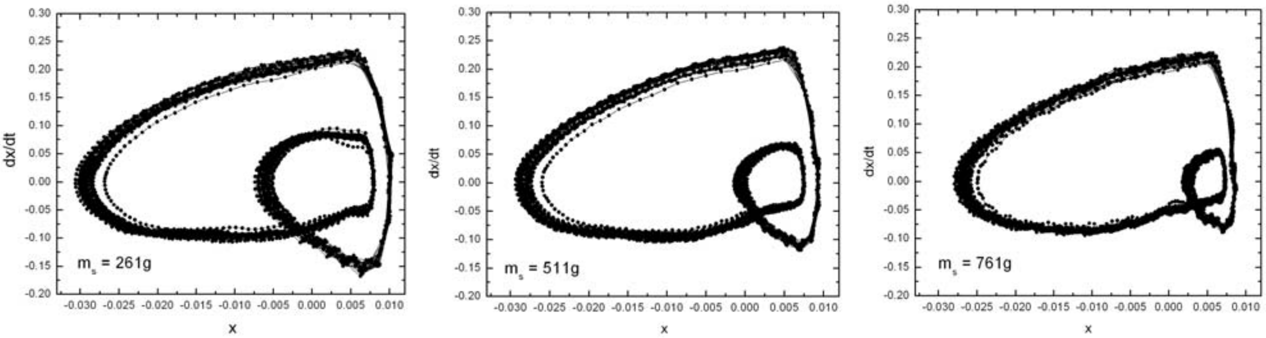

Figure 12. Comparison of the system dynamics for different values of the support inertia.

At this point, it is investigated the combination of the support inertia with the internal impact. It is assumed an oscillator with m = 0.471 kg and a support inertia of m_s = 0.761 kg. Moreover, g = 49.7mm, ρ = 0.65N and ω = 0.94 Hz. Figure 13 shows the difference between the system dynamics with and without internal impact. The left side picture presents the dynamics related to the non-impact system while the right side presents the system where the impact mass is now free to move through the guide. Notice that the internal impact tends to increase the response complexity being related to a chaotic-like response.

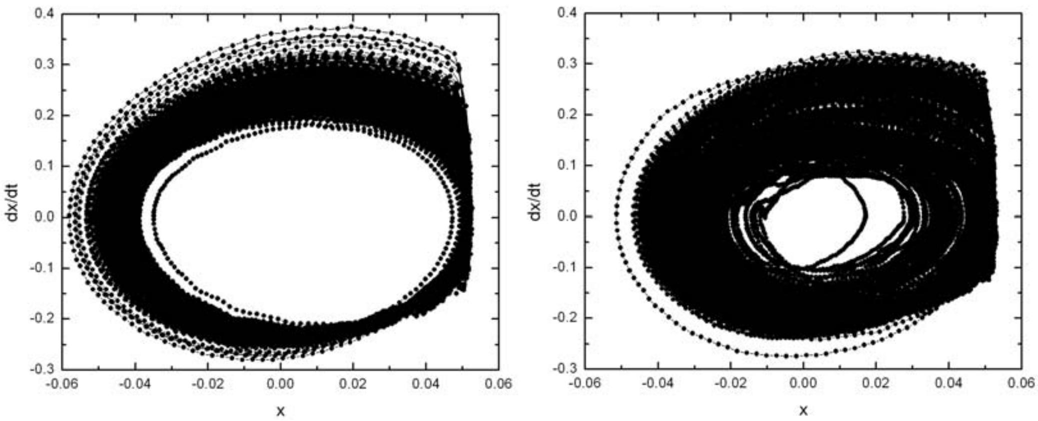

Figure 13. Comparison of the system dynamics with support inertia and internal impact.

INFLUENCE OF TWO SUPPPORTS

At this point, the experimental set up is altered in order to consider two supports. At first, the influence of different support stiffness combinations is of concern assuming ρ = 0.79 N, ω = 1.03 Hz, g^L = –9.5 mm, g^R = 9.5 mm and m = 838 g. The analysis considers the same left support, using three different right supports, basically associated with low, intermediate and high stiffness springs. Results considering these three configurations are presented in Figure 14 showing that, as expected, the increase in support stiffness reduces the phase space since the mass is not capable to reach certain positions in the contact region. Besides, this can change the dynamical response causing period-2 response presented by the high stiffness spring responses.

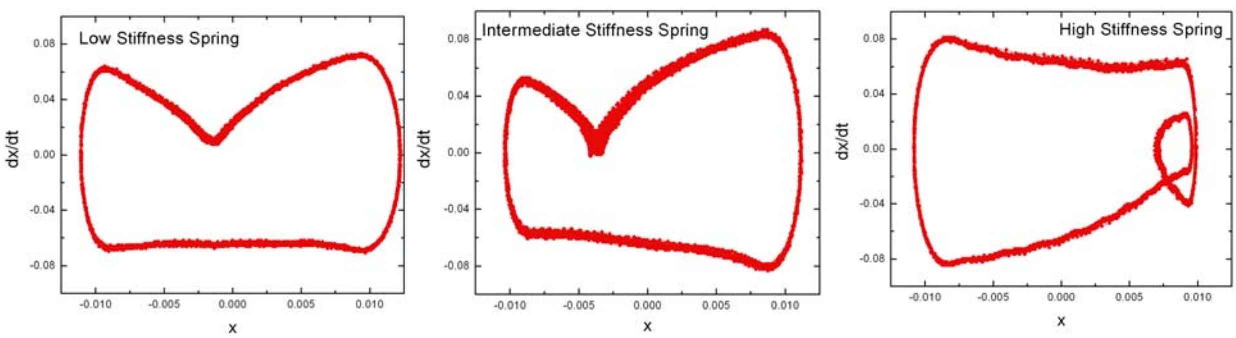

Figure 14. Comparison of the system dynamics with two supports and different stiffness.

The gap influence is now evaluated considering ρ =0.79 N and ω =1.53 Hz. The left side support is fixed with a gap g^L = –23.95mm, while the right side support gap is varied assuming different values: –3.6 mm, 2.1 mm, 5.8 mm, 9.3 mm. Results obtained from these configurations are presented in Figure 15. Notice that, as the gap increases, the complexity of the system response decreases. For the first gap, there is a chaotic-like response and after that, the gap decrease causes a period-4, period-2 and finally, a period-1 response. These results show a qualitative change on system dynamics, being related to bifurcations among these results.

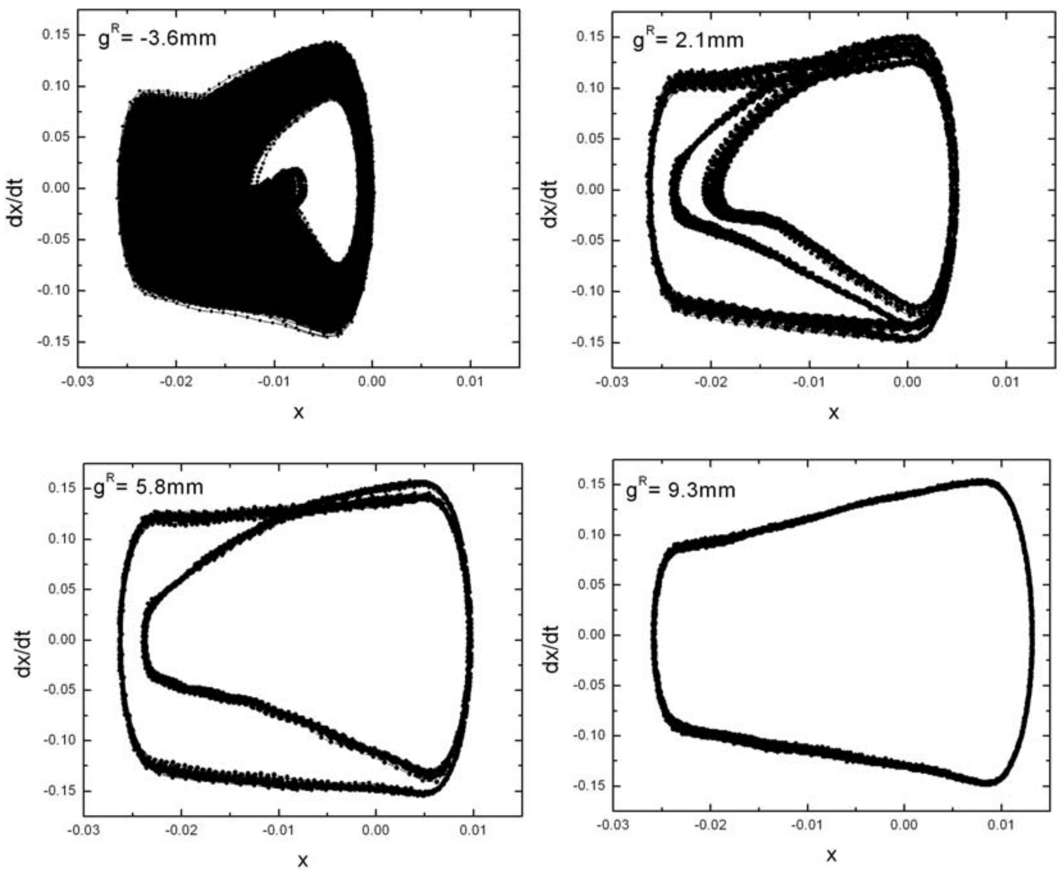

Figure 15. Comparison of the system dynamics with two supports and different gaps.

The influence of internal impacts on a system with two supports is now focused on. Therefore, it is considered situations with and without internal impact and different gaps, assuming ρ = 0.79 N and ω =1.44 Hz. The right side support is fixed with a gap g^R = 34.2 mm, while the left side support gap is varied assuming different values: –12.5 mm, –34.2 mm and –72.2 mm. Results obtained without internal impact are presented in Figure 16, while results with impact are presented in Figure 17. Notice that the impact influence on system dynamics depends on the gap. By considering large values of gap the system may present a different qualitative behavior.

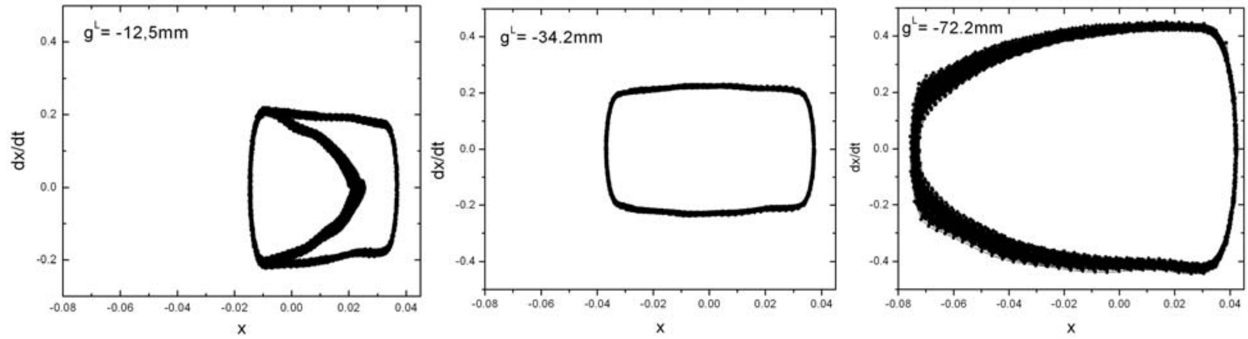

Figure 16. System dynamics without internal impact considering two supports and different gaps.

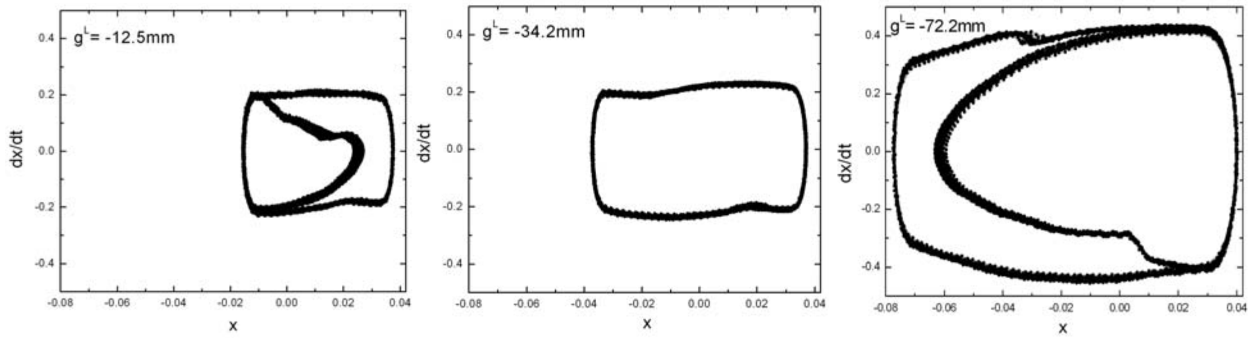

Figure 17. System dynamics with internal impact considering two supports and different gaps.

CONCLUSIONS

The experimental analysis of a non-smooth system with discontinuous support is presented in this contribution. An experimental apparatus is constructed in order to evaluate the nonlinear dynamics of this system, verifying the influence of the car characteristics and also the support in the system dynamics. In general, despite the deceiving simplicity of this system, its nonlinear dynamics is very rich, presenting dynamical jumps, bifurcations and chaos. With respect to the car characteristics, an internal impact is introduced considering a mass free to move through a guide. Concerning the support, different kinds are considered changing the stiffness and also the influence of support inertia. Besides, the system response with two supports is of concern, analyzing the influence of stiffness and gaps. These aspects can change the system dynamics in a qualitative point of view and also may be desirable in order to dissipate energy.

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