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THE IRRADIATION EFFICIENCY OF SIMPLY SUPPORTED BEAM-REINFORCED PLATES WITH EXPERIMENTAL VALIDATION

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Abstract. *The study of the irradiation efficiency of structural components has been subject of many researches along the years, since its precise understanding provides conditions for predicting the sound pressure field where these components are operating. This is the case of sound pressure generated at the accommodation area in offshore platforms, due to vibrations transmitted throughout the structure, excited by vibrations of large machines. It is known that reinforcing beams effects the irradiation efficiency of plates, which is increased since sound cancellations effects are interrupted in the vicinities of the beams. From the Rayleigh Integral, one can have the sound pressure field on the plate, which is necessary to calculate the irradiation efficiency. Fourier Transforms can solve this integral. When FFT algorithms are used, the integral is resolved in a very fast way, when compared with a numerical integration. This work presents results of irradiation efficiency of plane plates, considering the effects of the flexibility (web and flange own modes) of the reinforcing beams. The velocity distribution is obtained by Finite Elements and considers the real influence of the reinforcement. The results show that increases in the irradiation efficiency occur when the plate is reinforced. Comparisons of irradiation efficiencies will be shown, among simply supported plates and beam-reinforced plates. This work also presents the experimental set-up used for validation, and the comparison shows a good agreement between the numerical and experimental results.*

Keywords: *Irradiation Efficiency, Beam-reinforced plates, structural vibration*

1. Introduction

The sound irradiation and irradiation efficiency of structural components has been studied for some time, since its complete understanding gives an insight to predict the sound pressure in the environment surrounding these components. This is a consequence of more severe legislations regarding the health of the population, from the hearing point of view, especially for those people who work in noisy environments.

Most machines generate vibration while operating. As a consequence, structural components from the machine, foundation and floor irradiate noise. This is commonly observed in offshore platforms, which is composed basically by plates, beams and beam-reinforced plates.

Studies about radiation from beam-reinforced plates treat the discontinuities, as been ideal situations of supports and clamps. The beam effect has been considered in many articles (Maidanik, 1962; Lin e Hayek, 1972; Mace, 1980; e Berry e Nicolas, 1994), but only the rigidity and inertia of the reinforcements. Irradiation efficiency predictions for plates reinforced by beams, as made today have non-tolerable errors, since they do not consider the real effects of the beams in the velocity field of the plates.

The pressure field generated by plane vibrating surfaces can be obtained from the Rayleigh Integral (Williams, 1999). This integral can be solved by direct numerical integration or by means of Fourier Transforms (Williams e Maynard, 1982; e Williams, 1983). The Rayleigh Integral relates the pressure in a certain observation point with the normal velocity of each area element of the surface. Integrating all elements, all over the surface, we have the total pressure irradiated by the surface, calculated at the observation point.

To solve the Rayleigh Integral using Fourier Transforms we have to apply the transform to the velocity distribution of the plate. The pressure is obtained applying the inverse transform to the product between the transform of the velocity and the transform of the Green function for the plate. The use of Fourier Transforms can be very advantageous when FFT algorithms are used to calculate the direct and inverse transforms. The irradiation efficiency is given by (Williams, 1999):

$$\sigma_{rad} = \frac{W_{rad}}{\rho c \langle \bar{v}^2 \rangle} \tag{1}$$

where W_{rad} is the irradiated sound power of the surface, ρ is the density of the medium, c is the sound speed of the medium, S is the surface area, and $\langle \bar{v}^2 \rangle$ is the mean quadratic velocity, in time and space. Finite Elements will calculate the velocity distribution, since this method can incorporate the resonances of the beam in the velocity of the plate. This velocity will be used to calculate the sound power and the quadratic velocity, necessary to the efficiency. The sound pressure and sound power are calculated with FFT (Fast Fourier Transform) algorithms, (Press et al, 1992).

The choice for Finite Element Methods (FEM) for velocity distribution calculation is based on the fact that there is no analytical formulation that represents the real velocity of a beam reinforced plate. With FEM, one can obtain a velocity that considers the influence of the beams, since it is possible to model the beams in a way that the resonances, loads and displacements acting in the beam are accounted for in the velocity distribution of the plate. The software used was Ansys 5.4. Analyses where made until frequencies higher than the coincidence frequency, as the irradiation efficiency tends to be constant (equals one) for frequencies higher than the coincidence.

The objective of this work is to calculate and compare experimentally, the irradiation efficiency of beam reinforced, simply supported plates on all sides, considering the own modes of the web and flange of the beams, using FFT algorithms to solve the Rayleigh Integral. The irradiation efficiency of a non-reinforced plate and a plate with two equidistant beams is investigated, and the results show that the efficiency increases with the presence of the beams and the comparison between the numerical and the experimental data shows good agreement.

2. Basic Equations and Definitions

The sound pressure field generated by a vibrating surface can be obtained solving the wave equation, subjected to the surface boundary conditions. The wave equation and the boundary conditions can be combined in an integral equation, named Kirchhoff-Helmholtz Integral (Fahy, 1985). When the vibrating surface is plane, the sound pressure p(x,y,z) is obtained form the Rayleigh Integral, given by (Fahy, 1985, Williams, 1999):

$$p(x,y,z) = \frac{j w r}{2p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[v(x',y',z') \frac{e^{-ik|r-r'|}}{|r-r'|} \right] dx'dy' \tag{2}$$

where $|r-r'| = \left[(x-x')^2 + (y-y')^2 + z^2\right]^{1/2}$, j is a pure imaginary number, ω is the frequency, v is the velocity of the plate and k is the wavenumber.

With the knowledge of the pressure field and the velocity distribution of the surface, the irradiation efficiency can be calculated from Eq. (1).The irradiated sound power W_{rad}, is:

$$W_{rad} = \frac{1}{2} \iint_S \mathrm{Re}\left[p(x,y,0) v^*(x,y,0)\right] dS \tag{3}$$

and the mean square velocity is:

$$\langle \bar{v}(x,y)^2 \rangle = \frac{1}{2S} \int_S v(x,y)^2 \, dS \tag{4}$$

where Re( ) is the real part, * is the conjugate and S is the area of the plate

The sound radiation is a consequence from the cancellations that occur in the air movement over the plate, depending on the mode configuration (Beranek, 1988). Cancellation is a phenomenon where the air displaced outward by one section of the plate moves to occupy the space left by the motion of adjacent section, without being compressed and irradiating very little power. Finite plate's modes have a well-known behavior. For frequencies below the coincidence, which is the frequency where the bending speed of the plate is equal to the sound speed of the medium, corner and edge modes are present. For frequencies higher than the coincidence, surface modes occur. Corner and edge modes are poor irradiators, while surface modes are good irradiators.

3. Irradiation Efficiency and FFT

The Rayleigh Integral, with time dependence $e^{-i\omega t}$, is given by (Williams e Maynard, 1982):

$$p(x,y,z) = -i\omega\rho \int_{-\infty}^{\infty}\int v(x',y') \frac{e^{ikR}}{2\pi R} dx'dy' \tag{5}$$

where $R^2 = (x - x')^2 + (y - y')^2 + z^2$ and the integration is made over an infinite surface (z = 0). Equation (5) can be rewritten in a condensed form defining a kernel h:

$$h(x,y,d) = -i\omega\rho g(x,y,d) = -\frac{i\omega\rho}{2\pi} \frac{e^{ik(x^2+y^2+d^2)^{1/2}}}{(x^2 + y^2 + d^2)^{1/2}} \tag{6}$$

where g(x,y,d) is the Green function for the plate. It is possible to visualize Eq. (5) as being a convolution between v(x,y) and h(x,y,d), making z = d in p(x,y,z). The pressure, then, is:

$$p(x,y,d) = v(x,y) \circ h(x,y,d) \tag{7}$$

Taking the Fourier transform of Eq (7):

$$F[p(x,y,d)] = F[v(x,y) \circ (x,y,d)] \tag{8}$$

The Convolution Theorem (Bendat e Piersol, 1986) states that the Fourier Transform of the convolution of two functions is equal the product between the Fourier Transforms of the two functions. With that in mind, Eq. (8) can be changed to:

$$F[p(x,y,d)] = F[v(x,y)]F[h(x,y,d)] \tag{9}$$

Taking now the Inverse Fourier Transform of Eq. (9), we have:

$$p(x,y,d) = F^{-1}\left[\hat{V}(k_x,k_y)\hat{H}(k_x,k_y,d)\right] \tag{10}$$

where F^{-1} is the inverse Fourier transform and $\hat{V}(k_x,k_y)$ and $\hat{H}(k_x,k_y,d)$ are the Fourier transforms of v (velocity) and h (kernel), respectively.

The algorithm to calculate p(x,y,d), knowing v(x,y) and using Fast Fourier Transforms is implemented this way:

(1) calculate the Discrete Fourier Transform (DFT) of v(x,y), using FFT, and call it $\hat{V}_D$.

(2) calculate the analytical form of $\hat{H}(k_x,k_y,d)$.

(3) multiply the results from (1) and (2) and calculate the Inverse Discrete Fourier Transform of the result.

Symbolically speaking, we have:

$$p_D(x,y,d) = D^{-1}\left[\hat{V}_D(k_x,k_y)\hat{H}(k_x,k_y,d)\right] \tag{11}$$

where the subscript D refers to calculations via DFT and D^{-1} represents the inverse DFT.

Equations 2 to 4, along with the FFT technique, were implemented in executable program (Fiates, 2003). The program uses a FFT algorithm based in the Cooley approach (Cooley e Tuckey, 1960), where the function to be transformed must have 2^n points. As we are interested in finite plated and due to stationary waves in this kind of plates, the most part of vibratory energy is irradiated in the resonance frequencies. So, the program calculates the efficiency only for these frequencies (modes) of the plates.

The program has as entry data the velocity field for each mode, and all calculations are repeated for each mode, in a frequency loop. The resonance frequencies values are obtained from the finite element analysis and are also an entry data for the program. The program calculates, then, the mean square velocity, (Eq. 4), which is stored for later use, in the calculation of the sound power (Eq.3). But most of the calculations are made to obtain the sound pressure (Eq. 10).

The velocity matrix is patched with zeros to simulate a baffle condition and to improve the FFT accuracy. The Fourier Transform of the velocity matrix and the Green function are calculated, on the same points of the discretizated plate. These results are multiplied and the inverse transform is applied, resulting in the sound pressure. The zeros are taken off and the sound pressure in the plate is obtained. With the pressure and the velocity, the sound power is, then, calculated. The efficiency is determined from the power recently calculated and the mean square velocity calculated in the beginning of the loop. The calculation time for a typical plate was around five minutes, while solving the Rayleigh Integral by direct numeric integration took about one hour.

The Finite Element Method is used in part of this work to obtain the plate velocity, with or without reinforcements. As the efficiency will be calculated for each resonance frequency, it is necessary to know the velocity for each correspondent mode. Ansys offers an easy way to accomplish that, performing only a modal analysis of the structure. The shell element used was Shell63, an element with bending and membrane capabilities, four nodes and six degrees of freedom (three displacements and three rotations) per node. The analyses were made until frequencies above the coincidence and the number of elements obeyed the rule of a minimum of 12 elements for wavelength

4. Experimental Setup

To obtain the irradiation efficiency experimentally, one has to know the mean square velocity, in space and time, measured at various points on the plate, and the irradiated power. The sound power is calculated from sound pressure levels, measured in a controlled environment (reverberation room). The experiment was conducted in the reverberation room of the Laboratório de Vibrações e Acústica (LVA) – UFSC. For comparison, was used an aluminum plate with dimensions 1,0 m x 0,8 m x 3,0 mm.

The first part of the experimental analysis was the construction of a device to simulate the simply supported boundary condition for the plates. The simply supported condition implies in null displacement in all edges. A wood structure, as seen in Fig. 1 was built. In the structure where attached two triangular supports, per edge, and the plate was placed between them. The plate was fixed in place by a series of screws. This can be seen in Fig. 2.

Figure 1: Wood structure built for the simply supported plate.

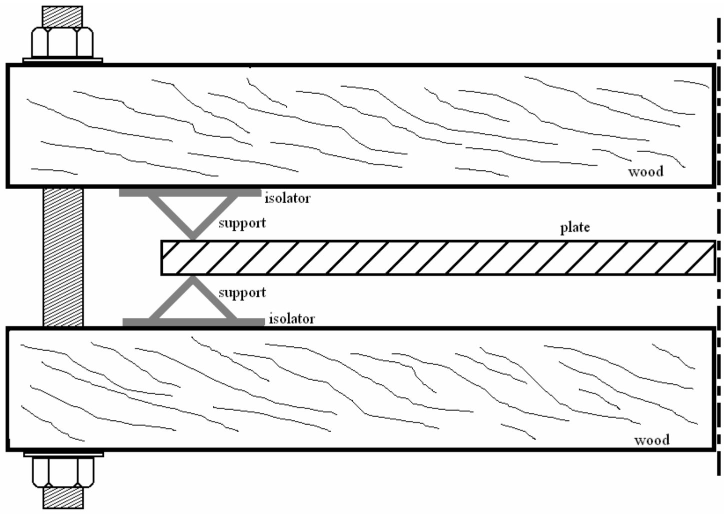

Figure 2: Schematic representation of the support system.

When the plate vibrates, it radiates sound energy in the two directions. We need to measure the sound energy only in one direction, and, for this reason, a wood box with absorptive material was built to eliminate the radiation from one side of the plate. The box was composed by two sheets of wood, with 2,0 cm thickness, glued together, and opened in the top. All the interior of the box was covered two layers with 5,0 cm Sonex foams. On the top of the box was placed the wood structure with the plate. Figure 3 shows this.

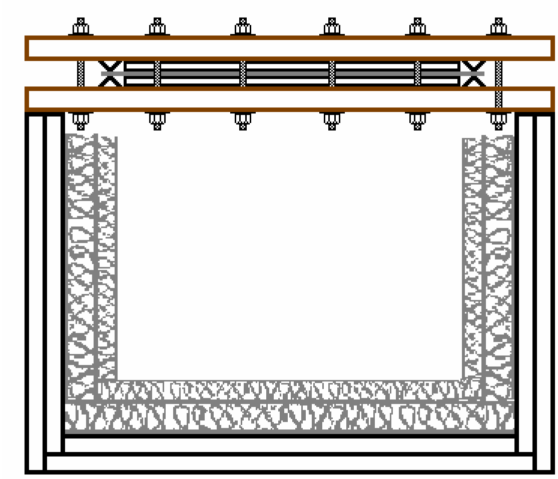

Figure 3: Representation of the apparatus built for experimentation.

4.1. Sound Power Determination

The reverberation room of LVA has 7,4 m of length, 7,4 m of width and 2,6 m of height, with a volume of 144 m^3 . The sound power was determinated by the comparison method, (ISO 3741) using a reference sound source, B&K 4204. This reference source has known sound power levels, supplied by the manufactor.

The procedure consists in measuring the sound pressure level of the reference source, remove the reference source and measure the sound pressure of the source we are interested in. By a simple calculation, the sound power level of our source (plate) is obtained. The relation between the sound power level and the sound pressure level in dB is (Beranek, 1988):

$$SWL = SPL - 10 \cdot \log T + 10 \cdot \log V + 10 \cdot \log\left(1 + \frac{S \cdot \lambda}{8 \cdot V}\right) - 10 \cdot \log\left(\frac{B}{1000}\right) - 14 \tag{12}$$

where SPL is the sound pressure level; V is the volume of the chamber in m^3; T is the reverberation time in s; λ is the wavelength in the frequency band; S is the total sound absorption area in m^2; B is the ambient pressure in milibar. For the same room and same ambient conditions, one can have:

$$SWL = SPL + K \ \ (dB) \tag{13}$$

where K is called room constant. The source sound power level (SWL_S) is obtained, then:

$$SWL\_S = SWL\_R + SPL\_S - SPL\_R \tag{14}$$

where SWL_S is the sound power level of our source (plate); SWL_R is the sound power level of the reference source (known); SPL_S is the sound pressure level of our source (measured) and SPL_R is the sound pressure level of the reference source (measured).

4.2. Measurement of plate velocity

After measuring the reference source, a random force excited the plate and the sound pressure level in the chamber was measured by the microphone, attached to a rotatory table. At the same time, the plate velocity was measured, insuring the correlation between pressure levels and velocity levels. The plate was divided in 63 points and the velocity measured with an ICP accelerometer. The vibration was induced using a shaker, fed by a signal generator included in the PULSE analyzer. The equipments used in this part of the experiment were: B&K PULSE 3560 Signal Analyzer; Power Amplifier B&K 2706; Microphone ½” B&K 4189; Microphone pre-amplifier B&K 2671; Cell Force B&K 8200; Accelerometer PCB WA 353 B15; Shaker B&K 4810; Charge pre-amplifier B&K 2635. The plate was excited by the shaker and the velocity measured at two positions at the same time. Also the sound pressure was measured in two complete spins of the rotatory table, up to 6400 Hz in 1/3 octave bands. The Figure 4 shows the schematic view of the experiment.

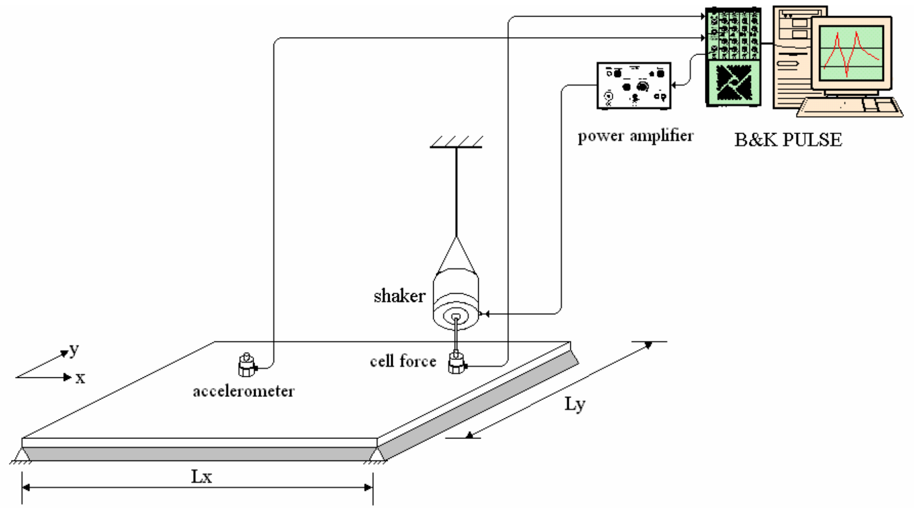

Figure 4: Representation of the experiment.

With the plate velocity and the sound pressure level measured, the spatial and temporal plate velocity was calculated and the sound power was also obtained. With all the data needed in hand, Eq. (1) was implemented and the experimental irradiation efficiency was obtained. Table 1 shows the sound pressure level generated by the plate and the background noise levels.

Table 1: Measured sound pressure levels generated by the plate and background noise levels.

Frequency [Hz]	Sound pressure level generated by the plate [dB]	Background Noise [dB]
50	66,0	25,0
63	54,2	25,2
80	54,7	25,3
100	58,0	31,6
125	65,3	23,7
160	67,9	25,2
200	60,3	23,0
250	62,4	24,9
315	70,5	23,4
400	73,6	24,7
500	73,5	25,8
630	75,2	25,9
800	72,1	26,9
1000	75,3	27,5
1250	77,0	28,3
1600	75,2	29,5
2000	76,7	30,3
2500	77,0	31,3
3150	81,6	33,2
4000	77,2	33,3

5. Results

The efficiency was calculated for two configurations of plate/beams, one a simply supported, non-reinforced plate, and a simply supported plate with two equally spaced reinforcing beams. The efficiency results are presented only in 1/3 octave bands, since the sound pressure levels were measured in these bands, and the irradiated power is also in 1/3 octave bands. For the two cases the plate was the same, of aluminum, with dimensions L_x = 1,0 m and L_y = 0,8 m; thickness h = 3,0 mm. The reinforcements were steel T inverted beams, with thickness h_T = 3,0 mm; web h_a = 2,2 cm and flange b_f = 2,2 cm, as seen in the Fig. 5. The reinforcing beams were attached to the plate with epoxy-based structural adhesive. In the numerical analysis the beams were also modeled with steel data.

The aluminum plate has a coincidence frequency of 4185,0 Hz, and the numerical calculations were made up to 4380,0 Hz, computing 360 modes. The numerical results were obtained using the methodology shown in section 2 and 3. A more detailed explanation of these results is presented in Fiates (2003) and in Fiates and Lenzi (2003).

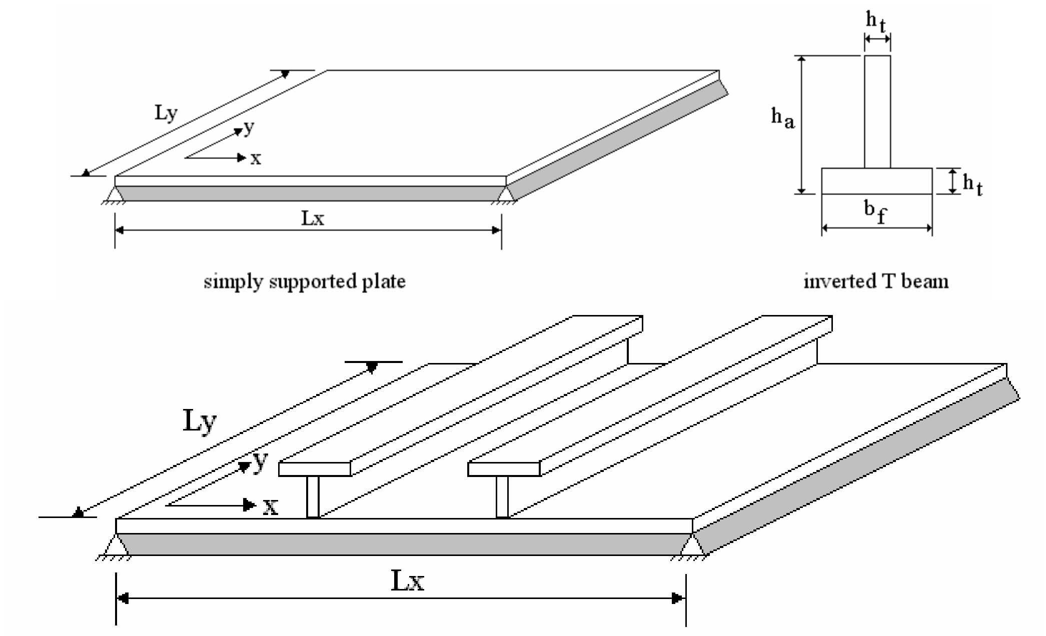

Figure 5: Scheme of the plate and beam used in the efficiency analysis.

For demonstrative objectives, Fig. 6 shows the efficiency results for the numerical analysis for the aluminum plate used in the experiment. The simply supported plate will be referred in the figures as “ss plate”. It is shown the results obtained with an approximation, by Ver (Beranek, 1988), and the numerical results. The irradiation efficiency is increased by the presence of the reinforcements. This can be seen more clearly in Fig. 7, where the results are plotted in 1/3 octave bands.

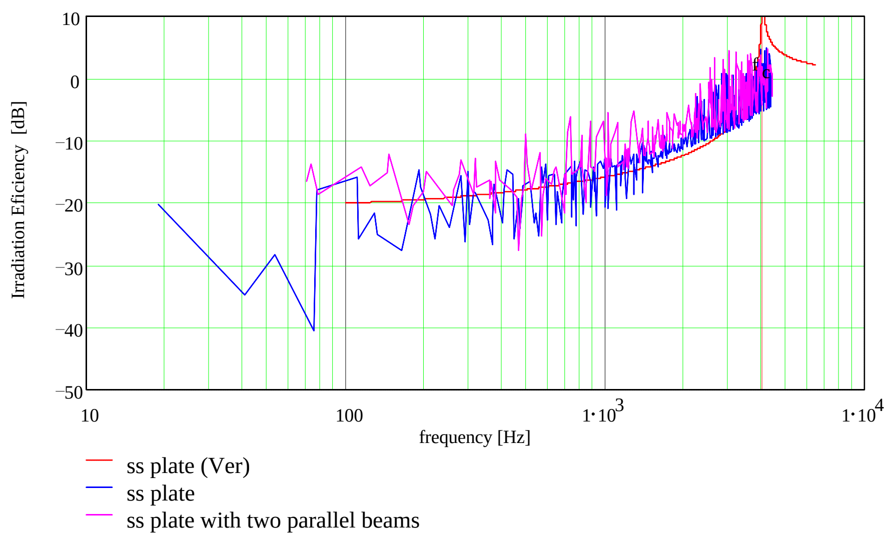

Figure 6: Irradiation efficiency for the plate used in the experiment. Numerical method, results for the resonance frequencies.

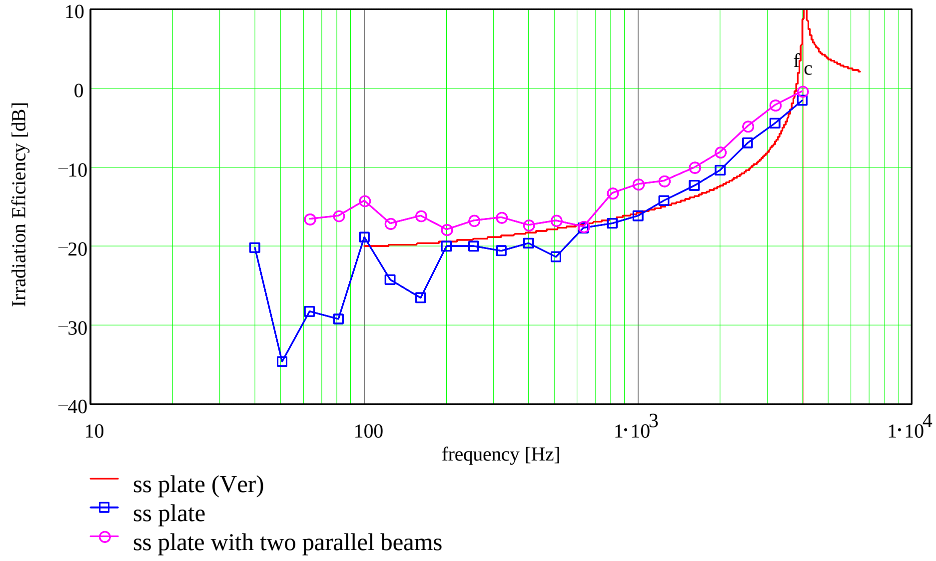

Figure 7: Irradiation efficiency for the plate used in the experiment. Numerical method, results for 1/3 octave bands.

Figures 8 and 9 show the efficiency results for the non-reinforced plate and the plate with two beams, in 1/3 octave bands. It can be seen in both figures a good agreement for bands higher than 125 Hz. This is a consequence of the volume of the reverberation room, which is not appropriate for measurements bellow this frequency. For frequencies higher than 125 Hz, the present methodology offers accurate results. Little discrepancies can be related to the experimental boundary condition of the plate, since a perfect simply supported condition is very difficult to be achieved. For the reinforced plate, (Fig. 9) the conclusions are similar, with valid results for frequencies above 125 Hz.

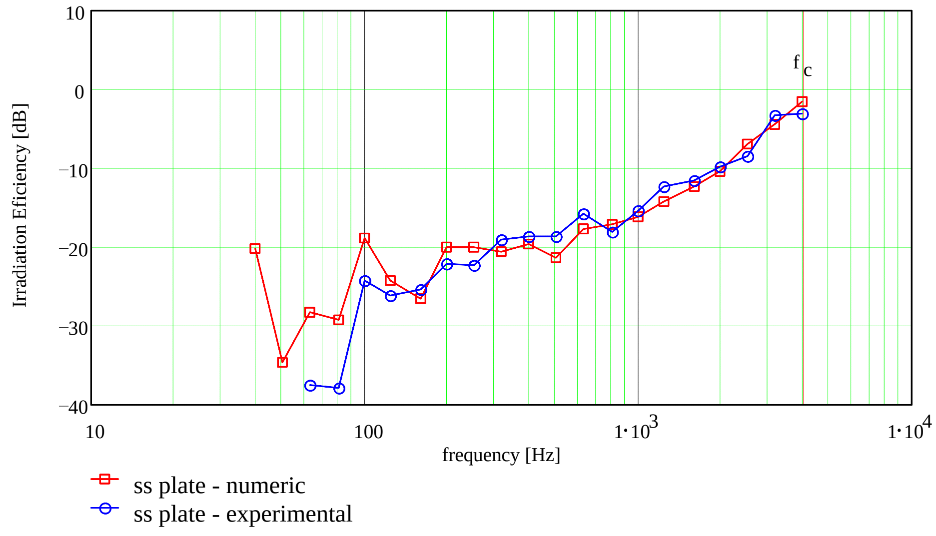

Figure 8: Irradiation efficiency for the non-reinforced plate used in the experiment. Numerical and experimental methods, results for 1/3 octave bands.

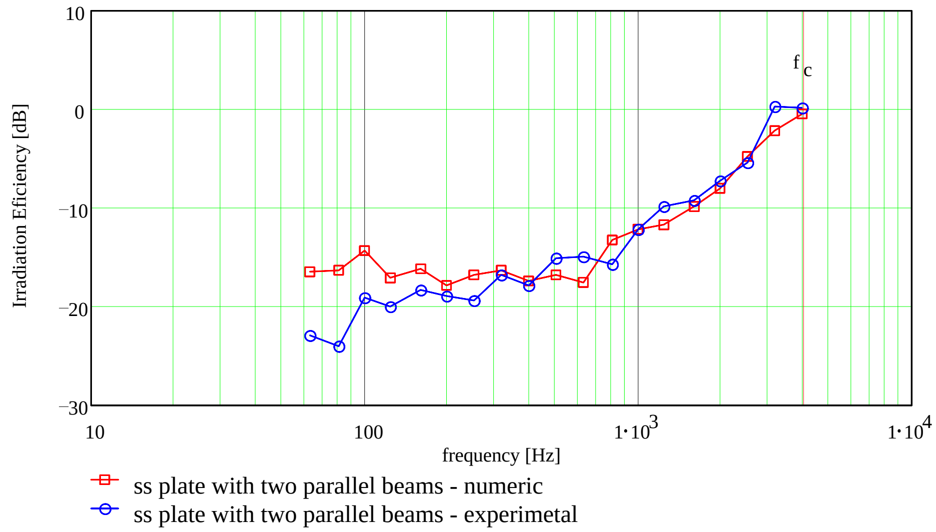

Figure 9: Irradiation efficiency for the reinforced plate used in the experiment. Numerical and experimental methods, results for 1/3 octave bands.

Figure 10 shows the four efficiency results superimposed. It can be seen that the reinforced plate has higher irradiation efficiency, both by the numerical and by the experimental method. The beam presence increases the efficiency, due to a higher number of non-cancelled areas on the plate, and this increase will be bigger as more disturbed the velocity field is by the beam.

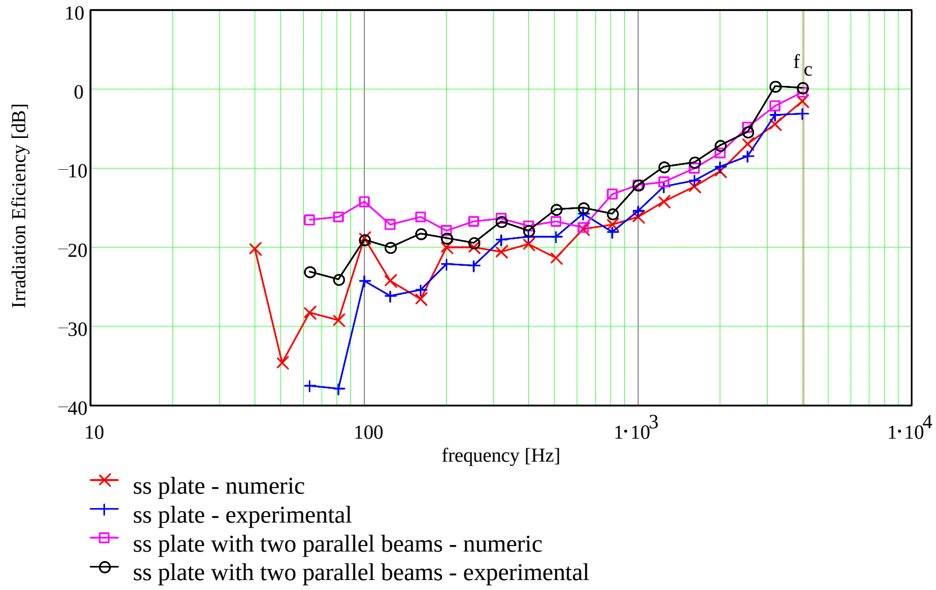

Figure 10: Irradiation efficiency for the reinforced plate and non-reinforced used in the experiment. Numerical and experimental methods, results for 1/3 octave bands.

6. Conclusions

A technique, based in the FFT, was implemented for the calculation of the irradiation efficiency of beam-reinforced plates and the results compared with an experimental analysis. By modeling the beams with shell elements, the modes of the beam are accounted for in the velocity distribution of the plate and more precise results are obtained.

The experimental set-up is explained and the comparisons show that the methodology can produce accurate results. Since the reverberation room has a volume suitable for measurements above 125 Hz, the efficiency results for frequencies bellow 125 Hz must not be considered.

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