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OPTIMIZATION OF THE PIEZOELECTRIC MATERIALS POSITIONS IN A FLEXIBLE STRUCTURE

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Abstract. *The purpose of this paper is to suggest a criterion for the optimal placement of collocated piezoelectric actuator-sensor pairs on a thin flexible plate using modal controllability and observability measures. A finite element approach is used for constructing the nominal model of the plant. The dimension of the model is then reduced by a combination of modal reduction techniques and system controllability and observability analysis. The optimum locations of the piezoelectric elements are then determinated by an optimization function that considers the participation of each vibration mode in the system response. The fundamental formulation of this optimization procedure is applied to a flexible thin plate type structure and the performance of the presented methodology is shown through numeric simulations. The work is concluded presenting the potentialities of the optimization methodology proposed and future developments to be implemented.*

Keywords: *Optimal placements, plate structure, piezoelectric materials, modal cost function.*

1. Introduction

The development of smart structures technology in recent years has provided numerous opportunities for vibration control applications. The use of piezoelectric ceramics has shown great promise in the development of this technology (Banks et al, 1996). The ability of these materials to convert mechanical strains into electrical voltage and vice versa allows them to be used as actuators and sensors once placed on flexible structures. The use of piezoelectric material as actuators in vibration control is also beneficial because these elements only excite the elastic modes of the structures without exciting the rigid body modes. This is important since very often only elastic motions of the structures need to be controlled. The continuous nature of structures allows one to choose where the piezoelectric patches are to be placed. Therefore, it is natural to ask where the actuators and sensors should be placed on a structure so that the performance of the composite structure is optimized. One can find locations on a structure where the controllability and observability measures of important modes are maximized (Moheimani and Ryall, 1999).

The choice of the actuator location is an important issue in the design of actively controlled structures. The actuators should be placed at the locations so that the desired modes are excited most effectively (Pota and Alberts, 1995). Various engineering applications using classical optimization and genetic algorithm schemes for the determination of optimal piezoelectric actuator placement have been reported. Kirby and Matic (1994) worked with genetic algorithms to determine optimal actuator size and location for two piezoelectric actuators bonded to a cantilever beam. Pereira and Steffen Jr. (2002) used a discrete-continuous optimization technique to determine the position of the actuators along the flexible structure and to obtain the controller gains. In that work the goal was to minimize the control effort applied to a beam type structure. Gawronski (1997) addresses the problem of actuator and sensor placement using their notion of modal controllability and observability. Also, Crawley and de Luis (1987) attempt to find the optimal placement for piezoelectric actuators by determining the location of high average strain on structures. Authors of (Hwang et al, 1997) find the placement for collocated piezoelectric actuator-sensor pairs on an all-clamped thin plate by determining the location of high position sensitivity of each mode. A number of other researchers (Fahroo and Wang, 1997; Demetriou, 2000) use the optimization of quadratic performance indexes to find optimal location for piezoelectric actuators and sensors for effective structural vibration suppression. These performance indexes are dependent on the choice of controllers. Therefore, while the final positions of the actuators and sensors may be optimal for one particular control law, it may not be a suitable choice for other compensators.

In this paper, the measures of controllability and observability are based on the modal cost function or modal cost analysis, as proposed by Skelton and Yousuff (1983). Such measures are used to guide the placement of sensors and actuators in flexible piezo-actuated structures. This paper is organized as follows: Section 2 discusses the modeling of a piezoelectric laminate plate. Section 3 describes the notion of modal cost function to find the optimal placement of piezoelectric actuators-sensors on the plate. Section 4 presents a numerical example of the application of the optimization procedure for a thin plate with simply supported boundary conditions. Section 5 gives overall conclusions of the paper.

2. Finite Element Discretization

In the present formulation, the following assumptions (Reddy, 1999) are considered:

- the piezoelectric layers are perfectly bonded together;
- the formulation is restricted to linear elastic material behavior (small displacement and strains);
- this formulation uses the Kirchhoff assumption (thin plate) in which the transverse normal remains straight after deformation and they also rotate such that they always remain perpendicular to the mid-surface.

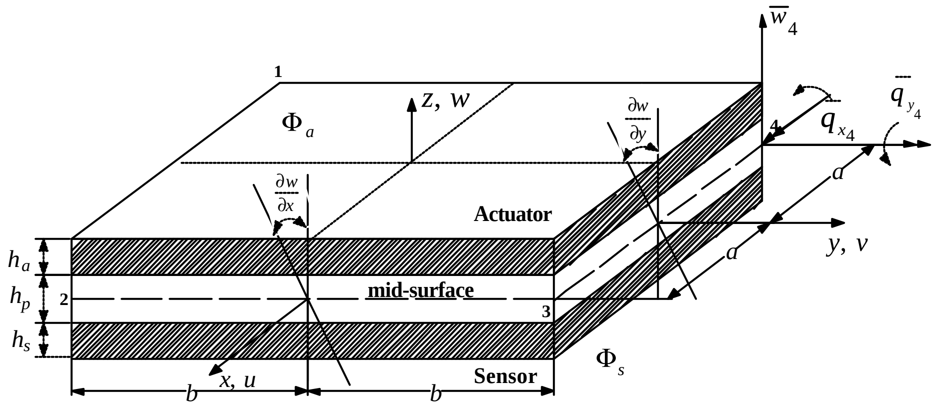

Figure 1. Coordinate system of a laminated finite element with integrated piezoelectric material.

In this work the following linear constitutive relations for piezoelectric materials are employed (Taylor et al, 1985):

$$\{s\} = \left[C^E\right]\{e\} - [e]\{E\} \tag{1}$$

$$\{D\} = [e]^T\{e\} + \left[x^S\right]\{E\} \tag{2}$$

where the superscript S means that the values are measured at constant strain and the superscript E means that the values are measured at constant electric field, $\{s\}$ is the stress tensor, $\{D\}$ is the electric displacement vector, $\{e\}$ is the strain tensor, $\{E\}$ is the electric field, $[C^E]$ is the elastic constants at constant electric field, $[e]$ denotes the piezoelectric stress coefficients, and $[x^S]$ is the dielectric tensor at constant mechanical strain.

The relation between [e] and [d], the piezoelectric strain coefficient, is:

$$[e] = \left[C^E\right][d] \tag{3}$$

The application of voltage to the element is analogous to the application of heat to a bimetallic strip. The voltage Φ_a across the bender element forces the bottom layer to expand, while the upper layer contracts, as depicted in Fig. 2.

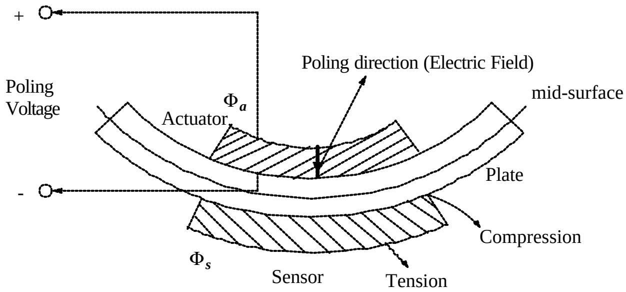

Figure 2. Curvature of a plate produced by the expansion of one piezoelectric layer and contraction of the other.

The result of these physical changes is a strong curvature; this implies in a large deflection at the tip when the other end is clamped (see Fig. 2). The tip deflection may be much larger than the change in length of either ceramic layer. Due to the reciprocity effect, deformation of the sensor will produce a charge across the sensor electrode, which is collected through the sensor surface as an electric voltage Φ_s.

When only the poling direction is taken into account, the applied or sensed electric potential through the actuator or sensor element is given by the following equation (Lopes et al., 2000):

$$\Phi_z = \left(\frac{z - \frac{h_p}{2}}{h} \right) \Phi \tag{4}$$

where h and Φ (see Fig. 2) are the thickness and the maximum electric potential at the external surface of the corresponding piezoelectric element (actuator and sensor), and z (z_a and z_s) is defined over the intervals:

$$\frac{h_p}{2} \leq z_a \leq \frac{h_p}{2} + h_a \tag{5}$$

$$-\frac{h_p}{2} \geq z_s \geq -\frac{h_p}{2} - h_s \tag{6}$$

Now, assuming that the electric field (E) is constant through the actuator and sensor elements thickness, the gradient operators are:

$$E = -\frac{d\Phi_z}{dz} = -B_z \Phi = -\frac{\Phi}{h} \tag{7}$$

2.1 Obtaining the Element Matrices

Hamilton's principle is employed here to derive the finite element equations.

$$\int_{t_1}^{t_2} \left[d\left(T - U + W_e - W_m\right) + dW \right] dt = 0 \tag{8}$$

where t_1 and t_2 are two arbitrary instants, T is the kinetic energy, U is the potential energy, W_e denotes the work done by electrical forces, and W_m is the work done by magnetic forces, which is negligible for piezoelectric materials. The work done by electrical forces and magnetic forces is given by:

$$W_e = \frac{1}{2} \int_V \{E\}^T \{D\} dV \tag{9}$$

$$d\,W = \int_V \{dq\}^T \{f_b\} dV + \int_A \{dq\}^T \{f_A\} dA - \int_A d\Phi s_q dA \tag{10}$$

where D is the electric displacement vector, f_b is the body force, f_A is the surface force, and s_q is the surface electrical stress.

Two equilibrium equations written in generalized coordinates are presented (Abreu et al, 2004) for the k-th element (see Fig. 1):

$$[M_{qq}^e]\{\ddot{q}_k\} + [K_{qq}^e]\{q_k\} + [K_{q\Phi}^e]\{\Phi\} - \{\bar{f}\} = 0 \tag{11}$$

$$\left[K_{\Phi q}^e\right]\{q_k\} + \left[K_{\Phi\Phi}^e\right]\{\Phi\} + \{Q_a\} = 0 \tag{12}$$

where $[K^e]$ is the extended element stiffness matrix and $[M_{qq}^e]$ is the element mass matrix.

The mechanical stiffness matrix $[K_{qq}^e]$ is given by (Abreu et al, 2004):

$$[K_{qq}^e] = \sum_{i=1}^{3} h_i [X]^{-T} \int_A [L_K]^T [D_i][L_K] dA [X]^{-1} \tag{13}$$

where [X] is a 12×12 matrix given by Eq. (14):

$$[X]=\begin{bmatrix}
1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 & x_1^3 & x_1^2y_1 & x_1y_1^2 & y_1^3 & x_1^3y_1 & x_1y_1^3 \\
0 & 0 & 1 & 0 & x_1 & 2y_1 & 0 & x_1^2 & 2x_1y_1 & 3y_1^2 & x_1^3 & 3x_1y_1^2 \\
0 & -1 & 0 & -2x_1 & -y_1 & 0 & -3x_1^2 & 2x_1y_1 & -y_1^2 & 0 & -3x_1^2y_1 & -y_1^3 \\
1 & x_2 & y_2 & x_2^2 & x_2y_2 & y_2^2 & x_2^3 & x_2^2y_2 & x_2y_2^2 & y_2^3 & x_2^3y_2 & x_2y_2^3 \\
0 & 0 & 1 & 0 & x_2 & 2y_2 & 0 & x_2^2 & 2x_2y_2 & 3y_2^2 & x_2^3 & 3x_2y_2^2 \\
0 & -1 & 0 & -2x_2 & -y_2 & 0 & -3x_2^2 & 2x_2y_2 & -y_2^2 & 0 & -3x_2^2y_2 & -y_2^3 \\
1 & x_3 & y_3 & x_3^2 & x_3y_3 & y_3^2 & x_3^3 & x_3^2y_3 & x_3y_3^2 & y_3^3 & x_3^3y_3 & x_3y_3^3 \\
0 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & x_3^2 & 2x_3y_3 & 3y_3^2 & x_3^3 & 3x_3y_3^2 \\
0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3^2 & 2x_3y_3 & -y_3^2 & 0 & -3x_3^2y_3 & -y_3^3 \\
1 & x_4 & y_4 & x_4^2 & x_4y_4 & y_4^2 & x_4^3 & x_4^2y_4 & x_4y_4^2 & y_4^3 & x_4^3y_4 & x_4y_4^3 \\
0 & 0 & 1 & 0 & x_4 & 2y_4 & 0 & x_4^2 & 2x_4y_4 & 3y_4^2 & x_4^3 & 3x_4y_4^2 \\
0 & -1 & 0 & -2x_4 & -y_4 & 0 & -3x_4^2 & 2x_4y_4 & -y_4^2 & 0 & -3x_4^2y_4 & -y_4^3
\end{bmatrix} \tag{14}$$

$[L_K]$ is given by:

$$[L_K]=\begin{bmatrix}
0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 & 0 & 6xy & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 6xy \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 6y^2
\end{bmatrix} \tag{15}$$

and h_i is given by:

$$h_1 = h_a\left(\frac{h_p}{2}+\frac{h_a}{2}\right)^2+\frac{h_a^3}{12} \tag{16}$$

$$h_2 = \frac{h_p^3}{12} \tag{17}$$

$$h_3 = h_s\left(\frac{h_p}{2}+\frac{h_s}{2}\right)^2+\frac{h_s^3}{12} \tag{18}$$

and $[D_i]$ or $[D_a]$, $[D_p]$, and $[D_s]$, for i = 1,2,3, are calculated by following equation for the piezoelectric and plate material properties, respectively, and dA is equal to $dxdy$.

$$[D_i]=\frac{E_i}{1-n_i^2}\begin{bmatrix}
1 & n_i & 0 \\
n_i & 1 & 0 \\
0 & 0 & (1-n_i)/2
\end{bmatrix} \tag{19}$$

where n is the Poisson ratio and E_i denotes the Young’s modulus of the structure, sensor or actuator.

The element mass matrix is given by (Abreu et al, 2004):

$$[M_{qq}^e]=\sum_{i=1}^{3} r_i[X]^{-T}\int_A [L_M]^T[H_i][L_M]dA[X]^{-1} \tag{20}$$

where $r_1 = r_a$, $r_2 = r_p$, $r_3 = r_s$, and $[H_i]$ (for i = 1,2,3) are:

$$[H_1]=[H_a]=\begin{bmatrix} h_a & 0 & 0 \\ 0 & h_1 & 0 \\ 0 & 0 & h_1 \end{bmatrix} \tag{21}$$

$$[H_2]=[H_p]=\begin{bmatrix} h_p & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_2 \end{bmatrix} \tag{22}$$

$$[H_3]=[H_s]=\begin{bmatrix} h_s & 0 & 0 \\ 0 & h_3 & 0 \\ 0 & 0 & h_3 \end{bmatrix} \tag{23}$$

The electrical-mechanical coupling stiffness matrix $[K^e_{q\Phi}]$ and dielectric stiffness matrix $[K^e_{\Phi\Phi}]$ are given by (Abreu et al, 2004):

$$\left[K^e_{q\Phi}\right]_a = -\frac{1}{2}\left(h_p h_a + h_a^2\right)[X]^{-T}\int_A [L_K]^T [e]_a{}^T B_z dA \tag{24}$$

$$\left[K^e_{\Phi\Phi}\right]_a = -\frac{4ab\left[x_a^S\right]}{h_a} \tag{25}$$

$$\left[K^e_{q\Phi}\right]_s = \frac{1}{2}\left(h_p h_s + h_s^2\right)[X]^{-T}\int_A [L_K]^T [e]_s{}^T B_z dA \tag{26}$$

$$\left[K^e_{\Phi\Phi}\right]_s = -\frac{4ab\left[x_s^S\right]}{h_s} \tag{27}$$

The Eqs. (24), (25), (26), and (27) are integrated numerically by using the Gauss-quadrature integration method (Bathe, 1982):

$$[K^e_{qq}] = \sum_{i=1}^{3} h_i [X]^{-T} \sum_h \sum_x [L_K]^T [D_i][L_K] W_x W h [X]^{-1} \tag{28}$$

$$[M^e_{qq}] = \sum_{i=1}^{3} r_i [X]^{-T} \sum_h \sum_x [L_M]^T [H_i][L_M] W_x W h [X]^{-1} \tag{29}$$

$$\left[K^e_{q\Phi}\right]_a = -\frac{1}{2}\left(h_p h_a + h_a^2\right)[X]^{-T}\sum_h \sum_x [L_K]^T [e]_a{}^T W_x W h B_z \tag{30}$$

$$\left[K^e_{q\Phi}\right]_s = \frac{1}{2}\left(h_p h_s + h_s^2\right)[X]^{-T}\sum_h \sum_x [L_K]^T [e]_s{}^T W_x W h B_z \tag{31}$$

where (x, h) are the Gaussian integration point coordinates and W_x and W_h are the associated weight factors.

2.2. Obtaining the Global Matrices

Each of these element matrices can be assembled into global matrices. The assemblage process to obtain the global matrices is written as:

$$[M] = \sum_{k=1}^{N} [T_k]^T \left[M^e_{qq}\right][T_k] \tag{32}$$

$$\left[K_{qq}\right] = \sum_{k=1}^{N} [T_k]^T \left[K^e_{qq}\right][T_k] \tag{33}$$

where N is the number of finite elements and $[T_k]$ is the distribution matrix defined by:

$$T_k(i, j) = \begin{cases} 0 & if \quad j \neq m_k(i) \\ 1 & if \quad j = m_k(i) \end{cases} \tag{34}$$

for i =1, 2, …, 12, and j = 1, 2, …, n_{dof}

where n_{dof} is the number of degrees of freedom of the entire structure, and m_k denotes the index vector containing the degrees of freedom (3 dof) of the n-th node (1, 2, 3 or 4 – see Fig. 1) in the k-th finite element given by:

$$m_k = \{3n_k - 2 \quad 3n_k - 1 \quad 3n_k\} \tag{35}$$

Considering that n_a actuators and n_s sensors are distributed in the plate, Eqs. (11) and (12) can be written in the global form:

$$[M]\{\ddot{z}\} + [K_{qq}]\{z\} + \sum_{k=1}^{n_{e_i}} [T_k]_i^T [K_{q\Phi}]_i \{\Phi\} - \{F\} = 0 \tag{36}$$

$$\sum_{k=1}^{n_{e_i}} \left(\left[K_{\Phi q}\right]_i [T_k]_i \{z\} + \left[K_{\Phi\Phi}\right]_i \{\Phi\} + Q_a\right) = 0 \tag{37}$$

where $[T_k]_i$ is the distribution matrix (Eq. 34) which shows the position of the k-th element in the plate structure by using zero-one inputs, where the zero input means that no piezoelectric actuator/sensor is present, and one input means that there is an actuator/sensor in that particular element position, n_{e_i} is the number of finite elements of the i-th piezoelectric actuator/sensor, and $\{z\}$ is the nodal displacement vector of the global structure.

In the piezoelectric sensor there is no voltage applied to the corresponding element ($Q_a = 0$), so that the electrical potential generated (sensor equation) is calculated by using Eq. (37), yielding:

$$\{\Phi_s\} = -\sum_{k=1}^{n_{e_i}} [K_{\Phi\Phi}]_{si}^{-1} [K_{\Phi q}]_{si} [T_k]_i \{z\} \text{ for } i = 1, 2, \ldots, n_s \tag{38}$$

The total voltage $\{\Phi\}$ is composed by the voltage $\{\Phi_s\}$ that is sensed by the sensor, the voltage $\{\Phi_{sa}\}$ that is sensed by the actuator (see Eq. 38), and by the applied voltage $\{\Phi_a\}$. Then, $\{\Phi\}$ can be expressed by:

$$\{\Phi\} = \{\Phi_s\} + \{\Phi_{sa}\} + \{\Phi_a\} \tag{39}$$

The global dynamic equation can be formed by substituting Eq. (38) into Eq. (39) and then into Eq. (36). Thus, moving the forces due to actuator together with the mechanical forces to the right hand side of the resulting equation, yields:

$$[M]\{\ddot{z}\} + [K_{qq}^*]\{z\} = \{F\} + \{F_{el}\}_j \tag{40}$$

where $[K_{qq}^*]$, and $\{F_{el}\}$ (electrical force) are given by, respectively:

$$[K_{qq}^*] = [K_{qq}] - [K_{el}] \tag{41}$$

and

$$\{F_{el}\}_j = -\sum_{k=1}^{n_{e_j}} [T_k]_j^T [K_{q\Phi}]_{aj} \Phi_{aj} \text{, for } j = 1, 2, \ldots, n_a \tag{42}$$

where $[K_{el}]$ is the electric stiffness written as:

$$[K_{el}] = \sum_{i=1}^{n_s} \sum_{k=1}^{n_{ei}} [T_k]_i^T [K_{q\Phi}]_{si} [K_{\Phi\Phi}]_{si}^{-1} [K_{\Phi q}]_{si} [T_k]_i + \sum_{j=1}^{n_a} \sum_{k=1}^{n_{e_j}} [T_k]_j^T [K_{q\Phi}]_{aj} [K_{\Phi\Phi}]_{aj}^{-1} [K_{\Phi q}]_{aj} [T_k]_j \tag{43}$$

The system static equation is written by using Eq. (40) as follows:

$$\{z\} = [K_{qq}^*]^{-1}(\{F\} + \{F_{el}\}) \tag{44}$$

Transforming Eq. (40) in state-space form, results:

$$\{\dot{\bar{x}}\} = [A]\{\bar{x}\} + [B]\{\Phi_a\} \tag{45}$$

$$\{\Phi_s\} = [C]\{\bar{x}\} \tag{46}$$

where $\{\Phi_a\} = \{\Phi_{a_1} \quad \Phi_{a_2} \quad \cdots \quad \Phi_{a_{n_a}}\}$ is the electric potential vector applied to n_a actuators and $\{\bar{x}\}$, $[A]$, $[B]$ and $[C]$ are:

$$\{\bar{x}\} = [j \quad j]^T \tag{47}$$

$$[A] = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -[\overline{K}_g] & -[\overline{C}_a] \end{bmatrix} \tag{48}$$

$$[B] = \begin{bmatrix} 0_{n\times n_a} \\ -[f]^T \sum_{k-1}^{n_{e_j}} [T_k]_j^T [K_{q\Phi}]_{aj} \end{bmatrix} \tag{49}$$

$$[C]=\begin{bmatrix} \dfrac{-\sum_{k=1}^{n_{e_i}}\left[K_{\Phi q}\right]_{si}\left[T_k\right]_i[f]}{\sum_{k=1}^{n_{e_i}}\left[K_{\Phi\Phi}\right]_{si}} & 0_{n_s \times n} \end{bmatrix} \tag{50}$$

where $[\overline{K}_g]$ and $[\overline{C}_a]$ are the stiffness and damping matrices obtained by the modal transformation of the global matrices ($[K^*_{qq}]$ and $[C_g]$) with the modal matrix $[f]$ (Thomson, 1973), n is the number of selected vibration modes, and $[C_g]$ is the global damping matrix, obtained by the relation:

$$[C_a]=a[M]+b\left[K^*_{qq}\right] \tag{51}$$

where a and b are the Rayleigh coefficients (Thomson, 1973).

3. Modal cost function

In this section the problem of choosing the optimum locations for the piezoelectric elements used in the active vibration control of the structure is discussed. For this purpose, modal cost techniques will be used, i.e., how the controllability/obsevability of the system is changed as the piezoelectric actuators/sensors move along the structure (Skelton and Yousuff, 1983).

Assuming the pair (A,B) controllable and (A, C) observable, the controllability and observability are measured according to the grammians, as defined in the following equations (Laub et al, 1973):

$$W_c\left(0,t_f\right)=\int_0^{t_f} e^{At}BB^t e^{A^T t}\,dt \tag{52}$$

$$W_o\left(0,t_f\right)=\int_0^{t_f} e^{A^T t}C^t C e^{At}\,dt \tag{53}$$

where t_f is some fixed final time and W_c and W_o satisfy the algebraic Lyapunov equations:

$$W_c A^T + AW_c + BB^T = 0 \tag{54}$$

$$A^T W_o + W_o A + C^T C = 0 \tag{55}$$

Skelton et al (1988) suggested a measure of the effect of the actuators/sensors positions in the dynamical system by using a quadratic cost function:

$$V=\sum_{i=1}^{n}\int_0^{\infty} y^{iT}(t)Q_c y^i(t)dt \tag{56}$$

where the vector $y^i(t)$ is composed of output variables x^i due to impulsive inputs $u_i(t) = \delta(t)$ (with $u_j(t) = 0$, $i \neq j$), applied at t 0, with zero initial conditions, and Q_c is a weight matrix.

Substituting the relation:

$$y^i(t)=Cx^i(t) \tag{57}$$

into Eq. (56), yields:

$$V=\sum_{i=1}^{n}\int_0^{\infty} x^{iT}(t)C^T Q_c C x^i(t)dt \tag{58}$$

The unit impulse response for zero initial condition in the ith-diretion is given by (Ogata, 2001):

$$x^i(t)=e^{At}x(0)+\int_0^t e^{A(t-t)}Bu_i(t)dt \tag{59}$$

Considering zero initial conditions, Eq. (59) is given by:

$$x^i(t) = e^{At}B \tag{60}$$

Substituting Eq. (60) into (58), yields:

$$V = Q_c C\left[\sum_{i=1}^{n}\int_0^{\infty} e^{At}BB^T e^{A^T t}\,dt\right]C^T \tag{61}$$

Substituting Eq. (50) into (59), the cost function V is derived as follows:

$$V = \mathbf{trace}\{Q_c C W_c C^T\} \tag{62}$$

where trace{ } denotes the trace (diagonal sum) of the matrix { }.

If the cost function (V) of a particular sensor/actuator location is zero, the sensor/actuator has no authority over those modes. On the other hand, if a given sensor/actuator location has a maximum cost value, the actuator/sensor has maximum authority over those modes.

Based on these principles it is possible (Skelton et al, 1988):

- to compare different sensor and actuator configurations and to choose the one that contributes the most to the cost function, and,
- to analyze the importance of the contribution of a given sensor/actuator in the cost function V with respect to the others.

4. Optimal placement of piezo sensors and actuators by using the modal cost function

In order to test the proposed optimization method to find the optimal placement of sensors and actuators, a simply supported plate containing three set of sensor/actuator PVDF/PZT ceramic piezoelectric elements bonded to the plate surface are considered (see Fig. 3).

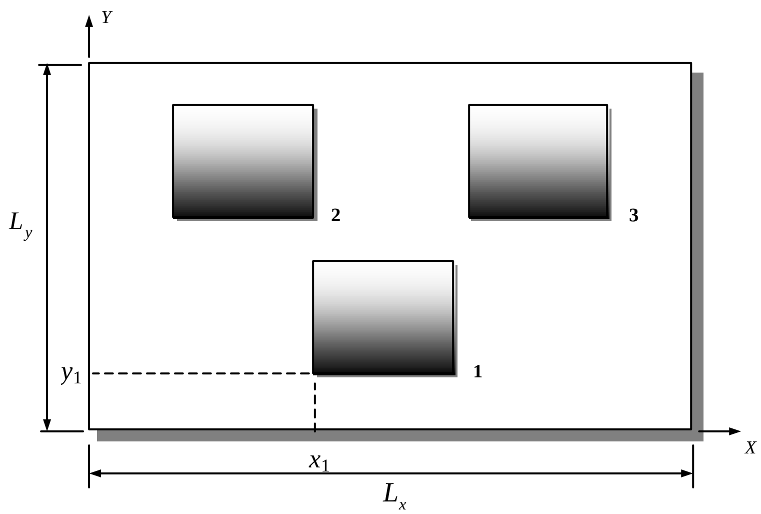

Figure 3. A thin plate with the piezoelectric patches attached.

The characteristics of the resulting mechatronic structure are shown in Table 1.

Table 1. Characteristics of the piezo-structure.

Properties	Units	Piezoelectric Sensor	Piezoelectric Actuator	Plate
E (Young's modulus)	Gpa	2	69	207
r (density)	Kg/m^3	1780	7700	7870
d_{31} (Charge constant)	$C/m^2/(N/m^2)$	23×10^{-12}	-179×10^{-12}	----
L_y (width)	$m\times10^{-3}$	100	100	400
L_x (length)	$m\times10^{-3}$	100	100	600
h (thickness)	$m\times10^{-3}$	0.205	0.254	1.0
k_d (dielectric constant)	----	12	1800	----

The main purpose of this section is to find the optimal placement of sensors and actuators (x_1, y_1) pairs bonded to the surfaces of the plate. The optimization procedure given in the previous section is used. In this case the five lowest frequency modes (n = 5) are used to calculate the cost function V. To diminish the computational effort, the structure was modeled by using a 6 × 4 finite element mesh.

The output observability/controllability, that is the participation of each sensor/actuator in the output cost function is computed by using Eq. (62). It means that the configuration presenting the largest cost function index is the one whose output is the largest. Three computed outputs (Eq. 62) of the system for different placements of the sensor and actuator (x_1, y_1) are summarized in Tab. 2, where the weight matrix Q_c = I (identity matrix), and the matrices A, B and C are given by Eqs. (48), (49) and (50), respectively.

Table 2. Piezoelectric positions (x_1, y_1) on the plate structure and the cost function V.

Piezoelectric Positions (x_1, y_1) 1	2	3	Function V
0.1; 0.1	0.1; 0.2	0.4; 0.2	0.47689
0.1; 0.1	0.3; 0.1	0.4; 0.2	0.40887
0.1; 0.1	0.4; 0.1	0.3; 0.2	0.40785

The optimum solution (largest V) (x_1, y_1) is equal to: (0.1; 0.1) for the first pair; (0.1; 0.2) for the second and (0.4; 0.2) for the third pair (see Fig. 4).

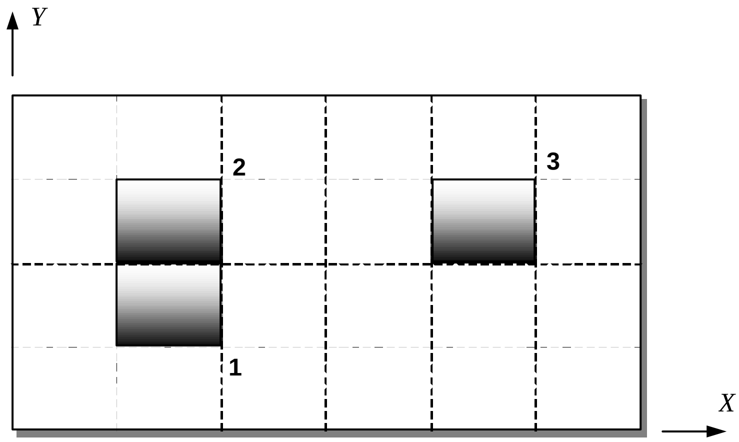

Figure 4. Optimal positions of the piezoelectric elements on the plate structure.

5. Conclusions

The optimization methodology allowed the placement of collocated actuator-sensor pairs for effective vibration reduction. The procedure used for placing sensors and actuators along the smart structure (modal cost technique) was proven to be effective. Besides, the modal cost technique has a strong intuitive appeal. It was shown that the developed methodology could be used for a collocated actuator-sensor system. The optimization methodology can also be used for more complicated flexible structures using modeling techniques such as finite element method.

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7. References

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