Proceedings of the XII International Symposium on Dynamic Problems of Mechanics (DINAME 2007),
P.S.Varoto and M.A.Trindade (editors), ABCM, Ilhabela, SP, Brazil, February 26 - March 2, 2007

Efficient Flutter Analyses Using CFD-Based Unsteady Aerodynamic Forces

Alexandre Noll Marques[1] **and João Luiz F. Azevedo**[2]

[1] Instituto Tecnológico de Aeronáutica, CTA/ITA, 12228-900 - São José dos Campos - SP - Brazil

[2] Instituto de Aeronáutica e Espaço, CTA/IAE/ASA-L, 12228-903 - São José dos Campos - SP - Brazil

Abstract: The present paper shows the development and applications of a methodology for efficient prediction of non-linear flutter instability onset. This methodology is based on a modified version of the indicial approach for the construction of the aerodynamic operator. This modified version employs smooth excitations, which are more suitable for use together with complex CFD solvers than the indicial or impulsive ones, and frequency domain analyses in order to predict the generalized aerodynamic forces in the Laplace domain. Hence, the aeroelastic problem of flutter prediction is reduced to a simple eigenvalue equation with the dynamic pressure as a parameter, for each Mach number. Moreover, the indicial approach is well known for being much more efficient when compared to the determination of aerodynamic forces in the frequency domain through harmonic oscillations. Initially, in the paper, the standard indicial approach is presented and the difficulties concerning its application to the CFD context are discussed. Afterwards, the alternative methodology is presented. Applications of the proposed methodology with a two-dimensional Euler solver for the aeroelastic analysis of typical sections are, then, discussed.

Keywords: Aeroelasticity, Finite Volume Technique, CFD, Discrete-Time, Indicial approach

INTRODUCTION

Aeroelasticity can be defined as the science which studies the mutual interaction between aerodynamic and dynamic forces. The analysis of dynamics characteristics of either complex or simple structures are quite developed nowadays as far as numerical and experimental methods are concerned. Hence, it is correct to state that reliability in aeroelastic calculations, for the problems of interest to the present authors, is strongly dependent on the correct evaluation of the aerodynamic operator.

Traditionally, the methods developed for determining the aerodynamic operator for subsonic and supersonic regimes are based on linearized formulations which do not present the same satisfactory results for the transonic range. According to Tijdeman (1977), this occurs due to the nonlinearity of transonic flows characterizing a significative alteration of the flow behavior, even when a profile is submitted to small perturbations. Along the last three decades, at least, there have been many attempts of numerically solving more elaborate aerodynamic models for the unsteady transonic regime. Beam and Warming (1974) presented one of the earliest papers on solving the unsteady Euler equations. They used an explicit, third-order, non centered, finite difference scheme to obtain indicial solutions for plunging flat plates and parabolic arc airfoils in the transonic regime. However, the boundary conditions were imposed in a small-disturbance fashion on a mean-surface approximation to the thick airfoil. This procedure was considered not to be satisfactory for more practical airfoils. This fact, together with memory and computational time limitations led to the development of numerical solvers based on the nonlinear transonic equations of the potential theory. Traci, Albano and Farr (1975) developed the codes STRANS2 and UTRANS2 to solve the transonic steady and unsteady equations by the relaxation method. The unsteady equation is time-linearized by treating the unsteady solution as a small linear harmonic perturbation about a nonlinear steady-state solution. This approach has the obvious drawback of not being able to represent solutions to indicial excitations, only to harmonic ones. Later, Ballhaus and Goorjian (1978) developed the code LTRANS2 to time-accurately solve the low-frequency transonic small-disturbance (TSD) formulation, which constitute a small-disturbance approximation of the transonic potential equations. This formulation is able to capture small shock motions. Yang, Guruswamy and Striz (1982) present a compilation of the results obtained with the application of these and others transonic codes to different cases of aeroelastic analyses.

Ashley (1980) reported the use of semi-empirical corrections to the linearized theory results as a mean of improving flutter predictions. Nevertheless, Ashley (1980) himself believed that really satisfactory aeroelastic quantitative predictions of the transonic regime should be possible only when accurate, three-dimensional, unsteady CFD codes were developed. Hence, the methodology here presented intends to obtain the aerodynamic operator for two-dimensional lifting surfaces employing modern CFD techniques.

Computational Fluid Dynamics (CFD) is a subject that has played an extremely important role in recent aerodynamics studies. The possibility of treating numerically a broad range of phenomena which occur in flows over bodies of practically any geometry has innumerous advantages over experimental determinations, such as greater flexibility together with time and financial resources savings. However, obtaining more reliable numerical results for a growing number of situations has been one of the major recent challenges in many science fields. Hirsch (1994) shows that particularly in aerodynamics, the general phenomena are governed by the Navier-Stokes equations, which constitute a system of coupled nonlinear partial differential equations that has no general analytical solution and that is of difficult algebraic manipulation. He also comments, among other issues concerning CFD techniques, on how to simplify the mathematical models conveniently in order to ease the numerical treatment of each case. Space and time discretization schemes, as well as convergence acceleration techniques, boundary condition establishment and other numerical integration tools are available and largely used in order to solve such models.

The development of the present CFD tool is a result of the increased demand for aerodynamic parameters that followed the evolution of the work and projects performed by CTA/IAE. Nevertheless, the application of CFD tools in these parametric analyses has always been limited by the need of adequate code development and the lack of computational resources compatible with the work to be performed. Therefore, a progressive approach has been adopted in the development of CFD tools in CTA/IAE and in ITA, as presented by Azevedo (1990), Oliveira (1993), Azevedo, Fico and Ortega(1995), Azevedo, Strauss and Ferrari (1997), Bigarelli, Mello and Azevedo (1999), Simões and Azevedo (1999), Bigarelli and Azevedo (2002), and Marques (2004).

Nevertheless, the use of indicial excitations with more complex CFD solvers has led to some misinterpretations in the beginning of the 90's. Time-marching schemes posses the ability to solve perturbations which advance through the flowfield with velocities up to some determinate limit. This limit depends on the scheme itself and mesh resolution (Hirsch, 1994). Consequently, care must be taken when evaluating indicial responses with CFD codes, once indicial excitations usually provoque great perturbation velocities. This is very well illustrated by Beam and Warming (1974) and Ballhaus and Goorjian (1978) for simpler codes, and more recently by Raveh (2001) for more sophisticated and modern solvers. But, despite of the difficulties of dealing with these velocities, they are finite in magnitude, opposite to what Bakhle et al. (1991) have affirmed. This misinterpretation is easily understood in light of the usual definition of the indicial function, or step function, in which a discontinuous change of the function value causes an infinite derivative, and consequently should lead to infinite velocities. Actually, the numerical implementation of such discontinuous functions can be quite controversial and some authors even stated it not to be feasible (Oliveira, 1993). However, recently Silva (1997) showed that, once the flow governing equations are discretized, the resultant system can no longer be viewed as a continuous-time one, but should be treated as a discrete-time system. Furthermore, linear discrete-time systems present properties that are very similar to those of linear continuous-time systems and, more importantly, the indicial method is valid (Oppenheim and Schafer, 1989). As will be exposed later, the indicial, or step, sequence definition for discrete systems is quite different from the continuous step function. Namely, it does not contain any singularity and its numerical implementation is straightforward. In fact, although it is a well-defined sequence for discrete-time systems and is by no means a mere approximation of the continuous step function, the discrete step sequence has been used as an approximation for the step excitation in many numerical evaluations along the years (Beam and Warming, 1974, Ballhaus and Goorjian, 1978). Hence, the new interpretation proposed by Silva (1997) comes to validate and reaffirm as a rigorous procedure what was thought an approximation until recently.

The difficulties of treating the high velocities induced by the indicial excitation can be overcome by the use of smoother excitations (Davies and Salmond, 1980, Bakhle et al., 1991, Oliveira, 1993). Although such smooth excitations do not posses all the characteristics of the indicial one neither are capable of exciting uniformly the entire frequency domain, they can be calibrated in order to excite certain frequency bands of interest for aeroelastic purposes. The convolution sum (Oppenheim and Schafer, 1989, Marques and Azevedo, 2006) establishes a relationship between the desired indicial response and the actually obtained with the smooth excitations for the frequency range of interest. Nevertheless, after appropriately defining the aerodynamic input function, Silva (1997) states that this approach is not valid and that it has presented reasonable results because the smooth sequences excite only low frequencies and this led to small errors.

The authors believe that they are finally able to present an unified view of the different approaches for obtaining indicial aerodynamic responses through the application of modern CFD solvers. In fact, this unified view should demonstrate that these different approaches are rigorously equivalent and, when correctly implemented, generate identical results within the numerical accuracy expected from any numerical methodology. Actually, the present work is based on the finite volume formulation, where a CFD tool is applied with two-dimensional unstructured meshes around lifting surfaces to acquire unsteady responses to harmonic, smooth pulse, discrete step and unit sample motions. The unit sample sequence will be defined latter as the discrete-time sequence equivalent to the continuous-time impulse function. Although this investigation is conducted with a particular CFD solver, it is believed that the results presented here are representative of most numerical schemes used in modern CFD solvers.

Furthermore, it is also the objective of the present work to perform aeroelastic stability analysis, or more specifically, to predict nonlinear flutter instability onset. For this purpose, the simple linear structural model of the typical section is employed. The fundamental time domain aerodynamic responses supply the generalized aerodynamic forces necessary as input to the aeroelastic model. The methodology here presented intends to efficiently obtain frequency domain aerodynamic responses from the solutions of the unit sample motion with a single expensive CFD run per structural mode, in contrast with obtaining several aerodynamic results for different harmonic oscillations. Finally, with that information, it is possible to determine the aeroelastic stability margin.

AEROELASTIC FORMULATION

The test case considered in the present work is widely known and reported in the literature (Bisplinghoff, Ashley and Halfman, 1955, Oliveira, 1993). The dynamic system represented in the typical section is a rigid airfoil section with two degrees of freedom, plunging and pitching, subjected to aerodynamic and elastic forces and moments. The governing equation of such dynamical system is given by

$$[M]\{\ddot{\eta}(t)\} + [K]\{\eta(t)\} = \{Qa(t)\}, \tag{1}$$

where the generalized mass and stiffness matrices are, respectively, given by

$$[M] = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}, \qquad [K] = \begin{bmatrix} \omega_h & 0 \\ 0 & r_\alpha^2 \omega_\alpha^2 \end{bmatrix}, \tag{2}$$

and the generalized coordinate and force vectors are, respectively,

$$\{\eta(t)\} = \left\{ \begin{array}{cc} \xi(t) & \alpha(t) \end{array} \right\}^T, \qquad \{Qa(t)\} = \left\{ \begin{array}{cc} \frac{Qa_h(t)}{mb} & \frac{Qa_\alpha(t)}{mb^2} \end{array} \right\}^T, \tag{3}$$

where $\xi = h/b$ is the plunge mode coordinate and α is the pitch mode one. In the previous equations, ω_h and ω_α are the free vibration circular frequencies of each mode, which are defined as

$$\omega_h = \sqrt{\frac{k_h}{m}}, \qquad \omega_\alpha = \sqrt{\frac{k_\alpha}{I_\alpha}}. \tag{4}$$

Moreover, I_α denotes the section moment of inertia with respect to the elastic axis over which pitching takes place. Finally, the radius of gyration is given by

$$r_\alpha = \sqrt{\frac{I_\alpha}{mb^2}}. \tag{5}$$

Considering the methodology proposed in this paper, this system can be more easily studied in the Laplace domain. Applying the Laplace transform to Eq. (1), one can obtain

$$s^2[M]\{\eta(s)\} + [K]\{\eta(s)\} = \{Qa(s)\}. \tag{6}$$

Therefore, in order to complete the aeroelastic system represented by Eq. (6), it is necessary to determine the generalized aerodynamic force vector $\{Qa(s)\}$ in the Laplace domain for an arbitrary structural behavior. As will be exposed later, this can be performed by evaluating this vector over the frequency range of interest and, by making use of the analytical continuation principle (Churchill, 1974), extending such result to the entire s-plane. As presented by Oliveira (1993), assuming linearity with regard to the modal motion, one can write

$$\{Qa(s)\} = \frac{U_\infty^2}{\pi \mu b^2}[A(s)]\{\eta(s)\}. \tag{7}$$

It is also common to write $\{Qa(s)\}$ as a function of the dimensionless velocity U^* or the dimensionless dynamic pressure Q^*, which are defined as

$$U^* = \frac{U_\infty}{b\omega_\alpha}, \qquad Q^* = \frac{U_\infty}{b^2\omega_\alpha^2\mu}. \tag{8}$$

The aerodynamic influence coefficients matrix, $[A(s)]$, is given by

$$[A(s)] = \begin{bmatrix} -Cl_h(s)/2 & -Cl_\alpha(s) \\ Cm_h(s) & 2Cm_\alpha(s) \end{bmatrix}, \tag{9}$$

where the α and h subscripts indicate the pitch and plunge mode contributions, respectively.

CONTINUOUS-TIME INDICIAL METHOD

Linear time-invariant continuous-time systems present the very interesting property of possessing elementar solutions which are representative of the entire frequency content of the corresponding problem. The most elementar solution is the impulse one. This is evident from the fact that every other continuous input can be seen as the integration of successive time-shifted and scaled impulse excitations. Hence, as a consequence of the superposition principle, the corresponding result is the integration of the responses of each of these impulses scaled by the input magnitude, leading to the convolution integral of the impulse response with the given input. In a frequency domain point of view, the Fourier transform of an impulse function is equal to unit over the entire domain. This means that the impulse function uniformly excites the entire frequency domain.

Similarly, the step, or indicial response is another elementar response. Ballhaus and Goorjian (1978) develop a graphical and intuitive demonstration of how a series of time-shifted and scaled step functions are equivalent to any continuous function. Therefore, just as reasoned before, the convolution integral of the indicial response, but at this time with the input's derivative function, is equal to the system response to that particular excitation. This convolution integral is called the Duhamel integral.

It is also important to notice that a simple mathematical manipulation shows that the derivative of the indicial response is equivalent to the impulse response. Therefore, from the convolution theorem for the Fourier transform (Brigham, 1988) and Eq. (7), it is clear that the aerodynamic influence coefficient matrix in the Laplace domain, $[A(s)]$, is given by the Laplace transform of the impulse response. Therefore, the widely known indicial approach consists of obtaining the aerodynamic response to a step change of each structural mode. The Laplace transform of the time derivative of such responses constitute each term of $[A(s)]$. Naturally, this can be equivalently done with the aerodynamic impulse responses.

Nevertheless, the impulse and step functions present discontinuities which cannot be implemented in numerical codes. The matter of obtaining such elementar responses with CFD solvers is discussed in the next section.

CFD SOLVER AS A DISCRETE-TIME SYSTEM

As aeronautical researchers are generally used to deal with continuous-time problems, it has been very common in the literature to look at CFD solvers as mere approximations to continuous-time systems. Therefore it is equally common to use continuous-time system properties and thinking when performing CFD simulations (Beam and Warming, 1974, Ballhaus and Goorjian, 1978, Davies and Salmond, 1980, Mohr, Batina and Yang, 1989, Bakhle et al. 1991, Oliveira, 1993, Silva, 1993a, Silva, 1993b). They are approximations, indeed, but discrete-time approximations. Hence, as shown by Silva (1997), once the governing equations have been discretized, the resulting numerical scheme is actually a discrete-time system which has its own properties and peculiarities.

This mistake has led many authors (Davies and Salmond, 1980, Mohr, Batina and Yang, 1989, Bakhle et al. 1991, Oliveira, 1993) to justify the use of smooth pulse excitations, since the theoretical continuous-time impulse and indicial excitations are not numerically feasible. Nevertheless, Silva (1997) has suggested the use of equivalent discrete-time excitations: unit sample and discrete step (Oppenheim and Schafer, 1989). The unit sample sequence is given by

$$\delta[n] = \begin{cases} 0, & n \neq 0, \\ 1, & n = 0, \end{cases} \tag{10}$$

and the discrete step sequence is

$$u[n] = \begin{cases} 0, & n \geq 0, \\ 1, & n < 0, \end{cases} \tag{11}$$

Such sequences formally hold properties very similar to the ones attributed to the impulse and step functions. Namely, if a linear time-invariant discrete system is submitted to a unit sample excitation, then the correspondent response will contain all the information about the system and the response to every other input is given by the convolution sum (Oppenheim and Schafer, 1989).

The discrete step response also characterizes a discrete system since one can reproduce the unit sample response from it (Marques and Azevedo, 2006). Thus, theoretically, it would be more convenient, and even computationally cheaper (the transient solution should die out more rapidly), to acquire CFD results submitting the body to either unit sample or discrete step type perturbations. As mentioned earlier, this has been accomplished by some authors (Beam and Warming, 1974, Ballhaus and Goorjian, 1978, Silva, 1997, Silva, 2001, and Raveh, 2001), but some numerical complications may arise when simulating such cases. Raveh (2001), particularly, has performed a through inspection of these responses and concluded that numerical errors occurs due to the large velocities induced by sharp motions, which may exceed the velocity the numerical time marching scheme can capture. Naturally, it all depends on the numerical formulation used in the CFD solver, input amplitude and time step. Additionally, Raveh (2001) states that simulations carried out with the discrete step excitation tend to be less sensitive to the simulation parameters than those using the unit sample.

Another question raised by Silva (1993b, 1997) concerns the correct definition of the aerodynamic input function. He proposes that this should be the downwash function, which, for the excitation of a given mode is written as (Silva, 1997)

$$w(x,y,t) = \phi'(x,y)\sigma(t) + \phi(x,y)\dot{\sigma}(t), \tag{12}$$

where $\phi(x,y)$ is the modeshape, $\phi'(x,y)$ is the slope of the modeshape, $\sigma(t)$ is the generalized coordinate, and $\dot{\sigma}(t)$ is its time derivative. Therefore, according to Silva (1997), the input excitation is actually a two-channel input, i.e., each term is a separate input channel. This means that each channel should be excited individually using a unit sample or discrete step input and, for the linear case, the final solution would come from the superposition of both answers. This argument ruins use the smooth excitation which is based on the single-input premise. This is because it is not possible to employ a smooth input function without exciting the derivative term. Actually, the procedure could eventually work when the input function used as σ is smooth enough so that its derivative is small. In such cases, neglecting the influence of the second term does not lead to relevant errors. But this is not true when the slopes of the modeshapes are zero, as in the case of the plunge mode in the typical section model. In that case, Silva (1997) argues that the mentioned procedure would be totally incorrect.

Fortunately, the present authors have shown in previous work (Marques, 2004, Marques and Azevedo, 2006) good results obtained applying the proposed methodology, even for the plunge mode. The explanation for such contradictory results lies on the fact that Silva's point of view about the correct input excitation is not entirely true. One cannot separate the effects one function and its derivative. In the frequency domain, it is easily seen through the Laplace transform of Eq. (12),

$$W(x,y,s) = \phi'(x,y)\sigma(s) + s\phi(x,y)\sigma(s) = (\phi'(x,y) + s\phi(x,y))\sigma(s). \tag{13}$$

Thus, the system depends only on the input σ. Hence, although the system can be viewed to present multiple-channel inputs, the interpretation of σ as a single-input is equivalent. In the "Results and Discussion" section of this paper some test-cases are presented proving this reasoning to be correct.

AERODYNAMIC SOLVER

The CFD tool applied in this work is based on the Euler equations for the two-dimensional case. The space discretization is based on a cell-centered finite volume scheme, in which the stored information is actually the variable average value throughout the entire control volume. The numerical solution is advanced in time using a second-order accurate, 5-stage, explicit, hybrid scheme which evolved from the consideration of Runge-Kutta time stepping schemes (Jameson, Schmidt and Turkel, 1981, Mavriplis, 1990). Batina (1989) presents this same scheme including the necessary terms to account for changes in cell area due to mesh motion or deformation.

Steady-state solutions for the mean flight condition of interest must be obtained before the unsteady calculation can be started. Therefore, it is also important to guarantee an acceptable efficiency for the code in steady-state mode. In the present work, both local time stepping and implicit residual smoothing (Jameson and Mavriplis, 1986, Jameson and Baker, 1983, Jameson and Baker, 1987) are employed to accelerate convergence to steady state.

The meshes used in the present work were generated with the commercial grid generator ICEM CFD©, a very powerful tool capable of creating sophisticated meshes with very good refinement and grid quality control. Unsteady calculations involve body motion and, therefore, the computational mesh should be somehow adjusted to take this motion into account. Two approaches were adopted. The first one is to keep the far field boundary fixed and to move the interior grid points in order to accommodate the prescribed body motion. This was done following the ideas presented by Batina (1989), and Rausch, Batina and Yang (1990), which assume that each side of the triangle is modeled as a spring with stiffness constant proportional to the length of the side. Hence, once points on the body surface have been moved and assuming that the far field boundary is fixed, a set of static equilibrium equations can be solved for the position of the interior nodes. The other way of accounting for the motion of the body was to rigidly move the mesh accordingly. Of course this last approach is less general since it can only be used when the structural modes involve rigid body motions, as in the typical section case. However, this was done in order to show the sensitivity of the solutions to the mesh deformation, as will be shown later. More details on the aerodynamic formulation, numerical schemes, convergence acceleration techniques and meshes are presented by Oliveira (1993) and Marques (2004).

AEROELASTIC ANALYSIS METHODOLOGY

The unsteady motions related to the aeroelastic phenomena, mainly flutter, can be represented by a series of harmonic motions. Therefore, the construction of the aerodynamic operator results from the evaluation of aerodynamic responses to harmonic excitations of various frequencies. However, instead of performing many expensive computational simulations for different frequencies, a large computational cost reduction can be obtained with the use of the indicial method, as previously presented. Therefore, the aerodynamic calculations for a determined flight condition are reduced to a single computational run for each structural mode. Moreover, the only hypothesis adopted is that the aerodynamic generalized forces are linear with regard to the displacement modes and amplitudes, which guarantees that this methodology captures the flow nonlinearities and dynamics according to the aerodynamic model applied. These hypothesis are very reasonable when small amplitude perturbations are employed, as will be shown next. As the Euler formulation is used in the present work, one cannot expect for nonlinearities related to viscous effects.

Nevertheless, as mentioned before, the application of the impulse or indicial inputs may lead to numerical problems. Hence, other smoother excitation functions have been employed (Davies and Salmon, 1980, Mohr, Batina and Yang, 1989) in order to overcome such numerical problems. The motion used here is an exponentially-shaped pulse suggested by Bakhle et al (1991).

As the exponentially-shaped input is not an unit sample excitation, the real unit sample response is evaluated using a well-known property of the convolution theorem (Oppenheim and Schafer, 1989),

$$g[n] = f_p[n] * i[n] \rightarrow G[n] = F_p[n]I[n], \tag{14}$$

$$I[n] = \frac{G[n]}{F_p[n]}, \tag{15}$$

where $i[n]$ represents the time response to a unit sample movement and $g[n]$ is the response to the sampled exponentially-shaped excitation resultant from sampling the smooth function employed. The sequences in capital letters are discrete Fourier transforms of the corresponding sequences in lower case letters. Therefore, after obtaining the fast Fourier transform (FFT) of the time responses, it has to be divided by the FFT of the input sequence in order to obtain frequency domain responses. Although the input is not the exact unit sample excitation, it is capable of exciting the reduced frequencies of interest in aeroelastic studies. The corresponding frequency domain points resulting from this procedure are presented by Oliveira (1993).

The frequency domain responses obtained by these steps consist in a set of numerical values, which are not convenient for the solution of Eq. (6). Therefore, it is necessary to approximate these data using interpolating polynomials, as previously stated. As proposed by Oliveira (1993) and Abel (1979), this polynomial, already in the Laplace domain, is given by

$$[A(s)] = [A_0] + [A_1]\left(\frac{b}{U_\infty}\right)s + [A_2]\left(\frac{b}{U_\infty}\right)^2 s^2 + \sum_{m=3}^{n} \frac{[A_m]\,s}{s + \frac{U_\infty}{b}\beta_{m-2}}, \tag{16}$$

where β_m's introduce the aerodynamic lags with respect to the structural modes, and they are arbitrarily selected from the range of reduced frequencies of interest. Moreover, $[A_m]$ are the approximating coefficient matrices given by a least squares optimization method, where $s = ikU_\infty/b$. Oliveira (1993) and Abel (1979) present different ways of evaluating these matrices based on the same optimization principle. Both ways result in the same interpolation polynomials when the same poles are used. There also are other suggestions of approximating polynomials, such as the approach given by Eversman and Tewari (1991), but they have not been tested by the present authors yet. By representing the aerodynamic behavior with the approximating polynomials, Eq. (6) results in a classical eigenvalue problem, which is presented by Oliveira (1993).

The methodology consists, then, in obtaining a transfer function in the frequency domain applicable to any desirable input. This transfer function is the frequency domain response to the unit sample excitation. Therefore, this is accomplished using the following steps:

- Obtain the steady-state aerodynamic solution for a given Mach number and angle of attack;
- Perform unsteady aerodynamic response evaluations with an exponentially-shaped pulse departing from the steady solution given in the previous item. This stage leads to time responses in terms of aerodynamic coefficients as a result to such smooth excitations of each of the modes;
- Obtain the discrete Fourier transform of both the time responses and the exponentially-shaped pulse applying a fast Fourier transform algorithm. This is done in the present work employing the FFT capability available in the commercial program Matlab©. The discrete Fourier transform of the corresponding unit sample response results from the term-by-term division of the two previous FFTs;
- Approximate the obtained data with an interpolating polynomial;
- Formulate of the corresponding eigenvalue problem, valid for a determined range of dimensionless velocities, and, finally, perform flutter velocity prediction through a root locus analysis.

RESULTS AND DISCUSSION

Aerodynamic Responses

Before attempting applications of the proposed methodology, some validation simulations were performed with the CFD tool. This has been done throughout the entire development of this code as can be seen in the work of Azevedo (1992), Oliveira (1993), Simões and Azevedo (1999), and Marques and Azevedo (2006). Once the CFD tool was tested and proved to be a reliable one, the next step was to proceed in obtaining the unsteady responses of interest. The approach selected was to reproduce the frequency domain results presented by Rausch, Batina and Yang (1990) for a NACA 0012 airfoil at freestream Mach number $M_\infty = 0.8$ and zero degree angle of attack. Rausch, Batina and Yang (1990) obtained the present data with a the smooth excitation approach and a numerical solver for the Euler equations very similar to the one employed by the present authors. However, they do not mention any further details on the excitation used.

In order to demonstrate what is exposed in the previous sections of this paper, solutions for this same problem were obtained with the harmonic (H), exponentially-shaped pulse (EP), discrete step (Step), and unit sample (US) excitations.

Naturally, differently from the other cases, for the harmonic oscillations, one CFD run is required for each reduced frequency. Figures 1 and 2 show a comparison among such results and the literature data. Note that all the aerodynamic moment coefficients are evaluated with respect to the quarter-chord point of the airfoil. It can be directly verified that the three different approaches for representing the aerodynamic elementar response are completely equivalent. More importantly, the responses obtained with the modified indicial approach are very close to the ones corresponding to harmonic oscillations. This fact corroborates the linearity of the aerodynamic behavior regarding small disturbances even when the governing equations are completely nonlinear, as the Euler equations are. Furthermore, the solutions presented by the present authors agree very well with those given by Rausch, Batina and Yang (1990).

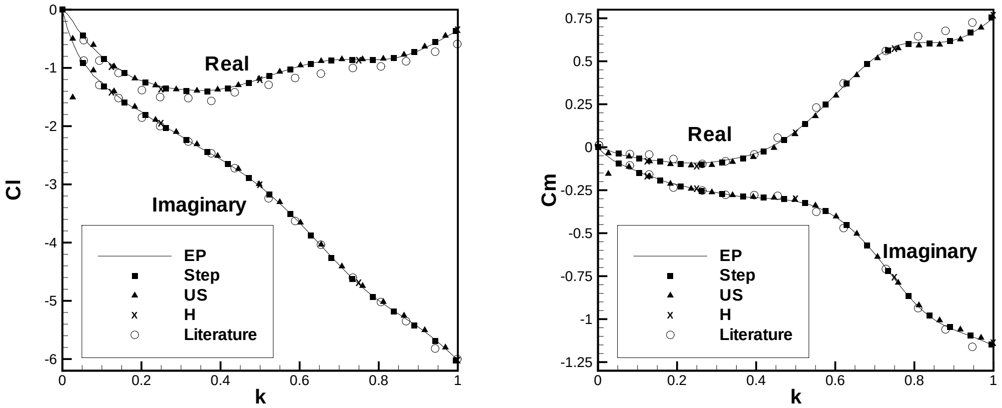

Figure 1 – Frequency domain Cl and Cm responses due to plunging excitation.

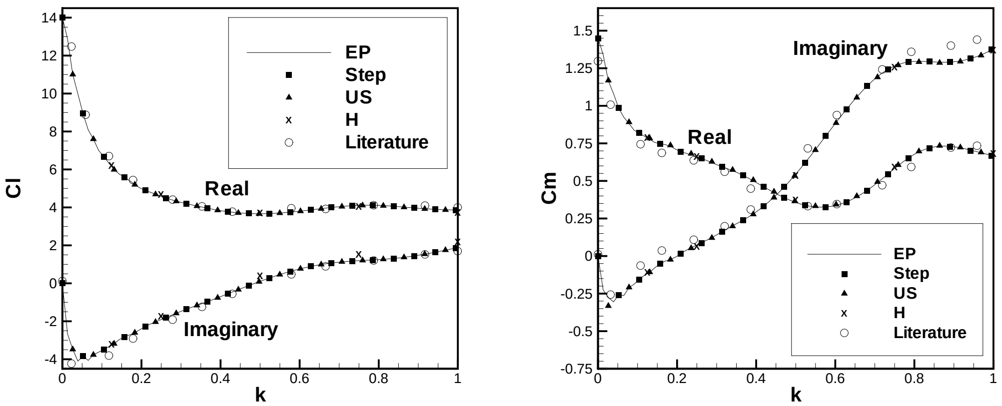

Figure 2 – Frequency domain Cl and Cm responses due to pitching excitation.

As mentioned before, there are many aspects which may lead to numerical errors in the solutions of the discrete step and unit sample cases, such as input amplitude, time step, mesh resolution and numerical scheme. The authors have studied the influence of some of these parameters, but, due to space limitations, this paper only presents the influence of another factor the authors do not know any references of in the literature. This parameter is mesh deformation. The results results presented in Figs. 1 and 2 were obtained with an algorithm that moves the mesh rigidly together with the body. The authors also applied the same solver with a dynamic mesh algorithm that deforms the mesh to account for the body motion. Figure 3 compares the results for the rigid-mesh (RM) and deformable-mesh (DM) solvers for the calculations of aerodynamic responses for the plunge mode excitation. As one can see, the deformable-mesh solutions do not agree as the rigid-mesh ones, although the smooth solutions are identical in both cases. Therefore, the mesh deformation affects mainly the very low reduced frequencies for sharp excitations. This can be explained by the steady-state solution fluctuations due to a mesh deformation. These fluctuations overlap the real transient solution and alter the low frequency contend of the answer. It is also important to note that the input amplitude limitations imposed by the discrete step and unit sample excitations force very small inputs. Therefore, even these extremely small fluctuations created by the mesh deformation become significative when divided by the input amplitude. This effect is more accentuated in the plunge mode because the very low frequency answer should be zero, but it also occurs in the pitch mode. Furthermore, since higher amplitude smooth excitations can be used, these fluctuation effects become negligible. Hence this modified indicial approach has shown to be more robust for deforming meshes than the classical ones, which is very important since rigid body motions are not significative to most aeroelastic problems.

Aeroelastic Analysis

After obtaining the frequency domain aerodynamic results given in last subsection, the authors proceeded with the aeroelastic analysis by interpolating them with the use of 15 poles distributed in the reduced frequency range up to

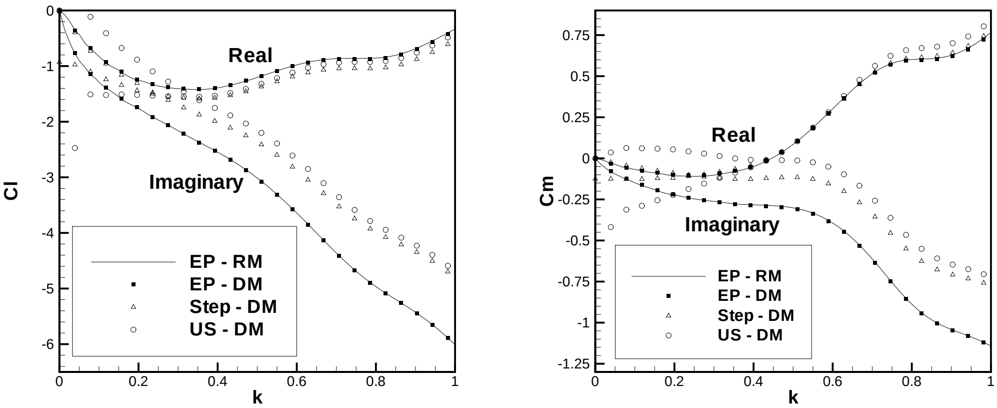

Figure 3 – Comparisons, between rigid-mesh (RM) and deformable-mesh (DM) algorithms, of frequency domain Cl and Cm responses due to plunging excitation.

0.40. The corresponding curves are shown in Figs. 4 and 5. The polynomial interpolation process has shown to provide excellent results. Additionally, one may notice that, although the reduced frequency range where the poles were allocated does not cover the entire range of approximated points, the resulting polynomial still holds as a close approximation. This occurs because all points are used in the least squares optimization. Therefore, one may concentrate the poles in the most convenient regions without compromising the overall result, just as was done in this case.

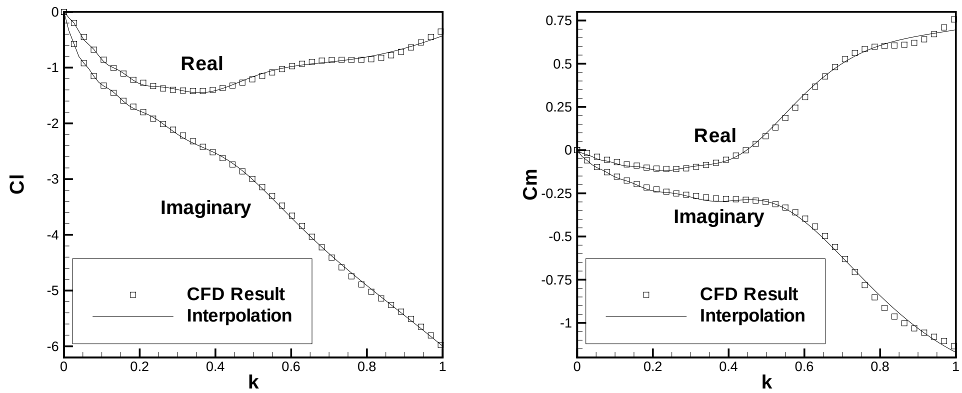

Figure 4 – Approximating polynomials for the frequency domain Cl and Cm responses due to plunging excitation.

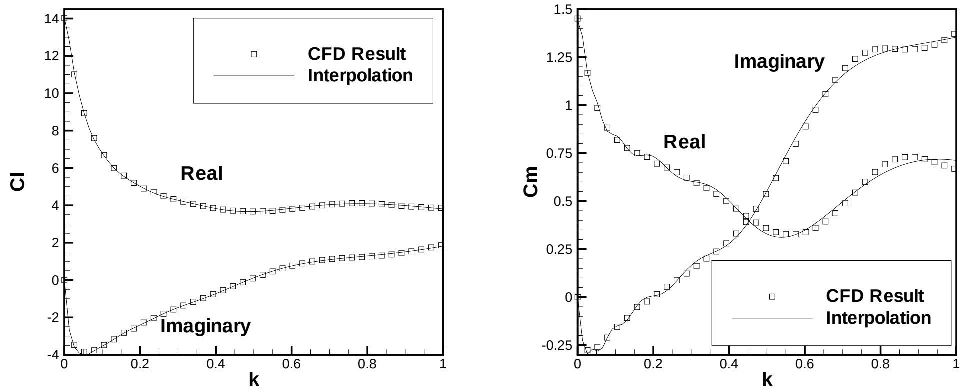

Figure 5 – Approximating polynomials for the frequency domain Cl and Cm responses due to pitching excitation.

Finally, with these results, it was possible to solve an aeroelastic problem submitted to a transonic flow. The resulting root loci are presented in Fig. 6 with the dimensionless dynamic pressure Q^* as parameter. The points of the present results differ from each other in $Q^* = 0.1$. Data found in the literature (Rausch, Batina and Yang, 1990) are also included. The agreement between the aeroelastic stability analyses is very good, at least for the plunge mode. A larger difference is seen in the pitch mode. However, it could be expected due to the differences presented in Fig. 2, specially in the moment coefficient, i.e., the pitch mode generalized force due to a pitching excitation. Furthermore, the flutter points are indicated in Table 1. Such values reinforce the solution agreement. It is important to notice the difference between flutter points indicated with the linear and Euler formulations. This is a typical example of the well-known "transonic dip" phenomenon, where the flutter condition is unsafely over-predicted by the linear theory. The ability to correctly predict the flutter condition within the transonic dip range of Mach numbers is the major motivation for the development of the present methodology.

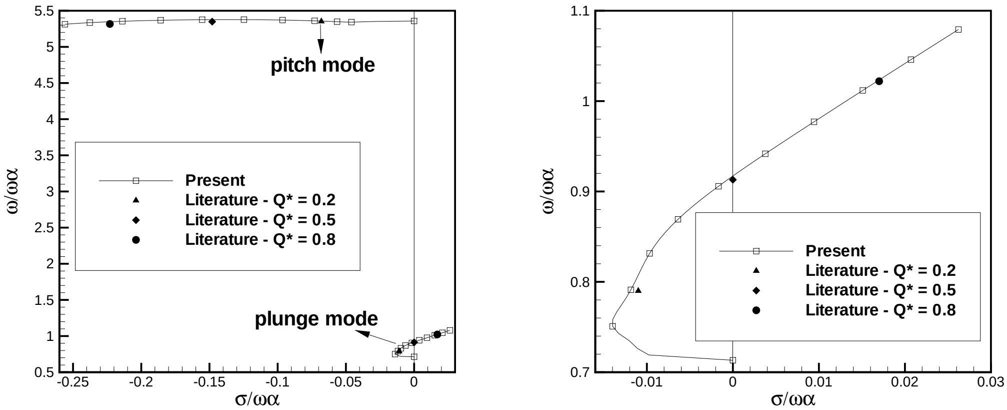

Figure 6 – Comparisons of dimensionless dynamic pressure root loci for the aeroelastic system. In the right, a zoom of the plunge mode root locus.

Table 1 – Comparisons of flutter points for the aeroelastic system.

REFERENCE	MODE	U^*	Q^*	$\omega/\omega\alpha$
Present	plunge	5.64	0.53	0.92
Literature - CFL3D (Euler)	plunge	5.37	0.48	not informed
Literature (Linear)	plunge	10.65	1.89	not informed

CONCLUDING REMARKS

The paper has shown successful aeroelastic analysis results obtained using the proposed indicial methodology with smooth excitations. The results are a demonstration of the appropriateness of the proposed formulation and of the correct implementation of the entire methodology. Furthermore, the use of smooth excitations has proved to be more robust and effective for general aeroelastic applications than the indicial and unit sample sequences. Hence, the present development provides the required capabilities to efficiently study aeroelastic stability problems using modern CFD tools. This is especially important for transonic cases, in which the linear theory may overpredict the flutter velocity. Therefore, this work represents a fundamental evolution in the capability of performing numerical aeroelastic studies at CTA/IAE.

Moreover, the authors expect to have made contributions towards the solution of some of the current theoretical questions concerning this sort of approach in CFD-based analyses. The authors demonstrated an unified view of the indicial approach for acquiring unsteady aerodynamic results through CFD solvers. This new vision is a consequence of the correct interpretation of CFD solvers as discrete-time systems. An important result from that is the elimination of the need for a two-channel input formulation suggested in the literature. Furthermore, practical results corroborate these theoretical developments. It is also important to emphasize that putting such questions to rest is very relevant in many areas, for instance, in the development of reduced-order models for aeroservoelastic control laws.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the partial support for this research provided by Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, under the Integrated Project Research Grant No. 501200/2003-7. The first author was also supported by a FAPESP MS scholarship under Grant No. 04/11200-1.

REFERENCES

Ashley, H., 1980, “Role of Shocks in the “Sub-Transonic” Flutter Phenomenon,” *Journal of Aircraft*, Vol. 17, No. 3, pp. 187–197.

Azevedo, J.L.F., 1990, “Euler Solutions of Transonic Nozzle Flows,” Proceedings of the 3rd National Meeting on Thermal Sciences – III ENCIT, Itapema, SC, Brazil, pp. 243–248.

Azevedo, J.L.F., Fico, N.G.C.R., Jr. and Ortega, M.A., 1995, “Two-Dimensional and Axisymmetric Nozzle Flow Computation Using the Euler Equations,” *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 17, No. 2, pp. 147–170.

Azevedo, J.L.F., Strauss, D. and Ferrari, M.A.S., 1997, “Viscous Multiblock Simulations of Axisymmetric Launch Vehicle Flows,” AIAA Paper No. 97-2300, 15th AIAA Applied Aerodynamics Conference, Atlanta, GA.

Azevedo, J.L.F., 1992, "On the Development of Unstructured Grid Finite Volume Solvers for High Speed Flows," Report NT-075-ASE-N/92, Instituto de Aeronáutica e Espaço, São José dos Campos, SP (also available as Report No. 91/9, Institut für Strömungsmechanik, T.U. Braunschweig, F.R. Germany, Dec. 1992).

Bakhle, M.A., Mahjan, A.J., Keith, T.G., Jr. and Stefko, G.L., 1991, "Cascade Flutter Analysis with Transient Response Aerodynamics," *Computers and Structures*, Vol. 41, No. 5, pp. 1073–1085.

Ballhaus, W.F. and Goorjian, P.M., 1978, "Computation of Unsteady Transonic Flows by the Indicial Method," AIAA Journal, Vol. 16, pp. 117–124.

Batina, J.T., 1989, "Unsteady Euler Airfoil Solutions Using Unstructured Dynamic Meshes," AIAA Paper 89-0115, 27th Aerospace Sciences Meeting, Reno, NV.

Beam, R.M. and Warming, R.F., 1974, "Numerical Calculations of Two-Dimensional, Unsteady Transonic Flows with Circulation," NASA TN D-7605.

Bigarella, E.D.V. and Azevedo, J.L.F., 2005, "A Study of Convective Flux Computation Schemes for Aerodynamic Flows," AIAA Paper No. 2005-0633, 43rd AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV.

Bigarelli, E.D.V., Mello, O.A.F. and Azevedo, J.L.F., 1999, "Three Dimensional Flow Simulations for Typical Launch Vehicles at Angle of Attack," Proceedings of the 15th Brazilian Congress on Mechanical Engineering – COBEM 1999, Águas de Lindóia, SP.

Bigarelli, E.D.V. and Azevedo, J.L.F., 2002, "On Turbulence Models for 3-D Aerospace Applications," Proceedings of the 9th Brazilian Congress of Thermal Engineering and Sciences, Paper CIT02-0341, Caxambu, MG.

Bisplinghoff, H.L., Ashley, H. and Halfman, R.L., 1955, *Aeroelasticity*, Addison-Wesley, Cambridge, MA.

Brigham, E.O., 1988, *The Fast Fourier Transform and its Applications*, Prentice Hall, Englewood Cliffs, New Jersey.

Churchill R.V., Brown J.W. and Verhey, R.F., 1974, *Complex Variables and Applications*, 3rd ed., McGraw-Hill, New York, NY.

Davies, D. and Salmond, D.J., 1980, "Indicial Approach to Harmonic Perturbations in Transonic Flow," *AIAA Journal*, Vol. 18, No. 8, pp. 1012–1014.

Hirsch, C., 1988, *Numerical Computation of Internal and External Flows - Fundamental of Numerical Discretization*, Vol. I, John Wiley & Sons, Chichester, NY.

Jameson, A. and Mavriplis, D., 1986, "Finite Volume Solution of the Two-Dimensional Euler Equations on a Regular Triangular Mesh," *AIAA Journal*, Vol. 24, No. 4, pp. 611–618.

Jameson, A. and Baker, T.J., 1983, "Solution of the Euler Equation for Complex Configurations," AIAA Paper 83–1929.

Jameson, A. and Baker, T.J., 1987, "Improvements to the Aircraft Euler Method," AIAA Paper 87–0452, AIAA 25th Aerospace Sciences Meeting, Reno, NV.

Jameson, A., Schmidt, W. and Turkel, E., 1981, "Numerical Solution of the Euler equations by Finite Volume Methods Using Runge-Kutta Time-Stepping Schemes," AIAA Paper 81-1259, AIAA 14th Fluid and Plasma Dynamics Conference, Palo Alto, CA.

Marques, A.N., 2004, "Simulation of Unsteady Aerospace Flows Using Unstructured Meshes," Graduation Project, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil, Dec. 2004 (in Portuguese, original title is "Simulação de Escoamentos Aeroespaciais Não Estacionários Utilizando Malhas Não Estruturadas").

Marques, A.N. and Azevedo, J.L.F., 2006, "Application of CFD-based Unsteady Forces for Efficient Aeroelastic Stability Analyses," AIAA Paper No. 2006-250, 44th AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV.

Mavriplis, D.J., 1990, "Accurate Multigrid Solution of the Euler Equations on Unstructured and Adaptative Meshes," *AIAA Journal*, Vol. 28, No. 2, pp. 213–221.

Mohr, R.W., Batina, J.T. and Yang, H.T.Y., 1989, "Mach Number Effects on Transonic Aeroelastic Forces and Flutter Characteristics," *Journal of Aircraft*, Vol. 26, No. 11, pp. 1038–1046.

Oliveira, L.C., 1993, "A State-Space Aeroelastic Analysis Methodology Using Computational Aerodynamics Techniques," Master Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil (in Portuguese, original title is "Uma Metodologia de Análise Aeroelástica com Variáveis de Estado Utilizando Técnicas de Aerodinâmica Computacional").

Rausch, R.D., Batina, J.T. and Yang, H.T., 1990, "Euler Flutter Analysis of Airfoil Using Unstructured Dynamic Meshes," *Journal of Aircraft*, Vol. 27, No. 5, pp. 436–443.

Simões, C.F.C. and Azevedo, J.L.F., 1999, "The Influence of Numerical Parameters on Unsteady Airfoil Inviscid Flow Simulations Using Unstructured Dynamic Meshes," Proceedings of the 15th Brazilian Congress on Mechanical Engineering – COBEM 1999, Águas de Lindóia, SP.

Tijdeman, H., 1977, "Investigation of the Transonic Flow Around Oscillating Airfoils," NLR-TR-77090U, National Aerospace Laboratory, The Netherlands.

Traci, R.M., Albano, E.D. and Farr, J.L., 1975, "Small Disturbance Transonic Flows About Oscillating Airfoils and Planar Wings," AFFDL-TR-75-100.

Yang, T.Y., Guruswamy, P. and Striz, A.G., 1982, "Application of Transonic Codes to Flutter Analysis of Conventional and Supercritical Airfoils," Journal of Aircraft, Vol. 19, No. 3, pp. 211–220.

RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.