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TEMPORAL IDENTIFICATION IN A FLEXIBLE MULTIBODY SYSTEM: APLICATION TO BIOMECHANICS

Gouveia, G.M.
LMR – INSA de Rouen BP 08 Av de l'Université – Saint Etienne du Rouvray - France
giovanni.mamao@insa-rouen.fr

Lemosse D.
LMR – INSA de Rouen BP 08 Av de l'Université – Saint Etienne du Rouvray – France
didier.lemosse@insa-rouen.fr

Pagnacco, E.
LMR – INSA de Rouen BP 08 Av de l'Université – Saint Etienne du Rouvray – France
emmanuel.pagnacc@insa-rouen.fr

Souza de Cursi, E.
LMR – INSA de Rouen BP 08 Av de l'Université – Saint Etienne du Rouvray – France
eduardo.souza@insa-rouen.fr

Abstract. *The aim of this paper is to evaluate forces and torques in flexible joints of a flexible multi-body system, FMS. To represent such a FMS a 2D non-linear model based on finite element method is built. Its numerical form solution based on a stable integration method and on the non-linear Newton-Raphson scheme is described. As the evaluation procedure first requires that some model physical properties are obtained as identified parameters, two procedures are used: direct and indirect identification. They are based, respectively, on dynamic equilibrium verification and on comparison between simulated and measured kinematics of the proposed model. The development of the two procedures and a performance comparison between them is carried out.*

Keywords*: Dynamic simulation of flexible multi-body system, non-linear numeric resolution, temporal parameter identification.*

1. Introduction

Many studies have shown that some kinematic and dynamic factors play an important role in osteoarthritis of hip joint (Dujardin *et al*, 1997) , (Mejjad *et al*, 1998). In order to have a better comprehension of such disease evolution, its related causes, as well as a better understanding of worn phenomenon of femur implant, the dynamic efforts (forces and torques) arising in the contact between the head of the femur and the hip in a normal human activity are necessary. Nevertheless, the use of inner sensors would be extremely hard to implement not to mention the troublesome for the user. Then, the remained option is to evaluate those efforts by means of a mathematical model. In biomechanical studies, the used mechanical models are classically based on the rigid body motion hypothesis, which, in the case of dynamic flexible systems, are neither reliable nor ranged in terms of uncertainties. Nevertheless, with such rigid behavior, dynamic parameters can be evaluated directly through an inverse dynamic strategy. The method consists of introducing measured kinematics in the equilibrium dynamic equations and considering external load and other parameters as unknowns. This can be quite easily done because few kinematic parameters are necessary. The whole kinematic behavior of a rigid solid can be described by only six parameters: 3 displacements and 3 rotations. A similar approach, named direct identification procedure, may be followed in the case of flexible models. However, the method needs that all kinematics data during a human gait are obtained, what is a challenge. One has to notice that, in the case of flexible bodies, the whole kinematics is described by a continuous field. That means, at least, a large number of parameters are necessary to identify each body configuration which implies in a measure system able to capture the whole body kinematic. In order to answer this need, an indirect identification procedure, dealing with only a part of the kinematic data, is proposed. These two different procedures are described in the following sections. A comparison between both will be made later through a numerical study involving a non-linear model based on the concept of open-chain mechanisms, with flexible beams and imperfect joints in 2D space. Those procedures shall be able to evaluate some physical parameters of that model and have in common a main development axe which consists in the two folowing steps:

1. Write the differential algebraic equations DAE of the dynamic flexible system by Eq.(1), employing some supposed known parameters $\mathbf{p}_{sol}$ and using finite element approach. The terms in Eq.(1) will be explained in the next section.;

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}}(t) + \dot{\mathbf{M}}\dot{\mathbf{q}}(t) + \mathbf{f}_{\text{int}} + \mathbf{B}^{\mathbf{T}}\boldsymbol{\lambda} - \mathbf{f}_{\mathbf{ext}} - \mathbf{g} = \mathbf{0} \\ \boldsymbol{\Phi}(\mathbf{q}) = \mathbf{0} \end{cases} \tag{1}$$

2. Solve them by using a temporal resolution scheme which will allow to gather the data set or reference kinematic set of the problem. This reference kinematic set, represented by a bar placed over the variables in sections 4 to 7, means respectively, positions, velocities and accelerations of nodal variables and has been obtained by two different ways: i) using the simulated response for the solution parameters $\mathbf{p_{sol}}$; ii) evaluating velocities and accelerations by means of central finite difference method from the simulated position response $\mathbf{q}$ given in i).

2. Remarks about the development of DAE

To evaluate all terms of the dynamic and constraint equations of DAE, we need many structural data as geometric and material information of the problem. We also need the involved external forces. All those informations can be regarded as a set of system parameters $\mathbf{p}$. The development of DAE is made for the two basic elements employed in the problem: the co-rotational beam and the co-rotational linear and torsional spring elements. We also need to develop their stiffness elemental matrix as preparatory step to the solution of DAE.

The inertia $\mathbf{M}\ddot{\mathbf{q}}$ and gyroscopic $\dot{\mathbf{M}}\dot{\mathbf{q}}$ forces in Eq.(1) are evaluated from kinetic energy of a constant section beam. The complete mass matrix $\mathbf{M}$, related to inertia forces, can be obtained by assembling of the element mass matrix $\mathbf{M_i}$, which can be found in (Dhatt and Batoz, 1990) in their corresponded degrees of freedom. The $\mathbf{M_i}$, corresponding to co-rotational linear and torsion spring element, is set to null matrix due to hypothesis of its absence of mass. The mass temporal derivative $\dot{\mathbf{M}}$, related to gyroscopic forces, is obtained from a similar way with its elemental matrix $\dot{\mathbf{M}}_\mathbf{i}$ given by Eq.(2). $\mathbf{R}$ is the rotation operator describing the orientation of the element with respect to a inertial frame.

$$\dot{\mathbf{M}}_\mathbf{i} = \dot{\mathbf{R}}_\mathbf{i}^\mathbf{T}\mathbf{M}_\mathbf{i}\mathbf{R}_\mathbf{i} + \mathbf{R}_\mathbf{i}^\mathbf{T}\mathbf{M}_\mathbf{i}\dot{\mathbf{R}}_\mathbf{i} \tag{2}$$

The evaluation of inner forces $\mathbf{f}_{\text{int}}$ of beams and linear springs is made through a co-rotational element formulation using Kirchhoff theory described in (Crisfield, 1991). The aim of this formulation is to subtract the rigid displacements, translation and rotation, from the total displacement of an element. This operation allows us to keep only elastics displacements and to compare them to the undeformed element. After that, one can assign this filtered deformation to the developed internal forces and torques. The evaluation of inner forces $\mathbf{f}_{\text{int}}$ concerning the torsional spring element is straightforward because they are not under influence of the rigid displacements of the element.

The term $\mathbf{B}^\mathbf{T}\boldsymbol{\lambda}$ represents the reaction forces, i.e. the developed efforts to satisfy the constraint equations in the second equation of Eq. (1). Then all system constraint equations, $\boldsymbol{\Phi}$ vector, are written as functions of our predefined global nodal variables $\mathbf{q}$. Thus, $\mathbf{B}$ represents the gradient of $\boldsymbol{\Phi}$ vector related to $\mathbf{q}$. The vector $\boldsymbol{\lambda}$, quantifying the influence of each constraint in the system forces, is obtained during the temporal resolution of DAE.

The two last terms of Eq. (1) are the external forces $\mathbf{f_{ext}}$ and the field (gravity) force $\mathbf{g}$. Each nodal external force applied in the dynamic system must be placed in its corresponding degree of freedom in the global vector $\mathbf{f_{ext}}$. The vector $\mathbf{g}$ is built from the assembly of elemental field forces also in its degree of freedom. This elemental field force is defined as being the derivative of the elemental potential energy with respect to adopted coordinates $\mathbf{q}$. The elemental field forces corresponding to co-rotational linear and torsional spring, are set to null vector due to their absence of mass, as stated before.

3. Temporal resolution scheme of DAE

Finding the solution of Eq. (1) in time $t + \Delta t$ implies to find a variable set $\left(\ddot{\mathbf{q}}_{\mathbf{t+\Delta t}}, \dot{\mathbf{q}}_{\mathbf{t+\Delta t}}, \mathbf{q}_{\mathbf{t+\Delta t}}, \boldsymbol{\lambda}_{\mathbf{t+\Delta t}}\right)$ that verifies simultaneously its two equations. Then, near of the solution, the vectors $\mathbf{r}\left(\ddot{\mathbf{q}}_{\mathbf{t+\Delta t}}, \dot{\mathbf{q}}_{\mathbf{t+\Delta t}}, \mathbf{q}_{\mathbf{t+\Delta t}}, \boldsymbol{\lambda}_{\mathbf{t+\Delta t}}\right)$ and $\mathbf{r}_{\boldsymbol{\Phi}}\left(\mathbf{q}_{\mathbf{t+\Delta t}}\right)$, defined in Eq. (3) must tend to $\mathbf{0}$.

$$\begin{cases} \mathbf{r}\left(\ddot{\mathbf{q}}_{\mathbf{t+\Delta t}}, \dot{\mathbf{q}}_{\mathbf{t+\Delta t}}, \mathbf{q}_{\mathbf{t+\Delta t}}, \boldsymbol{\lambda}_{\mathbf{t+\Delta t}}\right) & = \mathbf{M}\ddot{\mathbf{q}} + \dot{\mathbf{M}}\dot{\mathbf{q}} + \mathbf{f}_{\text{int}} + \mathbf{B}^\mathbf{T}\boldsymbol{\lambda} - \mathbf{f_{ext}} - \mathbf{g} \\ \mathbf{r}_{\boldsymbol{\Phi}}\left(\mathbf{q}_{\mathbf{t+\Delta t}}\right) & = \boldsymbol{\Phi}(\mathbf{p}, \mathbf{q}) \end{cases} \tag{3}$$

However, the presence of constraints equations in DAE can cause severe instabilities in DAE responses. The importance of high frequencies in those responses are reduced in order to avoid those instabilities. This is done by the evaluation of the residual vectors through Eq. (4) and Eq. (5). That means that between two very close time, t and $t + \Delta t$, the accelaration is supposed constant.

$$\begin{aligned} \mathbf{r}\left(\mathbf{q}_{\mathbf{t+\Delta t}}, \mathbf{q}_\mathbf{t}\right) = {} & \mathbf{M}_{\mathbf{t+\Delta t}}\ddot{\mathbf{q}}_{\mathbf{t+\Delta t}} + (1+\alpha)\left(\dot{\mathbf{M}}_{\mathbf{t+\Delta t}} + \dot{\mathbf{q}}_{\mathbf{t+\Delta t}} + \mathbf{f}_{\text{int}}\left(\mathbf{q}_{\mathbf{t+\Delta t}}\right) + \mathbf{B}^\mathbf{T}_{\mathbf{t+\Delta t}}\boldsymbol{\lambda}_{\mathbf{t+\Delta t}} - \mathbf{g}_{\mathbf{t+\Delta t}}\right) + \\ & - \alpha\left(\mathbf{M}_\mathbf{t}\ddot{\mathbf{q}}_\mathbf{t} + \dot{\mathbf{M}}_\mathbf{t}\dot{\mathbf{q}}_\mathbf{t} + \mathbf{f}_{\text{int}}\left(\mathbf{q}_\mathbf{t}\right) + \mathbf{B}_\mathbf{t}{}^\mathbf{T}\boldsymbol{\lambda}_\mathbf{t} - \mathbf{g}_\mathbf{t}\right) - \mathbf{f_{ext}}\left(\mathbf{t}_\alpha\right) \end{aligned} \tag{4}$$

$$\mathbf{r_\Phi}(\mathbf{q_{t+\Delta t}}, \mathbf{q_t}) = (1+\alpha)\Phi(\mathbf{q_{t+\Delta t}}) - \alpha\Phi(\mathbf{q_t}) \tag{5}$$

This procedure, named HHT (Jalon and Bayo, 1994) imposes less loss of total energy during the numerical resolution procedure. The strategy starts from known equilibrium set $(\ddot{\mathbf{q}}_\mathbf{t}, \dot{\mathbf{q}}_\mathbf{t}, \mathbf{q}_\mathbf{t}, \boldsymbol{\lambda}_\mathbf{t})$ at the time *t*. Then equilibrium perturbation is imposed in this variable set at time $t + \Delta t$ by using Eq. (6).

$$\begin{aligned}
\mathbf{q}^{\mathbf{0}}_{\mathbf{t+\Delta t}} &= \mathbf{q_t} + \Delta t\dot{\mathbf{q}}_\mathbf{t} + \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{\mathbf{q}}_\mathbf{t} \\
\dot{\mathbf{q}}^{\mathbf{0}}_{\mathbf{t+\Delta t}} &= \dot{\mathbf{q}}_\mathbf{t} + (1-\gamma)\Delta t\ddot{\mathbf{q}}_\mathbf{t} \qquad \text{with } \gamma = \frac{1-2\alpha}{2} \text{ and } \beta = \frac{(1-\alpha)^2}{4} \\
\ddot{\mathbf{q}}^{\mathbf{0}}_{\mathbf{t+\Delta t}} &= \ddot{\mathbf{q}}_\mathbf{t} \\
\boldsymbol{\lambda}^{\mathbf{0}}_{\mathbf{t+\Delta t}} &= \boldsymbol{\lambda}_\mathbf{t}
\end{aligned} \tag{6}$$

The stability parameters, γ and β, are related to a numeric dumping parameter α . t_α is a time between *t* and $t + \Delta t$ defined by $t_\alpha = (1+\alpha)(t+\Delta t) - \alpha t$. The best choice of α is in [-1/3,0]. $\alpha = 0$ meaning that no numeric damping is being added to the solution to avoid numerical instabilities, corresponding to classical Newmark scheme. In the other hand, $\alpha = -1/3$ means a great numeric dumping and brings to the system solution a high loss of energy (Geradin and Cardona, 2000).

3.1. Non linear solution

The DAE we deal with is a non linear system of equations and imposes to solve an adapted procedure. If the classical Newton-Raphson NR scheme is chosen, the tangent matrix of the system has to be evaluated. It is given by the differential of the residual vector with respect to **q** in Eq. (7).

$$\mathbf{K_T} = \frac{\partial}{\partial \mathbf{q}}\left(\mathbf{M} + \dot{\mathbf{M}} - \mathbf{g} + \mathbf{f}_{\text{int}} - \mathbf{f}_{\text{ext}} + \mathbf{B}^{\mathbf{T}}\boldsymbol{\lambda}\right)_{\mathbf{t+\Delta t}} \tag{7}$$

The term concerning to element internal forces is the only to be evaluated and can be found in (Crisfield, 1991). The differential with respect to **q** of the two first terms of Eq. (7) are very costly to evaluate just for convergence (Geradin and Cardona, 2000). The remaining terms are constant with respect to **q**. To apply the classical NR scheme, Eq.(1) has to be linearized in Eq. (8) form, in order to take account the temporal scheme.

$$\begin{cases} \mathbf{M}^n\Delta\ddot{\mathbf{q}} + \dot{\mathbf{M}}^n\Delta\dot{\mathbf{q}} + \mathbf{K}^n_\mathbf{T}\mathbf{B}^{n^\mathbf{T}}\Delta\boldsymbol{\lambda} - \mathbf{G} &= -\mathbf{r}\left(\ddot{\mathbf{q}}^n_{\mathbf{t+\Delta t}}, \dot{\mathbf{q}}^n_{\mathbf{t+\Delta t}}, \mathbf{q}^n_{\mathbf{t+\Delta t}}, \boldsymbol{\lambda}^n_{\mathbf{t+\Delta t}}\right) \\ \mathbf{B}^n\Delta\mathbf{q} &= -\mathbf{r_\Phi}\left(\mathbf{q}^n_{\mathbf{t+\Delta t}}\right) \end{cases} \tag{8}$$

Equation (8) can be rewritten in matrix form of Eq. (9), corresponding to a linear system, which has to be solved.

$$\begin{bmatrix} \mathbf{S_T} & \mathbf{B}^\mathbf{T} \\ \mathbf{B} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \Delta\mathbf{q} \\ \Delta\boldsymbol{\lambda} \end{bmatrix} = -\begin{bmatrix} \mathbf{r} \\ \mathbf{r_\Phi} \end{bmatrix} \quad \text{with } \mathbf{S_T} = \frac{\mathbf{M}}{\beta\Delta t^2} + \frac{\gamma\dot{\mathbf{M}}}{\beta\Delta t} + \mathbf{K_T} \tag{9}$$

The exponent *n* means the NR iteration number. Variables are updated during the NR process by means of Eq. (10) The process stops when the convergence of the residual vectors, Eq. (4) and Eq. (5) evaluated by HHT scheme, is checked.

$$\begin{aligned}
\mathbf{q}^{n+1}_{\mathbf{t+\Delta t}} &= \mathbf{q}^n_{\mathbf{t+\Delta t}} + \frac{\Delta\mathbf{q}}{\boldsymbol{\beta}\Delta\mathbf{t}^2} \\
\dot{\mathbf{q}}^{n+1}_{\mathbf{t+\Delta t}} &= \dot{\mathbf{q}}^n_{\mathbf{t+\Delta t}} + \frac{\boldsymbol{\gamma}\Delta\mathbf{q}}{\boldsymbol{\beta}\Delta\mathbf{t}} \\
\ddot{\mathbf{q}}^{n+1}_{\mathbf{t+\Delta t}} &= \ddot{\mathbf{q}}^n_{\mathbf{t+\Delta t}} + \Delta\mathbf{q} \\
\boldsymbol{\lambda}^{n+1}_{\mathbf{t+\Delta t}} &= \boldsymbol{\lambda}^n_{\mathbf{t+\Delta t}} + \Delta\boldsymbol{\lambda}
\end{aligned} \tag{10}$$

4. Direct Temporal Identification

The direct strategy is based on the dynamic equilibrium equation Eq. (1). If a complete set of variable kinematic response $(\ddot{\mathbf{q}}_\mathbf{t}, \dot{\mathbf{q}}_\mathbf{t}, \mathbf{q}_\mathbf{t})$ and a set of initial parameters $(\mathbf{p_0}, \boldsymbol{\lambda_0})$ of a dynamic system are avalaible, the residual equilibrium of those parameter dependent forces and torques of Eq.(1) can be evaluated by minimizing Eq. (11).

$$\mathbf{z}(\mathbf{p},\boldsymbol{\lambda},t)=\begin{bmatrix}\mathbf{r}(\mathbf{p},\boldsymbol{\lambda},t)\\ \mathbf{r}_{\boldsymbol{\Phi}}(\mathbf{p},t)\end{bmatrix}=\begin{bmatrix}\mathbf{M}(\mathbf{p},\bar{\mathbf{q}})\ddot{\bar{\mathbf{q}}}+\dot{\mathbf{M}}(\mathbf{p},\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}}(t)+\mathbf{f}_{\text{int}}(\mathbf{p},\bar{\mathbf{q}})+\mathbf{B}^{\mathrm{T}}\boldsymbol{\lambda}-\mathbf{f}_{\text{ext}}-\mathbf{g}\\ \boldsymbol{\Phi}(\mathbf{p},\mathbf{q})\end{bmatrix} \tag{11}$$

Then, the proposed solution can be regarded as an optmization procedure declared by means of Eq. (12), which search for a parameter list $\mathbf{p}_{\text{sol}}$ that minimize residues of Eq. (11),.

$$\min_{\mathbf{p}} \mathbf{J}=\sum \mathbf{z}^{\mathrm{T}}\mathbf{z} \tag{12}$$

At any studied time *t*, this **z** residual vector, which is non linear dependent from **p** and $\boldsymbol{\lambda}$, has to be equalized to zero. This can be done by a classical less square method and a first order Newton-Raphson strategy. Starting from a given $(\mathbf{p}_0,\boldsymbol{\lambda}_0)$, parameters are interactively modified through Eq. (13) until convergence is reached.

$$\begin{pmatrix}\mathbf{p}\\ \boldsymbol{\lambda}\end{pmatrix}^{n+1}=\begin{pmatrix}\mathbf{p}\\ \boldsymbol{\lambda}\end{pmatrix}^{n}-\frac{\partial \mathbf{z}}{\partial \mathbf{p}}^{+}\mathbf{z} \tag{13}$$

In that expression, $\frac{\partial \mathbf{z}}{\partial \mathbf{p}}^{+}$ is called (Moore-Penrose) pseudo-inverse matrix of residual gradient matrix $\frac{\partial \mathbf{z}}{\partial \mathbf{p}}$. It is numerically evaluated by a central finite difference procedure.

This strategy is numerically efficient because it is based on an interactive procedure, i.e. no temporal resolution is needed. Nevertheless, it shows a high sensibility to introduced measures and specially to accelerations. The main difficulty stays in the need of having measure data for all degrees of freedom introduced in the model. If this can be hardly done for displacement variables, it is almost impossible for rotations variables, much less in the context of bio-mechanic study, in which humans have to be instrumented. In order to avoid this difficulty an indirect procedure using only available kinematic variables have been proposed.

5. Indirect Temporal Identification

The proposed procedure consists in reducing the difference between reference kinematics or data and evaluated variables, which are obtained through the solution of DAE Eq. (1). As in the previous section, the proposed solution can also be regarded as an optimization procedure, but declared by means of Eq. (14), where a function is minimized by a specific parameter list $\mathbf{p}_{\text{sol}}$. The Greek variables represent the "influence" weights of each difference term.

$$\min_{\mathbf{p}} \mathbf{J}=\sum\left\{\phi(\ddot{\mathbf{q}}-\ddot{\bar{\mathbf{q}}})^{\mathrm{T}}(\ddot{\mathbf{q}}-\ddot{\bar{\mathbf{q}}})+\varphi(\dot{\mathbf{q}}-\dot{\bar{\mathbf{q}}})^{\mathrm{T}}(\dot{\mathbf{q}}-\dot{\bar{\mathbf{q}}})+\psi(\mathbf{q}-\bar{\mathbf{q}})(\mathbf{q}-\bar{\mathbf{q}})^{\mathrm{T}}\right\} \tag{14}$$

With the parameter list $\mathbf{p}_{\text{sol}}$, which also includes the set of Lagrange parameters $\boldsymbol{\lambda}$, the dynamics efforts in the joints and the reaction forces can be obtained through Eq (1), respectively, by correspondent components of $\mathbf{f}_{\text{int}}$ and $\mathbf{B}^{\mathrm{T}}\boldsymbol{\lambda}$.

The main advantage of this procedure is that the residue vector does not have to be composed of all kinematic data, but only of a part of them. With such a choice, velocity, acceleration as well as rotational data are not employed in the comparison vector **z** Eq. (15).

$$\mathbf{z}=[\bar{\mathbf{q}}(t)-\mathbf{q}(t)] \tag{15}$$

Even though, two tests were made using the whole kinematics data, then Eq. (16).

$$\mathbf{z}=\begin{bmatrix}\ddot{\bar{\mathbf{q}}}(t)-\ddot{\mathbf{q}}(t)\\ \dot{\bar{\mathbf{q}}}(t)-\dot{\mathbf{q}}(t)\\ \bar{\mathbf{q}}(t)-\mathbf{q}(t)\end{bmatrix} \tag{16}$$

Starting by simulating the movement with a initial given set of parameters $(\mathbf{p}_0,\boldsymbol{\lambda}_0)$ until the comparison time *t* is reached, the kinematics variable set $(\ddot{\mathbf{q}}_t,\dot{\mathbf{q}}_t,\mathbf{q}_t,\boldsymbol{\lambda}_t)$ is obtained. The residue vector **z** is evaluated and then, through the optimization procedure, updated until convergence. As the system is highly non linear, it also imposes a numerical evaluation of the gradient matrix, necessary for the optimization procedure and described in the previous section. This fact implies to run the whole simulation from initial time until time *t* for each disturbance introduced in each unknown parameter. For long time simulation, this procedure becomes calculating time expensive.

6. Numerical evaluation

Before to face the proposed biomechanical problem, some tests are made on a simpler model where the formulation and results can be, respectively, easier and more conclusive. This proposed model example is shown in Fig. 1. Despite its geometric simplicity, it can bring all sort of difficulties expected in a biomechanical model. It's composed of two flexible bars linked by torsional and linear spring element attached to a fixed point through this same element, but with different stiffness that will be the parameters **p** to identify.

The following data were given to this problem: bar's Young modulus $E=2.1\text{x}10^{10}$Pa, section area $A=25\text{x}10^{-6}$ m^2, inertia section $I=52.1\text{x}10^{-12}m^4$, bar's volume density $\rho=7800$ kg/m^3, bar's initial length $L_b=0.9$ m, non extended spring length $L_s=0.1$ m, k torsional = 0.5 Nm/rad, k linear first element = 700 N/m, k linear third element = 400 N/m.

Letting this whole multi-body system fall freely from a same initial angle of bars and springs with respect to a vertical line equal to $\pi/6$. Thus, the DAE are integrated for 3*s*, using HHT scheme, and all nodal kinematics responses $\left[\ddot{\overline{\mathbf{q}}}(t), \dot{\overline{\mathbf{q}}}(t), \overline{\mathbf{q}}(t)\right]$ are registered as our simulated set. Then, using just $\overline{\mathbf{q}}(t)$, we generate our measured set by finite central difference method. At times chosen by the greatest difference between simulated and measured responses, obviously found only for $\left[\ddot{\overline{\mathbf{q}}}(t), \dot{\overline{\mathbf{q}}}(t)\right]$, we decided to apply the two temporal identification methods starting from a vicinity of our known parameter solution $\mathbf{p_{sol}} = [700 \;\; 0.5 \;\; 400 \;\; 0.5]^{\mathbf{T}}$.

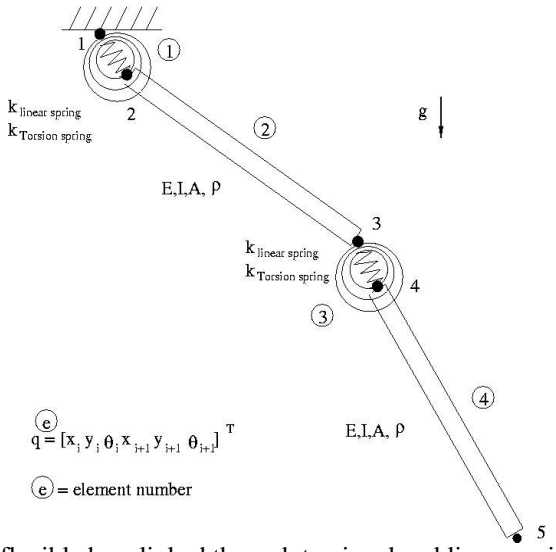

Figure 1. Two flexible bars linked through torsional and linear spring elements

7. Results and Conclusions

The two identification procedures were evaluated at times t_1=0,69 *s* and t_2=2,83 *s*. These times were close and far, respectively, from the starting state. They were also defined according to the criteria of bad numerical evaluation, established in the previous section.

The simulated reference values were obtained during the resolution of the system, with a parameter set $\mathbf{p_{sol}}$ that are the solution of our problem of identification. The measured velocities and accelerations employed as reference values were obtained applying a central finite difference procedure over the simulated reference positions.

The indirect identification procedure was performed with two residues. These last are resumed in Eq. (15) and Eq. (16). In Eq. (15) only the nodal position variables were used in $\overline{q}(t)$ and $q(t)$, as for example in $\mathbf{z} = \left[\text{x}_2 \;\; \text{y}_4 \;\; \text{x}_5 \;\; \text{y}_\text{n}\right]^\text{T} - \left[\overline{\text{x}}_2 \;\; \overline{\text{y}}_4 \;\; \overline{\text{x}}_5 \;\; \overline{\text{y}}_\text{n}\right]^\mathbf{T}$. This has been done in order to evaluate the capabilities of the procedure when dealing with a reduced residue vector.

Identification evaluations were summarized in Tab. 1. Its terms named "Good convergence", "Right convergence" and "Reasonable convergence" refer, respectively, to $\mathbf{p_{sol}}$, $\mathbf{p1} = [663 \;\; 0.49 \;\; 379 \;\; 0.49]^\mathbf{T}$ and $\mathbf{p2} = [757 \;\; 0.50 \;\; 465 \;\; 0.52]^\mathbf{T}$ sets. For node 5, Fig 1, the sensibility of evaluated kinematics to those parameter sets were shown in Fig. 2, Fig 3 and Fig. 4. The results have shown that the direct and indirect identification procedures are very sensible to acceleration modified values, but not to velocities and less to displacements. Figure 5 shows the sensibility of forces and torque to $\mathbf{p_{sol}}$, **p1** and **p2**, in the first element spring.

The simulation time also plays an important role in the results obtained. We could verify that the more the simulation time is big the more the starting point has to be near of the solution to convergence. This can be explained by the great differences, specially in trajectory accelerations, generated by different parameter sets. Moreover, the time processing is strongly linked to time simulation for the indirect procedure.

The direct identification procedure will be abandoned because it is not adapted to data characteristics. In the other hand, the strategy of using the reduced simulated set in indirect procedure has revealed a good alternative to follow due to its similarity with the available data of the original biomechanical problem. However, more powerful search methods

Table 1. Tests results

Identif	Reference Data	$t_1 = 0.69$ s	$t_2 = 2.83$
Direct	Simulated set	Good convergence for huge perturbations in departure point	Same results for t_1
	Simulated set except speed that comes from measured set	Right convergence for huge perturbation in departure point	Same results for t_1
	Measured set	Bad convergence even starting from solution parameter set	Same results for t_1
Indirect	Simulated set with (16)	Good convergence starting from small perturbations (0.93 to 1.10) in parameter solution set	Same results for t_1 but (0.90 to 1.02)
	Positions and accelerations from simulated set with (16). Velocities from measured set	Good convergence starting from small perturbations (0.93 to 1.10) in parameter solution set	Same results for t_1 but (0.90 to 1.02)
	Measured set with (16)	Bad convergence even starting from parameter solution set	Same results for t_1
	Simulated set with (15)	Good convergence starting from small perturbation (0.90 to 1.18) in parameter solution set	Reasonable convergence but for tiny perturbations (0.98 to 1.02)

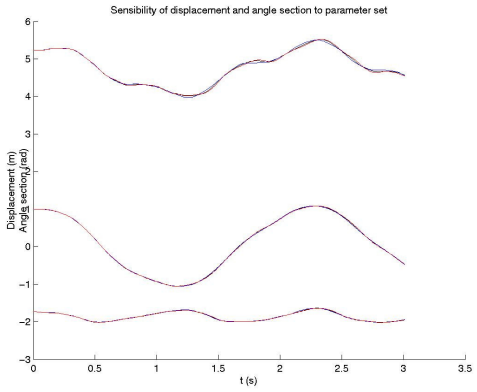

Figure 2. Displacement sensibility to $\mathbf{p_{sol}}$, **p1** and **p2**.

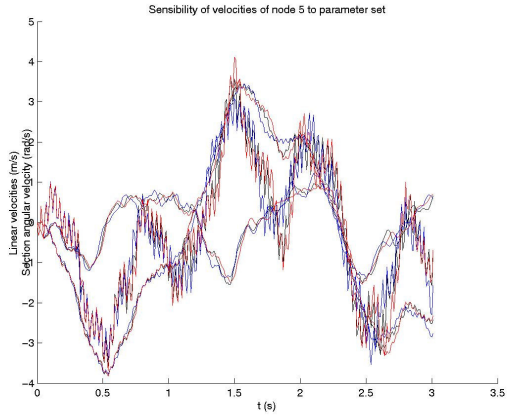

Figure 3. Velocity sensibility to $\mathbf{p_{sol}}$, **p1** and **p2**.

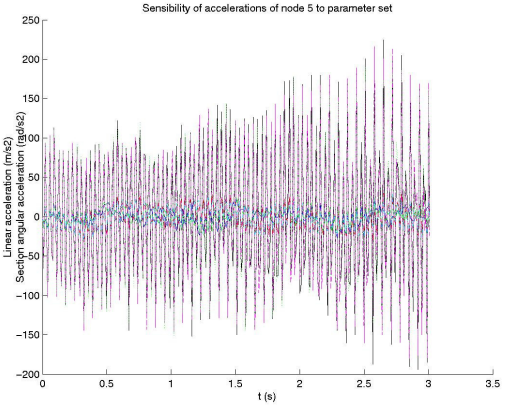

Figure 4. Acceleration sensibility to $\mathbf{p_{sol}}$, **p1** and **p2**.

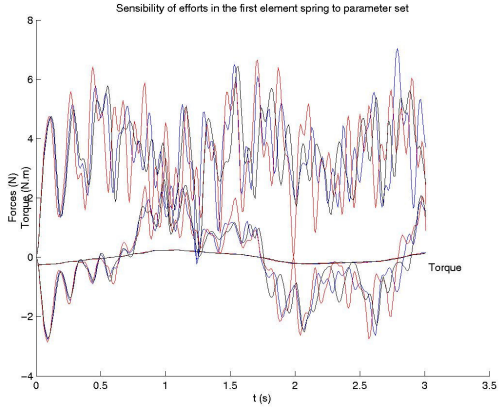

Figure 5. Force sensibility to $\mathbf{p_{sol}}$, **p1** and **p2**.

are required to overcome the uncertainties of the departure searching point when the answer is not known. Future tests, using many reference times and introduced noise in position reference data, will be performed to verify how those changes can affect the response quality and the departure point range of parameter set.

8. Acknwledgements

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