Proceedings of the XI DINAME, 28th February-4th March, 2005 - Ouro Preto - MG - Brazil
Edited by D.A. Rade and V. Steffen Jr.

OPERATIONAL VIBRO-ACOUSTIC MODAL ANALYSIS: MODE SHAPE NORMALISATION

Gert De Sitter
Department of Mechanical Engineering
Vrije Universiteit Brussel (VUB)
Pleinlaan 2, 1050 Brussels, Belgium
email: Gert.De.Sitter@vub.ac.be

Eli Parloo
Department of Mechanical Engineering
Vrije Universiteit Brussel (VUB)
Pleinlaan 2, 1050 Brussels, Belgium

Patrick Guillaume
Department of Mechanical Engineering
Vrije Universiteit Brussel (VUB)
Pleinlaan 2, 1050 Brussels, Belgium

Abstract. *For some years Operational Modal Analysis (OMA) has been used to get the mode shapes, damping factors and resonance frequencies of structures. One of the main advantages of OMA is that the test is done in operational conditions: the model will be linearised in much more representative working points compared to structures tested in laboratory conditions. Using a sensitivity rescaling technique it also became possible to estimate the correct scaling factors for structural modal analysis. Recently this rescaling technique used to estimate the correctly scaled structural mode shapes in operational conditions has been extended to the acoustic domain. Now it is possible to get a scaled acoustic modal model without the use of a volume-acceleration sound source. For coupled vibro-acoustic modes both methods can be used to rescale these modes. In this contribution the combination of the structural rescaling technique and the acoustic technique will be explained and validated by means of experimental results.*

Keywords. *operational modal analysis, vibro-acoustics, sensitivity analysis.*

1 Introduction

During the last years Operational Modal Analysis (OMA) has been frequently used to estimate the resonance frequencies, the damping factors and unscaled mode shapes of many structures (Hermans and Van der Auweraer, 1999; Parloo et al., 2003). One of the reasons is that the real boundary conditions are present. They often differ significantly from the ones in laboratory testing. As all real-world systems are to a certain extent non-linear, the models obtained under real boundary conditions will be linearised for much more representative working points. Another advantage is that the measurements can be performed in situ.

More recently one has developed a technique which is based on a sensitivity analysis that makes it possible to get the correctly scaled mode shapes. The technique can for example be used for large civil structures. This has been proved by an experiment on a bridge (Parloo et al., 2004). Concerning this application OMA has the big advantage that one uses the ambient forces to excite the structure and one doesn't need drop-masses or other exciters which one still needs for classical forced-vibration tests.

Because OMA has proved to be useful for structural problems, it was interesting to check if there wasn't an acoustic equivalent. An important argument for developing such an acoustic equivalent was that for the classical Acoustic Modal Analysis (AMA) one needs volume acceleration sources which aren't widely available. Note also that when one uses sound sources (by performing a classical Acoustic Modal Analysis) one alters the acoustic system that one wants to model. These two arguments make Acoustic Operational Modal Analysis (AOMA) even more interesting than Structural Operational Modal Analysis (SOMA). It turned out that the acoustic equivalent makes use of local volume changes where for structural OMA one uses local mass changes to perform a sensitivity analysis (De Sitter et al., 2004).

The identification of vibro-acoustic modes is important for several applications. The sound level in all kinds of cavities (the interior of a car, train, airplane, etc.) must be minimised as much as possible. Concerning vibro-acoustic problems the combination of the two operational techniques makes it possible to get all the scaled mode shapes. Vibration modes can be rescaled using the Structural Operational Modal Analysis (SOMA) while the acoustic modes can be rescaled using the Acoustic Operational Modal Analysis (AOMA). For coupled modes both techniques can be used.

In this contribution we will first focus on the theoretical aspects of the sensitivity-based mode shape normalisation. The equivalences between the structural and the acoustic method will be discussed. Later on some experiments will be dealt with. These experiments will focus on the rescaling of structural and acoustic mode shapes.

2 Theoretical Aspects

2.1 Analogy between acoustic and mechanical systems

Before we focus on the sensitivity based rescaling technique it is interesting to briefly recapitulate the analogy between acoustic and mechanical systems. In the frequency-domain we get for dynamic undamped mechanical systems:

$$\left(-\omega^2 \cdot M^s + K^s\right) \cdot X(\omega) = F(\omega) \tag{1a}$$

with

M^s	the structural mass matrix	
K^s	the structural stiffness matrix	
$X(\omega)$	the displacement vector	(1b)
$F(\omega)$	the force vector	

For dynamic undamped acoustic systems this results in:

$$\left(-\omega^2 \cdot M^f + K^f\right) \cdot P(\omega) = \dot{Q}(\omega) \tag{2a}$$

with

M^f	the acoustic equivalence of the mass matrix	
K^f	the acoustic equivalence of the stiffness matrix	
$P(\omega)$	the pressure vector	(2b)
$\dot{Q}(\omega)$	the volume acceleration vector	

Note that the acoustic equivalence of the structural mass matrix in fact consists of acoustic compliances. This acoustic compliance is defined as the ratio between the volume displacement and the pressure. (Augusztinovicz and Sas, 1996; Sas and Augusztinovicz, 1999)

$$M^f{}_{kl} = \begin{cases} 0 & \text{for } k \neq l \\ \frac{V_{kl}}{\rho \cdot c^2} & \text{for } k = l \end{cases} \tag{3a}$$

with

M^f	the acoustic equivalence of the mass matrix	
V_{kk}	the volume of element k of the cavity	(3b)
c	the speed of sound	

One can prove that the FRF-matrix of an undamped structural system $H^s(s)$ can be calculated as (Heylen et al., 1997):

$$H^s{}_{kl}(\omega) = \sum_{n=1}^{N_m} \frac{\Phi^s{}_{k,n} \cdot \Phi^s{}_{l,n}}{\bar{m}^s_n \cdot (\omega_n^2 - \omega^2)} \tag{4a}$$

with

$\bar{m}^s_n$	the structural modal mass of mode n $= \{\Phi^s{}_n\}^t \cdot M^s \cdot \{\Phi^s{}_n\}$	
$\Phi^s{}_{k,n}$	the structural mode shape value of mode n at point k	(4b)
ω_n	the resonance frequency of mode n	
N_m	the number of modes	

Table 1: Analogy between acoustic and mechanical systems

Variable for Structural Modal Analysis	Variable for Acoustic Modal Analysis
Φ^s	Φ^f
M^s	M^f
K^s	K^f
X	P
F	$\dot{Q}$
$\bar{m}^s$	$\bar{m}^f$

The acoustic equivalent equation is (Kuttruff, 1979; Augusztinovicz, 2000):

$$H^f{}_{kl}(\omega) = \sum_{n=1}^{N_m} \frac{\Phi^f{}_{k,n} \cdot \Phi^f{}_{l,n}}{\bar{m}_n^f \cdot (\omega_n^2 - \omega^2)} \tag{5a}$$

with

$\bar{m}_n^f$ the acoustic equivalence of the modal mass of mode n
$= \left\{\Phi^f{}_n\right\}^t \cdot M^f \cdot \left\{\Phi^f{}_n\right\}$
$\Phi^f{}_{k,n}$ the acoustic mode shape value of mode n at point k (5b)
ω_n the resonance frequency of mode n
N_m the number of modes

If one compares (4) to (5) one can conclude that the same formula's can be used for the sensitivity analysis if one uses the correct variables. Tabel 1 shows the equivalent variables.

Note that $\frac{\bar{m}^f}{\rho \cdot c^2}$ is nothing less than the volume matrix of the cavity.

2.2 Sensitivity-based rescaling

For structural operational modal analysis a rescaling method already has been developed. (Parloo et al., 2002) This method has the big advantage that one can calculate the scaled mode shapes while one doesn't need to know the sources. This technique already has been used for several applications. (Parloo et al., 2003; Parloo et al., 2004)

The technique is based on the sensitivity analysis of modal parameters. For example, the resonance frequencies of undamped structural systems will shift by changing the mass matrix locally (e.g. by adding some mass m_i at point i) (Heylen et al., 1997):

$$\frac{\partial \omega_n}{\partial m_i} = -\omega_n \frac{\Phi^s{}_{i,n}^2}{2 \cdot \bar{m}_n^s} \tag{6}$$

Because the operational mode shapes Ψ^s aren't correctly scaled, one can write that each scaled mode shape $\Phi^s{}_{i,n}$ equals the operational mode shape $\Psi^s{}_{i,n}$ multiplied by a scaling factor α_n:

$$\Phi^s{}_{i,n} = \Psi^s{}_{i,n} \cdot \alpha_n \tag{7}$$

One can linearise (6) and combine it with (7). The scaling factor can be calculated by:

$$\alpha_n = \sqrt{\frac{-2 \cdot \bar{m}_n^s \cdot \Delta\omega_n}{\omega_n \cdot \left(\sum_{i=1}^{N_c} \Psi^s{}_{i,n}^2 \cdot \Delta M^s{}_i\right)}} \tag{8}$$

If one wants to use a mass-normalised scaling-scheme (8) becomes:

$$\alpha_n = \sqrt{\frac{-2 \cdot \Delta\omega_n}{\omega_n \cdot \left(\sum_{i=1}^{N_c} \Psi^s{}_{i,n}^2 \cdot \Delta M^s{}_i\right)}} \tag{9}$$

For Acoustic Operational Modal Analysis the same technique can be used. If one uses the acoustic equivalent variables that are listed in Table 1 equation (8) becomes:

$$\alpha_n = \sqrt{\frac{-2 \cdot \bar{m}_n^f \cdot \Delta\omega_n}{\omega_n \cdot \left(\sum_{i=1}^{N_c} {\Psi^f}_{i,n}^2 \cdot \Delta {M^f}_i\right)}} \tag{10a}$$

$$= \sqrt{\frac{-2 \cdot \frac{\bar{V}_n^f}{\cancel{\rho c^2}} \cdot \Delta\omega_n}{\omega_n \cdot \left(\sum_{i=1}^{N_c} {\Psi^f}_{i,n}^2 \cdot \Delta \frac{V_i}{\cancel{\rho c^2}}\right)}} \tag{10b}$$

$$= \sqrt{\frac{-2 \cdot \bar{V}_n^f \cdot \Delta\omega_n}{\omega_n \cdot \left(\sum_{i=1}^{N_c} {\Psi^f}_{i,n}^2 \cdot \Delta V_i\right)}} \tag{10c}$$

with

$$\bar{V}_n^f = \left\{{\Phi^f}_n\right\}^t \cdot V \cdot \left\{{\Phi^f}_n\right\} \tag{10d}$$

Now one can use for acoustic mode shapes the volume-normalised scaling-scheme. Equation (10) becomes:

$$\alpha_n = \sqrt{\frac{-2 \cdot \Delta\omega_n}{\omega_n \cdot \left(\sum_{i=1}^{N_c} {\Psi^f}_{i,n}^2 \cdot \Delta V_i\right)}} \tag{11}$$

From (11) one can conclude that one now needs to add or subtract some volume instead of some local mass. Doing this and measuring the frequency shift of the resonance frequencies one is able to calculate the scaling factors of the acoustic mode shapes. Note that the second measurement is only needed to estimate the frequency shift and that no mode shape values are needed. So it isn't necessary to measure in all the points! Note also that with this technique it is possible to calculate the FRF's but without using a volumetric acceleration sound source. In the next section some simulations and experiments will validate this technique.

3 Vibro-acoustic experiment

3.1 Vibro-acoustic set-up and results of first analysis

The structure used for the experiments was a box of dimensions $1.24 \times 0.36 \times 0.30$ m. Five surfaces of the box were made of wood while the sixth surface was an aluminium plate (3 mm). The front of the structure has been excited by a pneumatic jet. The vibrations of the aluminium plate were measured by a scanning laser vibrometer (PSV-300 Polytec). An accelerometer was used to measure the reference signal. Finally, the acoustical grid consisted of 40 points. A Maximum Likelihood Estimator was used to calculate the modal parameters. An overview of the natural frequencies is

(a) Outside (b) Inside (c) Measurement grid

Figure 1: Experimental setup

given in Table 2.

The estimated mode shapes show clearly that some modes are linked to the acoustical part of the system while other modes are the result of the vibrating plate. For instance, the mode found at 382 Hz is obviously a vibration mode (see Figure 2) while the mode at 278 Hz is an acoustic mode (see Figure 2). Some modes are clearly coupled modes, for example, the modes at 132 Hz, 148 Hz and 573 Hz (see Figure 2).

Table 2: Natural frequencies of the modes

Mode	Natural frequency found by output-only measurements (Hz)
1	131.6
2	148.1
3	277.5
4	299.6
5	381.8
6	423.4
7	474.1
8	499.5
9	542.5
10	550.2
11	564.0
12	573.2

3.2 Structural sensitivity analysis and results of second analysis

To estimate the scaled vibration mode shapes we added some mass and performed a second measurement. Using equation (9) we then found the mass-normalised mode shapes. In this experiment a mass of 10.24×10^{-3}kg was added at the front of the plate at the same location of the accelerometer, which was placed inside the structure.

The results can be found in Table 3. In this table we compared the estimated mode shape values in one point with the mode shape values which resulted from a classical structural modal analysis. The results confirmed that SOMA is an alternative to classical experimental modal analysis. In this experiment it turned out that the vibration modes and coupled acoustic modes could be scaled with an error smaller than 8 %.

Table 3: Results of the structural scaling procedure

Mode	mode frequency ω (Hz)	$\Delta\omega$ (Hz)	normalised mode shape (structural operational modal analysis)	normalised mode shape (experimental modal analysis)	error (%)
1	132.2	0.179	0.514	0.478	7.7
2	147.9	0.522	0.820	0.827	0.9
4	300.5	1.704	1.052	1.016	3.4
5	382.4	4.679	1.546	1.593	3.0

It is important to note that one has to consider which mass to use. A small mass will lead to very small shifts in frequency while heavy masses will induce important prediction errors since the first order approximation is only valid for small changes. Note that heavy masses also will change the stiffness (in theory the mass has to be located in a single point). During the experiment the mass led to a frequency shift of 0.18 Hz for the 132 Hz mode. This is to small in comparison with the estimation error. So it isn't surprising that the error of the calculated scaling factor is approximately 8 %. In fact different masses should be used for different frequency bands.

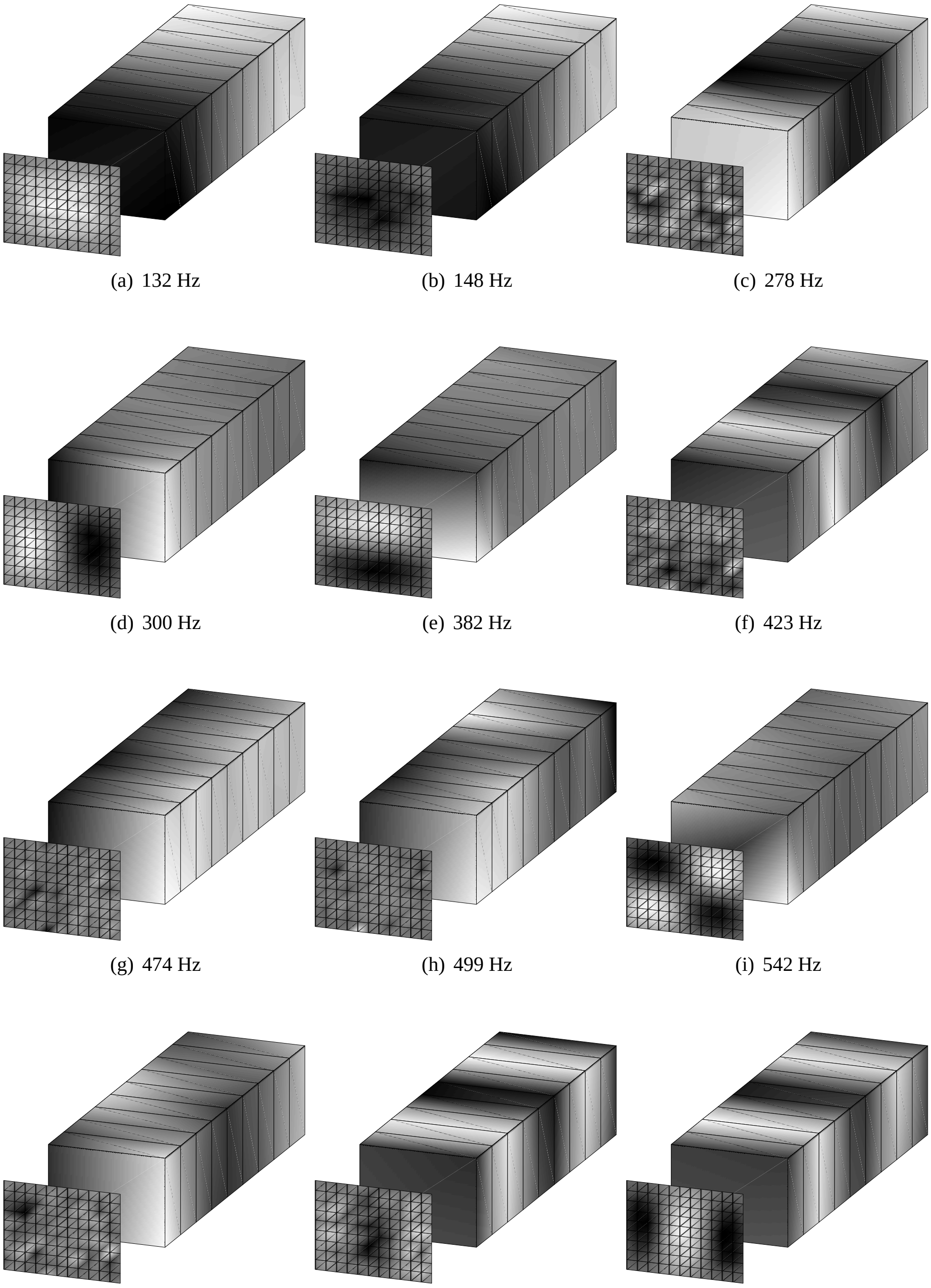
Another important remark follows from the observation that uncoupled acoustic modes (e.g. the mode of 278 Hz) cannot be rescaled by altering the structural system. To find the correctly scaled mode shapes we have to alter the acoustic system (by local volume changes).

3.3 Acoustic sensitivity analysis and results of third analysis

To estimate the scaled acoustic mode shape at 278 Hz we added some volume ($1.5 \times 10^{-3} m^3$) and performed a new measurement. The result of the rescaling is shown in Table 4. In this case the volume-normalised mode shapes were compared to the results of a FEM-analysis. the results confirmed that AOMA is an alternative to classical experimental acoustic modal analysis. In this experiment it turned out that the acoustic modes could be scaled with an error smaller than 2 %.

4 Conclusions

In this paper, it was shown that for vibro-acoustic analysis operational modal analysis is an alternative to classical modal analysis. The vibration, acoustic and combined modes can be rescaled using the specific methods. For vibration modes, rescaling is done by a sensitivity analysis which is based on altering the structural system by local masses while for acoustic modes it is based on changing the volume of the cavity. Some experiments showed that the errors are negligible



(a) 132 Hz (b) 148 Hz (c) 278 Hz

(d) 300 Hz (e) 382 Hz (f) 423 Hz

(g) 474 Hz (h) 499 Hz (i) 542 Hz

(j) 550 Hz (k) 564 Hz (l) 573 Hz

Figure 2: Modes between 0 Hz and 600 Hz

compared to the advantages. These advantages of OMA are that the real boundary conditions are present and that one doesn't need any known sources (e.g. volume acceleration sound sources).

5 Acknowledgements

This research has been supported by the Fund for Scientific Research - Flanders (Belgium) (FWO), the Institute for the Promotion of Innovation by Science and Technology in Flanders (IWT) and by the Research Council (OZR) of the Vrije Universiteit Brussel (VUB). Also the Concerted Research Action "OPTIMech" of the Flemish Community is gratefully acknowledged.

Table 4: Results of the acoustic scaling procedure

Mode	mode frequency ω (Hz)	$\Delta\omega$ (Hz)	normalised mode shape (acoustic operational modal analysis)	normalised mode shape (FEM)	error (%)
3	276.6	2.92	3.75	3.86	2.9

6 References

Augusztinovicz, F. (2000). *Isaac 12 course*, chapter Acoustic modal analysis. Katholieke Universiteit Leuven.

Augusztinovicz, F. and Sas, P. (1996). Acoustic modal analysis at low frequencies: similarities and differences in formulations. In *International Conference on Noise and Vibration Engineering (ISMA 21)*, pages 1685–1699.

De Sitter, G., Guillaume, P., and Parloo, E. (2004). Mode shape normalisation for operational acoustic modal analysis: theory and applications. In *Proceedings of the International Conference on Noise and Vibration Engineering (ISMA-2004)*.

Hermans, L. and Van der Auweraer, H. (1999). Modal testing and analysis of structures under operational conditions: Industrial applications. *Mechanical Systems and Signal Processing*, 13(2):193–216.

Heylen, W., Lammens, S., and Sas, P. (1997). *Modal Analysis Theory and Testing*. Katholieke Universiteit Leuven, Departement Werktuigkunde.

Kuttruff, H. (1979). *Room acoustics*, pages 309–309. Applied Science Publishers LTD, second edition.

Parloo, E., Cauberghe, B., Benedettini, F., Alaggio, R., and Guillaume, P. (2004). Sensitivity-based operational mode shape normalization: Application to a bridge. In *Proceedings of the 22th International Modal Analysis Conference*.

Parloo, E., Guillaume, P., Anthonis, J., Heylen, W., and Swevers, J. (2003). Modelling of sprayer boom dynamics by means of maximum likelihood identification techniques: Part 2: Sensitivity-based mode shape normalisation. *Biosystem Engineering*, 85(3):291–298.

Parloo, E., Guillaume, P., Van Overmeire, M., and Verboven, P. (2002). Sensitivity-based operational mode shape normalization. *Mechanical Systems and Signal Processing*, 16(5):757–767.

Sas, P. and Augusztinovicz, F. (1999). Acoustic modal analysis. *NATO Advanced Study Institute Series E applied sciences*, (363):487–506.

7 Responsibility notice

The authors are the only responsible for the printed material included in this paper.