Proceedings of the XI DINAME, 28th February-4th March, 2005 - Ouro Preto - MG - Brazil
Edited by D.A. Rade and V. Steffen Jr.

IMPACT FORCE IDENTIFICATION IN FLEXIBLE STRUCTURES BY USING P-I OBSERVER AND OPTIMAL CONTROL

Carlos E. Ingar Valer
Pontifícia Universidade Católica do Rio de Janeiro,
Departamento de Engenharia Mecânica
cingar@mec.puc-rio.br

Rubens Sampaio
Pontifícia Universidade Católica do Rio de Janeiro,
Departamento de Engenharia Mecânica
rsampaio@mec.puc-rio.br

Abstract. *In this paper we propose two new strategies to experimentally identify impact forces in flexible structures by analyzing the measured vibration response. The first strategy is based on the use of a proportional-integral observer (named also, P-I observer). The main feature of this non-conventional observer is its capacity to estimate not only state variables but also disturbances in dynamical systems. Application of this observer to the problem of impact in flexible structures have as advantage the ability to perform identification of impact forces in an online manner. The estimation by the P-I observer is compared with the estimation obtained by other proposed strategy which uses the linear optimal control theory results to develop an algorithm to estimate impact forces. With the purpose of verify experimentally the effectiveness of the proposed strategies, both methods were applied in an experimental test rig implemented at the Laboratory of Vibrations in the PUC-Rio University. The performances of both methods are compared and conclusions about advantages and disadvantages are underlined.*

Keywords*: impact, flexible structure, inverse problem, observers*

1. Introduction

In some engineering applications it is necessary to determinate experimentally impact forces in flexible structures in order to monitor structural damage and to characterize impact. However, sometimes it is not possible to instrument the impactor to measure directly the impact force and, because of this, the alternative is to determinate experimentally the impact force through the use of indirect methods, i.e. by analysis of the measured vibration responses of the flexible structure (Gaul and Hurlebaus, 1999). This problem is an Inverse Problem and could be stated as follow: Find the impact force (input) that gives a given response (output) for a flexible structure. It is important to remark that identification of impact forces is a hard problem mainly because of the short duration of the impact event.

In this paper we propose two new strategies to identify impact forces in flexible structures and they are applied in an experimental test rig. This work is organized as follows . Section 2 describes the first of the two proposed strategies. It is based on the use of a proportional-integral observer which is a robust observer that not only reconstruct the states of dynamical systems but also is able to estimate disturbances (Müller, 1995). Application of this observer to the problem of impact in flexible structures is new to the best of the authors knowledge. In section 3 the second strategy is explained, which consist of an algorithm based on some results of the linear optimal control theory. Section 4 describes the test rig constructed at the Laboratory of Vibrations in the PUC-Rio University to experimentally validate the methods developed in this paper and also discuss and compare the results obtained with the two methods. The advantages and disadvantages of both methods are discussed. Finally, in section 5, the main conclusions of this work are underlined.

2. Impact force identification by proportional-integral observer

State observers are auxiliary dynamical systems which are designed to reconstruct the full state vector of a given dynamic system from partial information of the state, i.e. some state variables. Observers are extensively used in modern control applications. The full-order Luenberguer observer (Ogata, 1993) is the kind of observer most commonly used in practice, however, in the literature, we can find many other kinds and variations of them (Valer, 1999). One of those is the proportional-integral observer (named also, P-I observer) which is a robust observer developed to reconstruct the state vector of a given dynamical system even in presence of some unknown inputs or disturbances (Linder, 1997 and Müller, 1995). As a consequence of the process of the robust state reconstruction, the P-I observer internally estimates and compensate the unknown inputs, i.e. the disturbances. The ability of this observer to estimate disturbances have been shown to be useful to compensate nonlinearities (like friction and backlash when modeled as disturbances) in some dynamical systems (Söffker and Müller, 1995). However, the P-I observer is relatively new when compared to the other kinds of observers and its potential have not been fully exploited yet, especially in mechanical engineering applications. We propose here the use of this observer to solve the problem of impact force identification in flexible structures.

2.1. The P-I Observer

Let us consider the following linear time-invariant dynamical system:

$$S: \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\ \mathbf{x} + \mathbf{B}\ \mathbf{u} + \mathbf{B_d}\ \mathbf{d} \\ \mathbf{y} = \mathbf{C}\ \mathbf{x} \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathrm{R}^n$ represents the state vector of the system, $\mathbf{y} \in \mathrm{R}^m$ $(m \le n)$ the vector of output variables or measurements, $\mathbf{u} \in \mathrm{R}^r$ the known input vector and $\mathbf{d} \in \mathrm{R}^s$ the unknown input vector, i.e. the disturbance vector. The system matrix $\mathbf{A}$, the input matrices $\mathbf{B}$, $\mathbf{B_d}$ and the output matrix $\mathbf{C}$ are of appropriate dimensions and they are assumed known. The Eq. (1) can represent the dynamics of a flexible structure (Meirovitch, 1990) where $\mathbf{y}$ represents the measured vibration response that can be obtained through the use of sensors (for example: strain gages, accelerometers, or proximity sensors like it was made in our experimental test rig). The impact force to identify is the unknown input, then it is represented by the vector $\mathbf{d}$. Note that, even though the disturbance vector is considered unknown, its location, given by the vector $\mathbf{B_d}$, is not. Based on the system defined by the Eq. (1), the corresponding P-I observer equations can be now written (Valer, 1999; Müller, 1995):

$$\hat{S}_{\mathrm{PI}}: \begin{cases} \dot{\hat{\mathbf{x}}} = \mathbf{A}\ \hat{\mathbf{x}} + \mathbf{B}\ \mathbf{u} + \mathbf{B_d}\ \hat{\mathbf{d}} + \mathbf{K_P}\ (\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \\ \dot{\hat{\mathbf{d}}} = \mathbf{K_I}\ (\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \end{cases} \tag{2}$$

where $\hat{\mathbf{x}} \in \mathrm{R}^n$ represents the reconstructed state vector for the system S and $\hat{\mathbf{d}} \in \mathrm{R}^s$ the estimated disturbance vector. Note that the observer only uses as input the available signals, i.e. the measurement vector $\mathbf{y}$ (partial state information) and the known input vector $\mathbf{u}$, if any. A schematic representation of structure of the P-I observer is showed on Fig. 1.

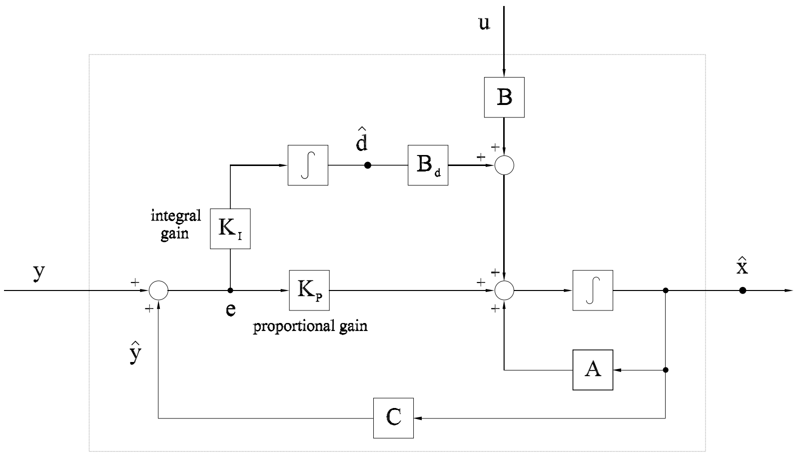

Figure 1. Structure of the P-I observer

As it can be seen from the Fig. 1, the structure of the P-I observer is similar to the classic full-state proportional observer (Friedland, 1996) but with an additional integral loop that allows an internal estimation and compensation of the disturbance vector in the state reconstruction process. A necessary condition for estimation of disturbances by using P-I observer is that the number of available independent measurements be greater or equal than the number of unknown inputs (Saif, 1997). In other words, if $\mathbf{C}$ and $\mathbf{B_d}$ are full rank matrices, the necessary condition is: $s \le m$. The matrices, $\mathbf{K_P}$ (proportional) and $\mathbf{K_I}$ (integral) are the observer gains and the determination of suitable values for them is part of the observer design. To make it easy, let us rewrite the Eq. (2) as follow:

$$\hat{S}_{\mathrm{PI}}: \; \left\{ \; \dot{\hat{\mathbf{x}}}_{\mathbf{a}} = \mathbf{A_a} \; \hat{\mathbf{x}} + \mathbf{B_a} \; \mathbf{u} + \mathbf{K_a} \; (\mathbf{y} - \mathbf{C_a} \; \hat{\mathbf{x}}) \right. \tag{3}$$

with

$$\hat{\mathbf{x}}_{\mathbf{a}} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{d}} \end{bmatrix}, \; \mathbf{A_a} = \begin{bmatrix} \mathbf{A} & \mathbf{B_d} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \; \mathbf{B_a} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \; \mathbf{C_a} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \tag{4}$$

$$\mathbf{K_a} = \begin{bmatrix} \mathbf{K_P} \\ \mathbf{K_I} \end{bmatrix} \tag{5}$$

This way, the P-I observer looks like a classical proportional observer. From here, it is clear that if the pair ($\mathbf{A_a}, \mathbf{C_a}$) is observable, the poles of the observer state matrix, which is defined by:

$$\hat{\mathbf{A}}_{\mathrm{PI}} = \mathbf{A_a} - \mathbf{K_a} \; \mathbf{C_a} = \begin{bmatrix} \mathbf{A} - \mathbf{K_P} \; \mathbf{C} & \mathbf{B_d} \\ -\mathbf{K_I} \; \mathbf{C} & \mathbf{0} \end{bmatrix} \tag{6}$$

can be allocated arbitrary by choosing the matrix $\mathbf{K_a}$. Then, either the conventional methods for pole allocation or the Kalman Filter method (Meirovitch, 1990) can be used to determinate an appropriate gain matrix $\mathbf{K_a}$ and, from Eq. (5), the proportional and integral gains, $\mathbf{K_P}$ and $\mathbf{K_I}$, are also determined. In the section 4, the P-I observer will be applied to experimentally estimate impact forces in a flexible structure when the impact forces are treated as disturbances.

3. Impact force identification by an algorithm based on the linear optimal control

By using some results of the linear optimal control theory it is possible to develop a algorithm that is able to estimate impact forces in flexible structures. To do it, first formulate the optimal tracking problem and then interpret it as an inverse problem. For simplicity, in this section, the algorithm will be explained by using discrete-time versions for the dynamical systems. Let us consider the following discrete-time dynamical system that represent the flexible structure:

$$D: \; \begin{cases} \mathbf{x}_{k+1} = \mathbf{A} \; \mathbf{x}_k + \mathbf{B} \; \mathbf{u}_k \; , \; k \geq 1 \\ \mathbf{y}_k \quad = \mathbf{C} \; \mathbf{x}_k \end{cases} \tag{7}$$

where $\mathbf{x}_k \in \mathrm{R}^n$, $\mathbf{y}_k \in \mathrm{R}^m$ and $\mathbf{u}_k \in \mathrm{R}^s$, for $k \geq 1$ are the state vector of the system, the vector of output variables and the input vector, respectively. The system matrix $\mathbf{A}$, the input matrix $\mathbf{B}$ and the output matrix $\mathbf{C}$ are of appropriate dimensions and they are assumed known. Additionally, let us consider a reference signal $\mathbf{y^r}_k$, $k \geq 1$ such that it is desired that the system response $\mathbf{y}_k$ track this reference signal. The optimal tracking problem consist of finding a input vector $\mathbf{u}_k$ for $k \geq 1$, that minimize the following quadratic performance measure:

$$J = \frac{1}{2} \sum_{k=1}^{N} \left(\mathbf{e}^T{}_k \; \mathbf{Q} \; \mathbf{e}^T{}_k + \mathbf{u}^T{}_k \; \mathbf{R} \; \mathbf{u}^T{}_k \right) \; + \; \frac{1}{2} \mathbf{e}^T{}_\mathbf{N} \; \mathbf{P} \; \mathbf{e}^T{}_\mathbf{N} \tag{8}$$

with

$$\mathbf{e}_k = \mathbf{y} - \mathbf{y^r}_k \; , \; k \geq 1 \tag{9}$$

where N is a integer number that represent the index of the final time. The matrices $\mathbf{Q}$, $\mathbf{R}$ are symmetric positive definite and $\mathbf{P}$ is symmetric positive semi-definite. They define the performance measure to be minimized in Eq. (8) and they are free parameters that must be chosen by the designer. The solution to the optimal tracking problem can be found in advanced books on control theory for example in Lin (1994). For simplicity we only present the results:

$$\mathbf{u}_k = -\mathbf{K}_k \ \mathbf{x}_k + \mathbf{K^v}_k \ \mathbf{v}_{k+1} \tag{10}$$

with

$$\mathbf{K^v}_k = \left(\mathbf{B} \ \mathbf{S}_{k+1} \ \mathbf{B} + \mathbf{R}\right)^{-1} \mathbf{B}^T \tag{11}$$

$$\mathbf{K}_k \ = \mathbf{K^v}_k \ \mathbf{S}_{k+1} \ \mathbf{A} \tag{12}$$

and the feed-forward vector $\mathbf{v}_k$ and the matrix $\mathbf{S}_k$ are determined by the following difference equations:

$$\mathbf{v}_k = \left(\mathbf{A} - \mathbf{B} \ \mathbf{K}_k\right) \ \mathbf{v}_{k+1} + \mathbf{C}^T \ \mathbf{Q} \ \mathbf{y^r}_k \quad , \quad \mathbf{v}_N = \mathbf{C}^T \ \mathbf{P} \ \mathbf{y^r}_N \tag{13}$$

$$\mathbf{S}_k \ = \mathbf{A}^T \ \mathbf{S}_{k+1} \ \left(\mathbf{A} - \mathbf{B} \ \mathbf{K}_k\right) + \mathbf{C}^T \ \mathbf{Q} \ \mathbf{C} \ , \quad \mathbf{S}_N = \mathbf{C}^T \ \mathbf{P} \ \mathbf{C} \tag{14}$$

Note that, these equations must be solved backward. By using Eq. (10) the optimal control input $\mathbf{u}_k$ can be computed but it requires the full state vector $\mathbf{x}_k$, which is, in most of the cases in practice, not available. To overcome this problem, we will use a classical observer which is defined by the following equation (Friedland, 1996):

$$\mathcal{D}: \ \left\{ \ \hat{\mathbf{x}}_{k+1} = \mathbf{A} \ \hat{\mathbf{x}}_k + \mathbf{B} \ \mathbf{u}_k + \mathbf{K_P} \ \left(\mathbf{y}_k - \mathbf{C} \ \hat{\mathbf{x}}_k\right), \ k \geq 1 \right. \tag{15}$$

then, the estimated state vector $\hat{\mathbf{x}}_k$ is used instead of $\mathbf{x}_k$ in Eq. (10). This way, we can compute the input $\mathbf{u}_k$ that drives the output of the system $\mathcal{D}$ along a desired trajectory $\mathbf{y^r}_k$ in a optimal way, to the minimize the performance measure given in Eq. (8). At this point, let us reinterpret the last sentence as follows: Given a measured system response $\mathbf{y^r}_k$ for the system $\mathcal{D}$ known on an interval $1 \leq k \leq N$, we can compute the input $\mathbf{u}_k$ that provoke such response. It is not hard to realize why Eqs. (10)-(15) can help to compute that input $\mathbf{u}_k$. Backing to our problem of the estimation of impact forces, we can consider that $\mathbf{y^r}_k$ is the measured vibration response of a flexible structure whose dynamics is represented by the Eq. (7) and the computed input $\mathbf{u}_k$ is the estimated impact force. In the following section we will test the effectiveness of this strategy.

4. Experimental setup and results

In order to experimentally validate the two methods described in the previous sections when applied to the problem of estimation of impact forces in flexible structures, it was constructed a test rig at the Laboratory of Vibration in the PUC-Rio. Fig. 2 shows a diagram for the experimental setup.

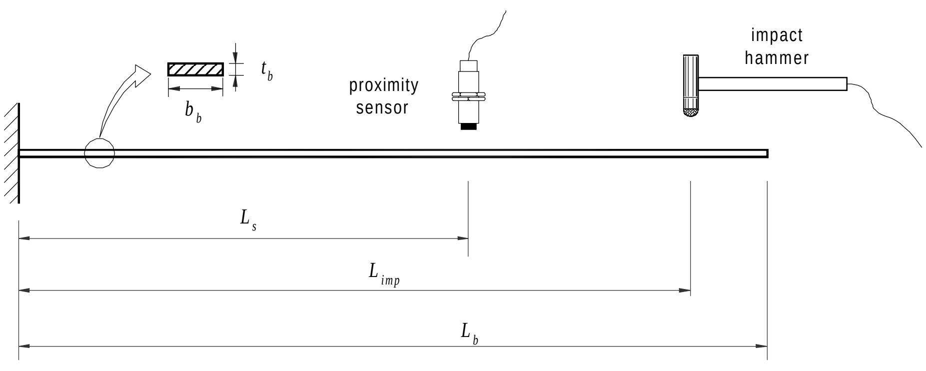

Figure 2. The experimental setup

As it can be seen, the system consist of a one-sided clamped flexible beam which is subject to an impact load near to its free end. The beam material is steel and the main dimensions of the test rig are presented on Tab. 1.

Table 1. Dimensions of the experimental test rig *

L_b	b_b	t_b	L_s	L_{imp}
0,510	0,025	0,005	0,320	0,390

* all dimensions in meters.

The impactor consist of an instrumented hammer Endevco Model 30927 which has a piezoelectric force sensor on the head (sensitivity 0,4363 mV/N). The measurements obtained by the force sensor will be useful to evaluate the performance of the methods used to estimate impact force. As vibration response sensor was used a inductive proximity sensor Balluff BAW 018PF1K located away from the impact point (bandwidth: 1,5 kHz). This sensor allows us to get information of the vibration of the flexible beam in the form of displacements.

4.1. Experimental results

The data were acquired through a spectral analyser equipment, Hewlett Packard model 3566A. The sampling rate used was 30μs. After the beam was impacted by the hammer, the acquisition data procedure, which triggered by the force detected by the sensor assembled on the head of the hammer, starts with a total acquisition time of 125 ms. The proximity sensor measures the displacements of a fixed point on the flexible beam to a fixed distance from the clamped end and the measurements fell inside the sensor linear range. Figure 3 shows the sort of data that were acquired by the analyser. As it can be seen from this figure, noise was detected on the measurements.

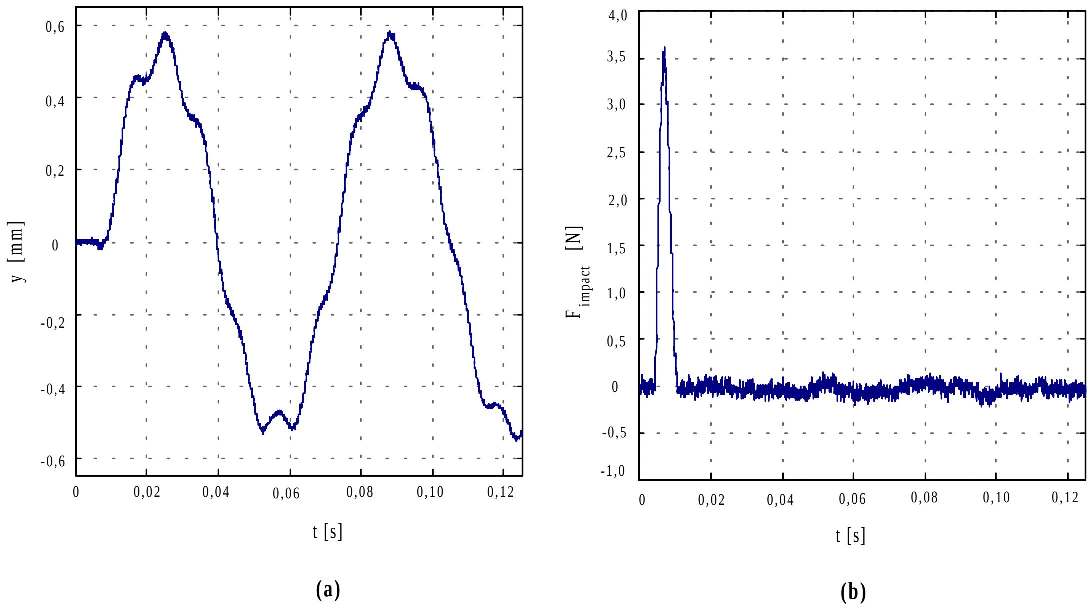

Figure 3. Experimentally measured vibration response and impact force

4.1.a. Results obtained with the P-I observer

Because the state observers are model-based estimators, prior to use the method, it is necessary to obtain a dynamical model of the flexible structure. To this, it was used the Finite Element Method (Inman, 1996) to model the one-side clamped flexible beam, considering 10 Euler-Bernoulli beam elements and no damping. The result was a 20 modes model. Later, this model was transformed to use modal coordinates and after that it was reduced to retain only the first five modes. Good agreement between the experimental and theoretically determined natural frequencies was verified. By expressing the dynamical model in the space-state form, we arrived to the form of the Eq. (1).

The next step was to implement the P-I observer given in Eq.(2). To do this, a program in the Simulink®/Matlab® environment was done and it is shown in the Fig. 4.

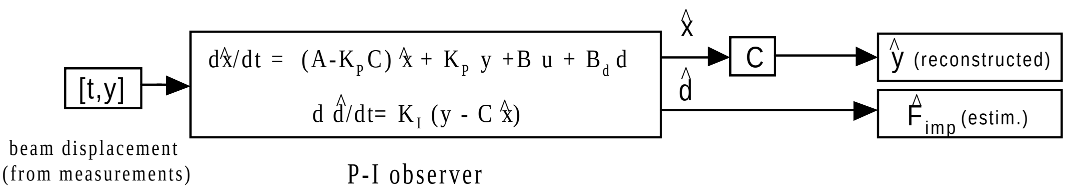

Figure 4. Simulink® program used to estimated impact forces by using the P-I observer technique

The observability condition of the pair $(\mathbf{A_a}, \mathbf{C_a})$ was verified and the observer gains $\mathbf{K_p}$ and $\mathbf{K_I}$ were determined by using the algorithm described in section 2 and the Ricatti equations resulting from the Kalman Filter method (Valer, 1999). After some trial and error tests, the final values of the gain matrices are chosen to the reconstruct satisfactory the measured displacement $\mathbf{y}$, i.e. to approximate $\mathbf{y}$ to $\hat{\mathbf{y}} = \mathbf{C}\,\hat{\mathbf{x}}$. The results are shown on Fig. 5. As can be seen, a very good agreement between the reconstructed variable and measured variable was achieved.

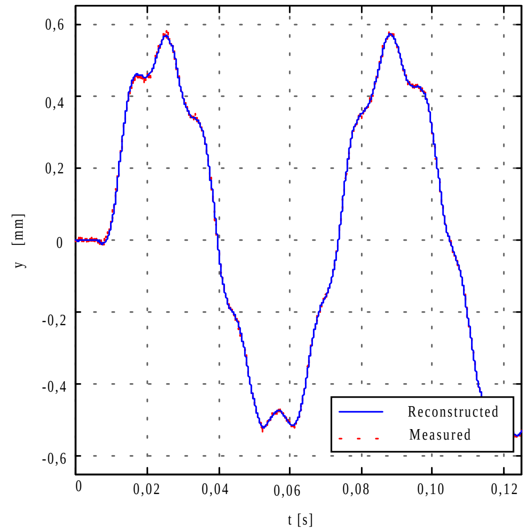

Figure 5. Reconstructed displacement by using the P-I observer technique

The estimated impact force obtained by the P-I observer is showed on Fig. 6. As it can be seen, there is a delay between the signals of the measured impact force and its estimated values which was expected because the integral action of the observed acts as a low pass filter introducing some phase-lag. However, in general terms, the performance of the P-I observer was acceptable because it was able to detect the impact force.

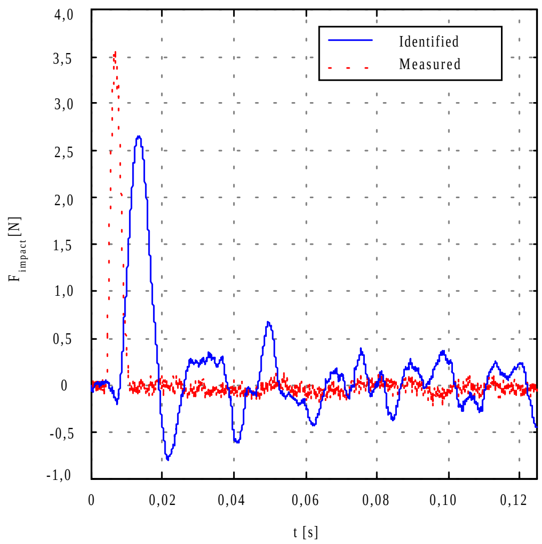

Figure 6. Impact force estimation by using the P-I observer technique

It is important to remark that, these results were obtained through offline computations, but the P-I observer, as showed in Fig. 4 can, without any changes, be implemented in an online manner (i.e. in real time). So, the presented method could be very useful to detect impact forces in some industrial or aerospatial applications, or even to implement impact/force control systems (Valer, 2004).

4.1.b. Results obtained with the algorithm based on optimal control

Now we apply the algorithm proposed on section 3 to our experimental test rig to test its performance. The same dynamical model for the flexible beam that was used to construct the P-I observer will be used here. As said in section 3, it is necessary to choose appropriate values for the symmetric matrices $\mathbf{Q} > \mathbf{0}$, $\mathbf{R} > \mathbf{0}$ and $\mathbf{P} \geq \mathbf{0}$. For simplicity these were chosen as follows:

$$\mathbf{Q} = q\ \mathbf{I}, \quad \mathbf{R} = \mathbf{I} \quad \text{and} \quad \mathbf{P} = \mathbf{0}$$

With this choice we have reduced the number of free parameters to only one, the scalar q. The equations (10)-(15) are used are implemented using Matlab®. It include the backward integration of equations (13)-(14). Due of this backward integration, the algorithm can only be implemented off-line, i.e. after that the vibration measures signals have been acquired and recorded in some interval time. The free parameter q was determined to achieve a good agreement between the measured beam displacement and the reconstructed signal $\mathbf{y}$ computed by Eqs. (7). After some trial and error tests, the value $q = 8 \times 10^{10}$ was chosen. The results for the reconstructed displacement is shown in Fig. 7 and the results for the estimated impact force is showed in Fig. 8.

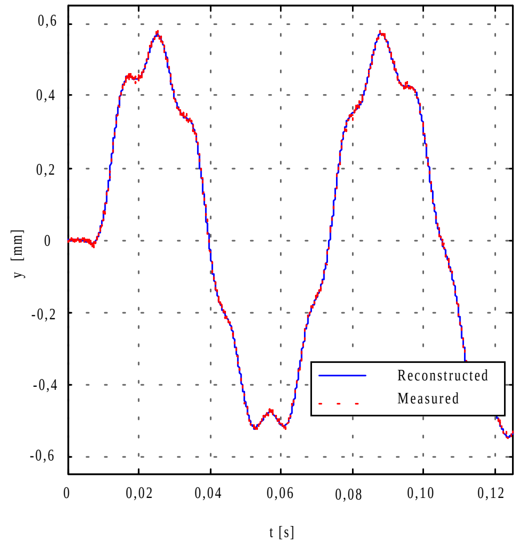

Figure 7. Reconstructed displacement by the optimal control method

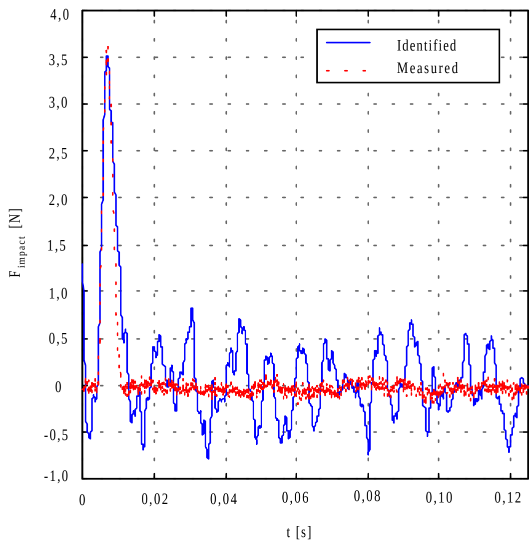

Figure 8. Impact force estimation by the optimal control method

As can be seen, there is a very good agreement between the measured impact force and the estimated one. It shows a superior performance that the P-I observer method but has the disadvantage of the offline implementation.

5. Conclusions

This work presented two new methods to estimate impact forces in flexible structures, which when implemented in an experimental test rig showed to be effective. These methods have advantages and disadvantages. The second method, based on some results of the linear optimal control, produces better estimates for impact forces histories than the first method, which is based on a P-I observer. However, the first method can be implemented through either online (real time) or offline computations but the second method can be only implemented through offline computations. Both methods seem promising to estimate impact forces in flexible structures. Future research will considerate applications in more complex flexible systems.

6. References

Dorf, R. C. and Bishop, R. H., 1995, “Modern Control Systems”, Ed. Addison-Wesley Publishing Company Inc., 811 p.

Friedland, B, 1996, “Observers”, The Control Handbook, Ed.CRC Press Inc., pp. 607-618.

Gaul, L. and Hurlebaus, S. 1999, “Determination of an Impact Force in a Plate by Piezoelectric Film Sensors”, Archive of Applied Mechanics, Vol.69, pp. 691-701.

Inman, D. J., 1996, “Engineering Vibration”, Ed. Prentice Hall International Inc., 560 p.

Lin, C. F. 1994, “Advanced Control Design”, Ed. Prentice Hall Series in Advanced Navigation, Guidance and Control and Their Applications.

Linder, S. P. 1997, “Robust Qualitative and Quantitative Methods for Disturbance Rejection and Fault Accommodation”, Ph.D. Thesis, Northeastern University, Department of Mechanical, Industrial and Manufacturing Engineering, Boston, Massachusetts.

Meirovitch, L., 1990, “Dynamics and Control of Structures”, Ed. John Wiley and Sons Inc., New York, 425p.

Müller, P. C., 1995, “Schätzung und Kompensation von Nichlinearitäten mit Störgrössenbeobachtern”, Entfurt Nichtlinearer Regelungen, Ed. R. Oldenbourg Verlag München Wien GmbH, pp. 142-161.

Ogata, K., 1993, “Engenharia de Controle Moderno”, Ed. Prentice Hall do Brasil Ltda, Rio de Janeiro, Brazil, 781 p.

Saif, M., 1997, “An Observed Based Approach for Fault Diagnosis”, IASTED Control ´97, Cancun, Mexico.

Söffker, D., and Yu, T.-J. and Müller, P. C., 1995, “State Estimation of Dynamical Systems with Nonlinearities by using Proportional-Integral Observer”, International Journal of Systems Science, Vol. 26, No.9, pp. 1571-1582.

Valer, C. I., 1999, “Uma Introdução ao Controle Robusto com Aplicações a Estruturas Flexíveis”, Master Thesis, Pontifícia Universidade Católica do Rio de Janeiro, Department of Mechanical Engineering, Rio de Janeiro, Brazil.

Valer, C. I., 2004, “Controle de Impacto em Manipuladores Robóticos”, Ph.D. Thesis, Pontifícia Universidade Católica do Rio de Janeiro, Department of Mechanical Engineering, Rio de Janeiro, Brazil.

7. Responsibility notice

The authors are the only responsible for the printed material included in this paper.