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Finite Element Analysis of Critical Speeds and Stability of Flexible Rotors Supported by Fluid Film Bearings

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Abstract: This work deals with a finite element procedure developed to perform the eigenvalue analysis of high-speed rotating machines supported on fluid film journal bearings. The rotor finite element model is based on the Timoshenko beam theory, in which the rotary inertia and gyroscopic moments are taken into account. The governing equations for the hydrodynamic journal bearing are obtained through the Galerkin weighted residual method applied to the classical Reynolds equation. A perturbation scheme on the fluid film governing equation permits to obtain the zero-th and first order lubrication equations for the bearings, which allow the computation of the dynamic force coefficients associated with the bearing stiffness and damping. The rotor-bearing system equation, which consists on a case of damped gyroscopic equation, is rewritten on state form to compute the complex eigenvalues. The natural frequencies at several rotating speeds and the stability maps are obtained for rotating machines operating at several operating conditions. The influence of the effective damping on the eigenvalue real part sign is analyzed for some examples of high-speed rotor-bearing systems. The procedure implemented in this work can provide useful guidelines and technical data about the selection of the more appropriate set of bearing parameters for rotating machines operating at stringent conditions.

Keywords: Flexible Rotors, Fluid Film Bearings, Rotor-Bearing Systems, Finite Element, Eigenvalue Problem.

INTRODUCTION

Researchers have been continuously devising experimental, analytical and computational procedures to analyze the several dynamic aspects associated with rotating shafts employed on high-speed machines. Since 1970, the finite element method has been largely used to develop models for flexible rotors and to perform analyses of balancing, stability and torsional vibration of rotating machinery (Childs, 1993; Vance, 1998; Nelson and McVaugh, 1976; Zorzi and Nelson, 1977; Nelson, 1980; Almeida Jr. and Faria, 2003; Miranda et al., 2005). For a rotating shaft, the Timoshenko beam theory has been employed to build finite element models very accurate to analyze the dynamics of flexible rotors (Nelson, 1980).

Computational procedures able to predict the dynamic response of high-speed rotors supported on fluid film bearings have been the goal of many turbomachinery manufacturers (Busse et al., 1980). Those procedures are very useful at the preliminary design stages and commissioning of industrial rotating machines employed on the oil industry and petrochemical plants (Sternlicht and Lewis, 1968). The eigenvalue analysis of rotating and stationary components of machines and mechanical equipments has been a basic step in any dynamic analysis of rotating systems (Lund, 1994).Vibration modes associated with the rotating shaft and bearing support have provided important subsides for the development of computational procedures on vibration analysis, balancing techniques and monitoring of high speed rotating machinery (Busse et al., 1980; Boedo and Booker, 1997).

On the eigenvalue analysis of structural dynamic systems, the governing equations are generally based on both the undamped and dampeg gyroscopic systems (Gupta, 1974). On the other hand, for industrial turbomachinery and rotating machines, the eigenvalue analysis has been carried out based on both the nongyroscopic and gyroscopic systems, taking into account or not the dissipative properties (Meirovitch and Ryland, 1979; Palazzolo et al., 1983; Faria and Barcellos, 1991). The eigenvalue problem is also important on the sensitivity analysis of the system dynamic response (Plaut and Huseyin, 1972; Lund, 1980; Rajan et al., 1986; Done and Hughes, 1975). The stability analysis and the dynamic response of gyroscopic systems can also be performed from the eigenvalue problem (Ehrich, 1992).

This work deals with a finite element procedure devised to perform the eigenvalue analysis of high-speed rotating machines supported on fluid film journal bearings. The Timoshenko beam theory is applied on the rotating shaft finite element modeling, accounting for the shear effects, the gyroscopic moments and the rotatory inertia. Lumped masses are used to model mechanical components rigidly attached to the rotating shaft, which may represent any rotating part of a turbomachine shaft, such as turbine wheels, compressor disks or pump impellers. The hydrodynamic journal bearing finite element modeling is based on the classical Reynolds equation. A linearized perturbation method is applied on the Reynolds equation to render the lubrication equations capable of predicting the eight linearized dynamic force coefficients associated with the bearing stiffness and damping. The rotor-bearing system equation, which consists on a case of damped gyroscopic equation, is rewritten on state form (Meirovitch,1974; Meirovitch,1980; Childs and Graviss,1982) to compute the complex eigenvalues. The natural frequencies at several rotating speeds and the stability maps are obtained for rotating machines operating at stringent conditions. The influence of the effective damping on the eigenvalue real part sign is analyzed for some examples of high-speed rotor-bearing systems. Also, the influence of the bearing dynamic force coefficients on the dynamic response and on the stability of flexible rotors is shown through some curves presented in this work. The effective damping of rotor-bearing systems is demonstrated to be a very important design parameter for high-speed rotating machinery.

FINITE ELEMENT EQUATIONS

The rotor-bearing system is modeled using finite element models for both the flexible shaft and the hydrodynamic journal bearings. A global equation of motion is obtained from the finite element matrices, where, [*M*] represents the global shaft translational inertia matrix, [*N*] represents the global rotatory inertia matrix, [*K*] the shaft and bearing stiffness matrix and [*C*] is the generalized shaft and bearing damping matrix, in which the shaft gyroscopic effects are included. The bearings stiffness [K_m] and damping [C_m] coefficients are included into the system matrices, in order to represent the fluid film resistance to the rotor displacement and to velocity, respectively. The rotor-bearing system equation is rewritten on state form to compute the complex eigenvalues. The complex eigenvalues associated with the system are separated to get the natural frequencies and information on the stability of the rotor-bearing system.

Shaft modeling

The finite element method is applied for the modeling of both the flexible shaft and the hydrodynamic journal bearings. Figure 1 depicts a schematic view of a flexible rotor supported on fluid film plain cylindrical journal bearings.

The finite element shaft modeling implemented in this work has been based on the special shape functions derived by Nelson (1980). Nelson (1980) employs the Timoshenko beam theory to derive the governing equations for a flexible circular shaft supported on elastic supports taking into account the shaft shear effects, gyroscopic moments and rotatory inertia. The system is represented schematically in Fig.1.

Two node beam finite elements with eight degrees-of-freedom are employed to model the lateral motion of flexible shafts. The journal bearing contributions to the rotor stiffness and damping coefficients are accounted for. The finite element procedure is based on the following global equation of motion

$$[M + N]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{R\} \tag{1}$$

where [*M*] represents the global shaft translational inertia matrix, [*N*] represents the global rotatory inertia matrix, [*K*] the shaft and bearing stiffness matrix and [*C*] is the generalized shaft and bearing damping matrix, which is expressed as $[C] = [C_1] - \Omega.[G]$, in which [*G*] is the shaft gyroscopic effects matrix. The matrix [C_1] represents the bearing damping. The shaft acceleration, velocity and displacement vectors are given, respectively, by $\{\ddot{U}\},\{\dot{U}\},\{U\}$, and Ω is the shaft rotating speed (rad/s). The external excitation force is represented by the vector {*R*}.

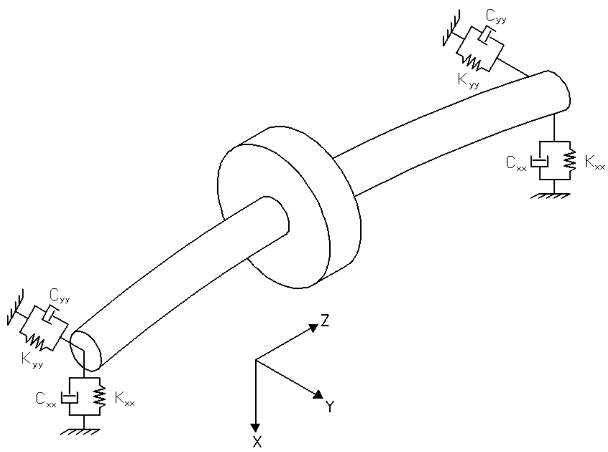

Figure 1 – Flexible shaft supported on fluid film journal bearings.

Bearing modeling

The journal bearing finite element model is developed based on the classical Reynolds equation for oil-lubricated plain cylindrical journal bearings (Childs, 1993). For the coordinates (X,Z), this equation is given by.

$$\frac{\partial}{\partial X}\left(\frac{h^3}{12\mu}\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{12\mu}\frac{\partial P}{\partial Z}\right)=\frac{\Omega R}{2}\frac{dh}{dX}+\frac{\partial h}{\partial t} \tag{2}$$

The journal rotational speed is denoted by Ω. Journal eccentricities on the vertical and horizontal directions are expressed as e_X and e_Y, respectively. The eccentricity ratio is defined as $\varepsilon = e/c$, where $e^2=e_X^2+e_Y^2$. The circumferential coordinate $X = R.\theta$ and R is the bearing radius. Fluid viscosity is given by μ, P represents the hydrodynamic pressure and h is the fluid film thickness. A linearized perturbation procedure is used in conjunction with Eq. (2) to render the zeroth- and first-order lubrication equations (Faria, 2001). These equations allow the computation of the bearing reaction forces and eight dynamic force coefficients. For brevity, these equations and the validation of the finite element procedure for the bearing dynamic coefficients are omitted in this work.

The dynamic force coefficients are represented in matrix form by the stiffness [K_m] and the damping [C_m] matrices as in Eq.(3). They stand for the fluid film resistance to the rotor displacement and velocity, respectively.

$$[K_m] = \begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix}; \qquad [C_m] = \begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix} \tag{3}$$

Figure 2 depicts the cross-section of a journal bearing and its linearized stiffness and damping coefficients along the X-axis and Y-axis.

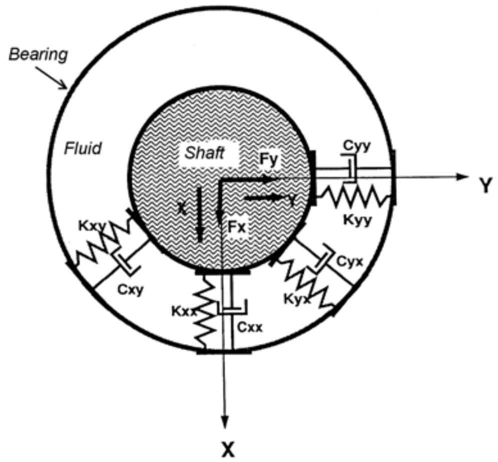

Figure 2 – Linearized stiffness and damping coefficients of the journal bearing

Eigenvalue Problem

A first vibration analysis of rotor-bearing systems can be carried out through computational procedures developed specially to predict the dynamic response and stability analysis of rotating shafts supported by fluid film bearings. At the preliminary design and commissioning stages of industrial turbomachinery, those procedures can bring important insights on the rotating system dynamic behavior.

The first step in the dynamic analysis consists on obtaining the system natural frequencies under several operating conditions. The free vibration problem associated with linear systems of differential equations leads naturally to the eigenvalue problem (James et al. 1994). For damped gyroscopic systems, the complex eigenvalues and eigenvectors provide very useful data about the mode shapes and stability of rotating systems.

The eigenvalue problem associated with Eq.(1) can be reduced to a standard form, following a procedure similar to that presented by Meirovitch (1980). A second order state vector $\{X\}$, defined in the following form, is used to rewrite the governing equation on state variables:

$$\{X\} = [\{\dot{U}\}^T \mid \{U\}^T]^T \tag{4}$$

The free vibration problem associated with Eq. (1) can be rewritten as follows

$$[M^*]\{\dot{X}\} + [C^*]\{X\} = \{0\} \tag{5}$$

where

$$[M^*] = \begin{bmatrix} [M]+[N] & 0 \\ 0 & [I] \end{bmatrix}; \qquad [C^*] = \begin{bmatrix} [C] & [K] \\ -[I] & 0 \end{bmatrix} \tag{6}$$

where $[I]$ is the identity matrix, with the same dimension as that of $[M]$, $[N]$, $[C]$ and $[K]$. The solution of Eq. (5) has the form

$$\{X\}(t) = e^{st}\{X_o\}(t) \tag{7}$$

and the associated eigenvalue problem can be stated as

$$s[M^*]\{X_o\} + [C^*]\{X_o\} = \{0\} \tag{8}$$

where s represent the system complex eigenvalues. These eigenvalues are composed by a real part "a" and an imaginary part "b", given by Eq. (9).

$$s = a \pm ib \tag{9}$$

The imaginary part "b" corresponds to the system natural frequency and the real part "a" gives information on the system stability, as shown in the following sections.

NUMERICAL RESULTS

Numerical results of some cases of rotors supported by fluid film bearings are obtained to validate the finite element procedure developed in this work and to perform the stability analysis of a damped gyroscopic system.

Example 1

The validation of the finite element procedure begins with an example of a uniform shaft supported at its ends on identical damped bearings, presented by Lund (1974). The baseline parameters of this first example are given on Tab.1.

Table 1 – System parameters for example 1

Parameter	Description	Value	Unit
L	shaft length	1,270	m
d	shaft diameter	0,1016	m
E	shaft Young module	207 x 10^9	Pa
ρ	shaft specific mass	7833	kg/m³
k_b	bearings direct stiffness coefficient	3,5024x 10^6	N/m
c_b	bearings direct damping coefficient	-	N.s/m

The complex eigenvalues lead to the damped natural frequencies of the system given by Tab. 1. Figures 3 and 4 depict the curves of natural frequencies in function of the bearing damping coefficient. The solid, dotted and dashed lines are associated with the results presented by Lund (1974), while the symbols indicate the predictions obtained by the finite element procedure developed in this work. The finite element predictions are represented by EV158, EV159, EV160, EV161 and EV162, which represent the 158th, 159th, 160th, 161st and 162nd system eigenvalues, respectively. This analysis shows the influence of the bearing damping on the natural frequencies of the rotor-bearing system. If the bearing parameters change, the damping may vary, changing the vibration behavior of the entire system. The stability can also be studied based on the eigenvalues. The computation of the damping exponent associated with part "a" of the complex eigenvalues renders results very similar to those obtained by Lund (1974), whose results are omitted here for brevity.

The numerical procedure using state variable form produces many eigenvalues, which are separated and organized. For this example, using a mesh with 80 beam elements, after filtering and organizing the eigenvalues, it can be observed in Fig. 3 that the 160th, 161st and 162nd eigenvalues represent, respectively, the 3rd, 2nd and 1st vibration modes of the rotor-bearing system. As the damping coefficient increases, the order of eigenvalues corresponding to the three first modeshapes changes, as shown in Fig. 4. The comparative results depicted on Fig. 3 and Fig. 4 show that the finite element procedure renders results in good agreement with those presented by Lund (1974).

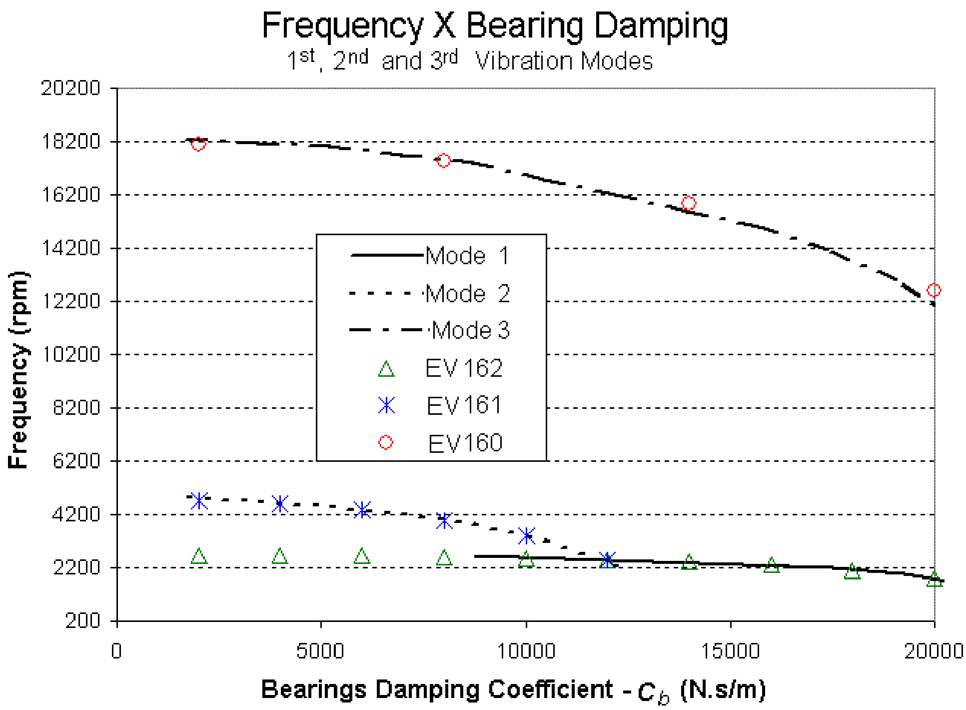

Figure 3 – Natural frequencies versus bearings damping coefficients, for the 1st, 2nd and 3rd vibration modes

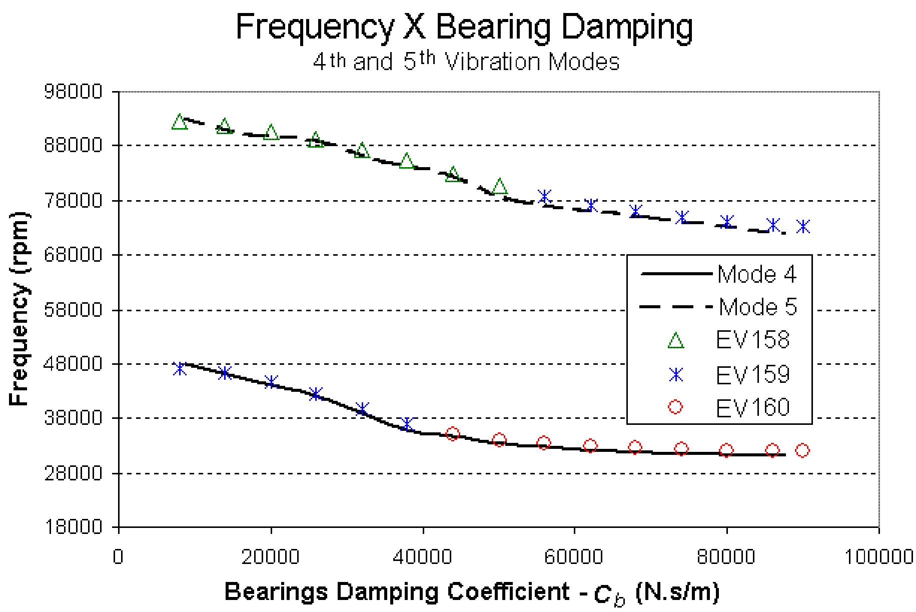

Figure 4 – Natural frequencies versus bearings damping coefficients, for the 4th and 5th vibration modes

Example 2

This second example consists on a rotor-bearing system composed by a shaft supported by two hydrodynamic journal bearings, similar to that shown in Fig.1. More details can be seen in the work of Miranda et al. (2005). The parameters of the system are shown in Tab.2.

The exponents associated with part "*a*" of the complex eigenvalues are computed for the rotating system with some sets of parameters associated with the hydrodynamic journal bearings. This analysis is performed to show the importance of the bearing effective damping $2\omega C_{xx}/K_{xy}$ (Vance, 1988) on the stabilization of the rotating system. In this example, the shaft rotating speed is represented by ω. C_{xx} is the bearing direct damping coefficient and K_{xy} is the bearing cross-coupled stiffness coefficient.

Basically, the bearing stiffness remains constant and only the damping coefficient varies at each rotating speed selected. Table 3 shows a summary of the stability analysis performed on the system given in Tab. 2, at three operating speeds – 750 rpm, 3200 rpm and 5600 rpm. For simplicity, the cross-coupled damping coefficients are neglected in this analysis.

Table 2 - System parameters for example 2

Parameter	Description	Value	Unit
L	shaft length	0,30	m
d	shaft diameter	0,015	m
l	bearings length	0,012	m
c_1	Bearing radial clearance	$34{,}5 \times 10^{-6}$	m
μ	lubricant viscosity	25×10^{-3}	Pa·s
ρ	lubricant mass density	892	kg/m³
E	shaft Young modulus	200×10^{9}	Pa
ν	shaft Poisson coefficient	0,3	-
ρ	shaft mass density	7870	kg/m³

The results obtained from this stability analysis show that an increase on the direct damping coefficient can stabilize a rotor operating unstably. A sign change on the eigenvalue exponent "*a*" indicates that the rotor is moving from unstable to stable conditions or vice-versa. The stability analysis based on the real part "*a*" of the complex eigenvalues associated with the rotor shown in Tab. 2 provides the same results than those based on the time integration of the rotor governing equation (Miranda et al., 2005). It is clear from this analysis that the bearing effective damping $2\omega C_{xx}/K_{xy}$ is a very important parameter for rotors operating on hydrodynamic bearings. When the effective damping approaches 0, the system tends to be unstable, and when it increases, the system tends to be stable. On Table 3, a system is considered unstable when $a > 0$, and stable when $a \leq 0$. The values shown on the second and third columns labeled $[K_m]$ and $[C_m]$ are the bearing dynamic force coefficients.

Table 3 – Numerical results for example 2 and stability parameters

Speed (rpm)	$[K_m]$ (N/m)	$[C_m]$ (N.s/m)	$2\omega C_{xx}/K_{xy}$	Stability	a
750	$10^4\text{x}\begin{bmatrix} 4{,}4150 & 4{,}4137 \\ -4{,}4137 & 4{,}4150 \end{bmatrix}$	$\begin{bmatrix} 29{,}81 & 0 \\ 0 & 29{,}81 \end{bmatrix}$	0,1	UNSTABLE	156
750	$10^4\text{x}\begin{bmatrix} 4{,}4150 & 4{,}4137 \\ -4{,}4137 & 4{,}4150 \end{bmatrix}$	$\begin{bmatrix} 1579{,}9 & 0 \\ 0 & 1579{,}9 \end{bmatrix}$	5,3	Stable	-260
5600	$10^6\text{x}\begin{bmatrix} 0{,}4092 & 2{,}3518 \\ -2{,}3518 & 0{,}4092 \end{bmatrix}$	$\begin{bmatrix} 200{,}2 & 0 \\ 0 & 200{,}2 \end{bmatrix}$	0,1	UNSTABLE	1767
5600	$10^6\text{x}\begin{bmatrix} 0{,}4092 & 2{,}3518 \\ -2{,}3518 & 0{,}4092 \end{bmatrix}$	$\begin{bmatrix} 4004 & 0 \\ 0 & 4004 \end{bmatrix}$	2,0	Stable	-130
3200	$10^6\text{x}\begin{bmatrix} 0{,}8111 & 1{,}3896 \\ -1{,}3896 & 0{,}8111 \end{bmatrix}$	$\begin{bmatrix} 208{,}4 & 0 \\ 0 & 208{,}4 \end{bmatrix}$	0,1	UNSTABLE	298
3200	$10^6\text{x}\begin{bmatrix} 0{,}8111 & 1{,}3896 \\ -1{,}3896 & 0{,}8111 \end{bmatrix}$	$\begin{bmatrix} 4168 & 0 \\ 0 & 4168 \end{bmatrix}$	2,0	Stable	-193

The appropriate choice of the hydrodynamic journal bearing for a high-speed rotating machine must consider carefully the bearing dynamic coefficients. These coefficients play a crucial role on the stability of the system, as shown in the second example. The finite element procedure developed in this work can used to determine the stability bounds for rotating shafts supported on fluid film cylindrical journal bearings.

Gyroscopic Effect

Accuracy and reliability on the computation of natural frequencies for rotating systems depend on the gyroscopic moments. The gyroscopic matrix [G] is proportional to the rotor velocity, as shown in Eq. (1), in which the equivalent damping matrix also takes into account the support damping [C_1]. The gyroscopic moments increase as the shaft rotating speed increases, affecting strongly the equivalent system damping matrix [C].

The influence of the gyroscopic effects on the natural frequencies of rotating systems is shown using the baseline parameters given in Tab. 1. A circular disk, which can represent an impeller or a wheel, is attached to the shaft midpoint. The disk diameter is 0.2032m and its thickness is 16mm. The first five natural frequencies (N.F.) of the rotating system are predicted at a wide range of rotational speeds. The bearing direct stiffness coefficient is set equal to K_b=10,51x10^6 N/m, following the same assumption made by Lund (1974).

Figure 5 depicts the variation of the first five undamped natural frequencies (N.F.) in function of the rotating speed. Moreover, the first four rotor critical speeds (C.S.) can be obtained by intercepting the natural frequencies curves by a straight line representing the synchronous whirl motion, which is drawn as a dashed line on Fig. 5. The four critical speeds obtained for the undamped system are respectively 7545 rpm, 9215 rpm, 35515 rpm and 61360 rpm. These critical speeds are associated with the forward whirling shaft motion.

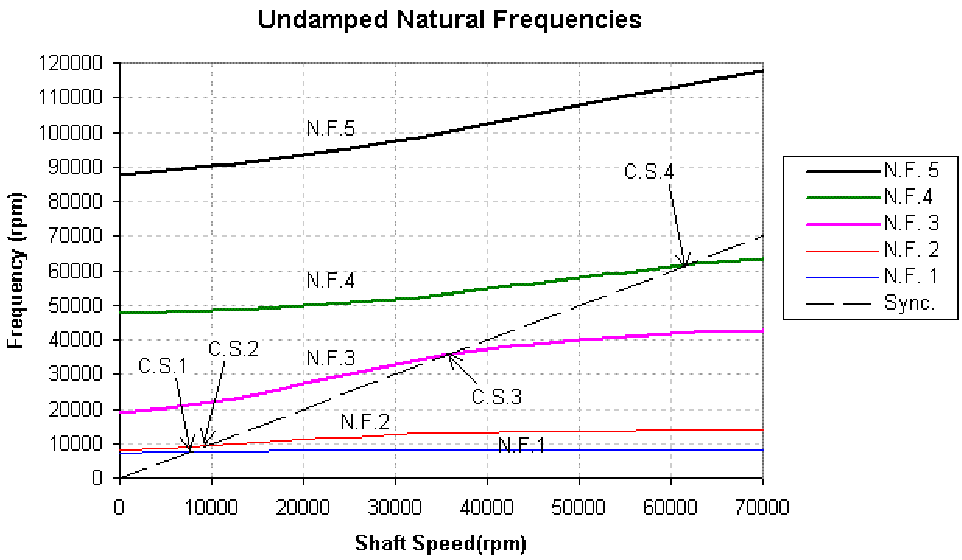

Figure 5 – Gyroscopic Effect – Undamped natural frequencies for 1 rpm to 70000 rpm shaft speed

Figure 6 shows the curves of damped natural frequencies versus rotating speed. The bearing damping coefficient is set equal to $C_{xx} = C_{yy} =$ 17512N.s/m. The critical speeds obtained for the damped system are, respectively, 6117 rpm, 8775 rpm, 31825 rpm and 58225 rpm.

CONCLUSIONS

The appropriate selection of a rotor supporting system is a fundamental step on the design and commissioning of industrial rotating machines. Dynamic force coefficients play a crucial role in the rotor capability to bear undesirable vibrations and to run under stable conditions. The results presented in this work show clearly the importance of selecting the more appropriate bearing configuration able to provide enough effective damping to bound the growth of the vibration response at critical operating conditions.

The finite element procedure has been implemented to analyze the stability of high speed turbomachines supported by hydrodynamic journal bearings. The numerical results presented in this work show that the computational procedure implemented for the eigenvalue analysis of damped gyroscopic systems is able to render reliable results, which are in good agreement with the results presented in the technical literature.

The finite element procedure can also be employed to evaluate design and operating changes in high-speed turbomachinery, in order to improve their dynamic response. From the parameters of the rotor-bearing system, its dynamic behavior can be studied and modified, for example, to avoid its operation near a critical speed, or to guarantee safe operation when traversing critical speeds.

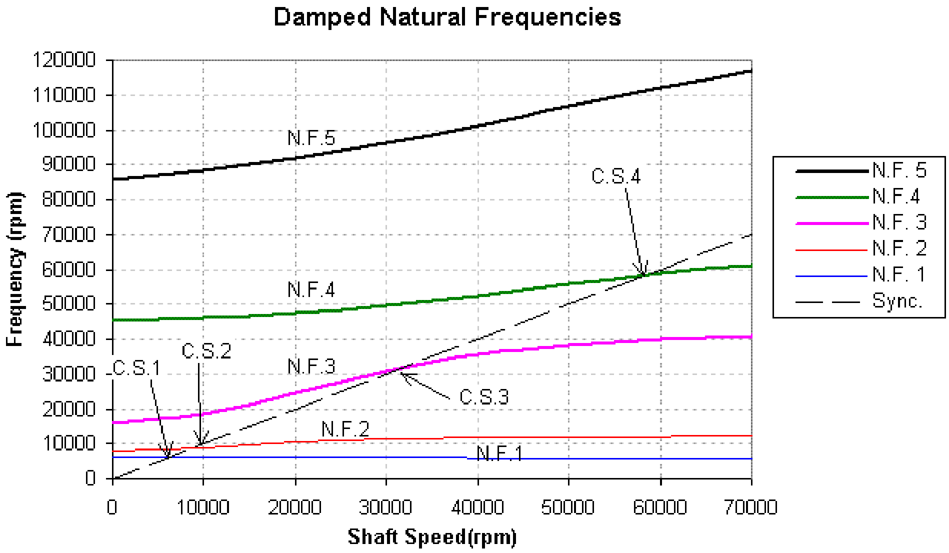

Figure 6 – Gyroscopic Effect – Damped natural frequencies for 1 rpm to 70000 rpm shaft speed

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