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NON-LINEAR SIGNAL ANALYSIS APPLIED TO SURFACE WEAR CONDITION MONITORING IN RECIPROCATING SLIDING TESTING MACHINES

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Abstract. . *When the surfaces of two elastic bodies present relative motions under certain amount of contact pressure the mechanical system can be instable. Experiments conducted on elastic bodies under friction forces shown that the dynamical system is self-excited by the non-linear behavior of frictional forces. These self-excited vibrations of reciprocating wear testing machines and automobile disk brakes can produce some noise, called squeal. The main objective of this paper is to study the vibrations signals measured on a reciprocating wear testing machine using non-linear signal analysis formulation. A two input and one output MISO model is proposed to represent the non-linear system dynamics. The global output is the sum of two outputs produced by one linear path associated in parallel with a non-linear path. This last path has a non-linear model that represents the friction force and another linear transfer function connected in series. Since the linear path is identified by traditional signal analysis, the non-linear function can be evaluated by the global input/output relationships, and can be correlated to wear conditions of the contact surfaces. Validation tests are conducted in a tribological system composed by a sphere in contact with and a prismatic body witch has an imposed harmonic motion. The global output force is simultaneously measured by piezoresistive and piezoelectric load cells. The sphere and prismatic body vibrations are measured by accelerometers and by a laser Doppler vibrometer. A microphone measures the generated sound level pressure and a resistive network determines the contact electric potential. All signals are digitalized with the same time base and the data is transferred to a microcomputer. The non-linear signal analysis technique uses this data to identify the friction force.*

***Keywords**. Non-linear signal analysis, friction force identification, wear, vibration*

1. Fundamentals of non-linear signal analysis

Physical parameters of the nonlinear system cannot be identified by traditional linear analysis since this technique implies that the probability density function of the inputs and outputs should have the same nature, condition not verified in nonlinear systems. (Bendat, 1997)

VOLTERRA series theory can be used to represent nonlinear systems using sets of high order polynomial functions assembled in parallel. Bendat (1992) proposed a new methodology that uses the well-known theory of multiple input/multiple output linear systems (MISO) modified to be applied to signal analysis of non-linear systems. Since the VOLTERRA methodology is highly dependent on the probability density function (PDF) of input signals, Bendat (1997) proposed a methodology that is independent of the PDF of the input signals and that produces results that are easy to interpret. This methodology is called nonlinear MISO signal analysis.

The nonlinear signal analysis is based on the hypothesis that the nonlinear characteristics of the system is additive to its linear characteristics and that the nonlinear effects acting on one input produces instantaneous output.

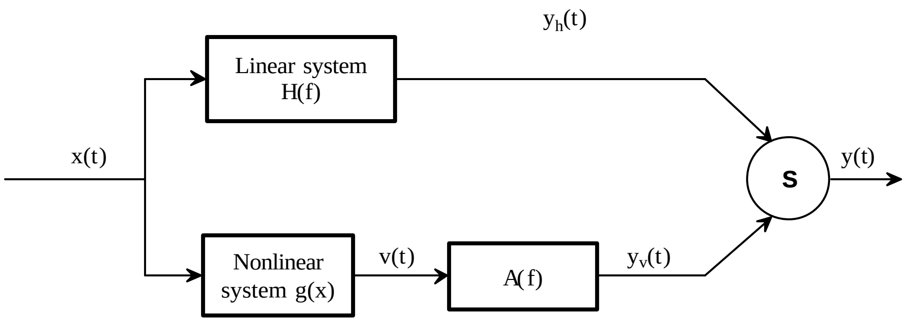

Figure 1. Nonlinear MISO systems block diagram.

The nonlinear system represented in the Fig. 1 do not have limitations regarding the characteristics of the signal input *x(t)* PDF, as imposed by VOLTERRA models. To use this methodology a prior knowledge of the nonlinear functions *g(x)* is required. As in the linear MISO theory, the system represented in the Fig.1, do not impose any restrictions on the correlation level of the input signals *x(t)* and *v(t)*, so the system can be represented by Fig.2, where *x(t)* and *u(t)* are uncorrelated.

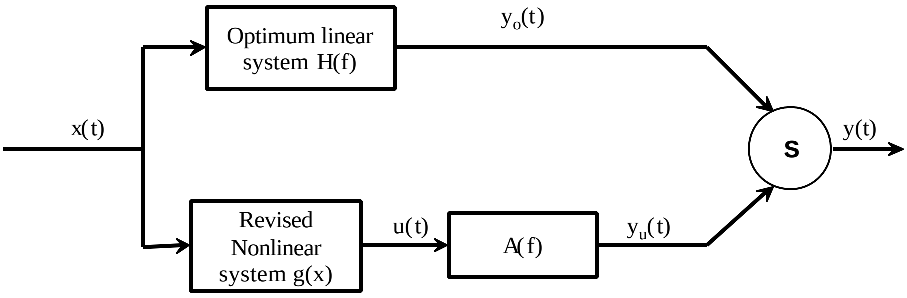

Figure 2. Nonlinear MISO system with uncorrelated inputs.

Spectral densities, transfer and coherence functions are calculated by:

$$L_{xv} = \frac{S_{xv}}{S_{xx}} \qquad S_{uy} = S_{vy} - L_{xv} * S_{xy} \qquad Ho = {S_{xy}}/{S_{xx}}$$
$$S_{y_o y_o} = \left|H_o\right|^2 * S_{xx} \qquad S_{uu} = S_{vv} * \left(1\text{-}\gamma_{xv}\right) \qquad A = {S_{uy}}/{S_{uu}}$$
$$H = H_o \text{-} \left({S_{xy}}/{S_{xx}}\right) * A \qquad S_{y_h y_h} = \left|H\right|^2 * S_{xx} \qquad S_{y_u y_u} = \left|A\right|^2 * S_{uu} \tag{1}$$
$$S_{y_v y_v} = \left|A\right|^2 * S_{vv} \qquad \gamma_{xv} = {\left|S_{xy}\right|^2}/{S_{xx} * S_{vv}}$$

The system presented by Fig.1 and Fig.2 have only one nonlinear function, however it should be noted that this methodology can represent systems with more than one nonlinear behavior. In this case the nonlinear MISO system representing each nonlinear function is preceded by a linear frequency response function (FRF), that represents the frequency dependency. The nonlinear MISO formulation permits an easy correlation of estimated parameters with physical properties of the assumed model of nonlinear systems.(Lépore, de Mello and Santos, 2004)

2. Vibratory systems with nonlinear properties

This nonlinear signal analysis methodology is applied to the mechanical system shown in Fig.3. The linear parameters m, c and k are the physical properties: mass, viscous damper and spring, respectively. A generic nonlinear device *g(x)* is connected in parallel to the suspension, and an external force F(t) excites the system. The time domain mathematical model is presented in Eq.2 and the correspondent frequency domain model of Eq.3 is obtained by taking the Fourier Transform of the Eq.2.

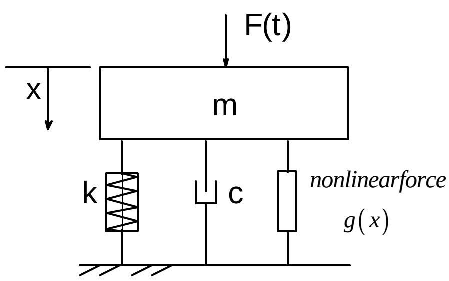

Figure 3. One degree of freedom mechanical system with a nonlinear device.

$$m\ddot{x} + c\dot{x} + kx + ag\left(x\right) = F\left(t\right) \tag{2}$$

$$F(f)=[H]\,X(f)+A(f)\boldsymbol{F}\left(g(x)\right) \tag{3}$$

The term *A(f)* represent the frequency dependencies of the nonlinear device and $\boldsymbol{F}\left(g(x)\right)$ is the Fourier transform of the nonlinear input *v(t)*. Eq. (2) and Eq. (3) show that there are no restriction on the nature of nonlinear function *g(x)* and that the nonlinear and linear terms are additive, according MISO theory. (Bendat, 1997)

The same procedure can be applied to systems with several degrees of freedom and several nonlinear effects without loss of generality. In this paper, the formulation is applied to identify the nonlinear force acting on a linear system, using only the measured system response. The analysis of linear systems submitted to friction forces are of great interest to tribology fields of research, since in the wear experiments the friction force between the specimens carries information about the surface condition. (Lépore, de Mello and Santos, 2004)

A generic linear system excited by a nonlinear force is represented in Fig.4 and its mathematical model is given by Eq.4.

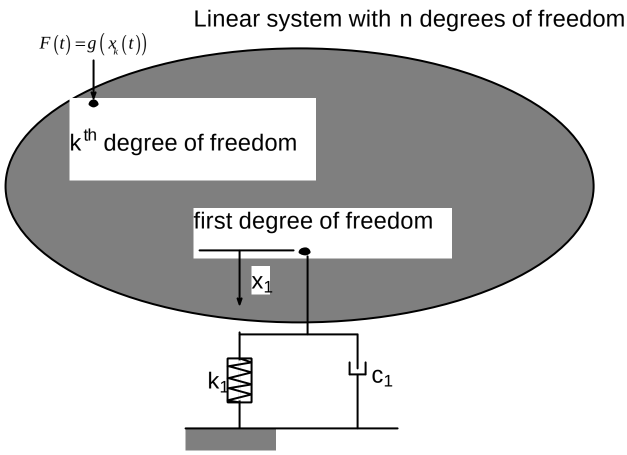

Figure 4. Linear system subjected to a nonlinear force.

$$[m]_{nxn}\{\ddot{x}\}_{nx1}+[c]_{nxn}\{\dot{x}\}_{nx1}+[k]_{nxn}\{x\}_{nx1}=\{F\}_{nx1} \tag{4}$$

The vector *{F}* includes the nonlinear forces. Using the same notation of Eq.(2) the nonlinear force can be written:

$$\{F\}_{nx1}=-[A]_{nxn}\{g(x)\}_{nx1} \tag{5}$$

Matrix *[A]* is diagonal with non-zero elements only at the positions corresponding to the degrees of freedom where the nonlinear forces act. The same condition is verified for the vector *{g(x)}* where no null nonlinear functions exist only at each corresponding degree of freedom. Assuming that the system is connected to the inertial reference by a spring and damper, located at the degree of freedom “1”, representing for example, a load cell that connects this degree of freedom to the ground, Eq.4 and Eq.5 can be rewritten as follows.

$$\begin{aligned}&[m]_{nxn}\{\ddot{x}\}_{nx1}+[c']_{nxn}\{\dot{x}\}_{nx1}+[k']_{nxn}\{x\}_{nx1}=-[A]_{nxn}\{g(x)\}_{nx1}+\{F_{load\_cell}\}\\&\{F_{load\_cell}\}=-[c'']_{nxn}\{\dot{x}\}_{nx1}-[k'']_{nxn}\{x\}_{nx1}\end{aligned} \tag{6}$$

where:

$$\begin{aligned}&[c']_{nxn}=[c]_{nxn}-[c'']_{nxn}\\&[k']_{nxn}=[k]_{nxn}-[k'']_{nxn}\\&[c'']_{nxn}=\begin{bmatrix}c_1&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&0\end{bmatrix}_{nxn}\\&[k'']_{nxn}=\begin{bmatrix}k_1&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&0\end{bmatrix}_{nxn}\end{aligned} \tag{7}$$

$$[m]_{nxn}\{\ddot{x}\}_{nx1}+[c']_{nxn}\{\dot{x}\}_{nx1}+[k']_{nxn}\{x\}_{nx1}+[A]_{nxn}\{g(x)\}_{nx1}=\{F_{load\_cell}\} \tag{8}$$

The frequency domain representation of the linear system, subjected to nonlinear forces, is finally obtained by taking the Fourier transform on both sides of Eq. (8).

$$\{F_{load\_cell}(f)\}_{nx1}=[H'(f)]_{nxn}\{X(f)\}_{nx1}+[A(f)]_{nxn}\{\boldsymbol{F}\ (g(x_k))\}_{nx1} \tag{9}$$

Using Eq.(9) and the concept presented in Fig. (1), the correct representation of a MISO system with nonlinear excitation force is constructed.

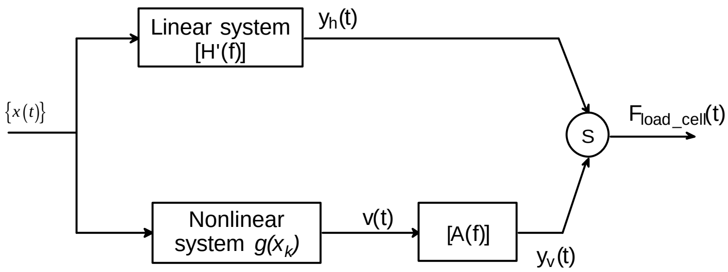

Figure 5. MISO system used to indirectly determine the nonlinear force.

Matrix *[H]* is replaced by matrix *[H']* to represent the FRF of the system in Fig.(4) with the terms k_1 and c_1 removed from stiffness and damping matrix and transferred to the right side of Eq.(8) representing the force measured by the load cell. It should be noted that the force measured by the load cell includes the linear component Sy_hy_h and the nonlinear component Sy_vy_v . Moreover, to obtain the true nonlinear force, by means of nonlinear MISO signal analysis, a prior knowledge of nonlinear function *g(x)* is required.

3. Nonlinear model of the friction force

The proposed model for the friction force between two surfaces is capable to represent stick and slip displacements and also its hysteretic nature. The Coulomb friction law is used to relate the normal and tangential forces at the contact surfaces. Figure 6 presents the physical behavior of contact model between one static surface and a second body in one cycle of harmonic imposed displacement. The mathematical model is given on Eq.(10) for each kinematics condition at the contact.

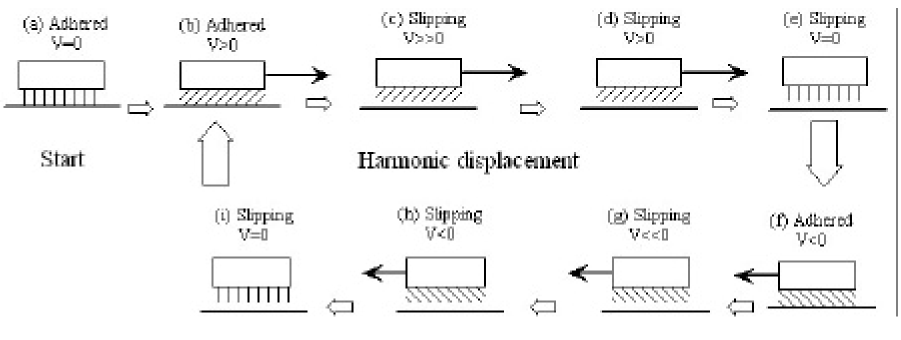

Figure 6. Schematic diagram of the friction model.

if the body is at rest and will start the movement

$$F_{Coulomb}=\mathrm{sgn}(v_{rel})*u_{rel}*K_{Coulomb}$$

if the body is slipping in the static surface

$$F_{Coulomb}=\mathrm{sgn}(v_{rel})*F_{Coulomb} \tag{10}$$

$$u_s=u_{rel}$$

if the body is adhered to surface after its slip in the static surface

$$F_{Coulomb}=\mathrm{sgn}(v_{rel})*(u_{rel}-u_s)*K_{Coulomb}$$

The slip displacement u_s do not change if the body is adhered to the static surface. Friction stiffness, related to elastic deformation of the asperities in contact, is defined as $K_{Coulomb.}$ The relative velocity and displacement are defined as v_{rel} and u_{rel} , respectively.

Using this model, the nonlinear function $g(x)$ of the MISO analysis, is completely defined. It can be verified that $F_{Coulomb}$ is function only of the state variables of the global system. (Lépore, de Mello and Santos, 2004)

Imposing $u_{rel}(t)$ as a sinusoidal displacement on Eq.(10), the normalized the friction force behavior is calculated and presented by Fig.7.

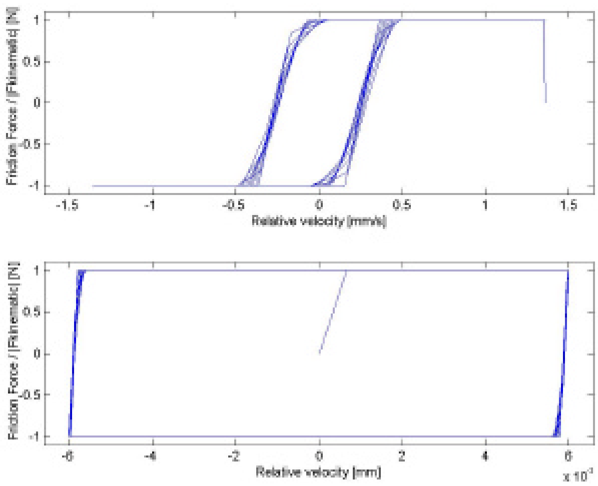

Figure 7. Behavior of nonlinear friction model.

The hysteretic and chaotic nature of the friction are evident. Comparing Fig.6 and Fig.7 should be noted that the elastic potential energy, stored in the deformed asperities in contact, was not released until the body and the surface adhered again. Moreover this condition is different from the start of movement, when the adhesion between the body and the surface occurs without any stored potential energy in the contact. Hysteresis is represented by transition from stage “h” to stage “b” and from stage “d” to stage “f”, shown in the Fig. 6.

4. Results and discussion

Application of the nonlinear MISO signal analysis in wear experiments is directly attained to the capacity of this methodology to provide the best estimate to friction force. Two classes of experiments were conducted to show the friction force identification capability.

A. One degree of freedom linear system with a Coulomb friction damper: In this group of experiments the vibratory system was excited by an harmonic force. Friction force due a Coulomb damper is obtained by MISO nonlinear signal analysis, then compared with the signal measured by a load cell placed between the mass and the Coulomb friction damper;
B. Reciprocating wear test in one specimen of aluminum covered by a small layer of bisulphate of molybdenum (BM): This group of experiments was conducted to verify if the friction force estimated by MISO nonlinear signal analysis is sensitive to changes of the tribological conditions on wear tests.

4.1. Case A: Coulomb damper on a one degree of freedom vibratory system.

The experimental apparatus used to analyze case A is shown in Fig. 8. However, the signal measured by the load cell 02 is not the actual friction force, since it is the summation of friction force and inertia force generated by the moving mass of the Coulomb damper.

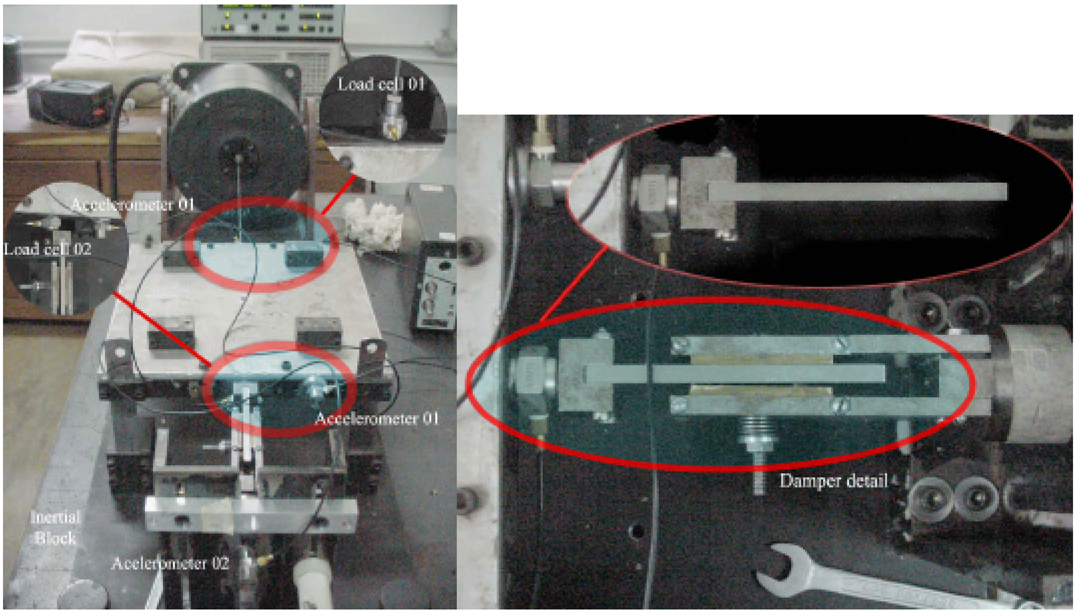

Figure 8. Experimental apparatus used in group tests A.

The nonlinear system represented in the Fig.3 is correlated to experimental apparatus as follow:

- Linear system: Vibratory table supported by four thin bars connected in parallel, with one degree of freedom at the frequency band of interest;
- Nonlinear device: Coulomb damper installed between the table and a fixed 'rigid' support.

As shown in the Fig. 8 the table movement was measured by the accelerometer 01, and the accelerometer 02 measured the small displacement of damper 'rigid' support. Load cell 01 measured the excitation force and load cell 02 measured the force due Coulomb friction damper plus the inertia force of damper moving part. So, the nonlinear MISO system was configured as follows:

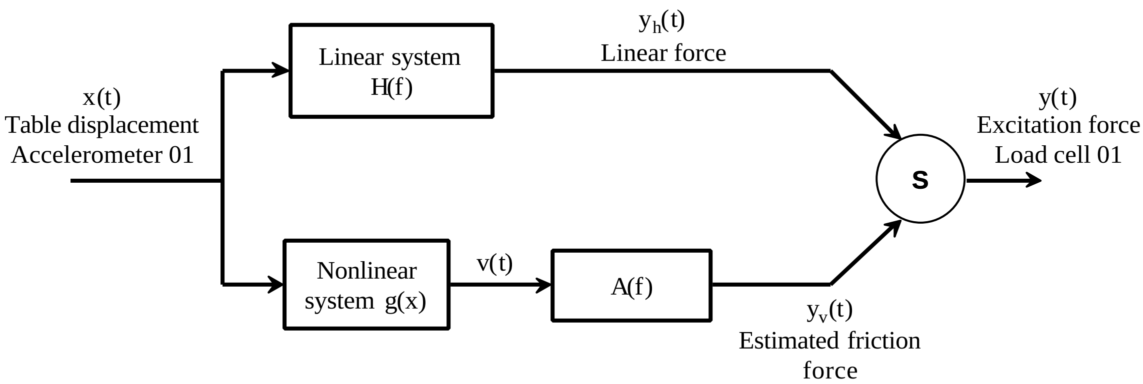

Figure 9. Nonlinear system configuration to group tests A.

Experiments used to validate the nonlinear MISO formulation were conducted with harmonic excitation forces with frequencies 5 Hz up and 18 Hz. These frequencies were chosen to assure that the force measured by load cell 02 be the best approximation of the real friction force, without the effect caused by the inertia force of the damper apparatus moving parts, and to minimize the vibrations of the fixed 'rigid' support. These frequencies were selected analyzing the cross correlation between the acceleration measured by accelerometer 02 with the force measured by the load cell 02, using a white noise force as excitation. This result is shown bellow:

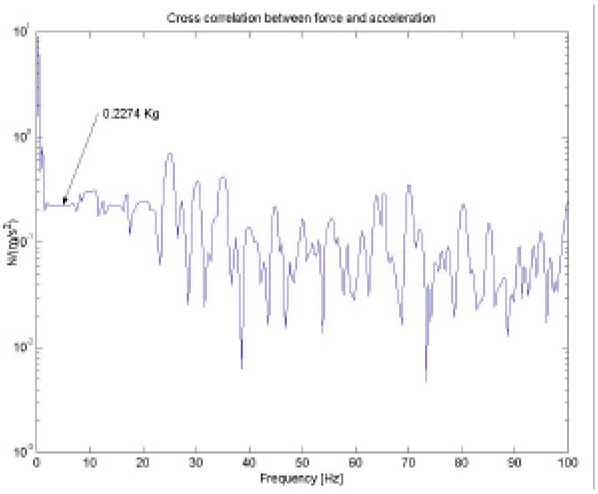

Figure 10. Cross correlation between the signals of accelerometer 01 and load cell 02.

As can be noted in the Fig. 10, the frequencies above 20 Hz have a great influence of inertia forces of the damper moving part. These influences can also be due to the bending effect in the load cell, small clearance of the joint parts and due small misalignment of the center of mass of moving parts with the load cell geometric center. Since inertia forces are directly proportional to acceleration, in this case is proportional to the square of frequency, the results agree with theory. When frequency increases, the inertia force is greater than the friction force. Additionally, the mass of the damper moving part was identified dynamically and statically and the results are 0.227 kg and 0.206 kg, respectively. This shows the influence of the inertia force.

The obtained results with harmonic forces are presented at the following figures: Fig.11, Fig.12a and Fig 12b for two excitation frequencies.

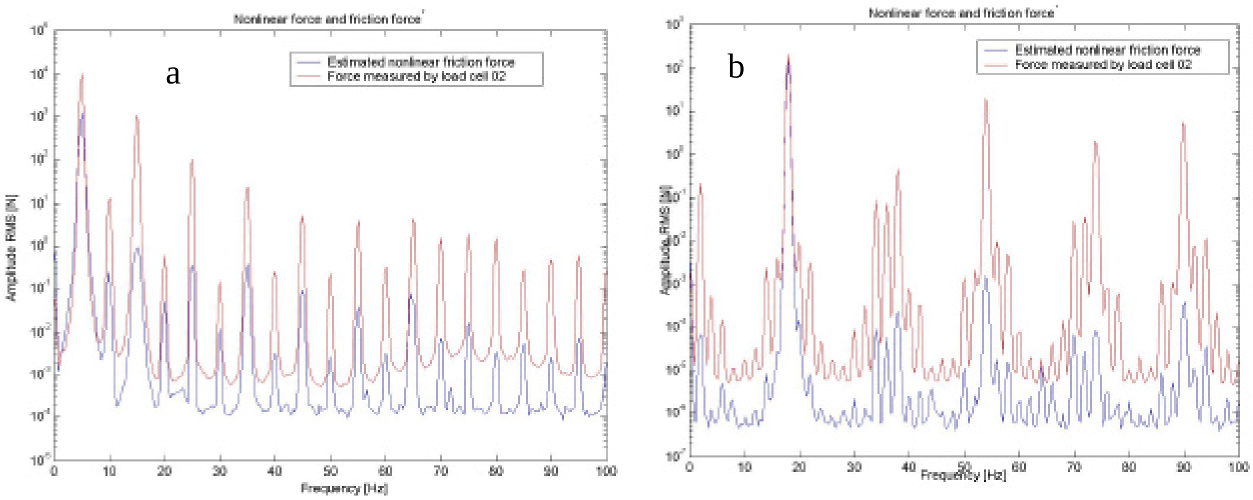

Figure 11. Results from group test A: (a) with 5 Hz and (b) with 18 Hz harmonic excitation forces

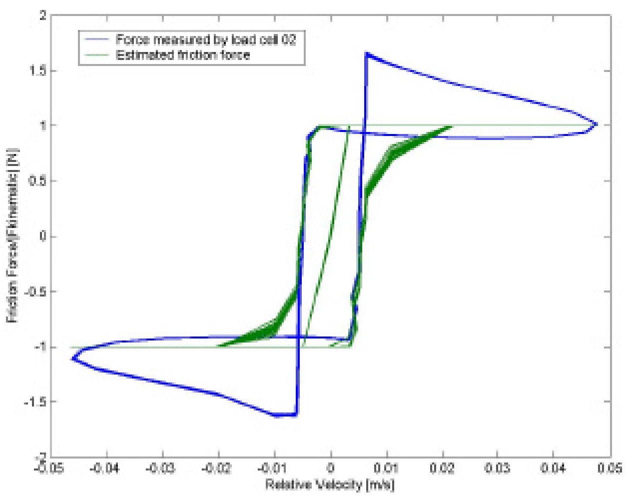

Figure 12. Behavior of nonlinear friction model and force measured by load cell 02 from group test A, with 18 Hz harmonic excitation force

At the excitation frequencies there is a small difference between the force measured by the load cell 02 and the estimated friction force due to the inertia force effect. Another source of error in the estimated friction force is the nonlinear function *g(x)* presented by Eq.(10) is not exact. This source of error appears since the theoretical friction force is assumed to be square, with a small distortion due hysteretic effect, but do not represent other effects, which can exist in the real friction force or in the measured force. As can be seem in the Fig. 12, the inertia effect promotes the lobes of the measured force for velocity modulus values greater than 0.01 m/s.

However, the proposed model is capable to represent the main effects and permits a good approximation to this type of force. Inertia effects are responsible by the greater difference present at higher harmonics of the excitation frequency, but they are extracted by the proposed methodology.. An amplitude modulation effect is observed in Fig.11b, generated by the motion of the large inertial block, upon which the vibratory system is mounted, whose natural frequencies are about 2.5 Hz.

4.2. Case B: Reciprocating wear test in one specimen of aluminum covered by a thin BM layer.

Experiments with the reciprocating wear test machine were conducted using the formulation presented by Eq.(9) and the nonlinear force model presented by Eq.(10). The diagram of the apparatus, the instrumentation chain and some of the test conditions are presented at Fig.13.

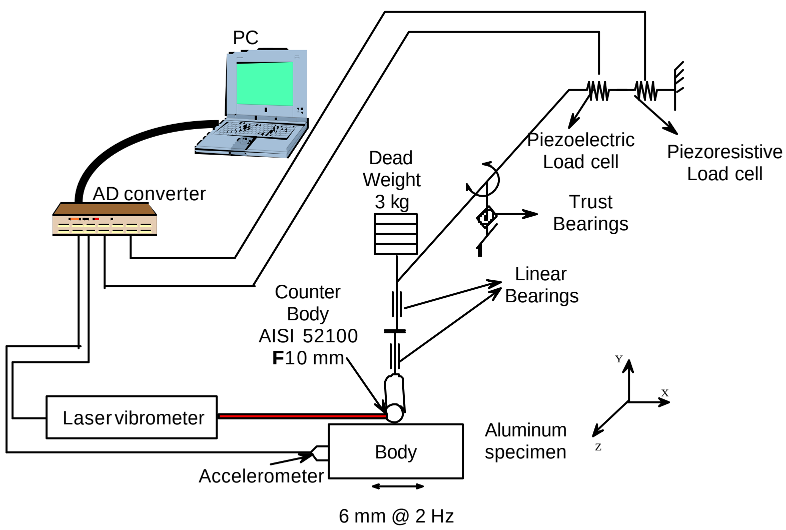

Figure 13. Experimental apparatus and the instrumentation chain used in reciprocating wear tests.

Based on Fig. 13 and Eq.(9) the block diagram associated with the nonlinear MISO system is constructed and presented in Fig.14.

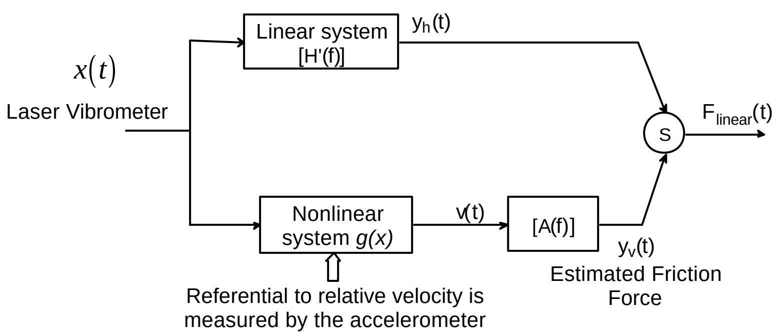

Figure 14. Nonlinear MISO system used to the reciprocating wear tests.

The reciprocating wear experiments take 2 hours to complete. The signals are acquired with 10 kHz rate using a simultaneously 16 channel AD converter with 12 bits of resolution. Averaging 10 blocks with 32768 points each, the results are stored in hard disk at 60 seconds interval. The tested specimens were analyzed in a scanning electron microscope (SEM) to compare the surface condition and the wear mechanism with the evolution of the friction force estimated by nonlinear MISO technique.

Three classes of experiments were carried out regarding the final surface condition of the aluminum specimen: I) the experiment was stopped before any damage occur to the BM layer; II) the experiment was stopped when the layer degradation is minimum; III) the experiment was conducted until the BM layer was completely removed.

Results obtained with nonlinear analysis are presented by the following figures, where the *x-axis* indicates the measurement time; the *y-axis* is the frequency band and the color scale is the *log* amplitude of the mean square of the nonlinear force identified signal.

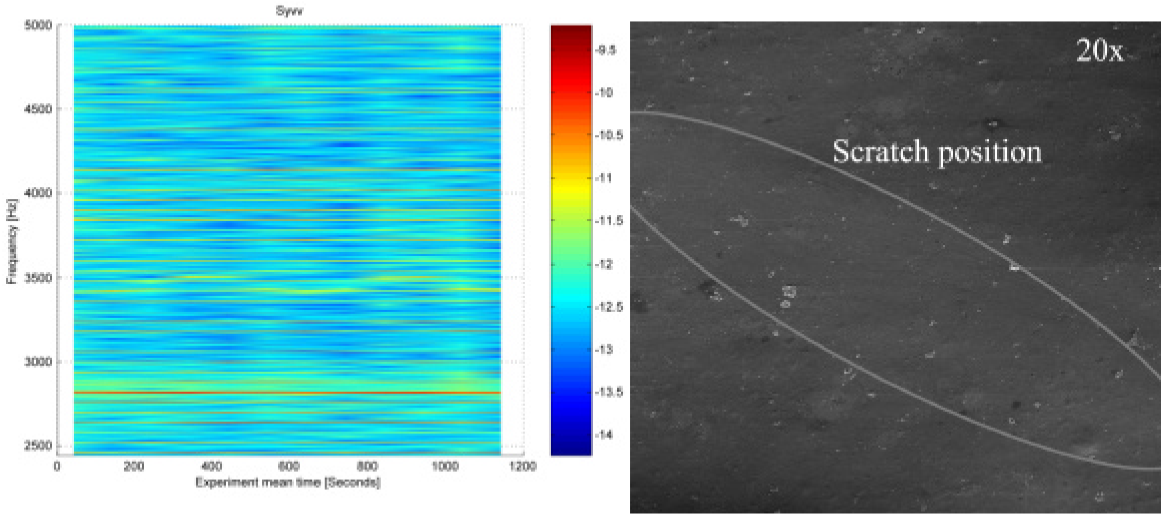

Figure 15. Nonlinear MISO results and SEM images from Class I experiments. (no damage at the BM)

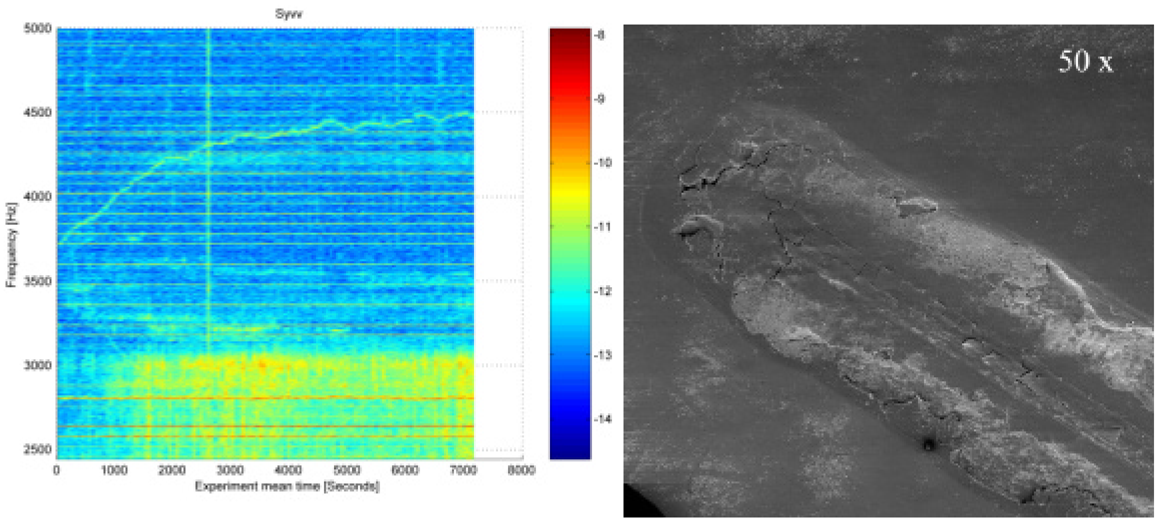

Figure 16. Nonlinear MISO results and SEM images from Class II experiments. (BM medium wear)

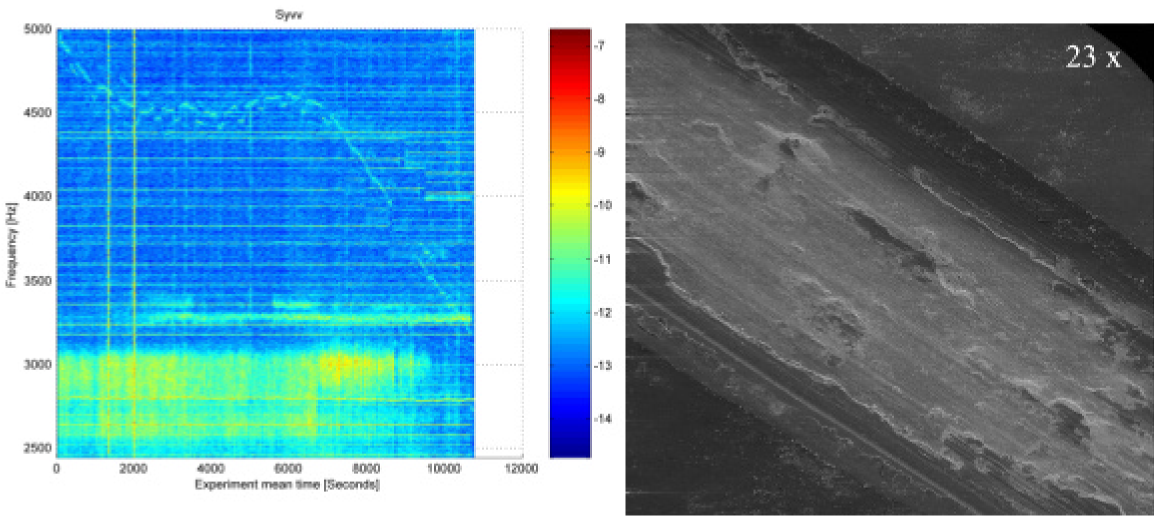

Figure 17. Nonlinear MISO results and SEM images from Class III experiments. (aluminum wear)

Figure 15 shows that there is no change of the estimated friction forces as well as in the surface condition along 1800 seconds of the experiment. However, Fig 16 and Fig. 17 in frequency band between 2500 Hz and 3000 Hz, present an increase on the mean square force amplitude, which is correlated to the surface conditions, indicated by the formation of some cracks on the BM layer, followed by its fragmentation promoted by the contact force. After fragmentation a small quantity of BM remains in the contact region, until it is completely removed (around 9000 seconds on Fig.17) and the signal energy in this frequency band decreases again. A detailed analysis of Fig. 17, using a bank of filters, is presented in Fig. 18b, where two peaks occur at the same time position (1642 and 2000 seconds) of the two vertical lines shown in Fig. 17. The same lines can be noted in the Fig. 16 and on its correspondent graphic on Fig.18a. Looking at the surface photography, the first peak seams to be related with the crack formation, and the second is associated to the fragmentation of the BM layer.

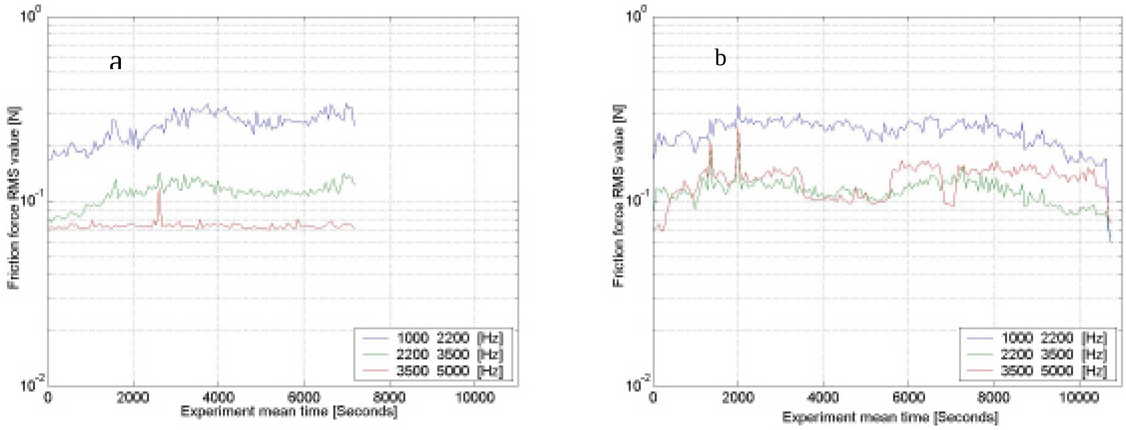

Figure 18. Filtered signal to experiments group B class II and III respectively.

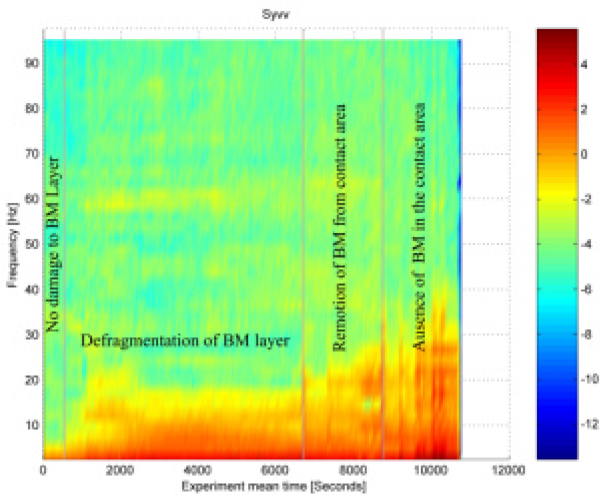

Figure 19. Frequency band from 0 to 100 Hz to experiment group B class III.

Figure 19 shows four stages in the experiment where the BM layer was completely removed. These stages are characterized by the increase of the friction force mean square values. As in higher frequency bands, it can be seem the correlation of estimated friction force with wear evolution. First some cracks appear at BM layer. The action of counter body fragments this cracked layer, after this the relative displacement removes the BM powder until the contact specimen area is only aluminum and the force level stabilizes again.

5 Conclusion

Analysis and interpretation of nonlinear behavior that occurs in linear systems is feasible by the usage of nonlinear MISO technique. The proposed methodology is useful to identify nonlinear effects that act on linear system without restrictions on the statistical nature of the measured signals. If the nonlinear behavior is well modeled, the proposed methodology is capable to identify and separate the exact nonlinear part from the global response of the system.

In case of mechanical systems with known linear properties the MISO representation is simple and easy to be physically interpreted than by other nonlinear analysis techniques. Since the methodology does not impose any restrictions on the nature of nonlinear functions, it finds usefulness in the identification of physical parameters of highly nonlinear mechanical systems.

A methodology to measure the friction force in the vibratory systems upon its response was presented. This methodology is capable to estimate the friction force without the influence of the system dynamics. The proposed methodology provides an estimative of friction force that is sensitive to changes in tribological conditions of a surface under wear tests. This characteristic is useful to identify specimens surface wear conditions as such the time of occurrence of discrete events, as observed on the experiment with bisulphate of molybdenum thin layer. The nonlinear MISO shows that it is possible to tribology researchers to study alternative mathematical formulations to represent friction between bodies. Since the friction force is identified, static and kinetic friction coefficients can be calculated if the normal forces between two bodies are measured.

The proposed methodology permits the identification of surface tribologic phenomena by analyzing the signals at higher or lower frequency bands. The best band may be selected depending on the correlation of the identified signal with the surface characteristics measured, for example, on a scanning electronic microscope. The higher frequency bands generally carry information about low energy surface tribology phenomena that produce small response amplitudes. This characteristic promotes the MISO nonlinear methodology as an alternative to monitor mechanical contact experiments where the acoustic emission technique is the only possible instrumentation. The lower frequency band carries information about the friction force as expected by the assumptions of the Coulomb theory. The proposed nonlinear friction model permits the experimentally identification of the contact stiffness. This characteristic is useful to find tribological properties of bodies in contact, as in the study of solid lubricants performance.

6 Acknowledgements

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