Proceedings of the XI DINAME, 28th February-4th March, 2005 - Ouro Preto - MG - Brazil
Edited by D.A. Rade and V. Steffen Jr.

ANALYTICAL AND NUMERICAL APPROACHES TO STUDY THE GRAVITATIONAL CAPTURE IN THE FOUR-BODY PROBLEM

Antonio Fernando Bertachini de Almeida Prado
Instituto Nacional de Pesquisas Espaciais – INPE, São José dos Campos - SP - 12227-010 - Brazil
PRADO@DEM.INPE.BR

Abstract. *The objective of this paper is to show a survey of results in the problem of gravitational capture in the bi-circular restricted four-body problem using numerical and analytical techniques. A gravitational capture occurs when a massless particle changes its two-body energy around one celestial body from positive to negative without the use of non-gravitational forces. In this paper it is mainly studied the effect of a fourth-body included in the dynamics. The Earth-Moon system with the inclusion of the Sun is used for the simulations. The results show the savings obtained in this more realistic model when compared with the more traditional restricted three-body problem model, as well as several examples of trajectories.*

Keywords: *astrodynamics, gravitational capture, celestial mechanics*

1. Introduction

The application of the phenomenon of gravitational capture in spacecraft trajectories has been studied in details in the literature in the last few years. Among the first studies are the ones performed by Belbruno (1987, 1990, 1992); Krish (1991); Krish, Belbruno and Hollister (1992); Miller and Belbruno (1991); Belbruno and Miller (1990). They all studied missions in the Earth-Moon system that use this technique to save fuel during the insertion of the spacecraft in its final orbit around the Moon. Another group of researches that made fundamental contributions in this field, also with the main objective of constructing real trajectories in the Earth-Moon system, are the Japaneses Yamakawa, Kawaguchi, Ishii and Matsuo (1992, 1993). In particular, Yamakawa wrote his Ph.D. Dissertation (Yamakawa, 1992) in this topic, with several important contributions in this field. A real application of those ideas was made during an emergency in a Japanese spacecraft (Belbruno and Miller, 1990). After that, some studies that consider the time required for this transfer appeared in the literature. Examples of this approach can be find in the papers by Vieira-Neto and Prado (1995, 1998). An extension of the dynamical model to consider the effects of the eccentricity of the primaries is also available in the literature (Vieira-Neto, 1999; Vieira-Neto and Prado, 1996). The present paper has the main goal of studying the potential savings obtained with the inclusion of a fourth body in the dynamics. Numerical simulations and analytical approximations are used.

In the bi-circular restricted four-body problem gravitational capture is assumed to occur when the massless particle stays close to one of the two primaries of the system for some time. A permanent capture is not required, because in this model it does not exist, and the phenomenon is always temporary, which means that after some time of the approximation the massless particle escapes from the neighborhood of the primary.

For practical purposes, the majority of the papers study this problem looking in the behavior of the two-body energy (E) of the spacecraft with respect to the Moon. From the value of this energy it is possible to know if the orbit is elliptical ($E < 0$), parabolic ($E = 0$) or hyperbolic ($E > 0$). Based in this definition, it is possible to see that the value of E is related to the velocity variation (ΔV) needed to insert the spacecraft in its final orbit around the Moon. In the case of a spacecraft approaching the Moon, if the gravitational force of the Earth and the Sun can be used to lower the value of E, the fuel consumption required to complete this maneuver is reduced. In that way, the search for trajectories that arrives at the Moon with the maximum possible value for the reduction of E is very important. In the majority of the studies relative to this problem, a numerical approach of verifying the values of E during the trajectory is used to identify useful trajectories. Since the trajectories are very sensitive to the initial conditions, it is more convenient to start the propagation of the trajectories at the point of the closest approach with the Moon and propagate the trajectories with a negative time step. If there is a change of sign from negative (closed trajectory) to positive (open trajectory), this trajectory is considered an escape in the backward sense of time. It means that a gravitational capture occurs in the positive sense of time and this particular trajectory can be used to reduce the amount of fuel in an Earth-Moon transfer.

2. Mathematical Model and Some Properties

The model used in this paper is the planar bi-circular restricted four-body problem. This model assumes that two main bodies (M_1 and M_2) are orbiting their common center of mass in circular Keplerian orbits and a third body (M_3) is orbiting these two primaries describing circular orbits around the center of mass of the first two primaries. Then, it is desired to study the motion of a fourth body, M_4, that is supposed to stay in the plane of the motion of the other three bodies and that is affected by all of them, but it does not affect their motion (Szebehely, 1967). The standard canonical system of units associated with this model is used (the unit of distance is the distance M_1-M_2 and the unit of time is chosen such that the period of the motion of M_2 around M_1 is 2π). Under this model, the equations of motion of the massless particle, in the inertial reference system, are:

$$\ddot{x} = -\mu_E \frac{(x - x_E)}{r_1^3} - \mu_M \frac{(x - x_M)}{r_2^3} - \mu_S \frac{(x - x_S)}{r_3^3} \tag{1}$$

$$\ddot{y} = -\mu_E \frac{(y - y_E)}{r_1^3} - \mu_M \frac{(y - y_M)}{r_2^3} - \mu_S \frac{(y - y_S)}{r_3^3} \tag{2}$$

where, $r_1 = \sqrt{(x - x_E)^2 + (y - y_E)^2}$, $r_2 = \sqrt{(x - x_M)^2 + (y - y_M)^2}$, $r_3 = \sqrt{(x - x_S)^2 + (y - y_S)^2}$, $x_E = -\mu_M \cos(t)$, $y_E = -\mu_M \sin(t)$, $x_M = \mu_E \cos(t)$, $y_M = \mu_E \sin(t)$, $x_S = R_S \cos(\psi)$, $y_S = R_S \sin(\psi)$, $\psi = \psi_0 + \omega_S t$, x,y are the coordinates of the massless particle, μ_E (0.9878715), μ_M (0.0121285), μ_S (328900.48) are the gravitational parameters of the Earth, Moon and Sun, respectively, (x_E,y_E) (x_M,y_M) (x_S,y_S) are the coordinates of the Earth, Moon and Sun, respectively, r_1 r_2 r_3 are the distances between the massless particle and the Earth, Moon and Sun, respectively, R_S (= 389.1723985) is the distance between the Sun and the origin of the reference system, ω_S (= 0.07480133) is the angular velocity of the Sun, t is the time. Figure 1 shows a sketch of the system considered.

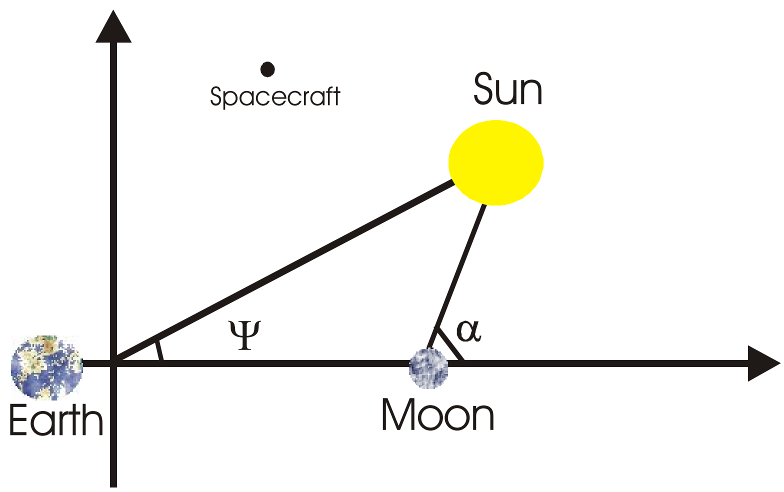

Figure 1 – Restricted four-body model (Cartesian Coordinate).

4. The Gravitational Capture

The “two-body” energy is no longer constant in the bi-circular four-body problem. Then, for some initial conditions, a spacecraft can alternate the sign of its energy from positive to negative or from negative to positive. When the variation is from positive to negative the maneuver is called a "gravitational capture", to emphasize that the spacecraft was captured by gravitational forces only, with no use of an external force, like the thrust of an engine. The opposite situation, when the energy changes from negative to positive is called a "gravitational escape". In the bi-circular four-body problem and in the restricted three-body problem there is no permanent gravitational capture. If the energy changes from positive to negative, it will change back to positive in the future.

One of the most important applications of the gravitational capture can be found in trajectories to the Moon. The concept of gravitational capture is used together with the basic ideas of the gravity-assisted maneuver and the bi-elliptic transfer orbit to generate a trajectory that requires a fuel consumption smaller than the one required by the Hohmann transfer. This maneuver consists of the following steps: i) the spacecraft is launched from an initial circular orbit with radius r_0 to an elliptic orbit that crosses the Moon's path; ii) a Swing-By with the Moon is used to increase the apoapsis of the elliptic orbit. This step completes the first part of the bi-elliptic transfer, with some savings in ΔV due to the energy gained from the Swing-By; iii) With the spacecraft at the apoapsis, a second very small impulse is applied to rise the periapsis to the Earth-Moon distance. Solar effects can reduce even more the magnitude of this impulse; iv) The transfer is completed with the gravitational capture of the spacecraft by the Moon.

5. Numerical Results

To quantify the "gravitational captures" we studied this problem under several different initial conditions. The assumptions made for the numerical examples presented are:

i) The system of primaries used is the Earth-Moon-Sun system;

ii) The motion is planar everywhere, so the Moon and the Sun are assumed to be in coplanar orbits;

iii) The starting point of each trajectory is 100 km from the surface of the Moon (r_p from the center of the Moon). Then, to specify the initial position completely it is necessary to give the value of one more variable. The variable used is the angle α, an angle measured from the Earth-Moon line, in the counter-clock-wise direction and starting in the side opposite to the Earth (see Fig. 2);

iv) The magnitude of the initial velocity is calculated from a given value of $C_3 = 2E = V^2 - \frac{2\mu}{r}$, where E is the two-body energy of the spacecraft with respect to the Moon, V is the velocity of the spacecraft, μ is the gravitational parameter of the Moon and r is the distance between the spacecraft and the center of the Moon. The direction of the velocity is assumed to be perpendicular to the line spacecraft-center of the Moon and pointing to the counter-clock-wise direction for a direct orbit and to the clock-wise direction for a retrograde orbit (see Fig. 2);

v) To consider that an escape occurred, we request, following the conditions used by Yamakawa (1992), that the spacecraft reaches a distance of 100000 km (0.26 canonical units) from the center of the Moon in a time shorter than 50 days. Figure 2 shows the point P where the escape occurs. The angle that specifies this point is called the "entry position angle" and it is designated with the letter β.

First of all, this paper studies numerically the effects of the main parameters of the system. First, the values for the angle ψ is fixed in zero, α in 180° and C_3 is assumed to have the values –0.6 (trajectory 1), –0.4 (trajectory 2), –0.2 (trajectory 3), 0.0 (trajectory 4). The trajectories are shown in Fig. 3. As expected, trajectories that offers smaller values for the savings (C_3 closer to zero) are closer to straight lines and trajectories that offers larges values for the savings (C_3 more negative) describes more loops and take more time to escape from the Moon in retrograde time.

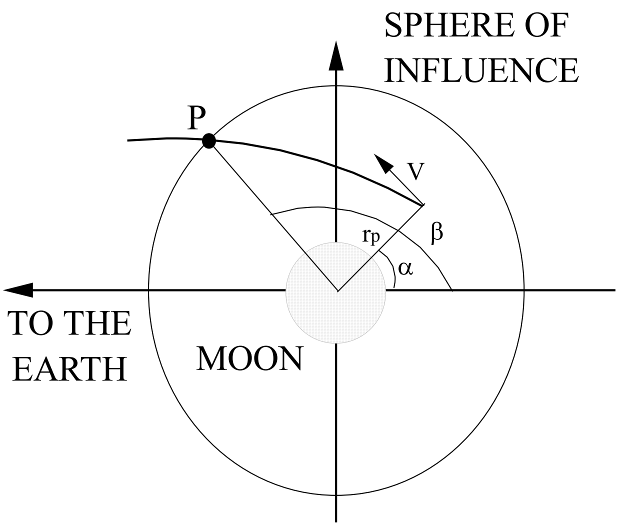

Fig. 2 - Variables to specify the initial conditions of the spacecraft.

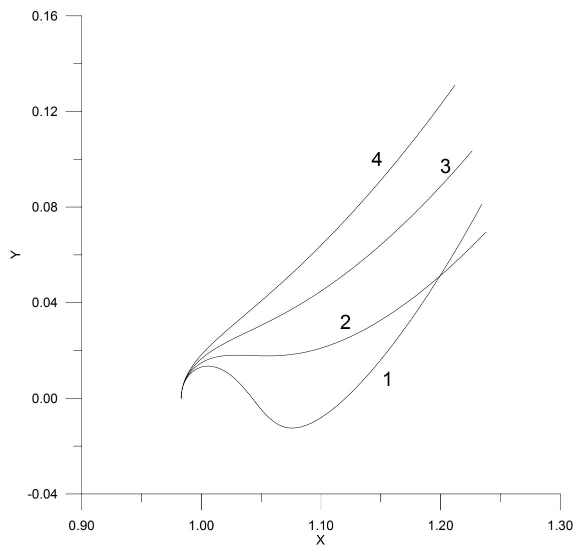

Fig. 3 – Trajectories for diferent values of C_3.

Then, the values for the angle ψ is fixed in zero, C_3 in –0.4 and α is assumed to have the values 90° (trajectory 1), 180° (trajectory 2), 270° (trajectory 3). The trajectories are shown in Fig. 4. They have different characteristics and time to reach gravitational capture.

To verify the influence of the angle ψ four more simulations are made. The value of C_3 is fixed in –0.15, α in 120° and ψ is varied in the values 0°, 90°, 180° and 270°. Figure 5 shows the four trajectories. The trajectories show that the spacecraft start in similar paths when close to the Moon, because the influence of the Sun is not very strong at this point, and then the trajectories deviate from each other, going to the direction of the Sun. It shows that the Sun start to have a very large effects in the trajectories when the spacecraft escape from the Moon.

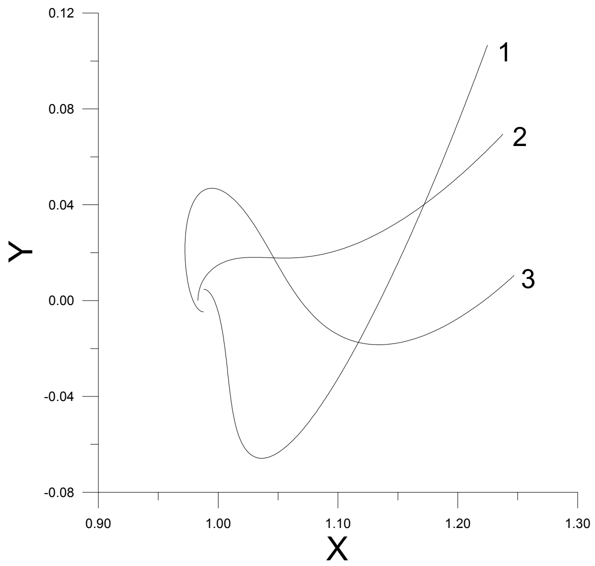

Fig. 4 – Trajectories for diferent values of α.

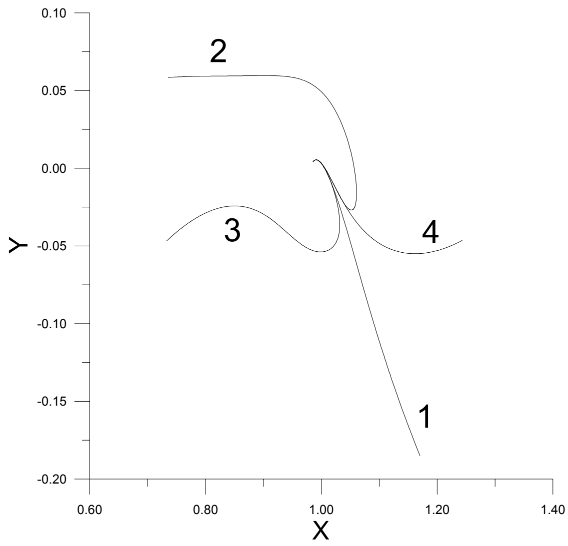

Fig. 5 – Trajectories for diferent values of ψ.

6. Analytical Analyses of the Forces

To understand better the physical reasons of this phenomenon, it is useful to calculate the forces acting over the massless particle. Figure 6 shows the gravitational force $\vec{F}_g$ of the Earth acting in a spacecraft M_3 that is approaching the Moon and Fig. 7 shows the gravitational force of the Sun and the centrifugal force acting in the same situation. There is also the Coriolis force, given by $-2\vec{\omega}_{E-M}\times\vec{v}$, where $\vec{v}$ is the velocity of the spacecraft. This force is not analyzed in detail because the main idea of this paper is to explain the ballistic gravitational capture as a result of perturbative forces acting in this direction and the Coriolis force acts perpendicular to the direction of motion of the spacecraft all the time. In this way, it does not contribute to the phenomenon studied here. The direction $\vec{r}$ points directly to the center of the Moon and the direction $\vec{p}$ is perpendicular to $\vec{r}$, pointing in the counter-clockwise direction. The distance between the spacecraft and the Earth is d, the angle formed by the line connecting the Earth to the spacecraft and the direction $\vec{r}$ is γ. The angle ϕ is used to define instantaneously the direction $\vec{r}$. From geometrical considerations shown in more detail in Prado (2002b), it is possible to write for the gravitational force:

$$\vec{F}_g = \frac{(1-\mu)(r+\cos\phi)}{d^3}\vec{r} + \frac{(1-\mu)\sin\phi}{d}\vec{p} = \frac{(1-\mu)(r+\cos\phi)}{(1+r^2+2r\cos\phi)^{3/2}}\vec{r} + \frac{(1-\mu)\sin\phi}{(1+r^2+2r\cos\phi)^{1/2}}\vec{p} \tag{3}$$

For the centrifugal force the expression is (Prado, 2002b):

$$\vec{F}_{ce} = -[r+(1-\mu)\cos\phi]\vec{r} + (\mu-1)\sin\phi\ \vec{p} \tag{4}$$

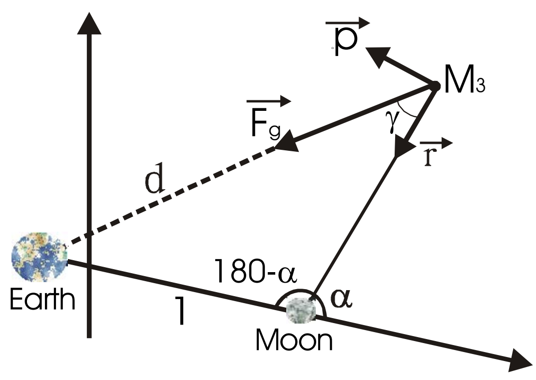

Fig. 6 – Gravitational force of the Earth.

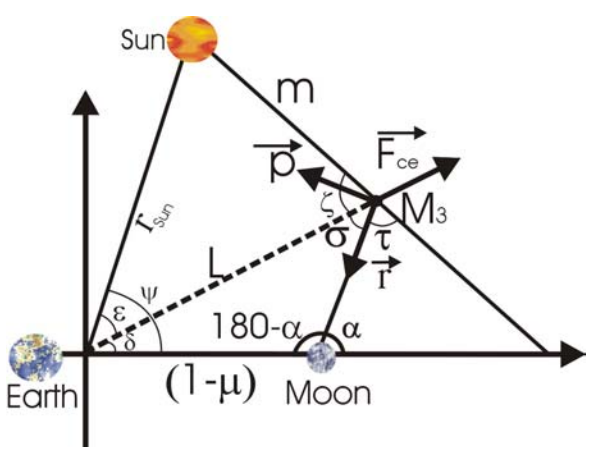

Fig. 7 – The gravitational force of the Sun and the centrifugal force.

Now, it is necessary to develop an equivalent equation for the gravitational force of the Sun ($\vec{F}_S$). From Fig. 6, it is possible to find the following relations, where $\vec{F}_{Sr}$ stands for the radial component and $\vec{F}_{Sp}$ stands for the perpendicular component:

$$\vec{F}_{Sr} = -\frac{\mu_{sun} \cos\tau}{m^2} \vec{r} , \tag{5}$$

$$\vec{F}_{Sp} = -\frac{\mu_{sun} \sin\tau}{m^2} \vec{p} \tag{6}$$

where $R_{sun}^2 = L^2 + m^2 - 2Lm\cos\zeta \Rightarrow \cos\zeta = \dfrac{R_{sun}^2 - L^2 - m^2}{-2Lm}$, $\tau = \pi - \sigma - \zeta$, $\delta = \alpha - \sigma$, $\varepsilon = \psi - \delta = \psi - \alpha + \sigma$, $m^2 = L^2 + R_{sun}^2 - 2LR_{sun}\cos\varepsilon$.

Those equantions can be used to find analytical equations for the radial and perpendicular components of the gravitational force of the Sun. More details can be found in Prado (2002a).

During the approach phase, when the spacecraft is close to the Moon, the force that dominates the dynamics is due to the central body (the Moon). All others forces are perturbations on the motion of the massless particle. In the model considered here, the perturbations are due to the gravitational force of the Earth and the Sun and the centrifugal force due to the rotation of the system. In that way, an approach to understand the behavior of the perturbing forces is to study the components of each force during the approach phase. This study is performed in Prado (2002b), that shows an equation that relates the reduction of C_3 with the integral of the forces over the time.

The next step of this research is to use the analytical expressions derived in Prado (2002b) for the effects of the gravitational force of the Earth and the centrifugal force to obtain an equivalent equation for the gravitational force of the Sun, in order to obtain an estimate of the effects of the forces studied. The main idea is to estimate the potential of the field around the Moon to reduce the value of the C_3 due to the Earth and the Sun and not to make predictions for a single trajectory. The analytical equations to measure the effects of this perturbation are derived under the assumption that the trajectory followed by the spacecraft is an idealized trajectory that does not deviate from the radial direction. The real trajectories are not radial, as can be seen in the references shown in this paper, but the equations derived under this assumption can be used to: i) estimate the values of the possible reductions in the value of C_3; ii) show the existence of directions of motion that results in larger reductions of C_3, so mapping analytically the decelerating field that exists in the neighborhood of the Moon; iii) estimate the effects of the periapsis distance and the size of the sphere of capture, since the equations derived are explicitly functions of those parameters; iv) to study the effect of the fourth body in the savings obtained in the gravitational capture. Another justification for the radial trajectories used to derive the equations is that the reduction of C_3 is a result of the effects of the forces in time during the whole trajectory and, even for trajectories that shows several loops before arriving at the periapsis, during most of the time the trajectory can be seen as composed by a set of trajectories close to radial.

For the derivation performed here, the component measured is the radial, because this is the direction of motion under the assumption used here. Then, assuming that the spacecraft is in free-fall (subject only to the gravitational and centrifugal forces) traveling with zero energy (parabolic trajectory) and that the trajectories do not deviate from a straight line, the result is:

$$\text{Total energy} = E = 0 = \frac{1}{2}V^2 - \frac{\mu}{r} \Rightarrow V = \sqrt{\frac{2\mu}{r}} = \frac{ds}{dt} \tag{7}$$

Where ds is the space traveled by the particle during the time dt. To obtain the integral of the effect of the perturbing forces with respect to time, it is possible to perform the calculations in terms of the radial distance, by making the substitution:

$$\int_{t_0}^{t_f} F\,dt = \int_{S_0}^{S_f} (F/V)ds = \int_{r_{min}}^{r_{max}} (F/V)dr \tag{8}$$

The extreme points of the integration changes position (S_0 by r_{min} and S_f by r_{max}) here and in all the following integrations to take into account that the positive sense of the radial direction points towards the Moon. Since the spacecraft is assumed to approach the Moon on a radial trajectory the result $\phi = \alpha = \beta$ is valid, and the variable α is used as the independent parameter. Then, for the radial component of the Earth's gravity, up to the first order, the integral is (Prado, 2002b):

$$F_1(\alpha)=\left[\frac{(1-\mu)(q+\cos\alpha)}{(2\mu/q)^{1/2}(1+q^2+2q\cos\alpha)^{3/2}}r+(1-\mu)\left(-\frac{3\,(q+\cos\alpha)^2}{(2\mu/q)^{1/2}(1+q^2+2q\cos\alpha)^{5/2}}+\right.\right.$$
$$\left.\left.+\frac{\mu(3q+\cos\alpha)}{q^2(2\mu/q)^{3/2}(1+q^2+2q\cos\alpha)^{3/2}}\right)\left(\frac{r^2}{2}-qr\right)\right]_{r_{min}}^{r_{max}} \tag{9}$$

For the radial component of the centrifugal force, the integral is (Prado, 2002b):

$$\int_{r_{min}}^{r_{max}}(F_{ce}/V)ds=\int_{r_{min}}^{r_{max}}((\mu-1)\cos\alpha+r)(2\mu/r)^{-1/2}dr=\left[\left(-0.4\,r^2+\frac{2}{3}r(\mu-1)\cos\alpha\right)(2\mu/r)^{-1/2}\right]_{r_{min}}^{r_{max}} \tag{10}$$

Repeating the process for the gravitational force due to the Sun, we have (Prado, 2002a):

$$F_3^1(\alpha)=\left[\left(\frac{\mu_{sun}(q+\cos\alpha-\mu\cos\alpha-r_{sun}\cos(\alpha-\psi))}{\sqrt{2\mu/q}\left(1+q^2+r_{sun}^2-2\mu+\mu^2+2q(1-\mu)\cos\alpha-2qr_{sun}\cos(\alpha-\psi)-2r_{sun}\cos\psi+2r_{sun}\mu\cos\psi\right)^{3/2}}\right)r\right]_{r_{min}}^{r_{max}}$$
$$+\left[\left(-\frac{3(2q+2(1-\mu)\cos\alpha-2r_{sun}\cos(\alpha-\psi))(q+\cos\alpha-\mu\cos\alpha-r_{sun}\cos(\alpha-\psi))}{\sqrt{2\mu/q}\left(1+q^2+r_{sun}^2-2\mu+\mu^2+2q(1-\mu)\cos\alpha-2qr_{sun}\cos(\alpha-\psi)-2r_{sun}\cos\psi+2r_{sun}\mu\cos\psi\right)^{5/2}}+\right.\right. \tag{11}$$
$$\left.\left.+\frac{\sqrt{q/\mu}+\sqrt{q/\mu}\left(\frac{(q+\cos\alpha-\mu\cos\alpha-r_{sun}\cos(\alpha-\psi))}{2q}\right)}{\left(1+q^2+r_{sun}^2-2\mu+\mu^2+2q(1-\mu)\cos\alpha-2qr_{sun}\cos(\alpha-\psi)-2r_{sun}\cos\psi+2r_{sun}\mu\cos\psi\right)^{3/2}}\right)\left(\frac{\mu_{sun}}{\sqrt{2}}\right)\left(\frac{r^2}{2}-qr\right)\right]_{r_{min}}^{r_{max}}$$

For numerical simulations, the values used are: r_{min} = 1838/384400 (100 km above the lunar surface), r_{max} = 100000/384400 (100000 km above the lunar surface, the usual value for the sphere of capture of the Moon in the ballistic gravitational capture studies), μ = 0.0121 (Earth-Moon system) and $q=(r_{min}+r_{max})/2$ (the medium point of the trajectory).

The best result, regarding capture, occurs for $\alpha = \psi$, that means that the spacecraft is aligned with the Sun, what is an expected result. Figure 8 shows the effects of the gravitational force of the Sun as a function of α and ψ.

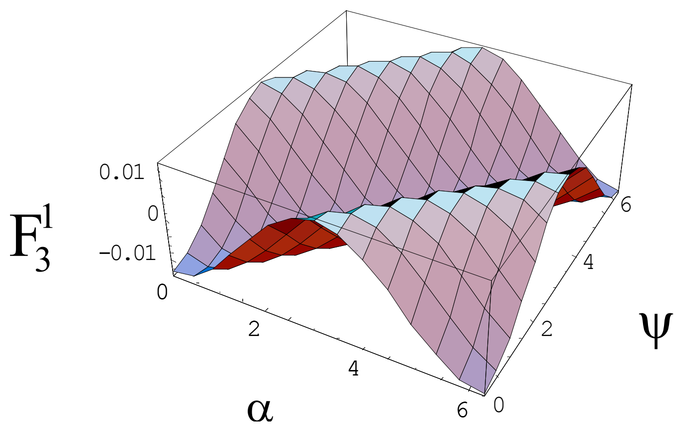

Fig. 8 – Effects of the gravitational force of the Sun as a function of α and ψ (Prado, 2002a).

Adding the radial effects of all the forces, the equation for the resultant force in the radial direction is obtained. This force will be called $F_r(\alpha)$. All the forces are plotted as a function of α in Fig. 9, for the case where $\psi = 0$. The numbers represents: 1 for the gravitational force due to the Earth; 2 for the centrifugal force; 3 for the gravitational force due to the Sun; 4 for the resultant force. From those results, it is clear that the integral of the total effect is always negative, which means that the spacecraft always has its velocity reduced by the perturbation. It is never increased. There are two points where the integral of the effect is null, which means that the two perturbing forces acting on the spacecraft cancel each other and it travels as if there were no perturbations at all. In this figure it is also possible to obtain the best point to perform the ballistic gravitational capture. This point is at $\alpha = 180°$, which has the strongest accumulated effect for the resultant force. Figure 6 shows the perturbation of the fourth body $F_{4b}(\alpha)$ in more detail, for the case where $\psi = 0$. Figure 7 shows the resultant forces acting in the motion of the spacecraft including and excluding the Sun. It is clear that the Sun helps to increase the effect of slowing down the spacecraft in a amount of the order of 3%.

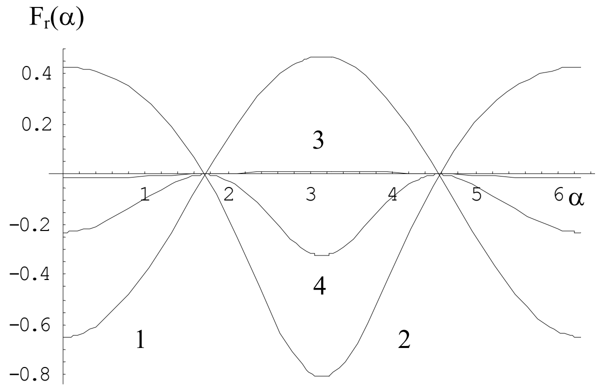

Fig. 5 – Integral of the disturbing radial forces (canonical units) vs. α (rad) for $\psi = 0$ (Prado, 2002a).

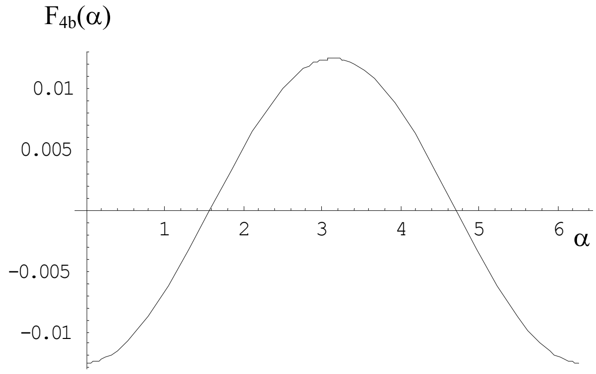

Fig. 6 – Effects of the gravitational force of the Sun as a function of α for $\psi = 0$ (Prado, 2002a).

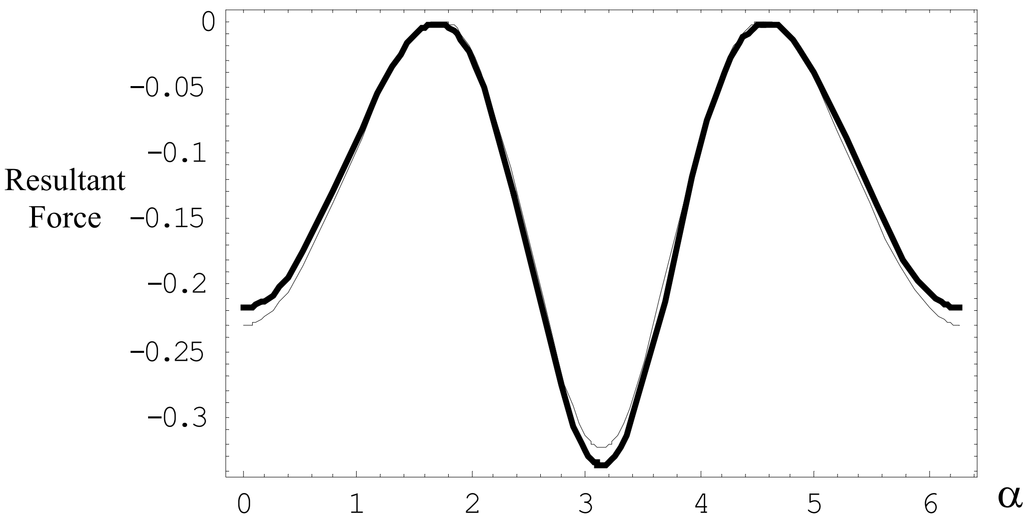

Fig. 7 – Integral of the resultant force including the Sun (dark line) and excluding the Sun (Prado, 2002a).

7. Conclusion

This paper studied the "gravitational capture" in the bi-circular restricted four-body problem by showing a survey of results related to this problem. Our numerical results showed some of the characteristics and concentrated in verifying the importance of the fourth body in this problem for Sun-Earth-Moon system.

The analytical results showed an explanation of the phenomenon based in the calculation of the forces involved in the dynamics as a function of time and in its integration with respect to time. It also derived analytical equations to study the effect of the fourth body, under the assumption of radial motion. The forces acting in the spacecraft can slow down its motion, working opposite to its motion. This is equivalent to applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to the Earth and the centrifugal force work in opposite directions, but the resultant force always works against the motion of the spacecraft, with the exception of two points where they cancel each other. Understanding these behaviors explains why a particle with a velocity slower than the escape velocity can escape from the Moon. The results also showed that the inclusion of the Sun in the dynamics could increase by about 3% the effects of the forces.

8. Acknowledgments

The author is grateful to the Foundation to Support Research in the São Paulo State (FAPESP) for the research grant received under Contract 03/03262-4 and to CNPq (National Council for Scientific and Technological Development) - Brazil for the contract 300828/2003-9.

9. References

Belbruno, E.A., 1987, "Lunar Capture Orbits, a Method of Constructing Earth Moon Trajectories and the Lunar Gas Mission", *AIAA-87-1054. In: 19th AIAA/DGLR/JSASS International Electric Propulsion Conference*, Colorado Springs, Colorado, May 1987.

Belbruno, E.A., 1990, "Examples of the Nonlinear Dynamics of Ballistic Capture and Escape in the Earth-Moon System", *AIAA-90-2896. In: AIAA Astrodynamics Conference*, Portland, Oregon, Aug. 1990.

Belbruno, E.A., 1992, "Ballistic Lunar Capture Transfer Using the Fuzzy Boundary and Solar Perturbations: a Survey", *In: Proceedings for the International Congress of SETI Sail and Astrodynamics*, Turin, Italy.

Belbruno, E.A., Miller, J.K., 1990, "A Ballistic Lunar Capture for the Lunar Observer", *Jet Propulsion Laboratory, JPL IOM 312/90.4-1752, Internal Document*, Pasadena, CA.

Belbruno, E.A., Miller, J.K., 1990, "A Ballistic Lunar Capture Trajectory for Japanese Spacecraft Hiten", *Jet Propulsion Lab., JPL IOM 312/90.4-1731, Internal Document*, Pasadena, CA.

Krish, V., 1991, "An Investigation Into Critical Aspects of a New Form of Low Energy Lunar Transfer, the Belbruno-Miller Trajectories", *Master's Dissertation, Massachusetts Institute of Technology*, Cambridge, MA.

Krish, V., Belbruno, E.A., Hollister, W.M., 1992, "An Investigation Into Critical Aspects of a New Form of Low Energy Lunar Transfer, the Belbruno-Miller Trajectories", AI*AA paper 92-4581-CP*.

Miller, J.K., Belbruno, E.A., 1991, "A Method for the Construction of a Lunar Transfer Trajectory Using Ballistic Capture", *AAS-91-100. In: AAS/AIAA Space Flight Mechanics Meeting*, Houston, Texas.

Prado, A.F.B.A., 2002a, “Analytical Approximations to Study the Gravitational Capture in the Four-Body Problem”. Anais do II Congresso Nacional de Engenharia Mecânica (CONEM), João Pessoa, 12 a 16 de agosto de 2002.

Prado, A.F.B.A., 2002b, Numerical Study and Analytic Estimation of Forces Acting in Ballistic Gravitational Capture, *Journal of Guidance, Control and Dynamics,* Vol. 25, No. 2, pp. 368-375.

Szebehely, V.G., 1967, "Theory of orbits", *Academic Press*, New York.

Vieira Neto, E., 1999, Estudo Numérico da Captura Gravitacional Temporária Utilizando o Problema Restrito de Três Corpos. *Ph.D. Dissertation*, Instituto Nacional de Pesquisas Espaciais, Brazil.

Vieira Neto, E., Prado, A.F.B.A., 1995, A Study of the Gravitational Capture in the Restricted-Problem. *Proceedings of the "International Symposium on Space Dynamics"* pg. 613-622. Toulouse, France, 19-23/June/1995.

Vieira Neto, E., Prado, A.F.B.A., 1996, Study of the Gravitational Capture in the Elliptical Restricted Three-Body Problem. *Proceedings of the "International Symposium on Space Dynamics"* pg. 202-207. Gifu, Japan, 19-25/May/1996.

Vieira Neto, E., Prado, A.F.B.A., 1998, Time-of-Flight Analyses for the Gravitational Capture Maneuver. *Journal of Guidance, Control and Dynamics,* Vol. 21, No. 1, pp. 122-126.

Yamakawa, H., 1992, On Earth-Moon Transfer Trajectory with Gravitational Capture. *Ph.D. Dissertation,* University of Tokyo.

Yamakawa, H., Kawaguchi, J., Ishii, N., Matsuo, H., 1992, "A Numerical Study of Gravitational Capture Orbit in Earth-Moon System", *AAS paper 92-186, AAS/AIAA Spaceflight Mechanics Meeting*, Colorado Springs, Colorado.

Yamakawa, H., Kawaguchi, J., Ishii, N., Matsuo, H., 1993, "On Earth-Moon transfer trajectory with gravitational capture", *AAS paper 93-633, AAS/AIAA Astrodynamics Specialist Conference*, Victoria, CA.

10. Responsibility notice

The author is the only responsible for the printed material included in this paper.