Proceedings of the XI DINAME, 28th February-4th March, 2005 - Ouro Preto - MG - Brazil

MODELING OF DYNAMIC ROTORS WITH FLEXIBLE BEARINGS USING VISCOELASTIC MATERIALS

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Abstract. *Nowadays the rotating machines produce or absorb large amounts of power in relatively small physical packages. The fact those machines work with large density of energy and flows is associated to the high speeds of rotation of the axis, implying in high inertia loads and deformations of the axis, vibrations and dynamic instabilities. Viscoelastic materials are broadly used in vibration and noise control of dynamic rotors to increase the area of stability, due to their high capacity of dissipation of vibratory energy. The widespread model, used to describe the real dynamic behavior of this type of materials, is the fractional derivative model. By using the finite element method it is possible to describe the viscoelastic dynamics of the rotor. In general, the stiffness matrix is composed by the stiffness of the shaft and bearings. In this particular case, this matrix is complex and frequency dependent because of the characteristics of the viscoelastic material that is part of the bearings. A clear and simple numerical code is proposed to calculate the modal parameters of a simple rotor mounted on viscoelastic bearings. A methodology to build a Campbell diagram (natural frequency versus rotation frequency) is presented. It must be built through an internal Campbell diagram (natural frequency versus variable frequency), for which the stiffness matrix is a function of the frequency.*

Keywords: dynamic rotor, viscoelastic material, Campbell diagram, critical rotations.

1. Introduction

Nowadays the rotative machines produce or absorb larger and larger amounts of power in relatively small physical packages. The fact those machines work with large density of flows of energy is associated to the high speeds of rotor rotation. It implies in high inertia loads and potential problems with deformations of the axis or axle, high levels of vibrations and dynamic instabilities.

Rotative machines often have problems of instability when working at high rotations, which can result in sudden failures of the whole system or parts of it. This problem can be solved by including damping in the bearings. In general, the area of stability can be enlarged with this type of control; also, the vibration levels can be reduced as well.

Viscoelastic materials are thoroughly used to control vibration and noise due to their high capacity to dissipate vibratory energy (see Espíndola et al., 2003). In order to do so, accurate knowledge of their dynamic properties is essential to desing the control devices.

Several papers can be found in the literature, with the purpose of modeling simple rotors mounted on elastomeric materials or on bearings made of this type of material. Generally, these papers use the model of Kelvin-Voigt, as proposed by Shabaneh and Jean (1999), where the viscoelastic material is put under the bearings. This model has a difficulty to represent the dynamic characteristics of most viscoelastic materials used in practice, when analyzed in a wide frequency band (Pritz, 1996; Bagley and Torvik, 1983). It is noticed that this model is described by a differential equation of integer order.

In the study accomplished by Marynowski and Kapitaniak (2002), the models of Kelvin-Voigt and Bürgers are compared in their capacity to describe the behavior of a viscoelastic material. The first is a model with two parameters (spring and viscous shock absorber in parallel), and the second is described by four parameters. Similar results were obtained for small values of internal shock absorption, but for the materials with larger coefficients of reduction the model of Bürgers proved itself more appropriate.

In Panda and Dutt (1999), polymeric materials are placed in the bearings. Using nonlinear optimization techniques, it is possible to find the optimal dimensions to reduce the vibratory response of the system to unbalance excitations. In Dutt and Toi (2002), models with three and four spring-shock absorber elements and integer order derivatives are used to predict the behavior of a viscoelastic material that is part of a dynamic rotor. In that paper the aim was to study the reduction of vibration and the changes in rotor dynamic behavior caused by the viscoelastic material.

In most of the papers mentioned above, the employed models of viscoelastic material cannot reproduce their dynamic characteristics faithfully in a wide frequency band.

Here it will be presented a numerical model capable of predicting the dynamic response of a simple rotor in steady state, whose bearings are made of layers of viscoelastic material. The model used for the polymeric material is the fractional derivative model with four parameters, due to its capacity to describe the real dynamic behavior of the material (Pritz, 1996). For this purpose, the characteristics of the viscoelastic material were determined by using the methodology proposed in Espíndola et al. (2003), Lopes et al. (2004) and Silva Neto (2004).

To describe the dynamic behavior of the system by Lagrange's equations, it is used the finite element method. By this way, the inertia (symmetrical and with constant coefficients), the gyroscopic (skew-symmetrical and a function of the rotating speed), and the complex stiffness (composed by the stiffness of the axis and the bearings, which is frequency and temperature dependent due to the viscoelastic material) matrices are obtained.

A simple strategy is proposed to calculate the modal parameters of the system. In this numeric implementation, a Campbell diagram is generated, through which it is possible to determine the critical rotations of the dynamic viscoelastic rotor. Due to the characteristics of the system - the complex stiffness matrix is a function of the frequency - the final Campbell diagram should be obtained through another internal Campbell diagram. That is, when the rotation of the axis is established, the inertia and the gyroscopic matrices are constant, but the stiffness matrix is a function of the frequency for a given temperature. For each rotation, the natural frequencies of the system are functions of the frequency, therefore, they should be found through another internal Campbell diagram (natural frequency versus variable frequency). These steps follow the sequence presented in Espíndola and Floody (1999), when the dynamic behavior of a sandwich beam (steel – viscoelastic material - steel) was studied.

To validate the numerical model, a simulation will be accomplished on a simple rotor system.

2. Elements of the rotor

The simple rotor in study is basically composed by an axis, one or more disks and several flexible bearings, made of viscoelastic material. The force upon the rotor can be caused by unbalances (synchronous excitation $\Omega = \Omega_{rpm}$), instabilities of hydrodynamic bearings (asynchronous excitation $\Omega \cong 0,5\Omega_{rpm}$) or excitation through to the base by $\Omega \neq \Omega_{rpm}$. This paper will consider only unbalance load.

The general equations of the rotor can be introduced through Lagrange's equations (see Eq. (1)). So it is necessary to define the kinetic energy T and the potential energy U of each element of the rotor, besides the virtual work done by an external force upon the bearings.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = F_{q_i} \tag{1}$$

In equation (1), q_i is the i-th generalized coordinate and F_{q_i} is the i-th generalized force. Then, using the finite element method, it is possible to describe the rotor dynamics.

2.1 The disk

The disk is assumed to be rigid, being characterized only by the kinetic energy. Therefore, $R_0\ (X, Y, Z)$ is defined as the inertial coordinate system, coincident with $R\ (x^i, y^i, z^i)$, a fixed system in the center of the disk (see "Fig. 1"). The coordinates XYZ and $x^i y^i z^i$ are related by three angles, ψ, θ and ϕ. To describe the rigid body rotation of the rotor concerning the axis X, Y or Z, the Euler angles are considered (see "Fig. 1").

The instantaneous angular speed vector of the disk can be written in the system of reference R as (Lalanne, 1990):

$$\vec{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \cong \begin{bmatrix} \dot{\psi}\ sen\ \phi\ \cos\theta + \dot{\theta}\cos\phi \\ -\dot{\psi}\ sen\ \theta + \dot{\phi} \\ \dot{\psi}\cos\phi\cos\theta - \dot{\theta}\ sen\ \phi \end{bmatrix} = \begin{bmatrix} \dot{\psi}\ sen\ \phi + \dot{\theta}\ \cos\phi \\ -\dot{\psi}\ \theta + \Omega \\ \dot{\psi}\cos\ \phi - \dot{\theta}\ sen\ \phi \end{bmatrix} \tag{2}$$

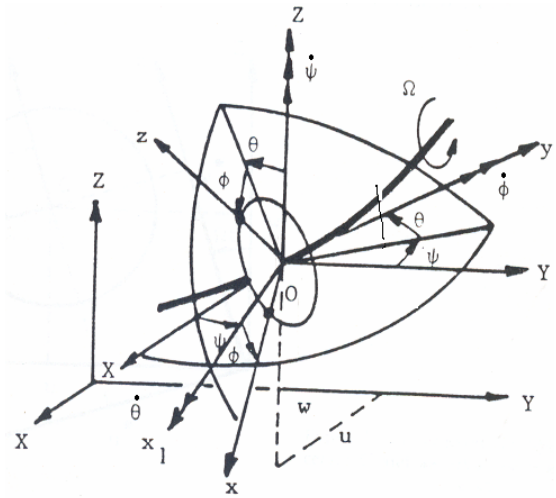

Figure 1. Systems of coordinates of the disk rotating around a flexible axis.

Considering linearity, the angles θ (rotation around axis X) and ψ (rotation around axis Z) are considered small, so that:

$$\cos\theta \cong 1 \;\; and \;\; \text{sen}\,\theta \cong \theta \tag{3}$$

The kinetic energy of the disk is given by the equation:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\left(I_{Dx}\omega_x^2 + I_{Dy}\omega_y^2 + I_{Dz}\omega_z^2\right) \tag{4}$$

where M_D is the mass of the disk, u and w are, respectively, the displacements in the X and Z-direction (see "Fig. 1"), and, n this particular case, the disk is symmetrical, being verified that $I_{Dx}=I_{Dz}$, where I_{Dx} and I_{Dz} are the transverse inertia in the X and Z directions. On the other hand, it is supposed that the angular speed remains constant, so $\dot{\phi} = \Omega =$ constant. Therefore, the expression of the kinetic energy of the disk is:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2} I_{Dx}\left(\dot{\theta}^2 + \dot{\psi}^2\right) + \frac{1}{2} I_{Dy}\left(\Omega^2 + 2\Omega\dot{\psi}\theta + \dot{\psi}^2\theta^2\right) \tag{5}$$

and, neglecting the term $\dot{\psi}^2\theta^2$ for being of second-order,

$$T_D \cong \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2} I_{Dx}\left(\dot{\theta}^2 + \dot{\psi}^2\right) + \frac{1}{2} I_{Dy}\left(\Omega^2 + 2\Omega\dot{\psi}\theta\right) \tag{6}$$

In the previous expression, it is observed that the term $(1/2)\, I_{Dy}\, \Omega^2$ is constant, not having any influence in Lagrange's equations. The last term, $I_{Dy}\Omega\dot{\psi}\theta$, represents the gyroscopic effect.

2.2. The Shaft

The shat is characterized by the potential and kinetic energy. The expression for the kinetic energy of the shat is the result of an extension of the kinetic energy of the disk (see Eq. (4)). If the element has length L, its kinetic energy can be expressed by the following equation:

$$T_E = \frac{\rho S}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{\rho I}{2}\int_0^L \left(\dot{\psi}^2 + \dot{\theta}^2\right) dy + \rho I L \Omega^2 + 2\rho I \Omega \int_0^L \dot{\psi}\,\theta\, dy \tag{7}$$

where I is the transverse inertia, ρ is the density and S is the transverse area.

Considering the symmetry of the axis *(Ix = Iz = I)* and neglecting the effects of axial forces, the expression for potential energy is defined by:

$$U_E = \frac{EI}{2}\int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dy \tag{8}$$

2.3. The bearings

It is composed by two parts: the bearing itself and the viscoelastic material. The viscoelastic material can be added between the external layer of the bearing roller and the bearing carcass or under its own base, as shown in “Fig. 2a” and “Fig. 2c”. In the first situation, the inertia of the bearing can be neglected, while in the second, the inertia of the bearing must be considered. “Figures 2b and 2d” show the simplified outlines for both situations mentioned above.

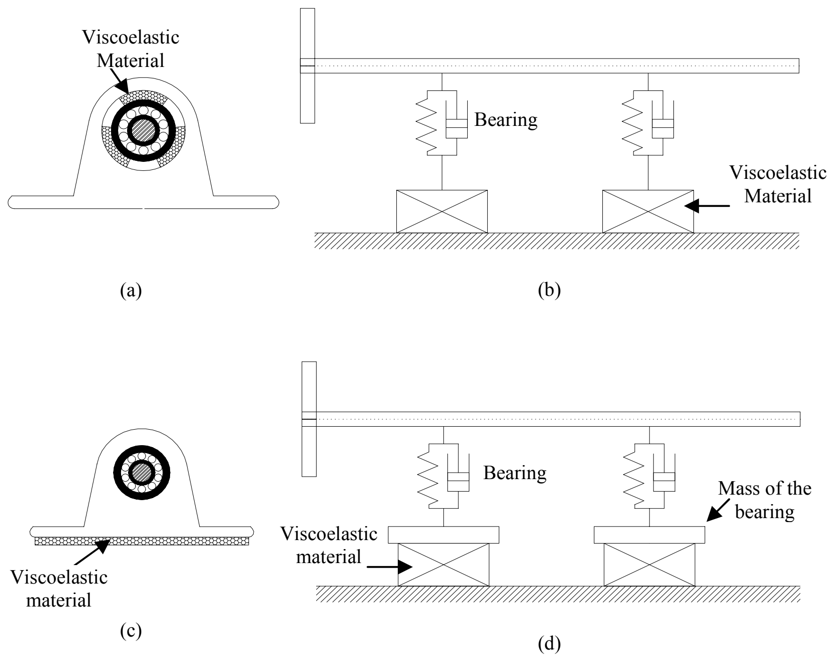

Figure 2. Model of bearings with viscoelastic material.

The viscoelastic material and the bearing are placed in sequency. Because of that, the stiffness of the bearing with roller is higher than the stiffness of the viscoelastic material layer and the resulting equivalent stiffness will be that of the viscoelastic material. The model used to describe the real dynamic behavior of the viscoelastic material is the four parameter fractional derivative model. According to Bagley and Torvik (1983), the unidimensional constitutive equation in fractional derivatives is:

$$\sigma(t) + \sum_{m=1}^{M} b_m D^{\beta_m} \sigma(t) = E_0 \varepsilon(t) + \sum_{n=1}^{N} E_n D^{\alpha_n} \varepsilon(t) \tag{9}$$

where b_m, β_m, α_n, E_0 and E_n, are the parameters of the considered material. The operators D^{β_m} and D^{α_n} represent fractional derivatives. When $M=N=1$ and $\alpha = \beta$, the equation above is denominated the four parameter fractional derivative model.

$$\sigma(t) + b_1 D^{\alpha}[\sigma(t)] = E_0 \varepsilon(t) + E_1 D^{\alpha}[\varepsilon(t)] \tag{10}$$

Applying the Fourier transform in Eq. (10), the result is:

$$\sigma(\Omega) + b_1 (i\Omega)^{\alpha} \sigma(\Omega) = E_0 \varepsilon(\Omega) + E_1 (i\Omega)^{\alpha} \varepsilon(\Omega) \tag{11}$$

The relation $\sigma(\Omega)/\varepsilon(\Omega)$ is denominated the elasticity modulus of the material (see Eq. (12)),

$$E_c(\Omega) = \sigma(\Omega)/\varepsilon(\Omega) = \left[E_0 + E_1(i\Omega)^\alpha\right]/\left[1 + b_1(i\Omega)^\alpha\right] \tag{12}$$

or, alternatively, according to Pritz (1996),

$$E_c(\Omega) = \frac{E_0 + E_\infty(i\Omega b)^\alpha}{1 + (i\Omega b)^\alpha} \tag{13}$$

where $E_1 = E_\infty b_1$, $b_1 = b^\alpha$ and $E_c(\Omega)$ is the complex modulus of the material. In general, this modulus is a function of frequency and temperature. In this paper, the temperature will be considered constant, so, it will not be included as an independent variable. Still, the elasticity modulus $E_c(\Omega)$ can be written in a general way by:

$$E_c(\Omega) = E(\Omega)(1 + i\,\eta(\Omega)) \tag{14}$$

where $E(\Omega)$ is the real part of $E_c(\Omega)$, and $\eta(\Omega) = \mathrm{Im}(E_c(\Omega))/\mathrm{Re}(E_c(\Omega))$ the loss factor. E_0 and E_∞ represent the inferior and superior asymptotes of the dynamic modulus of elasticity. The exponent α represents the slope of a tangent straight line in frequency corresponding to the point of inflection of the curve $E(\Omega)$, or, also, to the maximum point of loss factor. The parameter b is the relaxation time.

The equation that represents the complex shear modulus is written in the following way:

$$G_c(\Omega) = \tau(\Omega)/\gamma(\Omega) = \left[G_0 + G_1(i\Omega)^\alpha\right]/\left[1 + b_1(i\Omega)^\alpha\right] \tag{15}$$

or, in the complex form, $G_c(\Omega) = G(\Omega)(1 + i\eta(\Omega))$, where $G(\Omega) = \mathrm{Re}(G_c(\Omega))$ and $\eta(\Omega) = \mathrm{Im}(G_c(\Omega))/\mathrm{Re}(G_c(\Omega))$. For elastomers, $\eta(\Omega) = \eta_E(\Omega) = \eta_G(\Omega)$ (Snowdon, 1968). In the model considered herein, where the viscoelastic material is added under the bearing ("Fig. 2c"), only the component of xx and yy of stiffness and damping will be considered. In *X* direction, the stiffness will be represented by the shear modulus and in *Z* by the elasticity modulus.

Disregarding the stiffness associated to the rotations ψ and θ (in *Z* and *X* direction, respectively) of the rolling bearing, the stiffness matrix of the viscoelastic material will be given by:

$$\begin{bmatrix} F_u \\ F_w \end{bmatrix} = -\begin{bmatrix} \bar{k}_{xx} & 0 \\ 0 & \bar{k}_{zz} \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} \tag{16}$$

where $\bar{k}_{xx} = LG_c(\Omega)$. In *Z* direction, the stiffness will be:

$$\bar{k}_{zz} = LE_a(\Omega) = LE_a(\Omega) \tag{17}$$

where $L = A/h$, A is the loaded area, h is the viscoelastic material thickness, and $E_a = E_c\,.k_T$ the apparent modulus of elasticity. According to Nashif (1985), the apparent modulus of elasticity is obtained through the modulus of elasticity and the form factor k_T. In this paper, the layer of viscoelastic material was assembled in a such way that lateral expansion is allowed, or $k_T \cong 1$, what implies $E_c = E_a$ (see "Fig. 3")

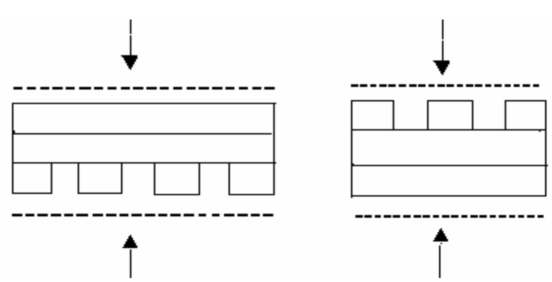

Figure 3. Form of the layer of viscoelastic material.

In the transition frequency of elastomers, it can be considered that Poisson's coefficient is approximately equal to 0,5 (Silva Neto, 2004), so $E_a = 3G_c$ and the Eq. (17) takes the following form:

$$\bar{k}_{zz} = 3\ L G_c(\Omega) \tag{18}$$

2.4. Matricial representation of the rotor

According to the finite element method, it is considered that each node of each element of the rotor has four degrees of freedom: two displacements u and w (in Z and X directions, respectively) and two rotations θ and ψ around the axes X and Z, respectively. Therefore, for node i, the generalized coordinate q_i is represented by:

$$q_i = [u_i, w_i, \theta_i, \psi_i]^T \tag{19}$$

The application of Lagrange's equations on the kinetic and potential energies of the elements of a simple rotor (neglecting effects of circulation and stiffness caused by rotation), and assembling each elementar matrix conveniently, results in the following differential equations (Lalanne, 1990):

$$M\,\ddot{q}(t) + \left(C + G(\Omega_{rpm})\right)\dot{q}(t) + \bar{K}(\Omega)q(t) = f(t) \tag{20}$$

where:
M is the inertia matrix (constant coeficient and symmetrical);
G is the gyroscopic matrix of the axis and disk (function of rotation and skew-symmetric);
C is the damping matrix of the axis and bearings (constant coeficient and symmetric);
$\bar{K}(\Omega)$ is the stiffness matrix of the dynamic rotor and viscoelastic bearings (symmetric, complex and frequency-temperature dependent). The stiffness of the viscoelastic layer is defined by:

$$\bar{K}(\Omega)_m = \begin{bmatrix} LG(\Omega)[1+i\eta(\Omega)] & 0 & 0 & 0 \\ 0 & 3LG(\Omega)[1+i\eta(\Omega)] & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

3. Resolution of the system of dynamic equations

The system of equations that represents the movement of a dynamic rotor is given by Eq. (20). This equation is valid for a sinusoidal excitation of frequency $\Omega = \Omega_{rpm}$. To generalize this expression, it is possible to represent it in the frequency domain, through Fourier transform. So,

$$[-\Omega^2 M + i\Omega\left(C + G(\Omega_{rpm})\right) + \bar{K}(\Omega)]Q(\Omega) = F(\Omega) \tag{22}$$

where *Q (Ω) and F (Ω)* are the Fourier transforms of *q (t)* and *f (t)*, respectively.

3.1. Eigenvalue Problem

Considering the solution of the homogeneous ($f(t) = \{0\}$), given by $q(t) = \{\phi\}e^{st} = \{\phi\}e^{i\Omega t}$, the following characteristic equation is obtained:

$$s^2 M + s\left(C + G(\Omega_{rpm})\right) + \bar{K}(\Omega) = 0\,, \tag{23}$$

which corresponds to a polynomial of s^{2n} order. This polynomial has the following characteristics: there will be 2n roots and those *2n* roots will be complex and distinct.

To solve the eigenvalue problem, a transformation of generalized coordinates for the state space, Ewins (1984) and Espíndola (1990), is proposed. A new *2n x 1* vector of coordinates *y(t)* is defined.

$$y(t) = \begin{Bmatrix} q(t) \\ \cdots \\ \dot{q}(t) \end{Bmatrix} \tag{24}$$

To represent the system of equations in the state space, consider the following equality:

$$M\dot{q}(t) - M\dot{q}(t) = 0 \tag{25}$$

resulting in

$$A\dot{y}(t) + By(t) = \{f_y(t)\} \tag{26}$$

where

$$A = \begin{bmatrix} C + G(\Omega_{rpm}) & \vdots & M \\ \cdots & \vdots & \cdots \\ M & \vdots & 0 \end{bmatrix}_{2nx2n}, \quad B = \begin{bmatrix} \overline{K}(\Omega) & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & -M \end{bmatrix}_{2nx2n} \quad and \quad \{f_y(t)\} = \begin{Bmatrix} f(t) \\ \cdots \\ 0 \end{Bmatrix}_{2nx1}.$$

Assuming that the homogeneous solution in the state space is

$$y(t) = \theta\, e^{st}$$

where $\theta = [\phi \;\; \vdots \;\; s\phi]^T = [\phi \;\; \vdots \;\; -\lambda\phi]^T$, and replacing it in Eq. (27), it follows that

$$[sA + B]\theta = 0\,. \tag{27}$$

Equation (27) represents an eigenvalue problem. Considering $\lambda = -s$, this problem is defined by:

$$B\,\theta = \lambda\, A\theta \tag{28}$$

where θ is denominated right eigenvector. Considering that A and/or B are not symmetrical matrices, the adjoint problem of eigenvalues must be solved:

$$B^T\, \psi = \lambda\, A^T\, \psi\,, \tag{29}$$

where $\lambda = -s = -i\overline{\Omega}$ and ψ the left eigenvector.

3.2. Orthogonality

It is shown that the following relations are satisfied:

$$[\psi_j]^T A[\theta_k] = a_j \delta_{jk} \tag{30}$$

$$[\psi_j]^T B[\theta_k] = b_j \delta_{jk} \tag{31}$$

with $\delta_{jk} \begin{cases} = 1 & se\ j = k \\ = 0 & se\ j \neq k \end{cases}$.

Starting from Eqs. (30) and (31), the following orthogonality properties in the configuration space are obtained (see Espíndola, 1990):

$$-\lambda_j \lambda_k \psi_j^T M\, \theta_K + \psi_j^T \overline{K} \theta_K = b_j \delta_{jk} \tag{32}$$

$$-(\lambda_j + \lambda_k)\psi_j^T M\, \theta_K + \psi_j^T \left(C + G\left(\Omega_{rpm}\right)\right)\theta_K = a_j \delta_{jk} \tag{33}$$

Equations (32) and (33) represent the orthogonality conditions as functions of matrices *M, C, G* and *K*. As the values of λ are complex, they can be represented by the real part δ_j and the imaginary part υ_j.

$$\lambda_j = \delta_j + i\upsilon_j \tag{34}$$

The eigenvalues are complex and different; on the other hand, they are in some way related for being in the state space. It is verified that the eigenvalues are formed by pairs λ_j and $-\lambda_j$. So, taking values of $j \neq k$, but with $\lambda_j = -\lambda_k$, and applying these values in the orthogonality relations above, the result is:

$$\lambda_j{}^2 = -\frac{\bar{k}_j}{m_j} = -\bar{\Omega}_j{}^2 = (i\bar{\Omega}_j)^2 \tag{35}$$

and, therefore, $\lambda_j = i\bar{\Omega}_j$. By definition:

$$-\lambda_j^2 = \bar{\Omega}_j^2 = \Omega_j^2\left(1 + i\eta_j\right) \tag{36}$$

So, $\lambda_j = \Omega_j\sqrt{1+i\eta_j}$, the natural frequency is $\Omega_j^2 = \mathrm{Re}\left(-\lambda_j^2\right)$ and the loss factor is $\eta_j = \mathrm{Im}\left(-\lambda_j^2\right)/\mathrm{Re}\left(-\lambda_j^2\right)$.

4. Campbell diagram

In this particular case, where the rotor is mounted on bearings with viscoelastic material, matrix A, which has the gyroscopic matrix, is a function of the rotation of the axis (Ω_{rpm}), and matrix B is complex and a function of the frequency (Ω). So, the eigenvalue problem is a function of the rotation and the frequency. That is, for a certain rotation of the rotor (Ω_{rpm} = cte), the eigenvalue problem is a function of the frequency and will be solved through an internal Campbell diagram, Ω_j x Ω, because $\bar{K}(\Omega) = K(\Omega)(1 + i\eta(\Omega))$. Starting from this internal Campbell diagram, considering $\Omega = \Omega_j$ and using a straight line that crosses the curves of the natural frequencies, the natural frequencies of the system are extracted in an equivalent way to Espíndola and Floody (1999). This process should be repeated for all the rotor rotations, resulting in a new final Campbell diagram, now (Ω_j x Ω_{rpm}), containing the critical rotations of the viscoelastic dynamic rotor. Starting from this final Campbell diagram, it is possible to determine the dynamic characteristics of the viscoelastic rotor system. "Figure 4" shows an outline of how the Campbell diagrams are built. "Figure 4b" represents the internal Campbell diagram and "Fig. 4a" the final result. As can be observed, to calculate the natural frequencies of the system for a constant rotation, it is necessary to solve an eigenvalue problem, function of the frequency due to the characteristic stiffness matrix.

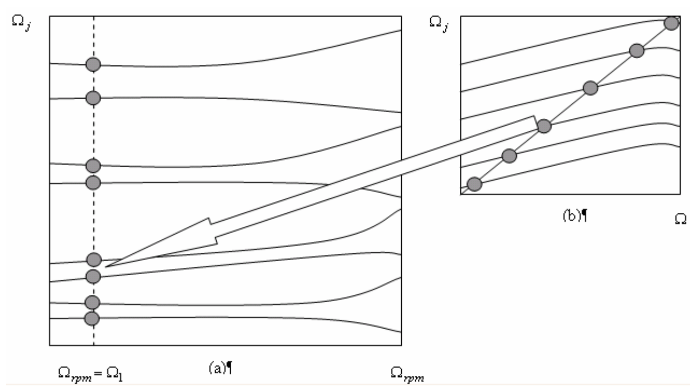

Figure 4. a) Final Campbell. b) Internal Campbell.

For the adjoint engenvalue problem it is necessary to consider that A and/or B are nonsymmetrical. The schematic diagram below shows how the eigenvalue problem and the adjoint engenvalue problem should be solved for a rotating system with viscoelastic bearings.

$$\left[\begin{array}{l} \quad \textit{Rotation loop } (\Omega_{rpm}) \\ \left[\begin{array}{l} \quad \textit{Frequency loop } (\Omega) \\ \qquad B\theta = \lambda A\theta \\ \qquad B^T\psi = \lambda A^T\psi \\ \textit{Internal Campbell diagram} \end{array}\right. \\ \quad \textit{Final Campbell diagram} \end{array}\right.$$

In the final Campbell diagram, two straight lines can be represented by by the following equations: $\Omega_1 = \Omega_{rpm} = \pi N/30$ and $\Omega_2 = \Omega_{rpm} = (\pi/2)(N/30)$, where N is the rotor spin velocity (rpm). These two straight lines, used to find out the critical rotations, represent the most frequent excitation forces that happen in practice: unbalanced mass, whose frequency matches the rotation of the axis (straight line Ω_1) and the excitation produced by the instability of the hydrodynamic bearings, which has a frequency approximately equal to the half of N (straight line Ω_2).

A numeric example of a simple dynamic rotor mounted on roller bearings and layers of viscoelastic material is presented. The dynamic characteristics of the viscoelastic material (pure butyl rubber) were determined in the Laboratory of Vibrations and Acoustics of the Federal University of Santa Catarina (LVA-UFSC/PISA). A fractional derivative model of four parameters was used to characterize dynamically the behavior of that pure butyl rubber. These parameters are, for temperature $T_0=273,0\ K$: $G_L = 1,53 \times 10^6\ Pa$; $G_H = 1,11 \times 10^8\ Pa$; $\alpha = 0,396$; and $b_1 = 1,34 \times 10^{-2}$.

"Figure 5" shows the dynamic shear modulus and of the loss factor as functions of frequency and temperature.

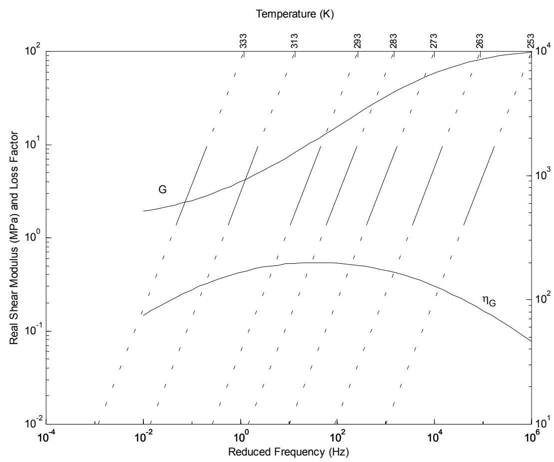

Figure 5. Dynamic properties – Pure butyl rubber.

Shaft data:	**Disk data:**
Total length, L = 1,001 m;	External radius, R_{ext} = 0.125 m;
Diameter, d = 0,0254 m;	Internal radius, R_{int} = d/2 m;
Modulus of Elasticity of the steel, E=210e9 MPa;	Density of the steel, ρ = 7800 Kg/m^3.
Coefficient of Poisson, nu = 0,3;	**Viscoelastic material data:**
Density of the steel, ρ= 7800 Kg/m^3.	Shear modulus and loss factor are given by Eqs. (15), (16), (17) and "Fig. 5".

"Figure 6" shows the final Campbell diagram and the straight line Ω_1 (unbalanced mass type excitation) that determines, amongst all the natural frequencies of the system, the ones that participate in the response to the unbalanced mass and are knwon as the critical rotations of the rotor. Given that the modal parameters of the system are cut by this straight line, it is possible to find out the response of the rotationg system for this kind of excitation. Therefore, it is observed that the proposed methology allows the determination of the dynamic characteristics of a rotor the stffness matrix of which varies with frequency. This variation with frequency is due to the introduction of the viscoelastic material under the bearings.

5. Conclusions

A simple and accurate methodology to find out the final Campbell diagram of a dynamic rotor, mounted on bearings made of viscoelastic material, was presented and implemented in a numeric example. Due to the characteristics of the stiffness matrix of this system, it is necessary to assembly two Campbell diagrams, one inside the other, to determine the overall dynamic behavior.

The viscoelastic material used in the bearings was represented by the four parameter fractional derivative model.

A numeric simulation showed the steps to be followed to obtain the Campbell diagram. Once the dynamic characteristics are known, it is possible to predict the response in distinct points and desing a control system, among other control actions.

6. Acknowledgements

This work was partially supported by WEG, to which the authors are grateful.

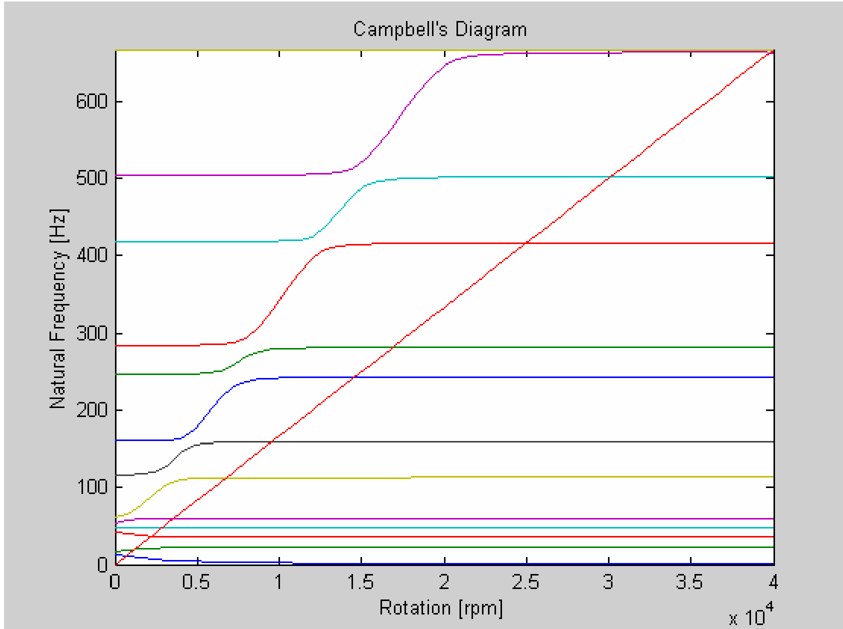

Figure 6. Final Campbell Diagram

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