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ON THE CHARACTERIZATION OF VIBRO-ACOUSTIC SOURCES

J. I. Piva
General Motors do Brasil, Laboratório de Ruídos e Vibrações - CPCA – Indaiatuba SP, Brasil
jose.piva@gm.com

J.R.F. Arruda
Dep. Mecânica Computacional, Universidade Estadual de Campinas - Campinas SP, Brasil
arruda@fem.unicamp.br

Abstract. *This paper reports an investigation on existing methods for the characterization of vibration sources. The concepts and techniques proposed in the literature are briefly reviewed, and results and conclusions obtained for the simple case of a source connected to the receptor structure at a single point in one direction (one degree of freedom) are presented. Random and deterministic force cases are treated. The source is an electromagnetic shaker and the receptor structure is either a beam or a plate with two different levels of damping. The difficulties in obtaining the mobility of the source, used in the calculation of the Characteristic Power (CP), are exposed. Three different methods for obtaining the source mobility are used. The methodology can be extend to multi-point, multi-degree-of-freedom connection cases.*

Keywords: *source characterization, structure-borne sound power, vibro-acoustic source, structural acoustic power transmission*

1. Introduction

The aim of this paper is to conduct a brief review of the concepts used in the characterization of vibration sources through mobility measurements of the contact point between source and receptor. The investigation is carried out for the simple case of a source connected to the receptor structure at a single point, in one direction, perpendicular to the contact surface, ignoring all other forces and moments in other directions, but it can be extended to multi-degree-of-freedom connection. Results are presented for simple experiments, in which an electrodynamic shaker was used as a source, with both deterministic and random signals, a beam and a plate with two different damping conditions were used as receptors.

Vibro-acoustic sources are normally classified with respect to their environment, or, more precisely, to the path through which energy is transported away from it, e.g., “airborne” or “structure-borne”.

People facing acoustic problems need reliable information about how noisy the sources that generate them are, so that they can compare: a) different sources, b) a source against a target curve, or c) predict the behavior of the source when it is connected to a receptor, d) quantify improvements implemented onto sources which are considered noisy.

Consequently, the concept used to characterize a source, and meet the needs mentioned above, must: a) characterize the ability of the source to produce structural sound power, b) constitute an intrinsic property of the source, c) be represented by a single value, d) constitute the basis for calculations of the delivered power when the source is connected to the receptor.

In most cases of sources with airborne energy propagation, the concept of sound power successfully fulfills all the above mentioned criteria. Unfortunately, because of the strong interaction between the source and the receptor, the concept used above cannot be easily applied to cases in which the energy is transported through the structure. Due to this fact, the power delivered by the source varies from installation to installation, and does not depend solely on the source, but on the receptor in which it is installed as well (Cremer at al. 1973).

To calculate the emission of vibro-acoustic energy, it is necessary to consider both the velocity vectors and effort components on the interface between source and receptor. At the contact points between the source and the receptor, as much as six distinct components or degrees of freedom are possible, in which force and moment efforts contribute to the final power emission result. The response at a given contact point results from the forces and moments at other points, and, therefore, it is necessary to consider not only the ordinary point mobility, but also the transfer-mobility between points and the cross-mobility among effort components (Petersson at al. 1982a, 1982b).

Two quantities are essential in the energy transfer process. The first is the vibration amplitude at the contact point, and the second is the active power delivered to the receptor. The active power is the power conveyed by the waves, which are transmitted to the receptor and may radiate far from the source and never return. Both vibration amplitude and active power may be obtained from the **complex power**, and, therefore, an adequate source characterization must be related to it.

A great number of source characterization methods have been suggested by many authors. Nonetheless, the most promising approach was proposed by Mondot and Petersson (1987), and expanded by Petersson and Gibbs (1993), Fulford and Gibbs (1997, 1999a, 1999b) for multiple points, who use the concept of “Source Descriptor”, which meets all the criteria above, except for receptor independence in the case of multi-point connection. Aiming at overcoming the limitations mentioned above, Moorhouse, Mondot and Gibbs (1997) proposed an extension of this concept, which was called "Characteristic Power" (CP).

2. Theoretical Basis

The fundamental concept of structure propagated noise is adequately described in terms of energy flow between source and receptor, and was initially presented by Cremer *et al.* (1973). Given by:

$$\overline{Q} = \frac{1}{2}\overline{F}^{*}\overline{v} \tag{1}$$

where F and v are force and velocity at contact point. The bar indicates a complex quantity and * the complex conjugate. The imaginary part of $\overline{Q}$ corresponds to reactive power, responsible for the so-called "near field", in which the energy is exchanged between source and receptor, and vice-versa, without net energy flow, and thus, affecting only the contact region. The real part of $\overline{Q}$ is called "active power", which corresponds to the propagative part of the energy, thus affecting receptor dynamics at positions that are distant from the contact point.

Source is assumed linear and its vibration results from the internal forces, which are considered to be independent from the connection to the receptor. Source activity is then considered to be unique and characterized by its free velocity $\overline{v}_{sf}$, measured under normal operating conditions, but free at the attachment point, and by its mobility $\overline{Y}_S$, at receptor contact point. Since dynamics at the excitation point is governed by the excitation effort and by the dynamics of the structure being excited, we can use the expressions below and rewrite complex power as follows (Mondot at al. 1987):

$$\overline{F} = \frac{\overline{v}_{sf}}{\overline{Y}_S + \overline{Y}_R} \qquad \text{and} \qquad \overline{v} = \frac{\overline{v}_{sf}}{\overline{Y}_S + \overline{Y}_R}\overline{Y}_R \tag{2, 3}$$

$$\overline{Q} = \frac{1}{2}\frac{\left|\overline{v}_{sf}\right|^2}{\left|\overline{Y}_S + \overline{Y}_R\right|^2}\overline{Y}_R \tag{4}$$

The complex power of a given source is maximized when source and receptor mobility magnitudes are equal, i.e., when $\left|\overline{Y}_R\right| = \left|\overline{Y}_S\right|$. This result can be obtained by transforming Eq. (4) into a polar format, as shown below, and subsequently making its derivative relative to $\left|\overline{Y}_R\right|$ equal to zero and considering $\left|\overline{Y}_S\right|$ as a constant.

$$\overline{Q} = \frac{1}{2}\left(\frac{\left|\overline{v}_{sf}\right|^2}{\left|\overline{Y}_S\right|^2 + \left|\overline{Y}_R\right|^2 + 2\left|\overline{Y}_S\right|\left|\overline{Y}_R\right|\cos\Delta\Theta}\left|\overline{Y}_R\right|e^{i\Theta_R}\right) \tag{5}$$

In the expression above, $\Delta\Theta = \Theta_R - \Theta_S$, where Θ_R is the receptor mobility phase angle and Θ_S is the source mobility phase angle. However, the maximum active power emitted by a source occurs when source mobility is the complex conjugate of the receptor mobility (Mondot at al. 1987, Moorhouse, 2001), i.e. $\overline{Y}_R = \overline{Y}_S^{*}$, or $\left|\overline{Y}_R\right| = \left|\overline{Y}_S\right|$ and $\Theta_R = -\Theta_S$. This may be obtained by equalizing the derivative relative to variable Θ_R of the real part of Eq. (5) to zero and considering Θ_S as a constant. Hence, if we substitute the condition $\overline{Y}_R = \overline{Y}_S^{*}$ in Eq. (4), we will have (Moorhouse, 2001):

$$\overline{Q}(\overline{Y}_R = \overline{Y}_S^{*}) = \overline{S}_a = \frac{1}{4}\frac{\left|\overline{v}_{sf}\right|^2\overline{Y}_S^{*}}{\left[\Re(\overline{Y}_S)\right]^2} \tag{6}$$

where $\Re$ denotes the real part of a complex quantity. Consequently, the real part of Eq.(6) is the maximum active power that any source will provide to a receptor through its contact point, called Maximum Available Power (MAP), and is given by the following expression (Moorhouse, 2001):

$$MAP = \mathrm{Re}\left(\overline{S}_a\right) = \frac{1}{8}\frac{\left|\overline{v}_{sf}\right|^2}{\Re\left(\overline{Y}_S\right)} \tag{7}$$

Another interesting condition occurs when $\left|\overline{Y}_R\right| = \left|\overline{Y}_S\right|$ and $\Theta_R = \Theta_S$. Then, if we substitute $\overline{Y}_R = \overline{Y}_S$ in Eq. (4), we will obtain the Mirror Power (MP) (Moorhouse, 2001):

$$MP = \overline{Q}(\overline{Y}_R = \overline{Y}_S) = \overline{S}_m = \frac{1}{8}\frac{\left|\overline{v}_{sf}\right|^2}{\overline{Y}_S^*} \tag{8}$$

Normally, the magnitude of MP is smaller than MAP, but, for the specific case in which the source mobility phase is equal to zero, we will have $MP = MAP$.

The last condition of interest in power transmission occurs when force at the contact point is equal to blocked source force ($\bar{f}_{bl} = \frac{\overline{v}_{sf}}{\overline{Y}_S}$) and velocity is equal to free velocity ($\overline{v} = \overline{v}_{sf}$). This is called Characteristic Power (CP), given by (Moorhouse, 2001):

$$CP = \overline{Q}(\bar{f} = \bar{f}_{bl}, \overline{v} = \overline{v}_{sf}) = \overline{S}_c = \frac{1}{2}\frac{\left|\overline{v}_{sf}\right|^2}{\overline{Y}_S^*} \tag{9}$$

It can be observed that $CP = 4MP$. Mondot and Petersson (1987) called this power the Source Descriptor. However, Moorhouse (2001) adopted the new term CP due to a generalization of the concept for multiple contact points between source and receptor, which is different from the concept used by Mondot and Petersson (1987).

These three concepts, MAP, MP and CP, are quantities which fully characterize a source in terms of power, as they do not depend on the receptor. Initially, MAP seems to be the most attractive concept, since it is the maximum limit for the power that a given source can deliver when connected to a receptor. Nevertheless, there are two disadvantages related to its dependency to the real part of mobility. The first is the existence of sources with purely imaginary mobility (mass-like and spring-like sources). In this case, MAP will be infinite, when, as a matter of fact, due to the condition assumed for maximum power, $\mathrm{Re}(\overline{Y}_S) = 0 \Rightarrow \mathrm{Re}(\overline{Y}_R) = 0$, i.e., the receiver cannot absorb energy and it would be impossible to inject any power into the receptor, thus violating the very definition of MAP. The second is the necessity of inverting the real part of the source mobility matrix, which is susceptible to measuring errors (Moorhouse, 2001). Both MP and CP are less sensitive to these types of errors and, because they are related solely by one constant, only one of the concepts is necessary. Thus, CP being the quantity normally used in the literature for the characterization of sources structure borne noise (Moorhouse, 2001), it is chosen as a source characterization parameter.

Even though in this paper we have developed and applied the concepts for a connection with one degree of freedom, in order to make them clearer, they can be easily extended to connections with multiple degrees of freedom. Nevertheless, it must be pointed out that there are important practical difficulties presented by the amount of experimental work needed for the determination of the impedances that are necessary to implement the formulation proposed by Moorhouse (2001).

3. Experimental Results

The concepts of power explained above were experimentally investigated using a setup where the source was an electromechanical shaker connected at a single point to a receiving structure, either a beam or a plate.

At all times, the shaker was excited by a signal generator fed with the same dynamic signal, in order to guarantee that the “source” under investigation would not vary under the many conditions to which it was submitted. In order to simulate two distinct sources, two different signals were used to excite the shaker, both of which were in the frequency range DC-1.6 kHz. The first was a periodic chirp. It was acquired with a total time per data block equal to the chirp period, such that leakage-free velocity and force spectra could be obtained via the discrete Fourier transform, without the need of windowing or averaging. The second was a random signal, and the number of averages was set to 100 and a Hanning window was used. To obtain the complex values of the free velocity ($\overline{v}_{sf}$), as well as blocked source force ($\bar{f}_{bl}$), it is necessary to use a reference signal so that the phase information is not lost, as these complex values are obtained from two distinct experiments. The complex amplitudes of velocity and force were calculated from the expressions in Eq. (10) and (11), respectively:

$$\overline{v} = \frac{\overline{S}rv}{\sqrt{Srr}} \quad \text{and} \quad \bar{f} = \frac{\overline{S}rf}{\sqrt{Srr}} \tag{10,11}$$

where $\bar{S}rv$ and $\bar{S}rf$ are the force and velocity cross spectra with respect to the reference signal, and Srr is the reference signal auto-spectrum. An impedance head was used for measuring force and velocity at the excitation point.

The free velocity was initially measured with the source under normal operation in free condition, as shown in Fig. (1a). Theoretically, in this condition, the measured force should be zero. Nonetheless, due to the mass and other dynamic properties of the sensor, force amplitude is not null, but very small, approximately 20 dB lower than the blocked condition force for both excitation signals, as shown in Fig. (2a) and (2b).

The shaker was subsequently connected to a fairly rigid structure as an attempt to obtain the force of the source for the blocked condition, as indicated in Fig. (1b). Theoretically, under this condition, the measured velocity should be zero, but, due to structure finite dynamic flexibility, the velocity is small but not null, as show in Fig. (3a) and (3b).

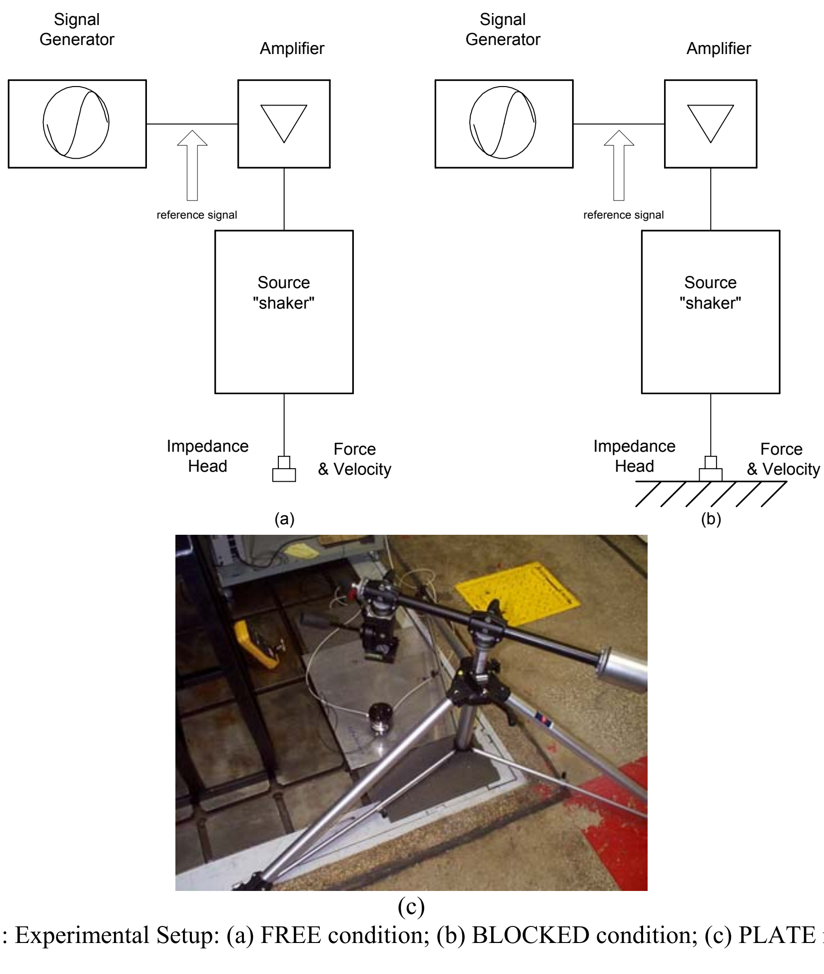
After those measurements, the sources were connected to two different receptors: 800x32x2 mm aluminum beam and a 520x450x2 mm aluminum plate (see Fig. (1c)), with two different damping levels. The first condition, which supplies very low damping, was achieved by supporting the receptors onto foam in a condition that could be considered quasi free along the edges. The second damping condition was obtained by placing the receptor structures in a sand box, which supported and partially covered them, in order to enhance receptor energy absorption.

In the case of deterministic excitation (periodic chirp), force and velocity spectra at the plate or beam excitation point were used to determine receptor mobility. The force spectra under blocked condition, as well as the spectra of velocity under free condition, were used for source mobility calculation, as seen below:

$$\bar{Y}_S = \frac{\bar{v}_{sf}}{\bar{f}_{bl}} \tag{12}$$

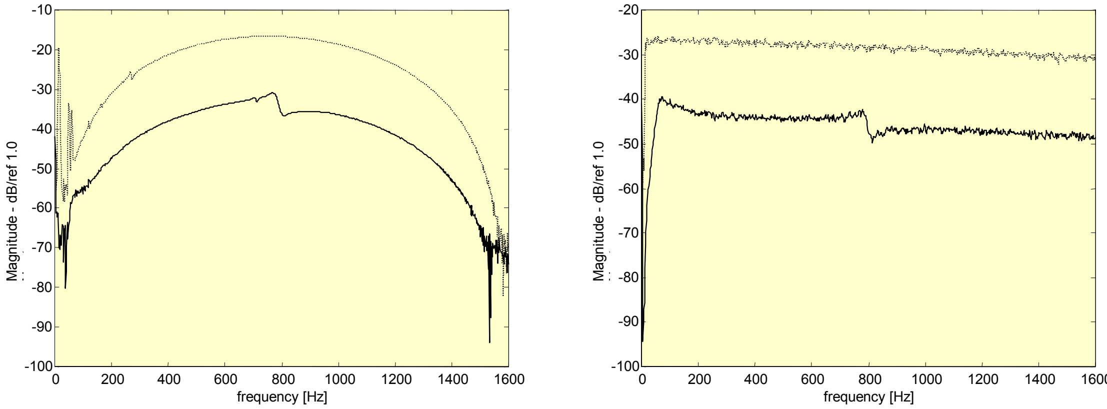
For random excitation, the average force and velocity spectra, which were used for calculing the receptors and source mobility, were obtained from cross spectra and auto-spectra of the reference signal, as shown by Eq. (10) and (10,11). It is interesting to notice that this procedure is not necessary to obtain the receptor mobility, as force and velocity at the excitation point can be acquired simultaneously.

The first comparison to be conducted is between the two calculation methods of the magnitude of the complex power input into the receptors: the first one uses Eq. (1) at the contact point between the source and the receptor, called $|\overline{Pin}|$ in Fig. (4a) through (5b), and the second uses Eq. (4), denominated $|\bar{Q}|$ in the graphs.



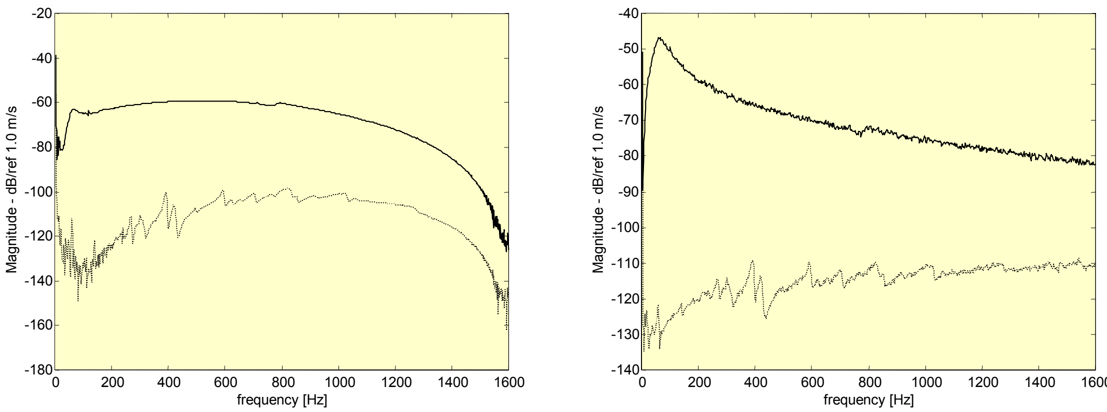
(c)

Figure 1: Experimental Setup: (a) FREE condition; (b) BLOCKED condition; (c) PLATE receptor



(a) (b)

Figure 2: Source force with:(a) periodic CHIRP signal and (b) RANDOM signal under:
– FREE and --- BLOCKED conditions



(a) (b)

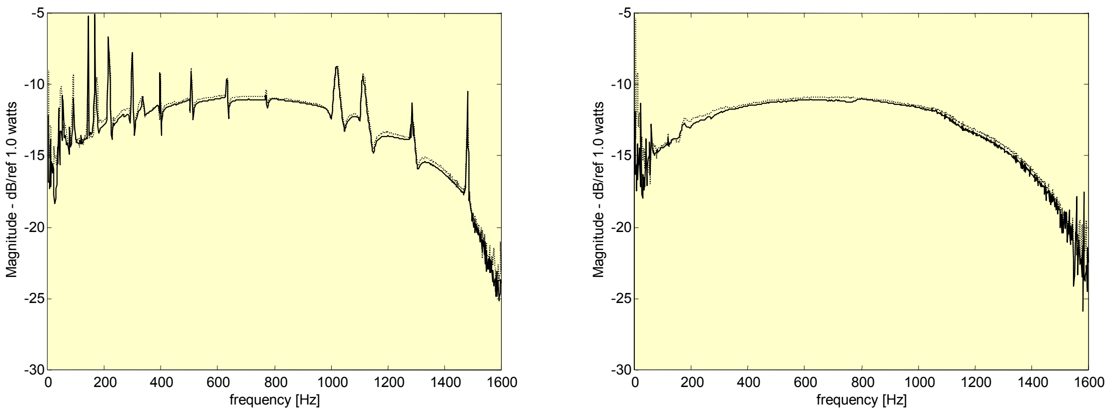
Figure 3: Source velocity with:(a) periodic CHIRP signal and (b) RANDOM signal under:
– FREE and --- BLOCKED conditions

As it can be observed in Fig. (4a) through (5b), there is a good consistency between these two parameters, except below 200 Hz and above 1400 Hz, which is more noticeable in the chirp excitation case, as shown in Fig. (4a-b). This discrepancy is justified by the noisy behavior of the velocity and force spectra obtained from the experiments in the free and blocked conditions in these frequency ranges (Fig. (2a) and (3a).

By further analyzing these figures, one can observe a slight tendency, for the power $\left|\overline{Q}\right|$, to underestimate $\left|\overline{Pin}\right|$. One of the causes for such differences may be a systematic overestimation of the source impedance due to a non-rigid connection in the blocked condition, which would cause an overestimation of $\left|\overline{Q}\right|$, as $\overline{Y}_r$ appears in the denominator of Eq. (4). In principle, $\left|\overline{Pin}\right|$ is a better estimate of input power, as it is calculated from a cross spectrum of force and velocity measured simultaneously, while $\left|\overline{Q}\right|$ is calculated from a velocity auto-spectrum and from mobilities obtained in different experiments.

With relation to the various power terms calculated for source characterization, some conclusions can be drawn. The first is that the shape of the curves does not display the dynamics of the receptor, i.e., it is independent of the receptor, as expected. This can be easily observed in Fig. (6a) and (7a) which represent the cases where the receptor is lightly damped and its natural modes are more evident. The same figures show that, if the magnitudes of MAP and $\overline{Q}$ are compared, in some receptor resonance frequencies, the value for $\left|\overline{Q}\right|$ surpasses the value of the $\left|MAP\right|$. This does not, however, invalidate the concept of MAP, since it has a physical meaning, and represents the maximum active power available at the source, while $\overline{Q}$ represents the complex power and, thus, for lightly damped receptors, its magnitude can be greater than that of MAP. This behavior is not observed in receptors with greater damping, as seen in Fig. (6b) and (7b).

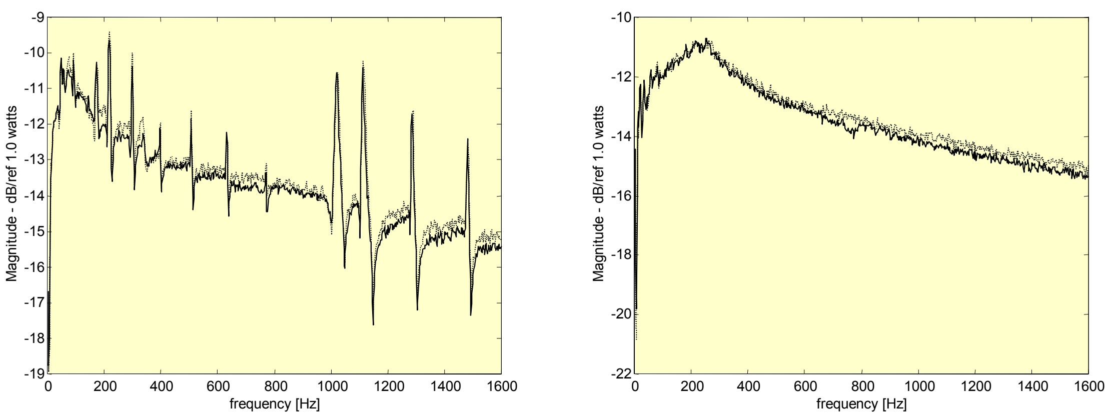
Another interesting consideration to be made is the comparison between MAP and MP magnitudes. As mentioned before in our theoretical review, $\left|MAP\right| \geq \left|MP\right|$; this can be corroborated by experimental results and observed in any of the Figs (6a) through (7b). Due to this theoretical relation, and since $CP = 4MP$, the CP parameter may surpass MAP , as observed at some points of the same figures, mainly the ones related to the random excitation, i.e., Fig. (7a-b).



(a) (b)

Figure 4: Power input into the BEAM with: (a) LIGHT damping and (b) HEAVY damping; source with periodic CHIRP signal: – $\left|\overline{Q}\right|$ Eq. (4), --- $\overline{|Pin|}$ Eq. (1).

In all the cases studied, and in almost every frequency range considered, the magnitude of MP follows $\left|\overline{Q}\right|$ magnitude curve mainly when the receptors are lightly damped, as shown in Fig. (6a) and (7a). The CP curve, is situated between the MP and MAP curves, except below 200 Hz and above 1400 Hz, probably for the same reason given before relative to the underestimation of $\left|\overline{Q}\right|$. However, this could also be particular to the experiments carried out, as it has not been formally proved.



(a) (b)

Figure 5: Power input into the BEAM with: (a) LIGHT damping and (b) HEAVY damping; source with RANDOM signal: – $\left|\overline{Q}\right|$ Eq. (4), --- $\overline{|Pin|}$ Eq. (1).

3.1 Difficulties in obtaining the source mobility

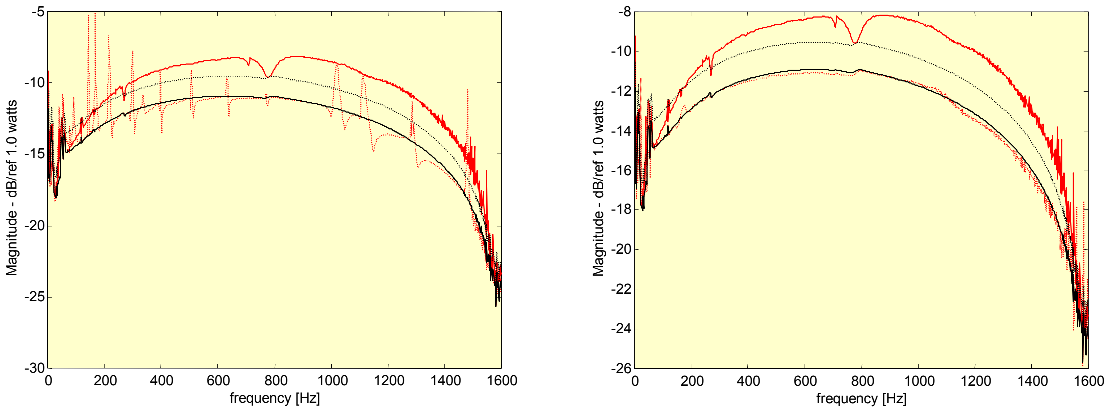
The source mobility estimate, given by Eq. (12), was used in order to calculate all the power terms compared before. This equation uses the force in blocked condition. However, there are practical difficulties in realizing the total blocking of the source to conduct the measurements and, for this reason, other source mobility measuring methods were investigated.

The first method, the "direct method", consists in obtaining the source mobility directly by using a shaker attached to the source, which is switched off (internal forces not present), as shown in Fig. (8a). By calculating the force and velocity measurements ratio, given by in this configuration, it is straightforward to obtain the source mobility.

Another method evaluated consisted in coupling the source to a known receptor, in this case a pendulum mass, as shown in Fig. (8b). Given that the mobilities are arranged in series, we will have:

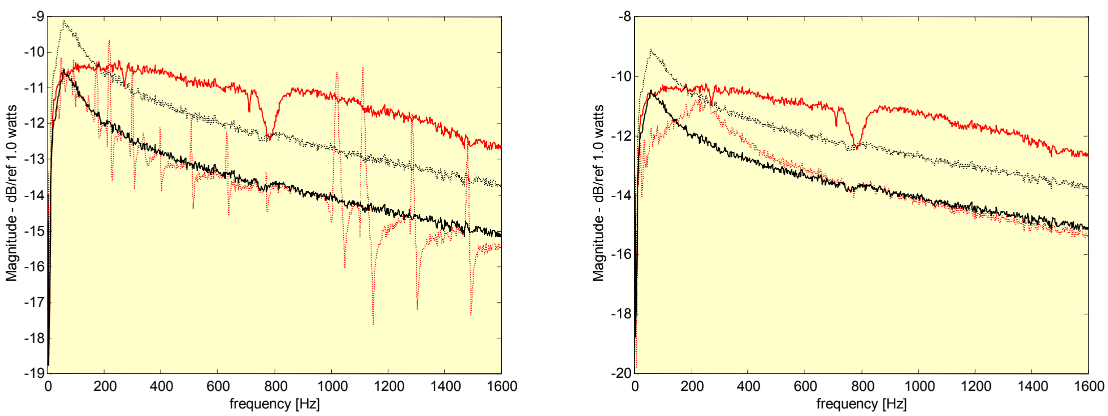
$$\bar{Y}_{measured} = \bar{Y}_s + \bar{Y}_{mass} = \frac{\bar{v}_{measured}}{\bar{f}_{measured}} \quad \text{where} \quad \bar{Y}_{mass} = \frac{1}{i\omega m} \quad \text{and } m \text{ is the pendulum mass.}$$

Results shown in Fig. (9a-b) indicate that the three methods used to estimate source mobility compare well. As mentioned before, the method that uses the free and blocked conditions tends to overestimate $\bar{Y}_S$, while the "direct method" yields a smooth curve, except for some frequencies below 200 Hz. The direct method is the easiest to implement in most practical cases.



(a) (b)

Figure 6: Power terms for the BEAM with:(a) LIGHT damping and (b) HEAVY damping; source with periodic CHIRP signal: --- $|\bar{Q}|$ input power, — $|MAP|$, — $|MP|$, --- $|CP|$



(a) (b)

Figure 7: Power terms for the BEAM with:(a) LIGHT damping and (b) HEAVY damping; source with RANDOM signal: --- $|\bar{Q}|$ input power, — $|MAP|$, — $|MP|$, --- $|CP|$

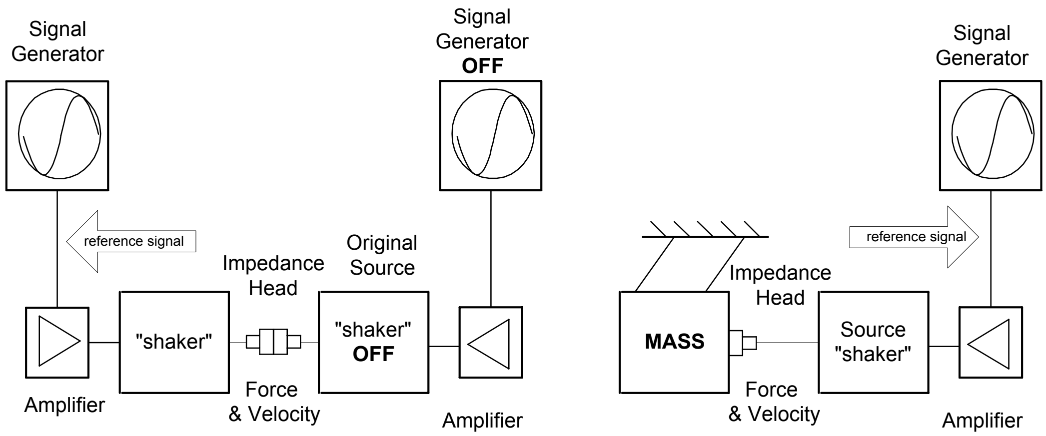
4. Conclusions

The expressions and concepts of power terms for the characterization of vibro-acoustic, structure-borne type sources developed by Moorhouse (2001) were briefly reviewed and applied to a single degree of freedom source/receptor connection. Different sources and receptors were simulated, and the experiments conducted described in detail. Two possible methods for obtaining the complex power delivered to the receptor were compared, with similar results. Due to the difficulties inherent to experimental determination of the source mobility, three alternative methods were investigated. Though all methods yielded similar curves, the "direct method" yielded the best practical results.

It was observed that the parameters MAP, MP and CP can be used to characterize vibro-acoustic sources, and that they present higher differences for lightly damped receptors. Although MAP represents the maximum power emission for a given source, it was possible to observe experimentally that $|\bar{Q}|$ and $|CP|$ may exceed $|MAP|$, and that $|MAP|$ is always equal or higher than $|MP|$, as theorized.

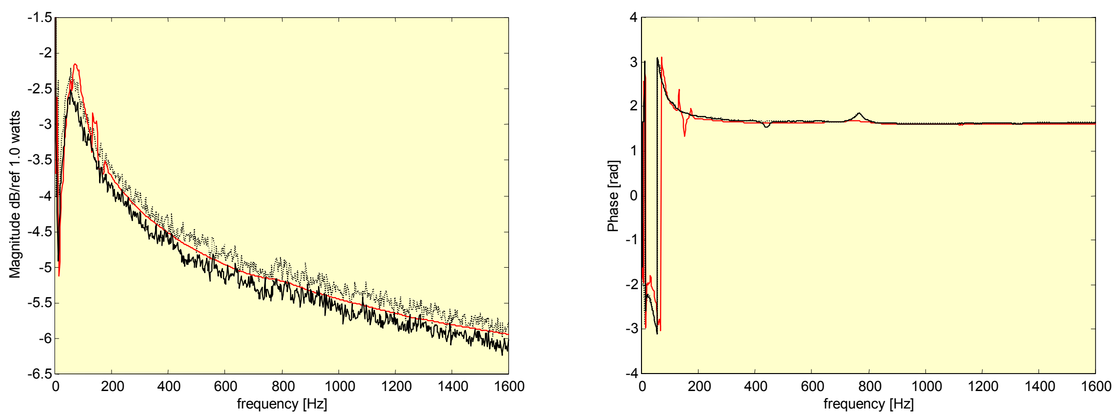
Finally, it was observed that the $|MP|$ curve, in the cases investigated, is the mid line of the $|\bar{Q}|$ curve. However, a more detailed theoretical study must be conducted in order to determine if this is a general or particular phenomenon.

Although all the conclusions presented here are valid for both beams and plates, only the results concerning beams were presented in this paper.



(a) (b)

Figure 8: Experimental Setup of Source Mobility: (a) “direct method”; (b) “mass method”



(a) (b)

Figure 9: Source mobility: (a) magnitude and (b) phase: --- $\bar{Y}_S = \dfrac{\bar{v}_{sf}}{\bar{f}_{bl}}$, – “direct method”, — “mass method”

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6. Responsibility notice

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