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STUDY OF EXPERIMENTAL MODEL OF SUPPORTING STRUCTURE OF ROTATING MACHINERY

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Abstract. *This works proposes a method to improve the identification of modal parameters extracted from measurements taken from a flexible foundation of a rotating machine. These parameters play a very important role in the simulation of a complete rotor-bearing-foudation system, since the behaviour of a flexible foundation affects the dynamic response of the rotor. The majority of the methods of modal parameter extraction can not deal with noisy data generating several modelling errors. Using a combination of simple optimization algorithms of direct search and the least square method, it is possible to improve the modelling minimizing the errors, which allows the full automatization of modal analysis process, making it faster and reliable. This method has been tested using the response of a finite element model of the foundation, and its robustness and efficiency have been proved to extract modal parameter in a wide range of conditions.*

Keywords: *Rotating Machinery Structure, Modal Analysis, Optimization Method.*

1. Introduction

Modal techniques are widely used to analyse and simulate behaviour of structures, however the quality of the measured data becomes very important depending on the technique applied. The methodology proposed in this work offers a more robust approach to determine the modal parameters, that makes the analysis faster and more reliable. Consequently, the experimental data can have a lower quality, without affecting the final results.

Rotating machinery has been studied since long time ago, and recent works (Feng, 1997 and Weiming, 1996) have been considering not only the dynamic of the rotor shaft, but also the behaviour of all elements in the system, such bearings, couplings and the foundation structure. Some analytic methodologies have been applied to the study of the dynamic behavior of structures and mechanical systems (Jainski, 1982). Computerized mathematical models are thoroughly developed, aiming to solve vibration problems in machines and their supporting structure. Such methodologies use refined models that do not always give quick answer due to the high computacional time involved in the process of obtaining the dynamic responses.

Cavalca (1992) proposed a methodology (Method of Mixed Coordinates) to include the support structure. This methodology requires the determination of the modal parameters of the foundation to describe its behaviour. The method presented in this work is the bridge between the requirements of the Mixed Coordinates Method and the experimental analysis of foundation structures, which allow the study of the influence of these structures on the rotor dynamic.

The methodology is based on modal techniques and there is an additional procedure to refine the modal parameters, based on two minimization methods: Golden Section Search and Least Square Method. A finite element model has been employed to simulate the data acquisition of a real structure, and the frequency response analysed refers to the rotor bearings location, which is necessary in the Method of Mixed Coordinates.

2. Modal Analysis

2.1. Frequency response function

To determine the natural frequencies, initially, the frequency response function has to be acquired and analysed for an harmonic excitation force applied to a specific node, and acting in vertical and horizontal directions, in a determined range of frequencies.

Although, to solve the equation of motion of the foundation, it is necessary to find the response of a damped system, subjected to an harmonic excitation force with an unitary amplitude. The response to a unitary impulse is the Fourier Transform of the frequency response:

$$\lfloor M_f \rfloor \cdot \ddot{X}_f + \lfloor R_f \rfloor \cdot \dot{X}_f + \lfloor K_f \rfloor \cdot X_f = F_f(t) \tag{1}$$

Considering a harmonic force, Eq. 1 becomes:

$$\left(-\Omega_e^2 \lfloor M_f \rfloor + i\Omega_e \lfloor R_f \rfloor + \lfloor K_f \rfloor\right) \cdot \{x_{fo}\} = \{F_{fo}\} \tag{2}$$

The solution of Eq. 2 is complex, and can be determined through the Fourier transform, using the Gauss algorithm. Then the frequency response function can be written as:

$$h_{kj}(\Omega_e, p) = \sum_{i=1}^{N} \frac{X_k^i X_j^i}{m_i\left(\omega_i^2 - \Omega_e^2 + 2i\zeta_i \omega_i \Omega_e\right)} \tag{3}$$

With the complex frequency response function determined, it is possible to include its effect on a rotor-bearing system and simulate the influence of the supporting structure on the dynamic behaviour of the rotor.

2.2 Natural frequencies

The natural frequencies can be determined by the analysis of the frequency spectrum, to calculate it, the imaginary part of the frequency response of each channel is extracted, then their square is calculated and summed for each frequency, in the range of frequencies analyzed, and finally the result divided by the number of channels.

The spectrum of the obtained frequencies shows peaks of frequency, which corresponds to the natural frequencies of the structure, in the analysed range.

Then, for a determined frequency:

$$\overline{A}_{\Omega_{e_n}} = \frac{1}{m} \sum_{j=1}^{m} \left(h_{I_j}\right)^2 \tag{4}$$

Where:

$\overline{A}_{\Omega_{e_n}}$ $\rightarrow$ Quadratic mean of the imaginary components of the transfer function, at the nth frequency (Ω_e).

Ω_{e_n} $\rightarrow$ Nth excitation frequency.

h_{I_j} $\rightarrow$ Imaginary component of the jth transfer function at the frequency (Ω_{e_n}).

$m \rightarrow$ Number of transfer functions.

$n \rightarrow$ Number of frequencies in the analysed range.

2.3 Damping coefficient

With the natural frequencies of the structure determined, they can be used to calculate the damping parameter of the structure. These parameters are determined through the derivative of the phase diagram, which has to be calculated near the natural frequencies of the structure. Thus if the coupling between modes is neglected, the transfer function phase presents a linear change of 180º, in an interval close to each ressonance. So the damping factor for the jth mode is defined as:

$$\xi_i = \left(\frac{\partial \omega_n}{\partial \psi}\right)_{\Omega_e = \omega_{n_i}} \tag{5}$$

Where:

$\xi_i \rightarrow$ Logarithm decrement or damping ratio

ω_{n_i} $\rightarrow$ Natural frequency of the ith mode.

$\Omega_e \rightarrow$ Excitation frequency.

$\psi \rightarrow$ Transfer function phase.

2.4 Mode shapes

The Nyquist diagram is used to determine the mode shapes, where the amplitude and transfer function phase are plotted. The method using the Nyquist diagram is based on the calculation of the circuference arc which aproaches the transfer function in the neighbourhood of each mode, considering the fact that the complex response vector represents the equation of a circle (Diana et al, 1988). Plotting the circunference, it is possible to determine the mode shapes analysing the coordinates of the center of the circle. Since the coupling between modes is neglected, the structure behaves as a SDOF system near each ressonance.

In the ressonance, the amplitude of the response becomes the circle diameter (Ewins, 1984) to a SDOF system:

$$D = \frac{1}{2\zeta_i}\left\| A_{kj_i} \right\|^2 \tag{6}$$

Where D is the diameter of the circle and :

$$A_{kj_i} = \frac{X_k^i X_j^i}{k_i} \tag{7}$$

The phase angle of the excited channel can be written as:

$$\alpha_j = \arctan\left(\frac{\mathrm{Re}\left(\left(i \cdot \bar{h}_{jj} \right)^{1/2} \right)}{\mathrm{Im}\left(\left(i \cdot \bar{h}_{jj} \right)^{1/2} \right)} \right) \tag{8}$$

Then the phase angle of the other channels become:

$$\alpha_j = \arctan\left(\frac{\mathrm{Re}\left(\frac{i \cdot \bar{h}_{kj}}{X_j^i} \right)}{\mathrm{Im}\left(\frac{i \cdot \bar{h}_{kj}}{X_j^i} \right)} \right) \tag{9}$$

2.5 Generalized mass

Once the mode shapes are determined, the modal masses are evaluated minimizing the error between the two transfer functions: the analytical transfer function, defined by the Eq. 3, and the approached transfer function. The analytical transfer function is calculated, determining the masses for all mode shapes. Then the masses are used to calculate the new transfer function (approached function (h)).

$$h_{kj} = \sum_{i=1}^{N} \frac{x_k^i x_j^i}{m_i\left(\omega_i^2 - \Omega_e^2 + 2i\zeta_i \omega_i \Omega_e \right)} \tag{10}$$

Thus, with the masses of Eq. 3 determined, the approached transfer function (calculated in the neighbourhood of the mode) is defined to a determined natural frequency, considering the n mode shapes:

$$h_{kj}(\omega_1) = \frac{1}{m_1}\left(\frac{X_k^{(1)} X_j^{(1)}}{2i\zeta_1 \omega_1} \right) + \ldots + \frac{1}{m_i}\left(\frac{X_k^{(i)} X_j^{(i)}}{\omega_i^2 + \omega_1^2 + 2i\zeta_i \omega_1} \right) + \ldots + \frac{1}{m_n}\left(\frac{X_k^{(n)} X_j^{(n)}}{\omega_n^2 + \omega_1^2 + 2i\zeta_n \omega_1} \right) \tag{11}$$

Where m_n is the generalized modal mass associated to the nth mode.

The solution of Eq. 11 determines the approached transfer function in the neighbourhood of each mode, and can be written as:

$$\begin{Bmatrix} h_{kj}(\omega_1) \\ \\ \cdots \\ \\ h_{kj}(\omega_n) \end{Bmatrix} = \begin{bmatrix} \dfrac{X_k^{(1)}X_j^{(1)}}{2i\zeta_1\omega_1} & \cdots & \dfrac{X_k^{(i)}X_j^{(i)}}{\omega_i^2-\omega_1^2+2i\zeta_i\omega_1} & \cdots & \dfrac{X_k^{(n)}X_j^{(n)}}{\omega_n^2-\omega_1^2+2i\zeta_n\omega_1} \\ \cdots & & \cdots & & \cdots \\ \dfrac{X_k^{(1)}X_j^{(1)}}{\omega_1^2-\omega_n^2+2i\zeta_1\omega_n} & \cdots & \dfrac{X_k^{(i)}X_j^{(i)}}{\omega_i^2-\omega_n^2+2i\zeta_i\omega_n} & \cdots & \dfrac{X_k^{(n)}X_j^{(n)}}{2i\zeta_n\omega_n} \end{bmatrix} \cdot \begin{Bmatrix} \dfrac{1}{m_1} \\ \cdots \\ \dfrac{1}{m_n} \end{Bmatrix} \tag{12}$$

$$h = [T] \cdot \frac{1}{m} \tag{13}$$

However, sometimes the number of frequencies, in the frequency range chosen, is different from the number of generalized masses. Then, to determine the generalized masses, an error function is defined, relating two transfer functions (analytical and approached), which minimized for the natural frequencies, will determine the generalized masses:

$$\frac{1}{m} = \left([T]^t [T]\right)^{-1} \cdot \left([T]^t \left[h^*\right]\right) \tag{14}$$

3. Optimization method

Sometimes the results obtained need to be refined, after the modal analysis. In order to solve this issue, an optimization method has been created, which uses the modal parameters calculated with the modal techniques presented, as the initial value for the optimization process.

The direct search method used is the Golden Section Search (Bunday, 1984). This method does not require a specified number of function evaluations at the outset, as the Fibonacci search. The name Golden Section search comes from the Golden ratio, which is the constant ratio obtained when the number of evaluations is infinite.

The Golden Section search is used to optimize the values of phase angle and the damping coefficient. To refine the generalized mass in each loop of the algorithm, a least squares method (Ruggiero et al, 1988) is used to minimize the error. This method is very fast and reliable, however its application is restrict, because it requires somes conditions of derivative of the equation to be minimized.

These are the steps of the optimization algorithm:

a) Read the modal parameters and the experimental data;
b) Set n=1 (number of loops);
c) Set i=1 (number of modes);
d) Determine the interval of the damping factor of the mode i to be searched;
e) Search the damping factor which minimizes the error (Golden Search);
f) Set j=1 (number of channels);
g) Determine the interval of the phase angle of channel j and mode i to be searched;
h) Search the phase angle which minimizes the error (Golden Search);
i) If j<number of channels, then $j=j+1$, and go back to step **g**;
j) Calculate the generalized mass of the mode i which minimizes the error (Least Squares);
k) If i<number of modes, then $i=i+1$, and go back to step **d**;
l) If n<maximum number of loop and error>minimum error determined, then $n=n+1$, and go back to step **c**.

4. Finite element model

To test the methodology, a structure of a rotating machine has been simulated, using the finite element method, and the frequency response function of two points (in both vertical and horizontal directions for each point) has been calculated.

The first bearing of the rotor shaft is located on the point 3, as it can be seen on fig.1, and the second bearing is located on the point 8. These are the points which the FRFs has been calculated for.

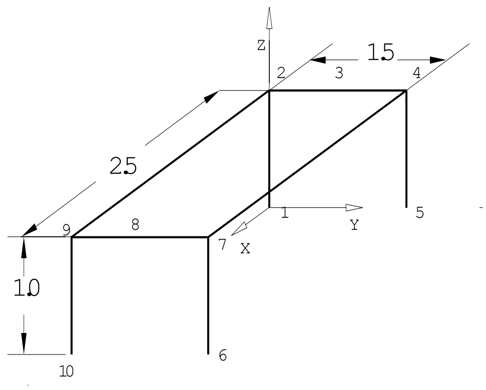

Figure 1. Foundation structure outline.

The finite element model has used beam elements, and steel has been considered as the material of the beams in this simulation. The horizontal direction of the point 3 has been excited with a harmonic signal.

5. Results obtained

Four channels have been used in this analysis, channel 1 and 2 refers to the vertical and horizontal directions of the bearing 1, point 3 in fig.1, channel 3 and 4 refers to the vertical and horizontal directions of the bearing 2, point 8 in fig.1. Table 1 presents the values of the modal parameters calculated through the methodology presented on section 2.

Few points were used because only the response of the points where the bearings are placed, are further used to simulate the behaviour of the rotor, however to get more realistic representation of the whole structure, for a deepest analysis of the foundation, a large number of nodes can be used.

Table 1. Modal parameters of the structure.

		Mode 1	Mode 2	Mode 3
Frequency [rad/s]		0.1987	0.2755	0.2473
Damping Coefficient		0.0015615	0.0028378	0.0048705
Generalized Mass		41868	30294	22187
Mode Shape (Channel)	**1**	-6.5713e-05 + 1.3516e-06i	-0.00014526 - 4.9664e-06i	-8.8638e-05 - 2.7765e-06i
	2	0.7066 - 0.014383i	0.70616 + 0.02578i	0.69961 + 0.023608i
	3	-3.3385e-05 + 6.4754e-07i	6.0629e-05 + 2.1155e-06i	5.0633e-05 + 1.5934e-06i
	4	0.70734 - 0.013676i	-0.70712 - 0.025398i	-0.71378 - 0.022542i

The three frequencies have been selected through the analysis of spectrum of the frequency response. It is important to notice that the frequencies selected refers to horizontal modes, excited by a harmonic force applied to point 3, the location of the bearing, in a horizonta1 direction.

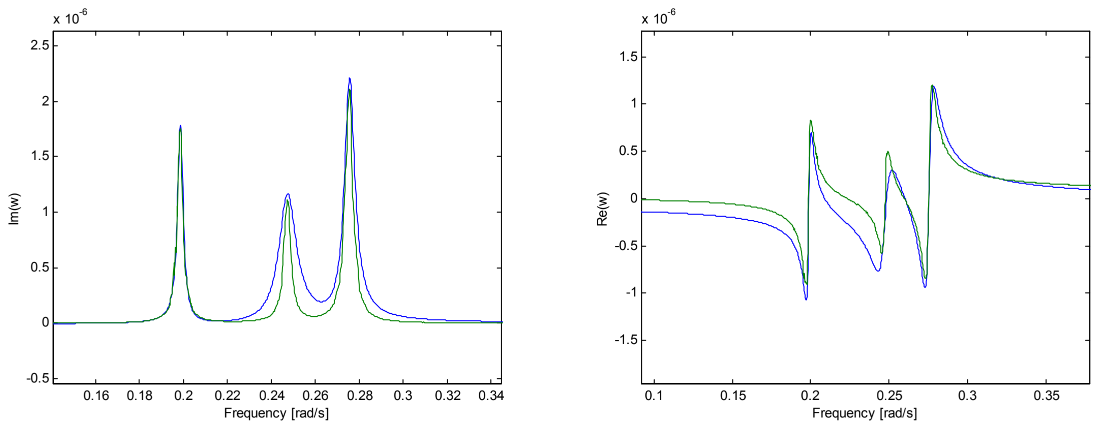

Figure 2. Comparison between frequency response functions of channel 1 (imaginary and real part) (green – experimental, blue – modelled)

Analysing fig. 2, it is noticiable the difference between the experimental and the modelled frequency response function concerning the third mode (0.24 rad/s). The coupling between the second and third mode has made the modal analysis less precise.

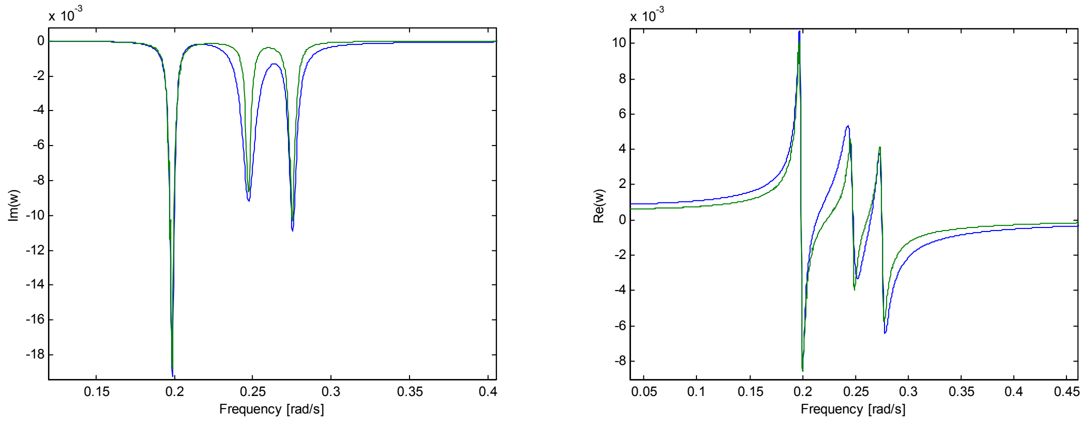

Figure 3. Comparison between frequency response functions of channel 2 (imaginary and real part) (green – experimental, blue – modelled)

In figure 3, the same event detected at channel 1 analysis takes place in the determination of the modal parameters of channel 2. The damping coefficient of third mode seems to be higher from what is expected, which can be visualized through the comparison of the imaginary parts.

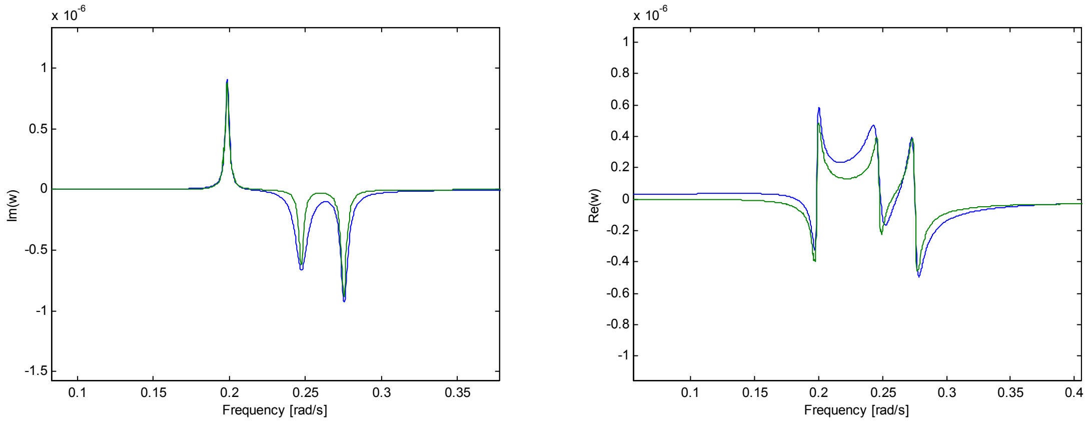

Figure 4. Comparison between frequency response functions of channel 3 (imaginary and real part) (green – experimental, blue – modelled)

Comparing the responses of channel 3 to channel 1, the negative influence of the calculation of the third mode becomes very clear.

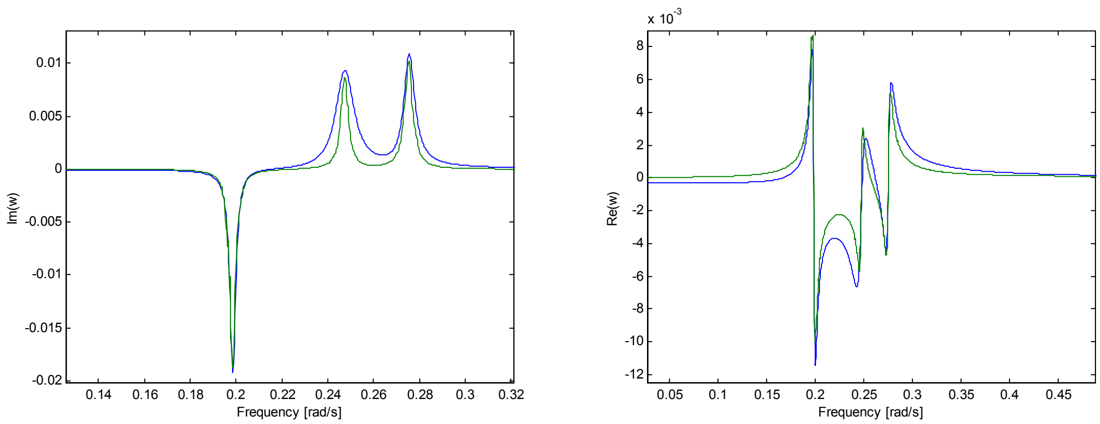

Figure 5. Comparison between frequency response functions of channel 4 (imaginary and real part) (green – experimental, blue – modelled)

The analysis of the experimental response compared to the modelled frequency response reveals that the modal technique used does not give precise results when there is a damping, which couples modes, even considering a light damping.

Table 2. Modal parameters optimized.

		Mode 1	Mode 2	Mode 3
Frequency [rad/s]		0.1987	0.2755	0.2473
Damping Coefficient		1.4990e-03	2.0433e-03	1.9023e-03
Generalized Mass		44599	43753	60435
Mode Shape (Channel)	**1**	-6.5716e-05 +1.1545e-06i	-0.00014504 +9.4154e-06i	-8.8671e-05 - 1.358e-06i
	2	0.70674 + 0.0025775i	0.7053 - 0.043437i	0.69987 + 0.013811i
	3	-3.3223e-05 +3.3467e-06i	6.0665e-05 -4.3201e-07i	5.0173e-05 +6.9917e-06i
	4	0.70513 - 0.057477i	-0.70754 - 0.007714i	-0.70529 - 0.11206i

Using the optimization procedure described on section 3, the modal parameters have been refined and the new values are presented on Tab. 2.

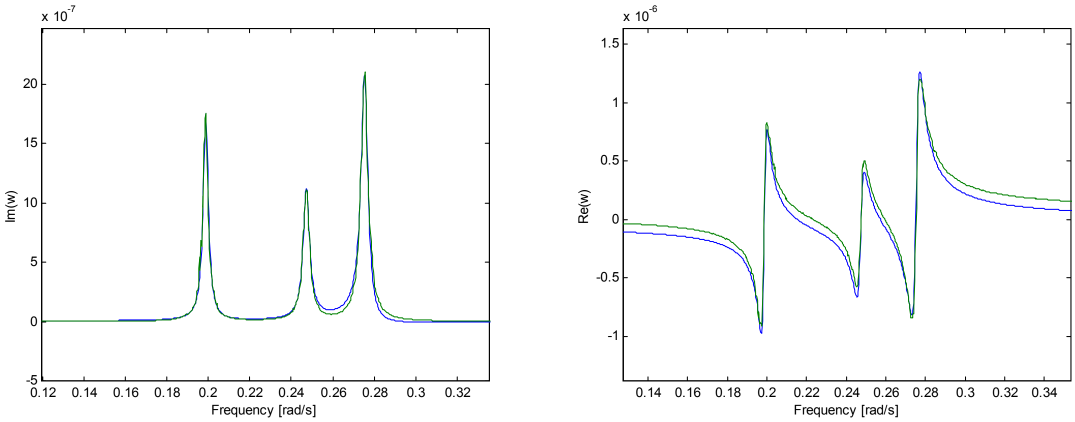

Figure 6. Comparison between frequency response functions of channel 1 (imaginary and real part) (green – experimental, blue – optimized)

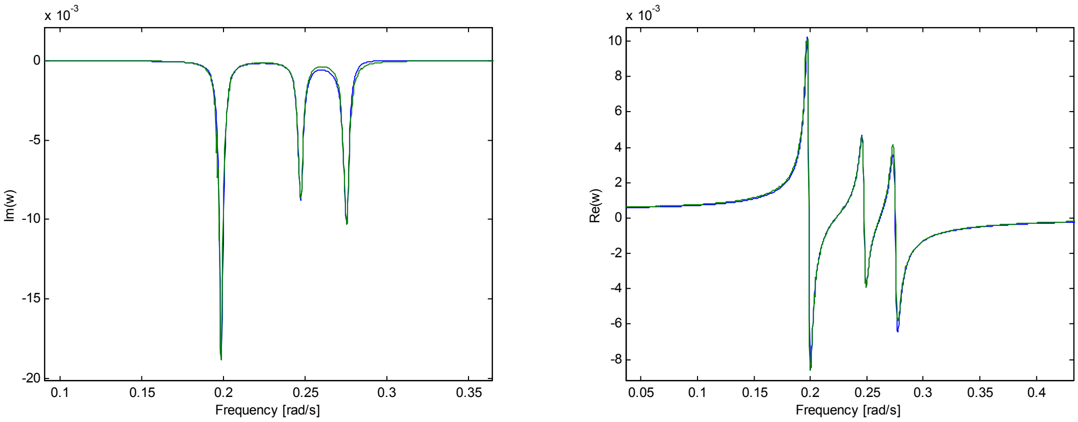

Figure 7. Comparison between frequency response functions of channel 2 (imaginary and real part) (green – experimental, blue – optimized)

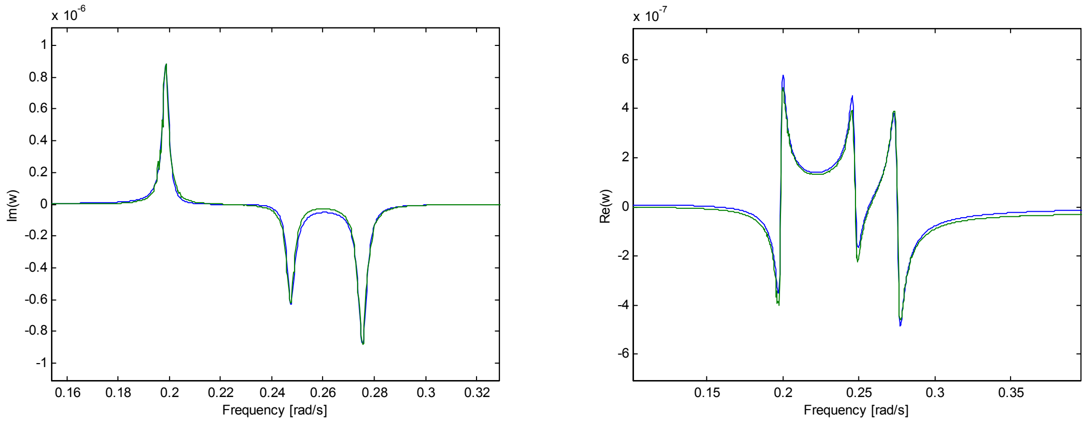

Figure 8. Comparison between frequency response functions of channel 3 (imaginary and real part) (green – experimental, blue – optimized)

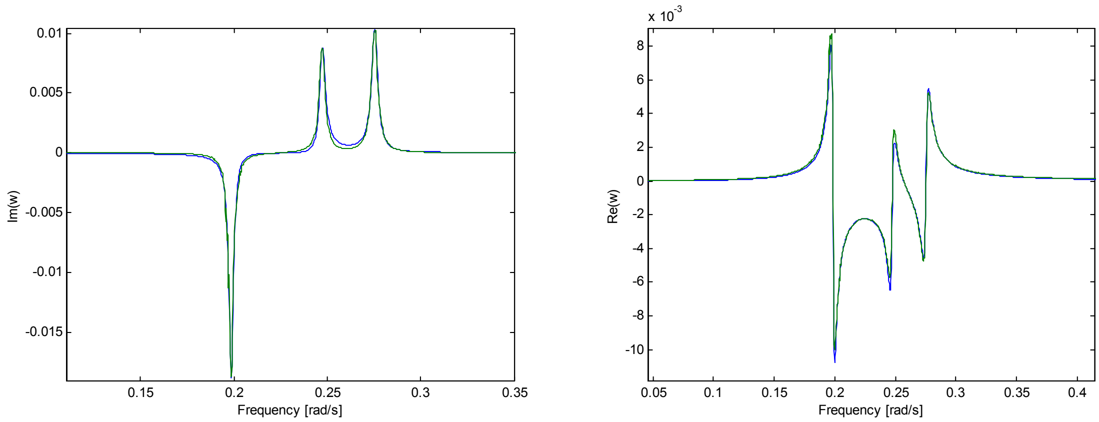

Figure 9. Comparison between frequency response functions of channel 4 (imaginary and real part) (green – experimental, blue – optimized)

Comparing the response optimized to the experimental response (Fig. 6, 7, 8 and 9), it is possible to notice an improvement on the results, particularly on the channels where the amplitude is higher (channels 2 and 4), which have a stronger influence on the error.

These optimized results could be improved if more modes were considered in the analysis. Considering the first three mode shapes the total error of the non optimized system is 64% of deviation from the experimental data. Otherwise, the optimized system error of the optimized system is 19%. Considering the seven first mode shapes the error becomes 7% considering the total range of frequencies acquired, and less than 3% considering the frequency range of the seven modes.

6. Conclusions

The use of simple optimization methods, as presented in this work, confirms that it brings benefits, specially when it takes the modal parameters calculated through the modal technique discussed on section 2. The computational effort of this methodology is minimal, if it is compared to multiple variable optimization methods, which depending on the number of variables and the quality of the experimental data, can take a long time to reach the global minimum.

This methodology is particularly useful to analyse experimental data with lower quality. Due to the wider frequency range that can be analyse during the optimization process, this range can be defined by the user, who selects the most appropriate range for each case.

Some improvements can be made considering the analysis of systems with higher damping coefficients, which makes more difficult the modal analysis and the optimization process. The use of multiple variable methods should be considered to refine the parameters in a small range of frequencie and with few variable to be optimized.

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9. Responsibility notice

The authors are the only responsible for the printed material included in this paper.