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NONLINEAR DIRECTIONAL CONTROL APPLIED TO A GROUND VEHICLE

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Abstract. *The primary objective of this paper is to present the initial steps towards an alternate control strategy for the well known vehicle steering problem. We begin by introducing the reasoning of a nonlinear four degrees of freedom dynamic model for a ground vehicle. Together, some simulations to validate the model are presented, reflecting the good decisions made during the modeling process. Once the model has been secured, an alternate control strategy based on feedback linearization of state variables is introduced. This step consists of theoretical development and stability analisys, seeking the validation of the controller designed. Once it has been proved efficient, some of the results attained can be then presented, and shall illustrate the true potentiality of such approach. Closing this first description presented here, conclusions and future work ideas are enlisted, giving a better notion of the hole development work yet to be done.*

Keywords*: Nonlinear Control, Vehicle Dynamics, Feedback Linearization, Dynamic Systems Modeling*

1. Introduction

The present work is based on a master dissertation result (Spinola, 2003), which combines vehicle dynamics and control theory for nonlinear systems. A study on vehicle dynamics modeling was done first, so that it was possible to achieve a simplified model still able of representing the characteristics of a real vehicle. For this first attempt, small angles and safe constant velocities were used, which allowed simplifications to the model. But the main information for a controller, the global positioning of the vehicle, was kept unchanged and resulted in nonlinear information that put away classical controls normally used in controlling lateral vehicle dynamics (Will and Zak, 1997) (Smith and Starkey, 1994 and 1995). A second part of the work suggests a control strategy able of solving the nonlinearities of the model. The choice has fallen to the Feedback Linearization of state variables approach, using Lie algebra (Slotine and Li, 1991) and the geometric concept of the Central Manifold theory (Isidori, 1989) as mathematical base. Once the control was built, computational simulations were prepared providing results that illustrate the potentiality of this technique. These very same results are presented throughout this paper.

The text will be divided into five main sections considering: a brief introduction; the vehicle model adopted; the control strategy used together with the control development; results showing the potentiality of the technique, introduced to illustrate what has been achieved so far; and at last, conclusions and some future work propositions giving the reader a perspective of what is still to come at this research.

2. Four Degrees of Freedom Model

The model used here considers an on-road vehicle that should have its lateral motion controlled. The first assumption to build the model consists of not considering pitch or bounce displacements, supposing that there aren't huge changes in longitudinal acceleration and the road is a smooth surface (Will and Zak, 1997). This simplifies the equations to be derived as well as decreases the degrees of freedom that represent the system. This particular model is composed of four degrees of freedom, namely lateral and longitudinal displacements together with yaw and roll angles.

The vehicle is modeled considering the presence of three distinct mass bodies, the chassis also called sprung mass or m_s, and the two masses corresponding to wheels, tires and some parts of the suspensions, at the front and rear of the vehicle. They are called unsprung masses and are represented by the constants m_{uf} and m_{ur}, for the front and rear respectively. The sum of all three mass bodies corresponds to the total mass of the vehicle, m_{tot}.

To obtain the equations that describe the dynamical behavior of the open loop system it is necessary to proceed with the balance of forces and moments, as illustrated in Figure 1.

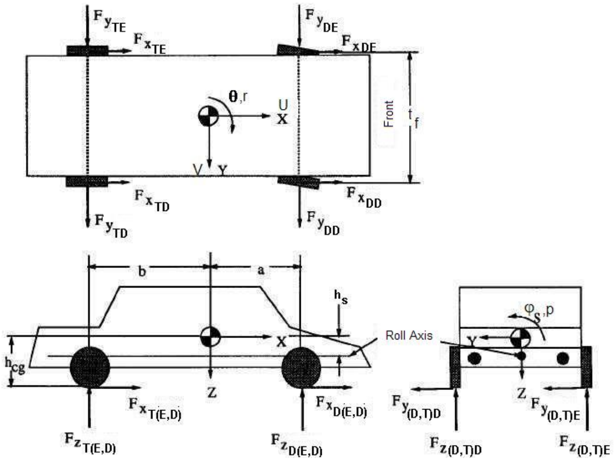

Figure 1. Distribution of forces and moments in a ground vehicle

The result of this equilibrium gives Eq. (1), the first model representation, where x represents the longitudinal displacement, y the lateral displacement, θ the yaw angle and φ the roll angle, all in local coordinates at the vehicle's center of gravity. It is possible to see at the fourth relation of Eq. (1) that there are a damping and a stiffness term, which represent the suspension effects. These effects are mostly due to the rebalance of the normal force at each tire and characterize some part of the vertical dynamics. There is also a term corresponding to gravitational influence as well as another one corresponding to yaw influence.

$$\begin{bmatrix} m_{tot} & 0 & 0 & 0 \\ 0 & m_{tot} & 0 & m_s h_s \\ 0 & 0 & I_z & 0 \\ 0 & m_s h_s & 0 & I_{roll} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} m_{tot}\dot{y}\dot{\theta} \\ -m_{tot}\dot{x}\dot{\theta} \\ 0 \\ -\beta_{roll}\dot{\varphi} - \kappa_{roll}\varphi + m_s g h_s \varphi - m_s h_s \dot{x}\dot{\theta} \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \\ \Gamma \\ 0 \end{bmatrix} \tag{1}$$

Still looking at Eq. (1), it can be seen that the system has three excitation functions, namely F_x, F_y and Γ. These inputs represent the reactions to the loads applied directly on the tires and to the tire-ground interaction, which are functions of the system inputs (Wong, 1993): the frontal steering angle (δ_f), the rear steering angle (δ_r) and brake pedal displacement (δ_b). The constants that appear in Eq. (1) represent the distance to roll axis from center of gravity (h_s), the moments of inertia around Z and roll axis (I_z and I_{roll}), the damping coefficient (β_{roll}), the stiffness coefficient (κ_{roll}) and gravitational acceleration (g). The equations for the excitation functions are seen in Eqs. (2), (3) and (4), where L_{DE}, L_{DD}, L_{TE} and L_{TD} are the lateral forces applied directly at the tires, C_{DE}, C_{DD}, C_{TE} and C_{TD} represent the cornering stiffness coefficients of the tires, t_f and t_r are the frontal and rear tracks and a and b are the frontal and rear distances from the center of gravity to the final drives.

$$F_x = -(C_{DE} + C_{DD} + C_{TE} + C_{TD})\delta_b - (L_{DE} + L_{DD})\delta_f - (L_{TE} + L_{TD})\delta_r \tag{2}$$

$$F_y = -(C_{DE} + C_{DD})\delta_b\delta_f - (C_{TE} + C_{TD})\delta_b\delta_r + (L_{DE} + L_{DD} + L_{TE} + L_{TD}) \tag{3}$$

$$\begin{aligned} \Gamma = &-\delta_b\left[C_{DE}\left(a\delta_f + \frac{t_f}{2}\right) + C_{DD}\left(a\delta_f - \frac{t_f}{2}\right) - C_{TE}\left(b\delta_r - \frac{t_r}{2}\right) - C_{TD}\left(b\delta_r + \frac{t_r}{2}\right)\right] \\ &+ a\left(L_{DE} + L_{DD}\right) - b\left(L_{TE} + L_{TD}\right) + \frac{t_f\delta_f}{2}\left(L_{DD} - L_{DE}\right) + \frac{t_r\delta_r}{2}\left(L_{TD} - L_{TE}\right) \end{aligned} \tag{4}$$

As the main goal of this study is to analyze the control strategy, more simplifications to the model are suggested and consists in considering the vehicle at a constant speed U. As the longitudinal velocity is constant, one of the inputs, relating to the brake pedal displacement, can be neglected, as no brake will be used. Also the heights for roll axis and unsprung masses are set to zero, making the vehicle similar to a rigid body on tires considering suspension stiffness and damping effects. And a last assumption, the vehicle can steer only the frontal wheels, taking another input, the rear steer angle, to zero. After these assumptions it is possible to rewrite the equations for the excitation functions, F_x, F_y and Γ, in Eqs. (5), (6) and (7).

$$F_x = -\delta_f 2C_f\left(\delta_f - \frac{v + ar}{U}\right) \tag{5}$$

$$F_y = 2C_f\left(\delta_f - \frac{v + ar}{U}\right) + 2C_r\left(\frac{br - v}{U}\right) \tag{6}$$

$$\Gamma = 2aC_f\left(\delta_f - \frac{v + ar}{U}\right) - 2bC_r\left(\frac{br - v}{U}\right) \tag{7}$$

where C_f and C_r stand for the frontal and rear cornering stiffness tire coefficients, v is the lateral velocity and r is the yaw angular velocity, used as state variables to complete a state-space description of the model.

Until now all model development has been done considering the local coordinates, placed at the vehicle's center of gravity. This coordinate system is not consistent for control application, as it does not allow the correct identification of the vehicle's position in space. It is necessary to take the vehicle to a global coordinate system, fixed in space, and described in Eq. (8).

$$\begin{cases} \dot{X} = U\cos\theta - v\sin\theta \\ \dot{Y} = -U\sin\theta - v\cos\theta \end{cases} \tag{8}$$

With the addition of these two state variables, describing the positioning of the vehicle at a global coordinate system, it is possible to write the differential equations that describe the dynamic behavior of the vehicular system developed so far.

$$\begin{bmatrix} \dot{v} \\ \dot{\theta} \\ \dot{r} \\ \dot{Y} \\ \dot{X} \end{bmatrix} = \begin{bmatrix} -Ur + \dfrac{F_y}{m_{tot}} \\ r \\ \dfrac{\Gamma}{I_z} \\ -U\sin\theta - v\cos\theta \\ U\cos\theta - v\sin\theta \end{bmatrix} \tag{9}$$

Equation (9) represents the model to be used at the nonlinear controller design, which will be seen in the next section. Note that Eq. (9) already presents the simplifications discussed earlier, which significantly reduced the size of the state-space model representation.

3. Nonlinear control: feedback linearization of state variables in the input-output sense

Here we begin to deal with the controller design itself, by using a well-known nonlinear technique based on concepts such as Lie algebra (Slotine and Li, 1991) and Center Manifold theory (Isidori 1989), that haven't yet been applied to ground vehicles' situations. Consider the system described in Eq. (10), where the terms that bind input and state variables *f(x)*, output and state variables *h(x)* and the input terms *g(x)* are clearly identified.

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{10}$$

The idea is to find a differential linear relation between the output y of the system and its input u, such that all nonlinear terms, present at the dynamical equation, are cancelled. To achieve this proposition it is necessary to differentiate *h(x)* a number of times so the input u explicitly appears at the new expression. Once an algebraic relation has been found, it is possible to design the input u to cancel the nonlinearities of *f(x)* and *g(x)*, of Eq. (10). The number of times *h(x)* is differentiated corresponds to the relative degree r of the system. In case r is equal to the system degree n, it is possible to say that all dynamics are observable and, therefore, controllable. If r is not equal to n, there are n-r degrees of freedom, representing an internal dynamics, which is not observable, and then cannot be controlled. For such cases it is necessary to perform a further stability analysis on the internal dynamics to verify whether it is stable or not. For the stability case, the controller can then be designed to control the observable states. If the internal dynamics is not stable, no matter what controller is designed, the system cannot be controlled.

We begin the feedback linearization analysis, in the input-output sense, by placing the system with known degrees of freedom in a region Ω_x, open at the state variables space. The next step is to find a linear input-output relation differentiating the output of Eq. (10).

$$\dot{y} = \nabla h(f + gu) = L_f h(x) + L_g h(x)u \tag{11}$$

For the case where $L_g h(x) \neq 0$ for any $x = x_0$ in Ω_x, then, by continuity, the relation is verified also at a finite neighborhood of x_0, Ω. The new input can then be represented by Eq. (12) and will result in a linear relation among $\dot{y}$ and $\eta(x)$, presented in Eq. (13).

$$u = \frac{1}{L_g h(x)}\left(-L_f h(x) + \eta(x)\right) \tag{12}$$

$$\dot{y} = \eta(x) \tag{13}$$

For the case where $L_g h(x) = 0$ for all x in Ω_x, more differentiations on Eq. (11) will be needed, until a direct and linear input-output relation is obtained. This procedure is illustrated in Eq. (14).

$$y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} h(x)u\,, \tag{14}$$

where the integer r represents the relative degree of the system, such that $L_g L_f^{r-1} h(x) \neq 0$ for some $x = x_0$, in Ω_x. So, by continuity, the linear relation is verified into a finite neighborhood of x_0, Ω. The control law for the system located in Ω can be expressed by Eq. (15).

$$u = \frac{1}{L_g L_f^{r-1} h(x)}\left(-L_f^r h(x) + \eta(x)\right) \tag{15}$$

Combining Eqs. (5), (6), (7) and (9) into (10) it is possible to write the system in the form of a vector field.

$$f(x) = \begin{bmatrix} v\left(\dfrac{-2C_f - 2C_r}{m_{tot}U}\right) + r\left(\dfrac{2bC_r - 2aC_f}{m_{tot}U} - U\right) \\ r \\ v\left(\dfrac{2bC_r - 2aC_f}{I_z U}\right) + r\left(\dfrac{-2b^2C_r - 2a^2C_f}{I_z U}\right) \\ -U\sin\theta - v\cos\theta \end{bmatrix} = \begin{bmatrix} va_1 + ra_2 \\ r \\ vb_1 + rb_2 \\ -U\sin\theta - v\cos\theta \end{bmatrix} \tag{16}$$

$$g(x) = \begin{bmatrix} \dfrac{2C_f}{m_{tot}} \\ 0 \\ \dfrac{2aC_f}{I_z} \\ 0 \end{bmatrix} = \begin{bmatrix} a_3 \\ 0 \\ b_3 \\ 0 \end{bmatrix} \tag{17}$$

$$h(x) = Y \tag{18}$$

Applying Eq. (9) it is possible to develop a control law to cancel the nonlinearities of the system. Deriving the output of the system allows achieving a relative degree ***r*** equal to 2. This reasoning is best seen along Eqs. (19) to (24). As the system's degree is 4, it is characterized that there exist an internal dynamics of 2nd order, which must be analyzed in order to check stability of the system.

$$y = h(x) = Y \tag{19}$$

$$\nabla h(x) = \begin{bmatrix} \dfrac{\partial h(x)}{\partial v} & \dfrac{\partial h(x)}{\partial \theta} & \dfrac{\partial h(x)}{\partial r} & \dfrac{\partial h(x)}{\partial Y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

$$\dot{y} = L_f h(x) + L_g h(x)u = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ -U\sin\theta - v\cos\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ 0 \end{bmatrix}u \tag{21}$$

$$\dot{y} = -U\sin\theta - v\cos\theta = L_f h(x) \tag{22}$$

$$\nabla L_f h(x) = \begin{bmatrix} \dfrac{\partial L_f h(x)}{\partial v} & \dfrac{\partial L_f h(x)}{\partial \theta} & \dfrac{\partial L_f h(x)}{\partial r} & \dfrac{\partial L_f h(x)}{\partial Y} \end{bmatrix} = \begin{bmatrix} -\cos\theta & -U\cos\theta + v\sin\theta & 0 & 0 \end{bmatrix} \tag{23}$$

$$\ddot{y} = L_f^2 h(x) + L_g L_f h(x) u = -\left[va_1 + \left(a_2 + U\right)r\right]\cos\theta + vr\sin\theta - a_3\cos\theta u \tag{24}$$

In order to define whether the system can be controlled, it is necessary to rewrite the system equations such that the internal dynamics becomes explicit. Then it is necessary to find a coordinate transformation in a neighborhood Ω of the origin point x_0, a diffeomorphism, that converts the system from its original form into a new one, called the normal form. This transformation consists of identifying the eigenvalues of the system that are linked to the internal dynamics, decoupling them from the rest of the system. To do so it is necessary to rewrite the system using the directional derivatives of the output function as part of the new state variables and, for the case where the relative degree is smaller than the system's degree, more *n-r* functions must be suggested to complete the normal form, as presented in Eqs. (25) and (26).

$$\dot{\mu} = \begin{bmatrix} \mu_2 \\ -\left[va_1 + \left(a_2 + U\right)r\right]\cos\theta + vr\sin\theta - a_3\cos\theta u \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} \tag{25}$$

$$\dot{\psi} = w(\mu, \psi) \tag{26}$$

The diffeomorphism is presented in Eq. (27).

$$\phi(x) = \begin{bmatrix} \mu_1 & \mu_2 & \psi_1 & \psi_2 \end{bmatrix}^T \tag{27}$$

To prove that $\phi(x)$ is a diffeomorphism it is necessary to verify if its Jacobian matrix defined in some region Ω of $\Re_4$ is invertible at a point $x = x_0$ of Ω. But first it is necessary to propose the gradients ψ_j and complete the diffeomorphism. These gradients need to be linearly independent and satisfy Eq. (28).

$$\nabla\psi_j g = 0 \quad 1 \le j \le n - r \quad \therefore \quad n - r = 4 - 2 = 2 \tag{28}$$

Also, Eq. (28) must comply with the so called curl conditions, which consists in a formal approximation to obtain a Lyapunov function through the adoption of a specific form for its gradient $\nabla V(x)$, instead of the function $V(x)$ itself. Suppose the scalar function $V(x)$ is related to its gradient by the integral expression presented in Eq. (29).

$$V(x) = \int_0^x \nabla V dx \quad \therefore \quad \nabla V = \left\{ \frac{\partial V}{\partial x_1} \quad \cdots \quad \frac{\partial V}{\partial x_n} \right\}^T \tag{29}$$

To make possible the statement of a unique scalar function $V(x)$, the gradient function must satisfy Eq. (30), which is the curl condition itself.

$$\frac{\partial V_i}{\partial x_j} = \frac{\partial V_j}{\partial x_i} \quad \left(i, j = 1,2,\ldots,n\right) \tag{30}$$

Applying Eqs. (28) and (30) to the model studied here, where $n\text{-}r = 2$, the gradient equations for the two modes representing the internal dynamics are presented in Eq. (31) bellow.

$$\begin{aligned} \nabla\psi_1 g = 0 \quad &\therefore \quad \nabla\psi_1 = a_{11}v + a_{12}\theta + a_{13}r + a_{14}Y \\ \nabla\psi_2 g = 0 \quad &\therefore \quad \nabla\psi_2 = a_{21}v + a_{22}\theta + a_{23}r + a_{24}Y \end{aligned} \tag{31}$$

where the coefficients a_{ij} are to be determined. If Eq. (31) is written in Lie algebra form, the reasoning to achieve a particular solution can be expressed and is presented along Eqs. (32) to (35).

$$L_g\psi_1 = 0 = \frac{\partial\psi_1}{\partial v}a_3 + \frac{\partial\psi_1}{\partial r}b_3 \tag{32}$$

$$\psi_1 = b_3 v - a_3 r \tag{33}$$

$$L_g \psi_2 = 0 = \frac{\partial \psi_2}{\partial \theta} 0 \tag{34}$$

$$\psi_2 = \theta \tag{35}$$

It is possible to write the diffeomorphism, expressed in Eq. (36), with the combination of Eqs. (25), (33) and (35), to begin the stability analysis.

$$\phi = \begin{cases} \mu_1 = Y \\ \mu_2 = -U sin\theta - v\cos\theta \\ \psi_1 = b_3 v - a_3 r \\ \psi_2 = \theta \end{cases} \tag{36}$$

The next step is to check if the Jacobian matrix of the diffeomorphism $\phi(x)$, presented in Eq. (37), is invertible, validating the coordinate transformation.

$$\frac{\partial \phi}{\partial x} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -\cos\theta & -U\cos\theta + v sin\theta & 0 & 0 \\ b_3 & 0 & -a_3 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{37}$$

As the reader can verify the Jacobian matrix in Eq. (37) is invertible and so $\phi(x)$ is a diffeomorphism. The original system described in Eq. (9) can then be written in normal form, presented in Eq. (38), which allows the stability test and the design of the control law that will lead the system to a desired trajectory.

$$\begin{cases} \dot{\mu}_1 = \mu_2 \\ \dot{\mu}_2 = \left[a_1 + \frac{b_3}{a_3}(a_2 + U)\right](U sen\psi_2 + \mu_2) + (\psi_1 - a_3\mu_1)\cos\psi_2 + \frac{sen\psi_2}{a_3}\left[b_3\left(\frac{U sen\psi_2 + \mu_2}{\cos\psi_2}\right)^2 + \psi_1\left(\frac{U sen\psi_2 + \mu_2}{\cos\psi_2}\right)\right] \\ \dot{\psi}_1 = \left[(a_3 b_1 - a_1 b_3) + (a_3 b_2 - a_2 b_3)\frac{b_3}{a_3}\right]\left(\frac{U sen\psi_2 + \mu_2}{\cos\psi_2}\right) - \frac{\psi_1}{a_3} \\ \dot{\psi}_2 = \frac{-b_3(U sen\psi_2 + \mu_2)}{a_3\cos\psi_2} - \frac{\psi_1}{a_3} \end{cases} \tag{38}$$

To perform the stability analysis the Center Manifold theory (Isidori, 1989) is used. According to it there exists a surface M_0, asymptotically stable, over which the internal dynamics modes can be positioned, after the coordinate transformation has been done. It is possible then to excite these modes and track their behavior along a period of time. If the trajectories generated at the surface M_0, with initial conditions at the surface, remain on the surface, the internal dynamics is said to be asymptotically stable and the system can be controlled. If not, the system is unstable and no controller can be designed. It gets easier to understand this approach if we understand that the diffeomorphism takes the original system to the surface M_0 itself. Once we've found a diffeomorphism that satisfies all conditions of linear independency and that also obeys to the curl conditions, the stability test becomes much simpler. Going back to the system defined in Eq. (38), the stability analysis consists in maintaining all $\mu(x)$ modes (the observable and therefore controllable modes) equal to zero while testing the $\psi(x)$ ones. Given an initial condition where $\psi_1(0) = \psi_2(0) = 0$, the system in normal form should remain at zero for all time instants while the test takes place (Spinola, 2003).

It is now possible to design the control law that will lead the lateral dynamics of the vehicle to a desired path. Going back to Eq. (15), the control law that cancels the nonlinearities of the system is presented in Eq. (39) and gives the linear relation among the input and output of the system presented in Eq. (40).

$$u = \frac{1}{L_g L_f}\left(-L_f^2 h(x) + \eta(x)\right) = \frac{-\left[v a_1 + (a_2 + U)r\right]\cos\theta + v r sin\theta - \eta(x)}{a_3\cos\theta} \tag{39}$$

$$\ddot{y} = \eta(x) \tag{40}$$

One last step is to define the polynomial $\eta(x)$, which must have all roots with negative real part as to keep the eigenvalues of the new closed loop system in the stability region. As the relative degree of the system is equal to 2, the polynomial needs to be of 2nd degree. Thus, only 2 roots need to be chosen. But before doing so, it must be clear that the control law expressed in Eq. (39) is responsible only for asymptotic stabilizing the system. The main objective of this work is to develop a controller to act on the ground vehicle and make it track a desired trajectory, i.e., a well-known road, street or even an off-road path.

Instead of stabilizing the output of the system, the controller needs to asymptotic stabilize the stationary error *e(t)* between a desired trajectory $y_d(t)$ and the system output *y(t)*.

$$e(t) = y(t) - y_d(t) \tag{41}$$

Replacing Eq. (41) into the control law obtained in Eq. (39), a new control law for tracking purposes is suggested. As the modification is promoted only at the polynomial $\eta(x)$, no other stability analysis is needed, which allows to write the new control law immediately.

$$u(x) = \frac{1}{L_g L_f y}\left[-L_f^2 y + \ddot{y}_d - k_1(\dot{y} - \dot{y}_d) - k_0(y - y_d)\right] \tag{42}$$

$$u(x) = \frac{-(a_1 + k_1)}{a_3} v - \frac{(a_2 + U)}{a_3} r + \frac{(vr - k_1 U)}{a_3} tg\theta + \frac{k_0(Y - y_d)}{a_3 \cos\theta} - \frac{k_1 \dot{y}_d}{a_3 \cos\theta} + \frac{\ddot{y}_d}{a_3 \cos\theta} \tag{43}$$

where k_1 and k_0 correspond to the sum and the product of the 2nd degree polynomial $\eta(x)$ whose roots are the closed loop system poles.

4. Simulation results

To validate the model and the controller developed here, some simulations were done. They consist in trying to force the vehicle into a lane change. For this test it was supposed a constant velocity of *18.3 m/s*, together with the coefficient values in Table 1.

Table 1. Constant values used during simulations

Constant	Value
C_f	20000 N/rad
C_r	20000 N/rad
m_{tot}	1280 kg
A	1.203 m
B	1.217 m
I_z	2500 kgm^2

The action of changing lanes corresponds to give successive but opposite steering angles with the same amplitude and as step inputs, just like illustrated in Figure 3. These same step inputs are used as excitations for the open loop model of Eq. (9) and should trigger a lane change to the right, if one is looking to the vehicle from its top view. As can be seen in Figure 2, the expected change occurs validating the model developed here. It is now ready for use in the controller simulations.

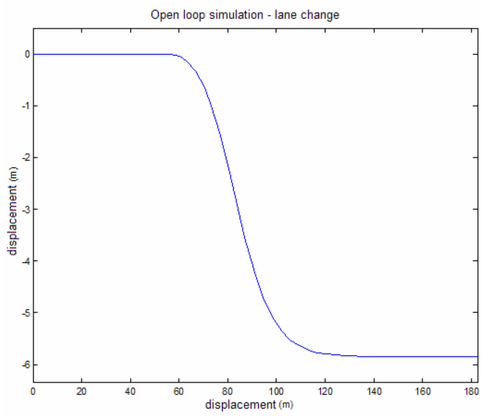

Figure 2. Simulation of the open loop model characterizing a right lane change

Another important condition for a smooth simulation is the regularity of the trajectory to be followed by the vehicle. This information will be used as an input to the controller so it can calculate the error. As the simulations consider a lane change maneuver, the input to the controller shall be as presented in Figure 4, which results from the simulations of a linear model simpler than the one of Eq. (9).

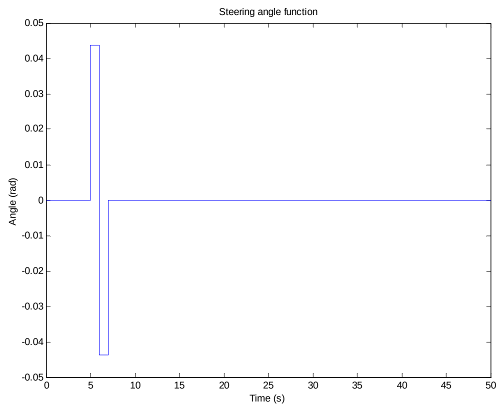

Figure 3. Input function for the frontal steering angle

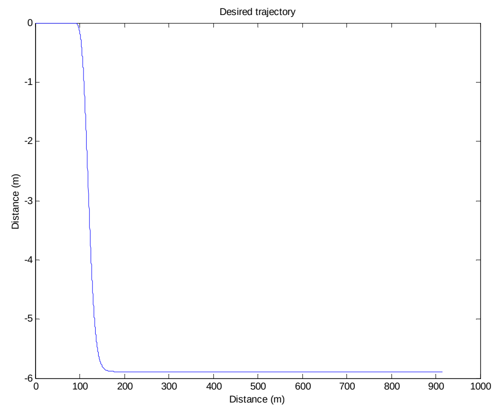

Figure 4. Function for the desired trajectory, input for the controller

The next step is to test the closed loop system, check if the controller works and if it is robust. For that, it is necessary to choose the 2 stable poles and complete the control law proposed in Eq. (43). For a first test the poles were placed at –2 and –5, producing a slow answer as can be seen in Figure 5.

To decrease the settling time of the system and enhance its response, the poles were chosen more to the left on the complex plane, which characterize a faster response. They were placed at –4 and –10, allowing a better but not yet optimal response, as can be seen in Figure 6.

A third test was done with poles at –16 and –40, which obtains the expected answer. It can be seen in Figure 7 that an optimal track of the desired trajectory was achieved, validating the technique discussed throughout this work.

To verify the robustness of the nonlinear controller developed in section 3, a last test was performed. It consists in adding gaussian white noise with zero mean to the signals originated in the controller and at the measurement of the state variables, the most probable places for noise to appear. The results of this test showed the robustness of the controller, which has filtered some of the applied noise. It can be seen that the biggest noise influence is on the state variables responsible for the nonlinearities of the system, which are canceled by the control law. In presence of noise this cancellation is inaccurate, resulting in model oscillations, as seen in Figure 8 and in Figure 9.

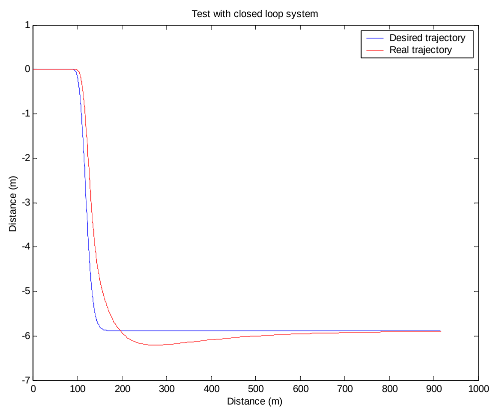

Figure 5. 1st test, with poles at –2 and –5

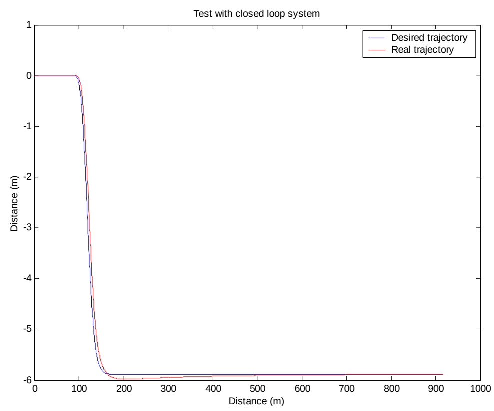

Figure 6. 2nd test with poles at –4 and –10

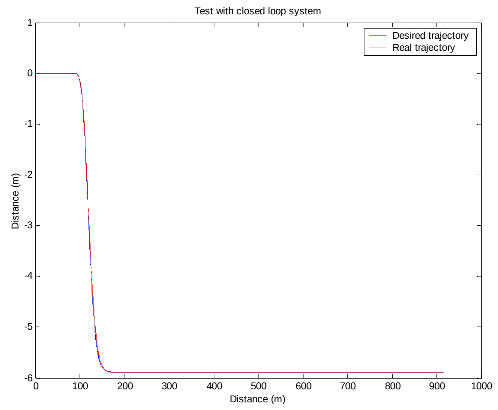

Figure 7. 3rd test with poles at –16 and –40

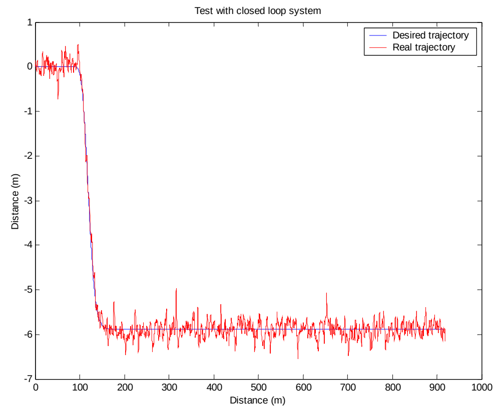

Figure 8. Test with noise at the measurement of state variable - SNR = -3dB

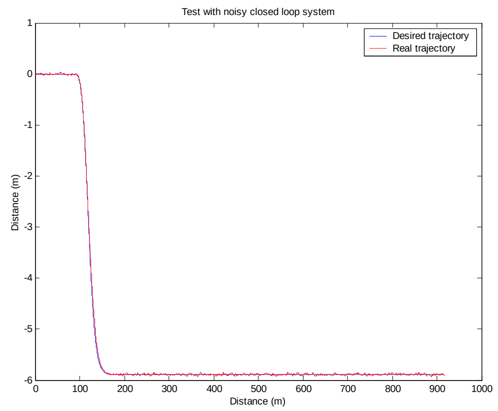

Figure 9. Test with noise at the controller output - SNR = -4dB

5. Conclusions and future works

The results attained here illustrate the feasibility of feedback linearization techniques into the development of nonlinear complex controllers, to be used widespread with all kinds of dynamic systems. More specifically for ground vehicles the results indicate very good tracking responses, even in presence of noise, making the controller capable of being spread to more complex dynamics. This is a very good perspective for, so far, situations up to now considered as difficult control tasks, like sudden change of lane, fast collision avoidance or very high-speed turning, were not dealt accordingly. Now there are good indications that those critical events could be dealt directly as described throughout this paper.

Future works on this subject comprise a deeper study of vehicle dynamics, development and validation of more realistic models, and the application of the feedback linearization technique to achieve more accurate control of complex dynamics.

6. References

Isidori, A., 1989, “Nonlinear Control Systems: an introduction”, Springer-Verlag, Berlin, Germany, 479p.

Slotine, J. –J. and Li, W., 1991, ”Applied Nonlinear Control”, Prentice Hall, Upper Saddle River, New Jersey, USA, 461p.

Smith, D. E. and Starkey, J. M., 1994, “Effects of Model Complexity on the Performance of Automated Vehicle Steering Controllers: Controller Development and Evaluation”, Vehicle System Dynamics, Vol. 23, pp. 627-645.

Smith, D. E. and Starkey, J. M., 1995, “Effects of Model Complexity on the Performance of Automated Vehicle Steering Controllers: Model Development, Validation and Comparison”, Vehicle System Dynamics, Vol. 24, pp. 163-181.

Spinola, A. L., 2003, “Modeling and Nonlinear Control of a Ground Vehicle’s Steering”, MSc Dissertation (in portuguese), Department of Electric Engineering of Pontifical Catholic University, Rio de Janeiro, Brazil, 140p.

Will, A. B. and Zak, S. H., 1997, “Modeling and Control of an Automated Vehicle”, Vehicle System Dynamics, Vol. 27, pp. 131-155.

Wong, J. Y., 2001, “Theory of Ground Vehicles”, John Wiley & Sons, 3rd edition, New York, USA, 560p.

7. Responsibility notice

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