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MODEL OF ENERGY HARVESTING USING A PIEZOELECTRIC TRANSDUCER

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Abstract: *Nowadays, a major concern is the need to develop new energy sources. In this context, a sector that has attracted much interest is one in which devices that are able convert other types of energy into electrical energy. This technique is known as energy harvesting and consists of energy capture and storage by, for example, solar, wind, thermal and kinetic sources. Of interest in this paper are piezoelectric transducers, which are able to convert mechanical vibrations into electrical energy. However, when an electrical circuit is coupled to the transducer the mechanical system is strongly influenced by it. This paper presents a model that considers the coupling influence between these systems. This model is based on The Impedance Method to get the Transduction Matrix. The structure modeled as a free sliding beam with a piezoelectric ceramic coupled to it and one electrical load. A program was developed to analyze the behavior of this system, as well as the optimal conditions for energy harvesting.*

Keywords: *Piezoelectric Material; Energy Harvesting; Transduction Matrix*

1. INTRODUCTION

The search for alternative sources of energy had been increasing. This has occurred for many reasons, among then the most important is the necessity to develop new sources of clean energy due environmental problems and the problem of exhaustible sources of energy due to growing demand.

Power Harvesting or Energy Harvesting is about the act of converting energy for electrical energy before it is wasted or lost. Normally, the electrical energy is stored in a battery to be used later. In this form, the Energy Harvesting may be a solution for charging batteries in many cases, mainly in remote applications where the connection with the electrical energy network is difficult.

The type of energy to be converted can be solar, wind, thermal and kinetic. In this paper the source is kinetic; specifically, vibration sources that can be anything that have periodic motion. For example the small vibrations of a machine, the motion of walking, even the motion of blood circulation. However, for this conversion to be possible the transducer should transform mechanical energy in electrical energy. The transducers mostly used for this are magnetic, electrostatic and piezoelectric. In this work, the harvesting of energy is through piezoelectric transducers, due to its ability to directly convert applied strain into electrical charge.

In this context, many studies had been conducted they: Sodano et al. (2002) performed a study to investigate the amount of power generated through the vibration of a piezoelectric plate, as well as methods of power storage; Lesieutre et al. (2002) investigated the damping added to a structure due to the removal of electrical energy from the system during power harvesting; Leffeuvre et al. (2005) constructed an electromechanical structure, trying to optimize the power flow of vibration-based piezoelectric energy-harvesters.

This paper describes a model of the interaction between electrical and mechanical systems proposed by Nakano et al (2007) using a two-port network model of a transducer. The methodology is applied to a piezobeam connected a resistive load and the behaviour of this system loads is studied. The optimum conditions for maximum power harvested of the system is also was investigated.

2. TWO-PORT NETWORK MODEL

To model the harvesting system a two-port network model of a transducer is used connected to the Thevenin equivalent for the vibrating structure and an electric load. This model was proposed by Nakano et al (2007) and can be seen in Fig. (1)

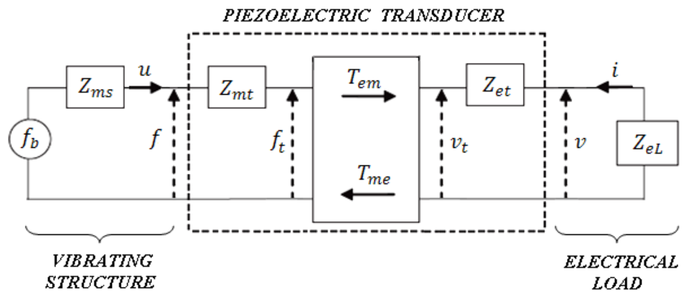


Figure 1. Two-port network model

In Fig. (1) f_b is the blocked force, Z_{ms} is the Mechanical Impedance of the system, u is the velocity, f is the force on transducer, Z_{mt} is the Mechanical Impedance of the transducer, Z_{et} is the Electrical Impedance of the transducer, Z_{eL} is the Impedance of the external load, i is the current, v is the voltage on load, T_{em} and T_{me} are the transduction coefficients. T_{me} describes the force produced per unit electric current and similarly T_{em} represents the voltage generated per unit velocity.

For the transducer the relationship between the mechanical and electrical variables is expressed by:

$$\begin{Bmatrix} f \\ v \end{Bmatrix} = \begin{bmatrix} Z_{mt} & T_{me} \\ T_{em} & Z_{et} \end{bmatrix} \begin{Bmatrix} u \\ i \end{Bmatrix} \tag{1}$$

where $|T_{me}| = |T_{em}|$ (2)

The voltage across the external load is given by:

$$v = -Z_{eL} i \tag{3}$$

Substituting Eq. (3) into the equation for voltage for the transducer given in Eq. (1) gives the current as function of velocity:

$$i = -\frac{T_{em}}{Z_{et}+Z_{eL}} u \tag{4}$$

Substituting Eq. (4) into the equation for the force in Eq. (1) gives the force as a function of velocity:

$$f = \left(Z_{mt} - \frac{T_{em}T_{me}}{Z_{et}+Z_{eL}}\right) u \tag{5}$$

The force in the transducer can be expressed as:

$$f = f_b - Z_{ms} u \tag{6}$$

Now, substituting Eq. (6) in (5) gives the expression for the velocity:

$$u = \frac{f_b}{\left(Z_m - \frac{T_{me}T_{em}}{Z_{et}+Z_{eL}}\right)} \tag{7}$$

where $Z_m = Z_{mt} + Z_{ms}$ (8)

Power harvested is considered as the power dissipated in the electric load. Under the harmonic excitation this power is:

$$P_h = \frac{1}{2} \mathrm{Re}[-iv^*] \tag{9}$$

where (*) means the conjugate complex number

From Eq. (3) and (4):

$$P_h = \frac{1}{2}\,\mathrm{Re}[Z_{eL}]\left|\frac{-T_{em}}{Z_{et}+Z_{eL}}u\right|^2 \qquad (10)$$

3. PIEZOELECTRIC TRANSDUCER

The piezoelectric transducer model showed in this work was based is previous study of Preumont (2006) and Nakano et al (2007). This model is used to find the mechanical and electrical impedances and transduction coefficients for this transducer.

Figure (2) shows the piezoelectric transducers used in this work. In this figure E_f is Electric Field, T is Stress, l_t, b_t and t_t are length, width and thickness of transducer.

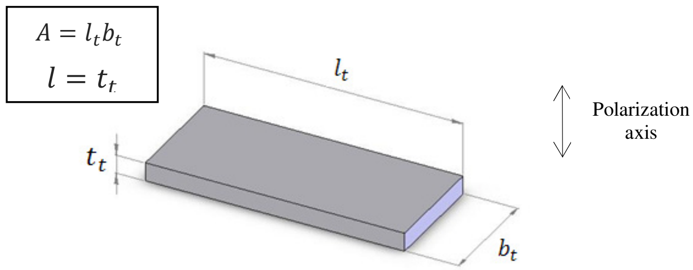


Figure 2. Piezoelectric transducer

A constitutive equation for a uniaxial piezoelectric transducer can be written as:

$$\begin{Bmatrix} Q \\ S \end{Bmatrix} = \begin{bmatrix} \varepsilon^T & d_{31} \\ d_{31} & s^E \end{bmatrix} \begin{Bmatrix} E_f \\ T \end{Bmatrix} \qquad (23)$$

where Q is Electric displacement, S is Strain, s^E is compliance of material under constant electric field, d_{31} is piezoelectric constant, ε^T is permittivity when the stress is constant.

Assuming a harmonic force the constitutive equation can be transformed to:

$$\begin{Bmatrix} f \\ v \end{Bmatrix} = \begin{bmatrix} \frac{1}{C_{mt}} & -D_{31} \\ -D_{31} & \frac{1}{C_{et}} \end{bmatrix} \begin{Bmatrix} q_{mt} \\ q_{et} \end{Bmatrix} \qquad (24)$$

where f is force, v is voltage, q_{mt} is mechanical deflection and q_{et} is electrical charge, D_{31} is the piezoelectric transducer constant, C_{mt} is mechanical compliance with open electrodes ($q_{et} = 0$) and C_{et} is electric capacitance of the transducer for a fixed geometry ($q_{mt} = 0$). Defining a coupling coefficient, κ, by:

$$\kappa = \frac{|d_{31}|}{\sqrt{s^E \varepsilon^T}} \qquad (25)$$

The others parameters of Eq. (24) are given by:

$$\frac{1}{C_{mt}} = \frac{K_a}{1-\kappa^2} \qquad (26)$$

$$\frac{1}{C_{et}} = \frac{1}{C\,(1-\kappa^2)} \qquad (27)$$

$$D_{31} = \frac{d_{31}\,K_a}{C\,(1-\kappa^2)} \qquad (28)$$

where $C = \frac{\varepsilon^T A}{l}$ (39)

and $K_a = \frac{A}{s^E\,l}$ (30)

In these equations C is the capacitance of the transducer with no external load ($f_t = 0$), K_a is the stiffness of the transducer with short-circuited electrodes ($v_t = 0$) and A is the cross section area.

Finally, the mechanical and electrical impedances and the transduction coefficients are given, respectively, by:

$$Z_{mt} = \frac{1}{j\omega\, C_{mt}}(1 + j\eta_{mt}) \tag{31}$$

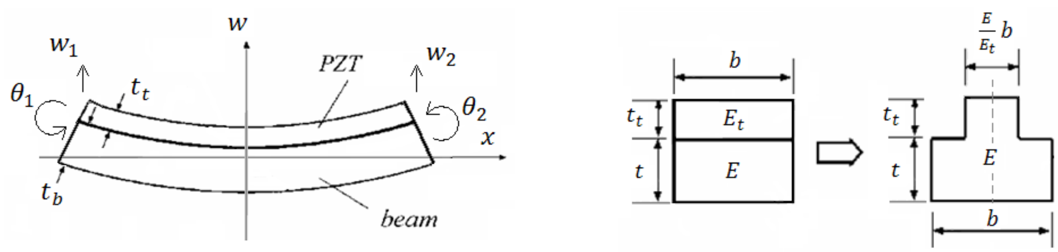
$$Z_{et} = \frac{1}{j\omega\, C_{et}}(1 + j\eta_{et}) \tag{32}$$

$$T_{me} = T_{em} = \frac{D_{31}}{j\omega} \tag{33}$$

where η_{mt} and η_{et} are the loss factors in the mechanical and electrical compliances.

4. MODEL OF FINITE PIEZOBEAM

The system investigated in this work was a Euler-Bernoulli beam with a piezoelectric patch bonded on a surface. For a finite beam with a piezoelectric element bounded is necessary to find the uniform equivalent beam for applying this theory. Fig. (3) shows the finite beam element and the cross-section before and after determines the equivalent beam.



(a) (b)

Figure 3. Piezobeam; (a) Finite element; (b) cross-section of beam and equivalent beam

The equation of motion for a uniform Euler-Bernoulli beam is derived by Linear Theory of Elasticity and for flexural vibration motion due to a transverse distributed force per unit length, $f_x(x,t)$, is given by Bishop and Johnson (1960).

$$EI\frac{\partial^4 w(x)}{\partial x^4} + \rho S\frac{\partial^2 w(x,t)}{\partial t^2} = f_x(x,t) \tag{34}$$

where E is the Young's modulus, I is the second moment of area, ρ is the density of the material, S is cross-sectional area and w is displacement. The solution for this equation in terms of trigonometric and hyperbolic functions is given by:

$$w(x) = A\sin kx + B\cos kx + C\sinh kx + D\cosh kx \tag{35}$$

where A, B, C and D are constants and k is the flexural wave number given by:

$$k = \left({}^{\rho S}/_{EI}\right)^{1/4}\,\omega^{1/2} \tag{36}$$

The mechanical Impedance of the beam depends of the boundary conditions and was determined using the methodology shown by Gardonio and Brennan (2004). The transfer impedance due a force excitation at x_i and a velocity response at x_j is given by the inverse of the mobility:

$$Z_{ij} = \frac{1}{Y_{ij}} \tag{37}$$

and $$Y_{ij} = j\omega\sum_{n=1}^{\infty}\frac{\psi_n(x_i)\,\psi_n(x_j)}{\rho Sl\,(\omega_n^2(1+j\eta)-\omega^2)} \tag{38}$$

where $\psi_n(x)$ is the nth natural mode, ω_n is the natural frequency for the nth natural mode, l is the length of the finite beam and η is the loss factor for the material of beam. The natural modes $\psi_n(x)$ can be obtained in many text books. Here we work with the approach developed by Gonçalves et al (2007).

5. NUMERICAL SIMULATION

Figure (4) shows the beam of interest and the Tab. (1) the properties of the system and the piezoelectric material. The input of the system was represented in the figure above too and it was a blocked force with unitary amplitude. The load connected was just a resistance representing a battery.

Figure 4. Piezobeam free-sliding with harmonic excitation

Table 1. Properties of the systems

Descriptions	Symbols	Values
Length of the beam and transducer	l e l_t	0.1 [m]
Width of the beam and transducer	b e b_t	0.02 [m]
Thickness of the transducer	t_t	0.00026 [m]
Thickness of the beam	t	0.002
Piezoelectric constant of material	d_{31}	-320 x10^{-12} [C/N]
Young's modulus of the transducer	$1/s^E$	62 [GPa]
Dielectric constant of the transducer	ε^T	3.36452x10^{-8} [F/m]
Electrical loss factor of the transducer	η_{et}	0.003
Mechanical loss factor of the transducer	η_{mt}	0.000056
Density of the transducer	ρ_t	7600 [m^3/kg]
Density of the beam	ρ	2700 [Ns/m]
Young's modulus of the beam	E	70 [GPa]
Mechanical loss factor of the beam	η	0.003

The input force in the transducer is different than input of the system and was found by this equation:

$$f_t = \frac{N\,I\,E\,\frac{\partial^2 w}{\partial x^2}}{q} \quad (39)$$

where q is the distance between the neutral axis and the upper surface of PZT and $N = {}^{E}/_{E_t}$.

Figure (5) shows the first fourth modes of vibration for the beam. The impedance of the beam was determined used the 200st first modes. The voltage and power for different resistances values can be seen in Figs. (6) and (7), respectively.

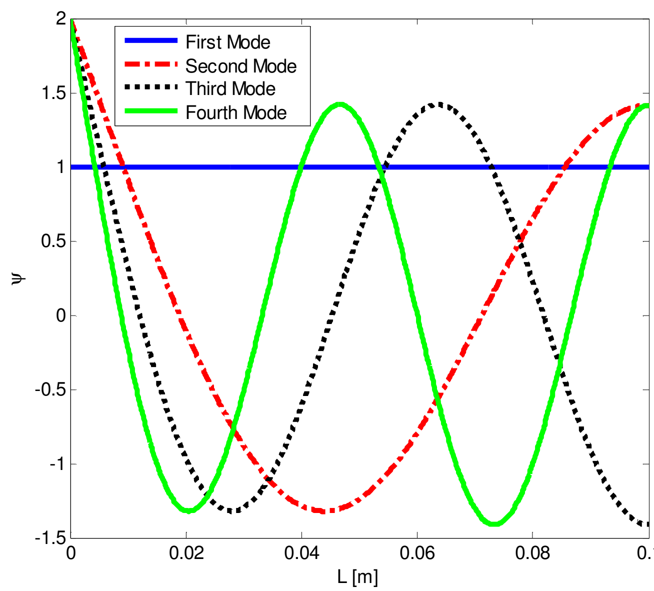


Figure 5. Modes Shape

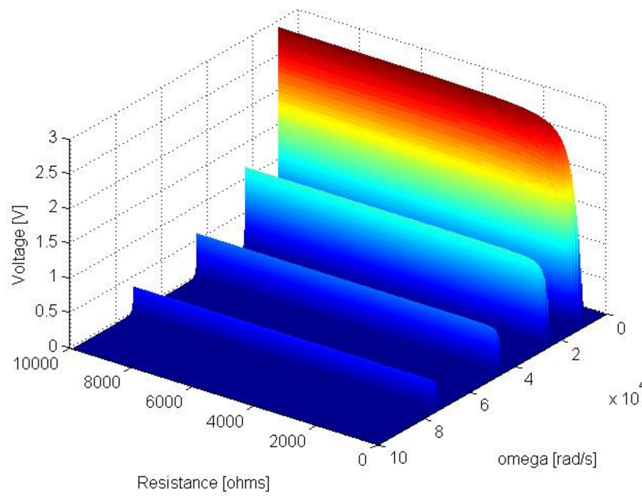


Figure 6. Amount of voltage as a function of frequency and load (resistance)

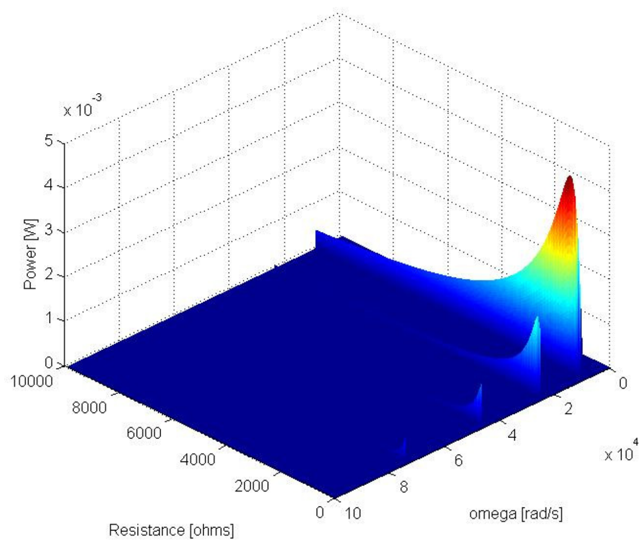


Figure 7. Amount of power as a function of frequency and load (resistance)

Observing Fig. (7) we can note the optimum region for the power as a function of the resistance and frequency. By examining this figure it can be seen that the frequency for the maximum power is the first natural frequency ($9.947 x 10^3$rad/s) disregarding the rigid body mode showed in the Fig. (5). How we can note more power is generated for a resistance of 500Ω, the power in this case reached around 4.482mW.

6. CONCLUSIONS

A study describing an electro-mechanical model for an energy harvesting device with a piezoelectric element has been undertaken to determine the optimum load conditions. When the load increase an open circuit rescuer the power becomes very small. For the opposite, when the load tends to zero, simulating a short circuit, the voltage and the power is smallest, tends to zero too. The optimization of the load for the system is extremity important to ensure efficiency of the harvesting system.

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