VI CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
VI NATIONAL CONGRESS OF MECHANICAL ENGINEERING
18 a 21 de agosto de 2010 – Campina Grande – Paraíba - Brasil
August 18 – 21, 2010 – Campina Grande – Paraíba – Brazil

ELECTROMECHANICAL RESPONSE OF 1-3 PIEZOELECTRIC FIBER COMPOSITES: A UNIT CELL APPROACH FOR NUMERICAL EVALUATION OF EFFECTIVE PROPERTIES

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Abstract. *Materials which exhibit piezoelectric behavior generate an electrical field in response to a mechanical deformation or alternatively undergo a mechanical deformation in response to an applied electrical field. This work presents the development of unit cell numerical models of 1-3 periodic composites, with piezoelectric fibers made of PZT embedded in a non-piezoelectric matrix. The common approach for estimating the macro-mechanical properties of 3D piezoelectric fiber composites is carried out by the unit cell approach, also called a representative volume element (RVE), which captures the major features of the underlying micro-structure. The main idea of this method consisting on evaluating a globally homogeneous medium equivalent to the original composite, where the strain energies stored in the two systems are approximately the same, with special emphasis placed on the formulation of suitable boundary conditions. The boundary conditions allow the simulation of all modes of the overall deformation arising from any arbitrary combination of mechanical and electrical loading. In the first instance, the unit cell is applied to prediction of the effective material coefficients of the transversely isotropic piezoelectric composite with circular cross section fibers. The numerical results(performed with the software ABAQUS®) are compared with methods reported in the literature and also to results previously published by the authors, in order to validate the proposed methodology. The second step of this work consist on applying the methodology to estimate the properties of composites with square cross section fibers (MFC – Macro Fiber Composites). The square arrangements of unidirectional piezoelectric fiber composites were used. These last models will be used to support further experimental tests of composites with active layers based in MFC fibers.*

Keywords*: Piezoelectric Fiber Composite, Active Fiber Composite (AFC), Macro Fiber Composite (MFC), Unit Cell*

1. INTRODUCTION

With the development of information industry and the appearance of smart materials and smart structures, it becomes more and more important to study the mechanical-electric coupled problems. Since piezoelectric composite materials are widely utilized in engineering, much research work has been done in the analysis and prediction of the effective electroelastic moduli of piezoelectric composite materials. Because, these analysis and prediction are based on mesomechanics, i.e., the problem consists on a piezoelectric inclusion in an infinite matrix. Piezoelectric materials have the property of converting electrical energy into mechanical energy, and vice versa. This capability makes possible the use of these smart materials as either sensors or actuators in several industrial fields, for example: noise and vibration control; acoustic speakers; precision position control and Systems of Health Monitoring (SHM). Thus, several rearch works have been developed, using analytical, numerical, and experimental or hybrid approaches. Chan and Unsworth (1989) as well as Smith and Auld (1991) were not capable of predicting the response to general loading, just in terms of loading cases applied by themselves. Dunn and Taya (1993) employed micro-mechanical theory coupled to the electro-elastic solution and studied ellipsoidal inclusions into an infinite piezoelectric medium. Rodriguez-Ramos et al. (2001) and Bravo-Castillero et al. (2001) applied the asymptotic homogenization to composites (piezoelectric or not) with fibers in square arrangement.

Finite element techniques using a representative volume element (RVE - unit cell) were employed by Gaudenzi (1997) to obtain the properties for piezo-composite patches applied on metallic plates. Poizat and Sester (1999) showed how to obtain two effective piezoelectric coefficients (longitudinal and transverse). Petterman and Suresh (2000) used unit cell models applied to 1-3 piezo-composites. Paradies and Melnykowycz (2007) studied the influence of interdigital electrodes over mechanical properties of PZT fibers.

After that, the research of Kar-Gupta and Venkatesh (2005, 2007a and 2007b) was about the influence of fiber distribution in 1-3 piezoelectric composites considering both, fiber and matrix, with piezoelectric properties. However, analytical techniques presented were not able to evaluate the influence of fiber distribution. Therefore, finite element analysis were presented and discussed by other researchers. Berger et al. (2005 and 2006) evaluated effective material properties of piezoelectric composites using analytical and numerical techniques. Azzouz et al. (2001) improved the properties of a finite element (three nodes aniso-parametric element) to take into account the modeling of AFC (active fiber composite) and MFC™ (macro fiber composite). Tan and Vu-Quoc (2005) presented a solid-shell element formulation to model active composite structures considering large deformation and displacements. The element has displacement and electrical degrees of freedom. The present authors ensured the efficiency and precision in the analysis of multilayer composite structures submitted to large deformation, including piezoelectric layers. Moreno et al. (2009 and 2010) investigated fibers with the same cross-sectional area (unimodal) and two different periodic fiber arrangement: the square arrangement and hexagonal arrangement. In the last paper, it was investigated the influence of applied boundary condition on the determination of effective material properties for active fiber composites. This paper, in fact, is a continuation of the last one. Thus, the FEM (Finite Element Method) is used in order to determine the effective properties of one ply made of unidirectional fibers from individual properties of the constituent materials (fiber and matrix) and composite characteristics. The procedure is based on the modeling of a RVE (unit cell), which is analyzed for different loading with different boundary conditions in order to evaluate the effective coefficients of transversely isotropic piezoelectric cylindrical and square fiber (1-3 periodic) composites. All analysis are carried out using the software ABAQUS®, where two different fiber geometry are studied: circular and square (MFC), both with square arrangement. Finally, RVE are chosen specifically for some analysis in order to make possible the use of suitable boundary conditions to represent the periodicity of the unit cell.

2. FORMULATION OF THE ELEMENT

2.1. Piezoelectricity and piezoelectric composites

The elastic and the dielectric responses are coupled in piezoelectric materials where the mechanical variables of stress and strain are related to each other as well as to the electric variables of electric field and electric displacement. The coupling between mechanical and electric fields is described by piezoelectric coefficients. This paper considers piezoelectric materials that respond linearly due to changes in the electric field, electric displacement, or mechanical stress and strain, thus, the constitutive response of piezoelectric materials in the linear elastic region is mathematically written in form of matrix as:

$$\begin{Bmatrix} \{T\} \\ \{D\} \end{Bmatrix} = \begin{bmatrix} [C] & [e] \\ [e]^t & -[\varepsilon] \end{bmatrix} \begin{Bmatrix} \{S\} \\ -\{E\} \end{Bmatrix} \tag{1}$$

where {T} denotes the stress tensor, {S} denotes the strain tensor, {E} denotes the electric potential field, {D} is the electrical displacement field, [C] denotes fourth-order elasticity tensor at constant electric field, [e] is the third-order piezoelectric coupling tensor and [ε] is the second-order dielectric tensor at constant strain field. The superscript t means the transpose of the matrix. For a transversely isotropic piezoelectric solid, the stiffness matrix, the piezoelectric matrix and the dielectric matrix have 11 independent coefficients and applying conditions for 1-3 piezoelectric composites, consequently, the constitutive Eq. (1) for the composite can be written in the matrix expanding form as:

$$\begin{Bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{12} \\ T_{23} \\ T_{31} \\ D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & 0 & 0 & e_{13} \\ C_{12} & C_{22} & C_{13} & 0 & 0 & 0 & 0 & 0 & e_{13} \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 & 0 & 0 & e_{33} \\ 0 & 0 & 0 & C_{66} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} & e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{15} & -\varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & 0 & -\varepsilon_{11} & 0 \\ e_{13} & e_{13} & e_{33} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{33} \end{bmatrix} \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{12} \\ S_{23} \\ S_{31} \\ -E_1 \\ -E_2 \\ -E_3 \end{Bmatrix} \tag{2}$$

The effective properties of the composites (subscript *eff*) can be defined by the average fields in the same form as Eq. (1), which can be written in a compact matrix form:

$$\begin{Bmatrix} \{\bar{T}\} \\ \{\bar{D}\} \end{Bmatrix} = \begin{bmatrix} [C]_{eff} & [e]_{eff} \\ [e]_{eff}^T & -[\varepsilon]_{eff} \end{bmatrix} \begin{Bmatrix} \{\bar{S}\} \\ -\{\bar{E}\} \end{Bmatrix} \tag{3}$$

It is assumed that the average mechanical and electrical properties of a unit cell are equal to the average properties of the particular composite. The average stresses, strains, electric fields and electrical displacements in the RVE are defined by:

$$\begin{aligned} \bar{T}_{ij} &= \frac{1}{V}\int T_{ij}\, dV & \qquad \bar{S}_{ij} &= \frac{1}{V}\int S_{ij}\, dV \\ \bar{D}_i &= \frac{1}{V}\int D_i\, dV & \qquad \bar{E}_i &= \frac{1}{V}\int E_i\, dV \end{aligned} \tag{4}$$

where V is the unit cell volume and the bar over the field component denotes the average value. Using the finite element approach, the average values can be post processed by:

$$\begin{aligned} \bar{T}_{ij} &= \frac{1}{V}\sum_{n=1}^{nel} T_{ij}^{(n)} V^{(n)} & \qquad \bar{S}_{ij} &= \frac{1}{V}\sum_{n=1}^{nel} S_{ij}^{(n)} V^{(n)} \\ \bar{D}_i &= \frac{1}{V}\sum_{n=1}^{nel} D_i^{(n)} V^{(n)} & \qquad \bar{E}_i &= \frac{1}{V}\sum_{n=1}^{nel} E_i^{(n)} V^{(n)} \end{aligned} \tag{5}$$

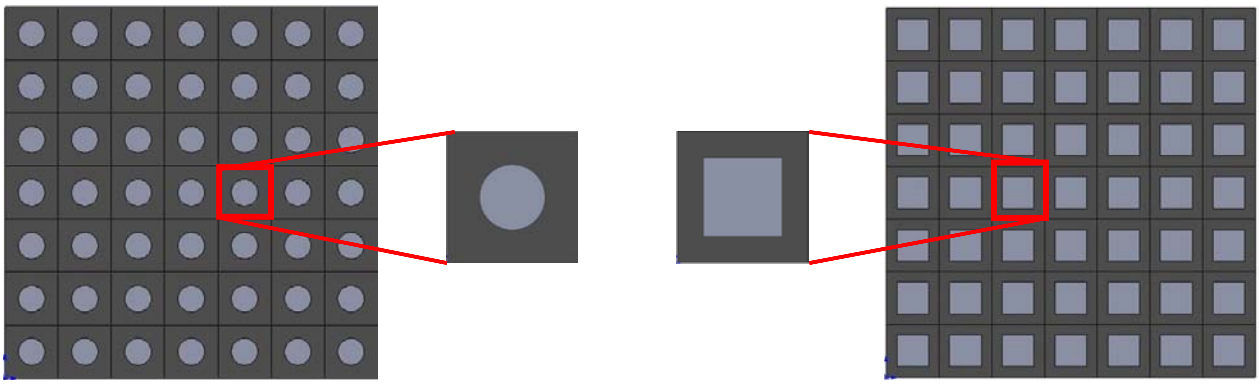
where V is the volume of the unit cell, *nel* is the number of elements modeling the unit cell, $V^{(n)}$ is the volume of the n-th element and $T^{(n)}$, $S^{(n)}$, $D^{(n)}$ and $E^{(n)}$ are the properties evaluated at the n-th element.

2.2. Macro fiber composite

The Macro Fiber Composite, developed at the NASA Langley Research Center, offers much higher flexibility and induced strain levels than monolithic piezoceramic. This increase of performance results from a laminated, piezoceramic fiber-reinforced construction and an interdigitated electrode pattern. The MFC is a laminate with a planar actuation device that employs rectangular cross-section, i.e., unidirectional piezoceramic fibers (PZT) are embedded in a thermosetting polymer matrix.

2.3. Representative volume element

Figure 1shows a composite with unidirectional fibers in square arrangement with correspondent unit cell (RVE), where the Fig. 1(a) shows a circular cross section and Fig. 1(b), a square cross section.



(a) (b)

Figure 1. Illustration indicating (a) circular cross section and (b) square cross section of fibers in a matrix as well as the corresponding unit cells used for the finite element modeling of 1-3 piezocomposites.

For example, regarding to a square arrangement, the unit cell is formed by the fiber at the center inner a cubic portion of matrix as showed in Fig. 2. The Fig. 2 also presents the designation given to the faces of RVE, adopted to help the explanation about loading and boundary conditions. According to its location, the faces of the RVE are designated as X+, X-, Y+, Y-, Z+ and Z-. In all analysis the fiber is continuous and orientated along the z-axis.

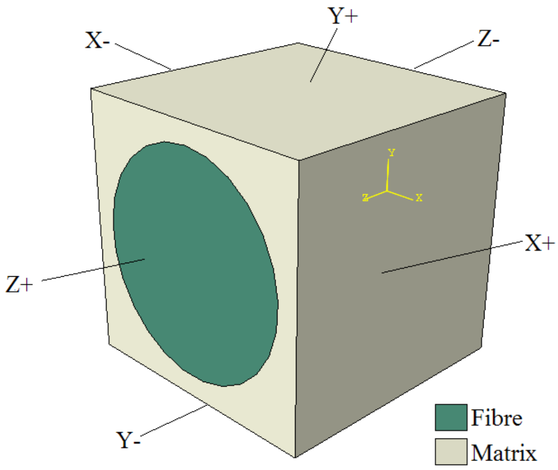


Figure 2. Square arrangement notations for surfaces of the unit cell

The microstructure shows locally a repetitive deformation that is modeled by the deformation of a microstructural RVE. It is assumed that the representative cell deforms in a repetitive way identical to its neighbors. In an RVE, the spatial periodicity conditions follow from compatibility demands with respect to the opposite edges. The demands enforce two adjacent RVEs to show identical deformations, while neither overlapping nor separation may occur. Considering two opposite point, A and B, and other set of opposite points, C and D, their displacements, u_i, respecting the periodicity of the RVE can be written in terms of the average unit cell strain (S_{ij}) as (Berger et al., 2005):

$$u_i^A = u_i^B + \overline{S}_{ij}\left(x_j^A - x_j^B\right) \tag{6}$$

$$u_i^C = u_i^D + \overline{S}_{ij}\left(x_j^C - x_j^D\right) \tag{7}$$

The same relations are also valid to the electrical degrees of freedom. Subtracting both equations and considering the average S_{ij} is the same in both equations, as well as, (x^A – x^B) is equal (x^C – x^D), the constraint equation can be rewritten as following, respectively to displacement and electrical potential degrees of freedom.

$$u_i^A - u_i^C = u_i^B - u_i^D \tag{8}$$

$$\varphi^A - \varphi^C = \varphi^B - \varphi^D \tag{9}$$

where φ is the electrical potential correspondent to the node indicated by the superscript index.

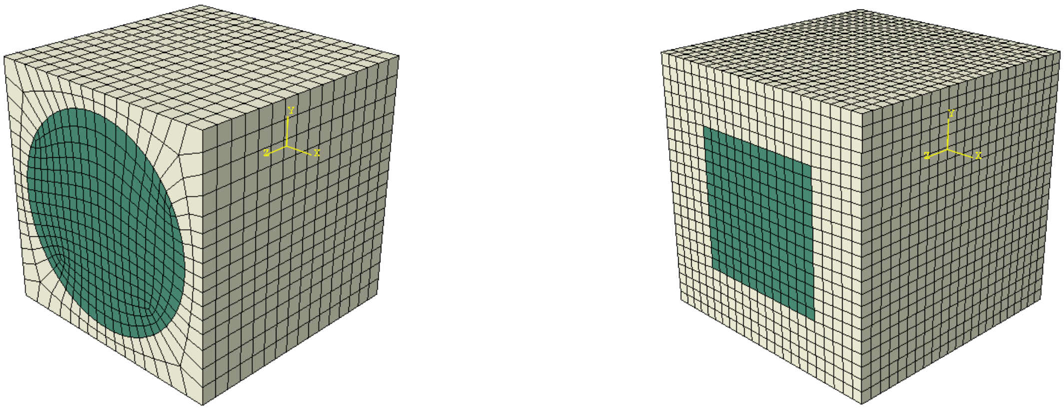
The last two equations represent a parallelism condition between the sides AC and BD. This condition must be applied for each pair of nodes in opposite sides of the unit cell (in vertical and horizontal directions) and must be repeated along the depth of the cell. In the analysis presented, it is not necessary to specify these conditions for all the cases, because sometimes the displacement and electrical boundary conditions already ensures this parallelism restriction. It is interesting to avoid the application of this condition, because there is a large number of equations that must be input. Automatic procedures to search opposite nodes and applying restrictions must be used. In the loading cases involving shear forces this procedure cannot be avoided, and the constraint equations have to be used at the sides submitted to the shear loading.

3. FINIT ELEMENT MODEL

3.1. Unit cell models for numerical homogenization methods

Several different unit cell configurations have been used according with the loading conditions and fiber arrangement. In this work, it was used square arrangement with circular and square cross section Fig. 3(a) and Fig. 3(b), respectively. The square arrangement was used for all loading cases.

All finite element calculations were made with the FE package ABAQUS™. Three-dimensional multi-field 20-node quadratic piezoelectric brick elements (C3D20E), with displacement degrees of freedom and an additional electric potential degree of freedom were used. These DOFs allow for fully coupled electromechanical analyses.



(a) (b)

Figure 3. FE Model (a) square arrangement with circular cross and (b) square arrangement with square cross section

3.2. Material properties

The elastic properties, piezoelectric constants and permittivity are given in 10^{10} Pa, $C.m^{-2}$, $nF.m^{-1}$, respectively. For the verification of the algorithm, it was considered a piezoceramic (PZT) fiber embedded in a non-piezoelectric material (epoxy – polymeric matrix). The material properties of the epoxy and PZT-5 were taken from Berger et al (2005) and showed in table 1. For the analysis presented, a fiber volume fraction of 55.5% was adopted.

Table 1. Material Properties for fiber and matrix and composite volume fraction

	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}	C_{66}	e_{13}	e_{15}	e_{33}	ε_{11}	ε_{33}
	$\times 10^{10}$ Pa						C / m^2			$\times 10^{-9}$ F / m	
Fiber	12.1	7.54	7.52	11.1	2.11	2.28	-5.4	12.3	15.8	8.11	7.35
Matrix	0.386	0.257	0.257	0.386	0.064	0.064	-	-	-	0.0797	0.0797

3.3. Boundary conditions for evaluation of the different effective coefficients

The prescribed boundary conditions will simplify the set of equations presented in Eq. (2) and it will be possible to evaluate the effective material properties. It can be seen that only six analyses are necessary to get all 11 effective coefficients. Most accurate results will be obtained when the loading is applied in fiber direction, here considered as z-direction.

1st Analysis: effective C_{13} and C_{33} calculation

- ✓ Normal displacements are set zero on surfaces X+, X-, Y+, Y- and Z- ($S_{11} = S_{22} = S_{12} = S_{23} = S_{31} = 0$);
- ✓ Positive displacements is prescribe on Z+ surface in Z direction; ($S_{33} \neq 0$);
- ✓ Electrical potential is set zero on all surfaces ({E}=0).

As just S_{33} is different of zero, first and third lines from Eq. (2) can be used to obtain C_{13} and C_{33}:

$$C_{13}^{eff} = \bar{T}_{11} / \bar{S}_{33} \tag{10}$$

$$C_{33}^{eff} = \bar{T}_{33} / \bar{S}_{33} \tag{11}$$

2nd Analysis: effective e_{13}, e_{33} and ε_{33} calculation

- ✓ Normal displacements are set zero on all surfaces ({S} = 0);
- ✓ Electrical potential is set zero on Z- surface;
- ✓ Electrical potential is applied to the Z+ surface;

So, from 1st, 3rd and last lines of Eq. (2), respectively the effective values of e_{13}, e_{33} and ε_{33} can be obtained:

$$e_{13}^{eff} = \bar{T}_{11} / \bar{E}_3 \tag{12}$$

$$e_{33}^{eff} = -\overline{T}_{33} / \overline{E}_3 \tag{13}$$

$$\varepsilon_{33}^{eff} = \overline{D}_3 / \overline{E}_3 \tag{14}$$

3rd Analysis: effective C_{11} and C_{12} calculation

- ✓ Normal displacements are set zero on surfaces X-, Y+, Y-, Z+ and Z- ($S_{22} = S_{12} = S_{23} = S_{31} = S_{33} = 0$);
- ✓ Positive displacements is prescribe on X+ surface in X direction; ($S_{11} \neq 0$);
- ✓ Electrical potential is set zero on all surfaces ({E}=0).

As just S_{11} is different of zero, first and second lines of Eq. (2) can be used to obtain C_{11} and C_{12}:

$$C_{11}^{eff} = \overline{T}_{11} / \overline{S}_{11} \tag{15}$$

$$C_{12}^{eff} = \overline{T}_{22} / \overline{S}_{11} \tag{16}$$

4th Analysis: effective ε_{11} calculation

- ✓ Normal displacements are set zero on all surfaces ({S} = 0);
- ✓ Electrical potential is set zero on X- surface;
- ✓ Electrical potential is applied to the X+ surface;

From the 7th line in Eq. (2):

$$\varepsilon_{11}^{eff} = \overline{D}_1 / \overline{E}_1 \tag{17}$$

5th Analysis: effective C_{66} calculation

- ✓ Z displacements are set zero on faces Z+ and Z-;
- ✓ All nodes from the center line perpendicular to the X-Y plane have the X and Y displacements set to zero;
- ✓ Two opposite nodes belonging to the fiber border are changed to cylindrical coordinate system and have their angular displacement constrained in order to avoid rigid body rotation;
- ✓ Electric potential is set to zero on all surfaces ({E} = 0);
- ✓ Shearing forces of same modulus and opposite orientation are applied on the surfaces Y+ and Y with X direction and on X+ and X- surfaces with Y direction, producing a pure X-Y shear state;
- ✓ The parallelism conditions deduced in Eqs. (8) and (9) must be applied between the pair of surfaces X+ and X-, and between Y+ and Y- surfaces.

These boundary conditions ensure the compatibility of the unit cell. As it was forced a pure shear state in X-Y plane, only the component S_{12} from {S} is different of zero. Therefore from the 4th line in Eq. (2):

$$C_{66}^{eff} = \overline{T}_{12} / \overline{S}_{12} \tag{18}$$

6th Analysis: effective e15 and C44 calculation

- ✓ X displacements are set as zero on faces X+ and X-;
- ✓ All nodes from the center line perpendicular to the Y-Z plane have the Y and Z displacements set to zero;
- ✓ Two opposite nodes belonging to the center of the fiber in faces Z+ and Z- (Figure 3(d)) are constrained in Y direction in order to avoid rigid body rotation;
- ✓ Electric potential is set to zero on X-, X+, Y- and Y+ surfaces;
- ✓ Shearing forces are applied in the surfaces Y+ and Y- with Z direction, same modulus and opposite orientation and on Z+ and Z- surfaces with Y orientation, producing a pure Y-Z shear state;
- ✓ The parallelism conditions deduced in Eqs. (8) and (9) must be applied between the pair of surfaces Z+ and Z-, and between Y+ and Y- surfaces.

These boundary conditions ensure the compatibility of the unit cell. Effective ε_{11} was obtained from Eq. (17). Effective values for C44 and e15 can be obtained from the 5th and 8th lines in Eq. (2):

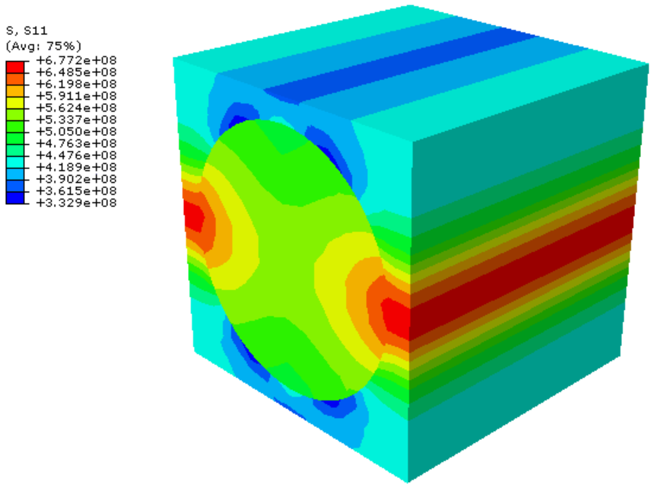
$$e_{15}^{eff} = \left(-\overline{E}_2 \cdot \varepsilon_{11} + \overline{D}_2\right) / \overline{S}_{23} \tag{19}$$

$$C_{44}^{eff} = \left(\overline{T}_{23} + \overline{E}_2 \cdot e_{15}^{eff}\right) / \overline{S}_{23} \tag{20}$$

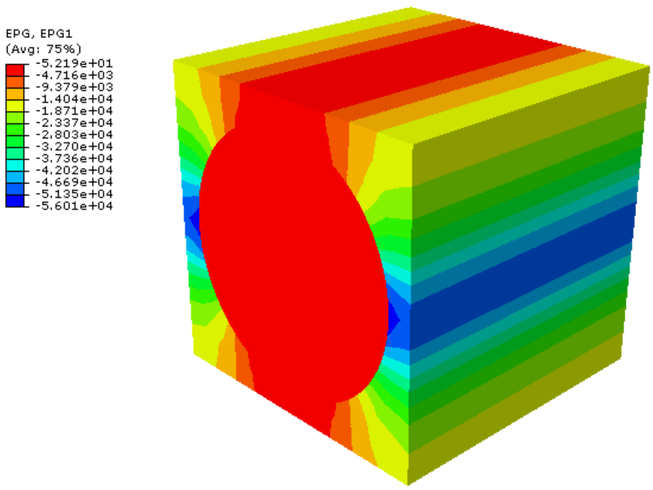
Finallly, a generalized procedure has been developed for calculating all effective coefficients for one volume fraction on the basis of the ABAQUS Python Language. This procedure reduces the manual work and save time, as well as, it can be used as a template for evaluating the effective coefficients of piezoelectric fiber composites with an arbitrary volume fraction of fibers.

4. RESULTS AND DISCUSION

According to the procedure discussed above, some results for the properties involved in the effective coefficients calculation are presented in Figures 4 and 5 for fibers with square arrangement and circular and square cross section, respectively.



(a)



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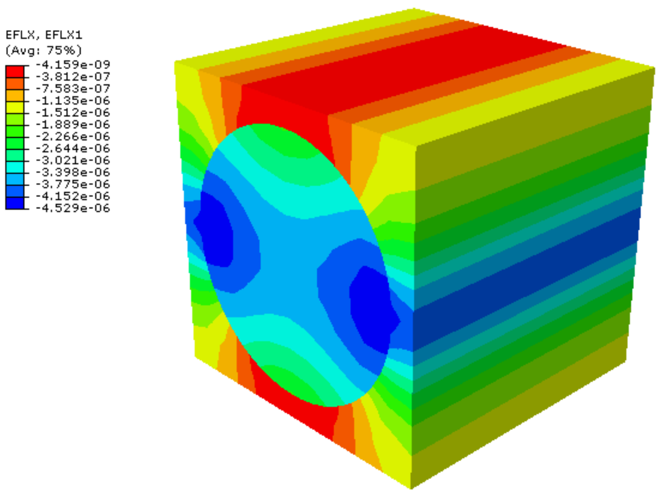
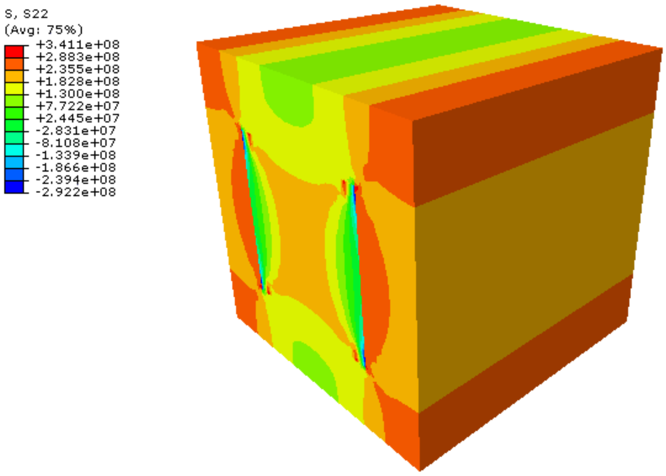
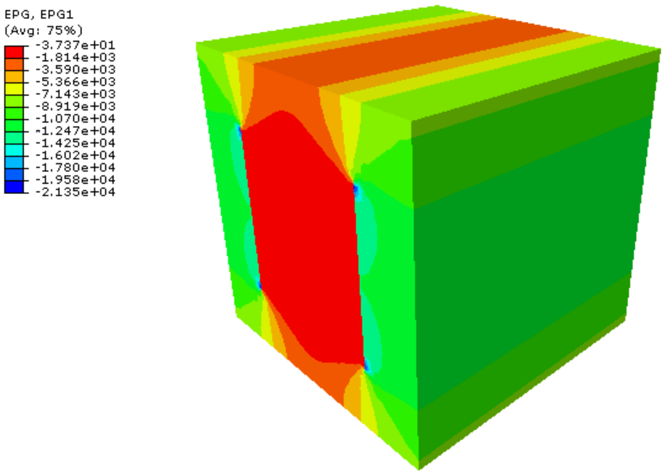


(c)

Figure 4. Circular cross section for square arrangement (a) First analysis T_{11} (b) Fourth analysis E_1 and (c) Fourth analysis D_1



(a)



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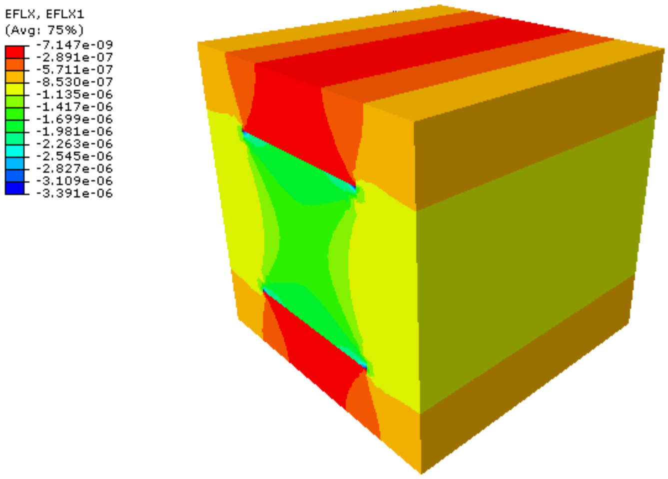

(c)

Figure 5. Square cross section for square arrangement (MFC) (a) Third analysisT_{22} (b) Fourth analysis E_1 and (c) Fourth analysis D_1

As commented before, all 11 effective coefficients have been calculated using FEM for one fiber volume fraction. In the first step, for the verification of the methodology, it was considered a piezoceramic (PZT) fiber with circular geometry embedded in a non-piezoelectric material (epoxy). From Eq. (5) and using FEM, it was calculated the average values. After that, Eqs. (10) to (20) were used to obtain the effectives coefficients. Thus, the results for a piezoceramic (PZT) fiber with circular geometry embedded in a non-piezoelectric material (epoxy) obtained in this work are compared to analytical and numerical results of the literature. After the verification of the methodology consistency, it was investigated the effective coefficients for a piezoceramic (PZT) fiber with square geometry embedded in a non-piezoelectric material (epoxy) in order to simulate MFC developed by NASA.

The results are summarized in the Table 2. The column designed as (1) refer to the result obtained by Berger et al. (2005), estimated from graphs presented in their papers. The column designed as (2) refer to the result obtained by Moreno et al (2009). The column (1) refers to analytical results obtained by asymptotic homogenization for circular cross section. The columns (2) summarize the coefficients obtained by the analysis using FEM for circular cross section obtained by Moreno et al (2009). The columns (3) and (4) summarize the coefficients obtained by the analysis procedure presented in this work. They refer to square fiber arrangement with circular and square cross section, respectively. The values obtained to the effective coefficients are compared using analytical and numerical results as reference for validation of the routines, and the diffence between results (Δ) is presented in the three last columns of Table 2. The first Δ is taken between the analytical and numerical results presented by Berger et al. (2005) and Moreno et. al. (2009), the second Δ is between analytical and present work (circular geometry), and the third Δ is between the numerical results from Moreno et al. (2009) and present work (circular geometry).

Table 2. Material Properties for fiber and matrix and composite volume fraction

Coefficient	Units	(1)	(2)	(3)	(4)	Δ_1 [%]	Δ_2 [%]	Δ_3 [%]
C_{11}	$\times 10^{10}$ Pa	0.95	1.088	1.085	0.640	14.5	14.2	0.3
C_{12}		0.56	0.465	0.467	0.336	16.9	16.6	0.3
C_{13}		0.60	0.604	0.604	0.384	0.7	0.7	0.0
C_{33}		3.50	3.525	3.513	2.071	0.7	0.4	0.3
C_{44}		0.22	0.215	0.190	0.129	2.3	13.6	11.3
C_{66}		0.20	0.154	0.151	0.0946	23.0	24.5	1.5
e_{13}	C / m^2	-0.26	-0.258	-0.258	-0.095	0.7	0.7	0.0
e_{15}		0.02	0.0241	0.0245	0.0167	20.5	22.5	2.0
e_{33}		11.0	10.86	10.864	6.067	1.3	1.2	0.1
ε_{11}	$\times 10^{-9}$ F / m	0.28	0.284	0.287	0.156	1.4	2.5	1.1
ε_{33}		4.20	4.270	4.270	2.408	1.6	1.6	0.0

Δ_1 - Comparing (1) and (2): Berger et al. (2005) x Moreno et al (2009)
Δ_2 - Comparing (1) and (3): Berger et al. (2005) x Present Work
Δ_3 - Comparing (2) and (3): Moreno et al (2009) x Present Work

5. CONCLUSION

A numerical approach (RVE) for predicting the homogenized properties of piezoelectric fiber composites has been presented. The numerical approach is based on the FEM analyses. Longitudinal and transversal elastic and piezoelectric effective coefficients for a piezoceramic (PZT) fiber with circular geometry embedded in a non-piezoelectric material (epoxy) have been calculated with the finite element numerical model and compared to analytical solutions based on the asymptotic homogenization method. The presented models showed, in general, a good agreement with analytical results obtained by the asymptotic homogenization. Numerical results presented here are very similar to results reported by Berger et al. (2005) and Moreno et al. (2009) for the square arrangement of fibers with circular geometry. Therefore, the proposed methodology to determine effective coefficients for PZT is very efficient. However, the boundary conditions have been applied with criterion for analyzing 1-3 piezoelectric composites in the both cases, circular and square cross section for square arrangement in order to avoid errors during numerical analyses. Thus, the determination of effective coefficients for MFC need to be checked using experimental results and/or analytical models.

6. ACKNOWLEDGEMENTS

The authors are thankful to FAPESP, as well as, CNPq and FAPEMIG for partially funding the present research work through the INCT-EIE. The authors also would like to thank Prof. Reginaldo Teixeira Coelho (EESC-USP) for the ABAQUS license.

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