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NUMERICAL INVESTIGATION OF STRESS DISTRIBUTION IN NET-SECTION OF A CENTER-CRACKED METALLIC SHEET

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Abstract: *This works deal with a stress distribution investigation in metallic sheets with strong stress concentrators. The numerical solution of boundary problems of the Theory of Elasticity was performed, through the Finite Element Method, in structures simulation software and follows the sequence: definition of material mechanical properties; creation of the finite element model, local constraints, external forces; simulation and post-analysis stress values.At second step, sheets with different ratio (length/width) were studied. The obtained results show that ratio greater than 3 is necessary to neglect the effect of boundary conditions on stress diagram in net-section. At next step, were studied sheets with fixed size and different lengths of central crack under uniform remote tension. The results were compared with asymptotic solution of linear fracture mechanics, in order to verify the zone extension of strong stress concentration.Then were performed the numerical analysis of plastic zone extension based on Irvin-type approach. Considering as elastic-plastic ideal, the difference between load flow in the zone where elastic solution results in values higher than yield stress, and load flow for real elastic-plastic material, is redistributed to former elastic zone. Due to effective load increase, stress in elastic zone increases too, needing a correction of initial estimation of plastic zone size, through comparison of stress diagram for elastic material and the yield stress of material. The implementation of this approach to numerical FEM analysis of stress distribution requires special iterative algorithm, aimed at determination of effective coefficient of stress increase in elastic zone, due to presence of plastic strain at crack tip. The obtained results show strong effect of both crack length to sheet width ratio and remote tension stress to yield stress ratio on plastic zone extension. The possible analytical approximation of numerical results is discussed.*

Keywords*: stress concentrators; cracks; finite elements method; fracture resistance; plastic zone.*

1. INTRODUCTION

In a simplified mode, in aeronautical structures a significant portion of panels are basically subject to cyclical strength, from pressurization of the aircraft. Searching simulate these conditions in panels with cracks (Figure 1), in experimental studies of crack propagation are used standardized samples, under tension normal to the crack surface, for example, the plate with central crack (Figure 2). The information of stress distribution on whole future trajectory of crack becomes important for applying the concept of cumulative damage in order to improve simulation methods of crack propagation (Anderson, 1995, Lee, 2005).

This study aims to investigate stress distribution in plates with strong stress concentrators (cracks), using the Finite Element Method, for solving the boundary problem of the linear theory of elasticity, as well as numerical methods of integration and iterative determination of zeros of real functions, to calculate auxiliary parameters.

The first objective of numerical research was to determine proportions, between length and width of the plate, that make negligible the effect of boundary conditions on the stress distribution in cross section with a crack. Then, the diagram of stress distribution obtained by Finite Element Method was checked in respect the tension flow in the minimum section. Checking the validity of terms of global boundary conditions and other properties of obtained diagrams, was prepared an analytical approach to stress distribution in the minimum section, simple and accurate, which reached the second objective.

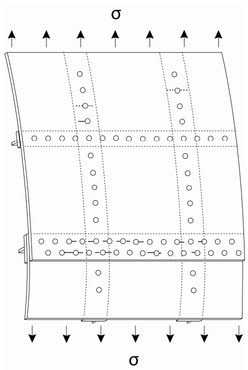

Figure 1: Aircraft panels subjected to mechanical stress and the presence of cracks

However, the direct application of this approach in simulation of crack propagation process would be restricted to cases of very low loads, when the plastic flow of material near the crack tip is negligible. These cases do not present great practical use, so the third goal of this work is the systematic study of the extension of plastic zone in the nearby of the cracks, due to the relative level of loading and relative length of crack, based on numerical results of the Theory of Elasticity problems.

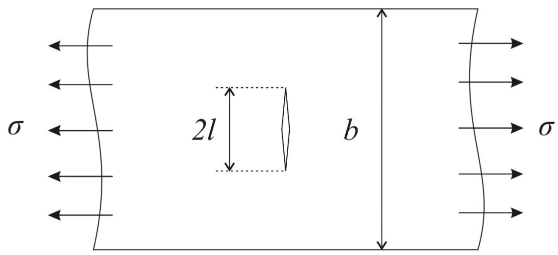

Figure 2: Panel with central crack

1.1. Theory of Elasticity and MEF

Considering the conditions in thin plate under tension in the plan, i.e. plane stress, the boundary problem of the theory of linear elasticity includes a system of eight partial differential equations for components of stress, strain and displacement, with respectives boundary conditions, represented by movement restrictions and external loads. The analytical solution of this class of problems is possible only in cases of relatively simple geometry (Timoshenko, 1951). In particular, in the presence of strong stress concentrators, these solutions are obtained on the basis of asymptotic methods ("small parameter") and are limited to the immediate vicinity of the concentrator (Pastoukhov, 1995). Therefore, to study the stress distribution across the section of the plate with a crack, it is necessary to apply numerical methods.

The main tool for numerical stress analysis, strain and displacement in elastic solid is the Finite Element Method (FEM) (Hartmann, 2007), widely used in development activities in mechanical design.

1.2. Fracture Mechanics

Approaching already the fundamentals of fracture mechanics, dealing with the stress distribution in bodies with cracks, the model is characterized by type of load, among the three primary factors, in search of stress intensity K_I, K_{II}, K_{III} dependents of external load, geometry of the body and the crack (Lee, 2005).

Considering solicitation cases of type I (normal traction to the crack), the asymptotic solution for stress distribution near the crack tip, considering only the minimum cross-section, has the following form (Anderson, 1995; Pastoukhov, 1995).

$$\sigma_{assint}(x) = \frac{K_I}{\sqrt{2\pi x}} \tag{1}$$

The stress intensity factor K_I is represented as:

$$K_I = \sigma \cdot \sqrt{\pi l} \cdot Y(\lambda) \tag{2}$$

Where:
σ_0: remote uniform stress;
l: crack length;
b: plate width;
$\lambda = l/b$.

In the case of symmetrical solids, as the plate with central crack, the parameter *l* corresponds to half of the crack (left or right), while the parameter *b* represents the full width of the plate.

For dimensionless geometric function *Y* there various approaches in terms of λ, obtained from discrete data, calculated numerically. In the case of central crack, we use the polynomial approximation:

$$Y(\lambda) = \frac{\left(1,77 + 0,454\lambda - 2,04\lambda^2 + 21,6\lambda^3\right)}{\sqrt{\pi}} \tag{3}$$

The parameter K_I, whose critical value is determined experimentally considering the thickness effect, is used in integrity criterion of the mechanical parts with cracks under static load (Lee, 2005; Pastoukhov, 1995).

The expression analysis shows that the stress tends to infinity when *x* distance of the cracks tends to zero. This means that metallic materials, whose mechanical behavior presents plastic flow when reach a critical stress level (elastic limit), there is a plastic zone surrounding the crack tip. The extent of this zone along the line of crack can be evaluated by equating the function on the right of equation (1) to the yield stress σ_e, resulting in:

$$R_{p_1} = \frac{K_I^{\ 2}}{2\pi\sigma_e^{\ 2}} \tag{4}$$

The first approach is imprecise because, when stress in the plastic zone equals to the yield stress (considering ideal elastic-plastic mechanical behavior), the tension flow in this area is lower than that determined by the solution of linear elasticity. Thus, a large part of the traction force is redistributed to the elastic zone, increasing the tension in this zone, so the yield will be achieved at some point farther from the crack tip than R_p.

Classical corrections of plastic zone extension, developed already half-century ago within in the same local approach, results in constant factor. For example, factor 2 obtained by Irwin (Irvin, 1957, 1960, Lee, 2005), or values close (Leonov-Panasyuk, 1959; Dugdale, 1960). These estimates can be used, due to safety factors, under certain conditions, when the main concern is the failure by generalized yield of cross section. However, the margins of applicability are subject to verification, whereas the degree of loading in relation to the yield stress strongly affects the redistribution of tension flow between the plastic and elastic zones. Another important parameter that should be considered is the geometric nature of crack (length) in relation to the plate width.

For use in crack propagation models, the stress diagrams should be investigated in the whole section with a crack, between the crack tip and the edge of the plate, which includes equations development of the extent of plastic zone in the parameters cited, rectifying thus, the classical estimates.

2. METHODOLOGY

2.1. Numerical solution of boundary problems

For the investigation of a problem through the finite element method, should be followed certain steps. First it's created a geometric model to be analyzed according to the particularities of the "pre-solvers" of FEM codes used (NASTRAN, ANSYS, COSMOS ... - (Hartmann, 2007)). After that, the characteristics of mechanical behavior of the material are defined. Then, for each part of the solid, appropriate parameters of elements are chosen and the discretization is performed. Then are applied loads and movement restrictions on the nodes or regions suitable to a real problem. From this stage, it has formulated the problem completely, and resolved according to the characteristics of the FEM algorithm used ("solver"). The results of resolution will be available for post-analysis, in numerical and graphical form.

The geometry, movement restrictions and loading for plates under traction normal to the surface of the central crack are shown in Figure 3. The research results will be executed initially on the basis of *h/b* and *λ=l/b.*

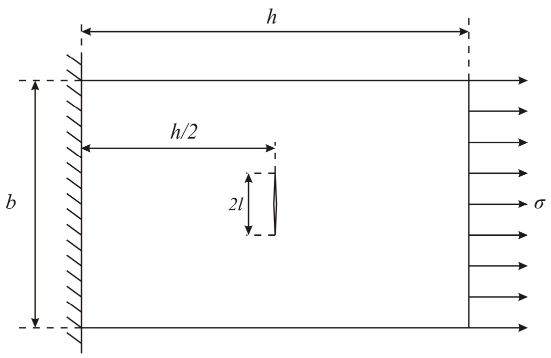

Figure 3: Schematic model of the plate with central crack under tension orthogonal to the crack

For the material properties, was adopted the equivalent to the properties of aeronautical aluminum Al 2024-T3 (*E*: 70GPa, ν: 0.3), and in the plate loading was applied reference tension σ_0 = 100MPa, i.e. "100%".

2.2. Computational model

For plate models analysis, was used structural analysis software based on finite element method, which incorporates the pre and post-processor in a single environment, known as LISA, developed by LISA Finite Elements Technologies, Ontario, Canada.

The conditions of mechanical solicitations and movement restrictions which the panel is subjected were created in a model of standard rectangular plate with homogeneous symmetric discretization. The creation of distributed load was done through the distribution of nodal loads, in order to obtain the same loading for each element, resulting in double amount of force to the nodes of the model, with respect to two nodes of each side of the plate width *b* (Figure 4). At the upper edge and in the nodes with loads, the movement was restricted to the orthogonal axes of force application, while the lower edge, the restriction was in all degrees of freedom.

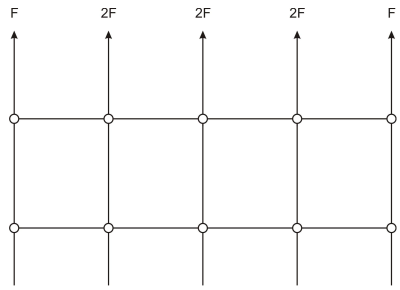

Figure 4: Nodal forces for the models of plates

After creating the stress concentrator in the symmetry axis of the plate, it has created a symmetrical mesh around the crack tips, capable of further local refinement. Then, the model was subjected to routine "*solver*" for resolution of boundary problem of linear elasticity. With tension value at each node in mesh collinear to the axis of crack, starting from the side of the plate to the crack tip, can be made a study of results in stretch of sharp increase of tension, continuing the mesh refinement until convergence is achieved.

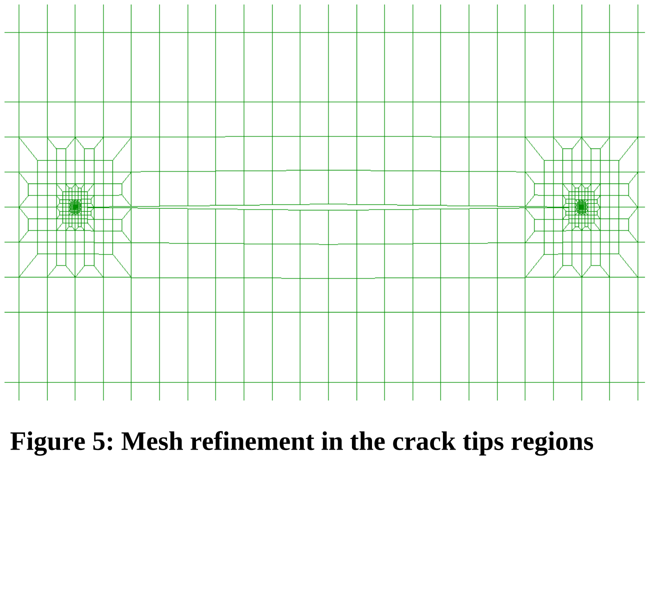

Figure 5: Mesh refinement in the crack tips regions

At this stage, with an initial model determinate, began the study of proportions effect of the plate, *h/b*. Then, for a standard ratio, calculations were performed for various values of geometrical parameter $\lambda=l/b$, in the range of practical importance.

2.3. Stress distribution analysis

The results analysis, obtained for normal-tension diagram in cross section with a crack, was focused on the following issues:

- Comparison with the asymptotic distribution according to equation (1), in order to determine the validity area of the same, in case of finite plate width;
- Stress distribution away from the crack, near the side edges;
- Transient behavior in the intermediate region;
- Tension flow, represented by integration of normal stress diagram.

Due to the symmetry of the problem and their results, these tests were performed only for one half of the plate (right).

2.3.1. Tension flow study

In integration process, was necessary to use the integrable singularity, due to the theoretical infinite values in crack tip, according to the asymptotic solution.

In stress analysis, was obtained the closest point (x_a) between the curves with a criterion of 5% difference between the values of two curves, which was observed very near the crack tip. Performing this, the curve was divides into two regions that have been integrated in different ways (Eq.5).

$$A=\left[A_1\right]+\left[In(x)\right]_{x_a}^{b/2} \tag{5}$$

Due the good correspondence between numerical results and asymptotic solution (1), the first region (A_1), starting at the crack tip and ending in x_a, was integrated analytically based on equation (1) to stress distribution, depending on the factor stress-intensity:

$$A_1=\int_0^{x_a}\frac{K_I}{\sqrt{2\pi x}}dx=\frac{K_I}{\sqrt{2\pi}}2\sqrt{x_a} \tag{6}$$

In the second region, beginning in the closest point (x_a) and ending in the lateral edge, due to the distance between the two curves the study of forces was made by numerical integration of the curve, represented by discrete numerical results. This integration was done following two known methods for approximate calculation of definite integral: Simpson 1/3, in stretches with an even number of equal steps, and the Trapezoidal Rule, in other stretches. Both methods have reliable estimates of error (Ruggiero, 1996). The stretches integrated by Simpson and Trapezoidal simple rules, have respectively the following forms:

$$I_S=\frac{h}{3}\left[\sigma(x_0)+4\sigma(x_1)+\sigma(x_2)\right] \tag{7}$$

$$I_T=\frac{h}{2}\left[\sigma(x_0)+\sigma(x_1)\right] \tag{8}$$

2.4. Analytical approach of stress diagram

Since the study of tension flow (as 2.3) confirmed the maintenance of overall equilibrium in terms of normal stress, the same condition was observed in development of an analytical approach to the diagram of normal tension in cross section of the crack. Other important observations were the aforementioned correlation between the numerical results with the asymptotic solution σ_{assint} (eq. 1) near the crack tip, and the tendency to value of remote tension σ_0 near the side edges. In all studied cases, the stress diagram showed characteristic monotonic (with first derivative in relation to the *x*-coordinate, negative), with drop ever lower farther from the crack (second derivative positive).

Therefore, was adopted the following type of approach. First, it separates the areas of strongest trends: the asymptotic, near the crack, and the remote tension, out of this one. As a point of separation, is adopted the point R_a, where asymptotic value equals the remote tension, $\sigma_{assint}(R_a)=\sigma_0$. Using equation (1):

$$R_a=\frac{K_I^{\,2}}{2\pi\,\sigma_0^{\,2}} \tag{9}$$

In the stretch $0 \le x \le R_a$, is adopted the asymptotic correction in the form of a linear function of the same, with coefficients ranging from 0 at the crack tip and a maximum value c_0 at the point $x=R_a$.

In the stretch $R_a \le x \le b/2-l$, remote tension σ_0 is considered as a basis and the fix also follows a linear function of the asymptotic $\sigma_{assint}(R_a)$. In this case, the coefficient ranges from 0 at the edge of plate ($x=b/2-l$) and a maximum value at the point $x=R_a$. Because of continuity of the diagram, and observing the condition $\sigma_{assint}(R_a)= \sigma_0$, this maximum value is also equal to c_0.

Thus, the approach of stress diagram will be searched in the form:

$$\sigma_d(x) = \begin{cases} \sigma_{assint}(x) \cdot \left(\dfrac{x}{R_a} c_0 + 1 \right), & 0 \le x \le R_a \\ \sigma_0 + \sigma_{assint}(x) \cdot c_0 \dfrac{x_* - x}{x_* - R_a}, & R_a \le x \le x_* \end{cases} \tag{10}$$

Where c_0 is a tuning parameter, determined from condition of global equilibrium:

$$\int_0^{b/2-a} \sigma_d(x) = \sigma_0 \frac{b}{2} \tag{11}$$

The value of this parameter only depends on the geometrical parameter $\lambda=l/b$.

2.5. Study of plastic zone extension

The study of plastic zone extension, near the crack tip, was based on the analytical approach of the stress diagrams obtained from numerical solutions of the problem of linear elasticity (10). With the normal tension equal to the yield limit σ_e within the plastic zone ($0 \le x \le R_p$), out of this area the redistribution of tension flow should be considered only in the asymptotic correction, since the trend to value in plate edge is not affected by local phenomena of crack area. Therefore, within the zone of stress concentration ($R_a \le x \le R_p$), the first part of equation (1) will increase proportionally ($p=cte.$ factor), which reflects the increase in external effective load in this area, due to relief in the plastic zone. Thus, there will also increase tension at the point $x=R_a$, which implies a corrective term to the correction equation for stretch ($R_p \le x \le x_a$), which represents the asymptotic contribution. We will adopt this fix also in proportional form, with coefficient $p_1=cte$, connected with p through the continuity condition at the point $x=R_a$. The approach of the stress diagram for elastic-plastic material takes the form:

$$\sigma_{dp}(x) = \begin{cases} \sigma_e, \quad 0 \le x \le R_p \\ \sigma_{assint}(x) \left(\dfrac{x}{R_a} c_0 + 1 \right) p, \quad R_p \le x \le R_a \\ \sigma_0 + \sigma_{assint}(x) \cdot c_0 \dfrac{x_* - x}{x_* - R_a} p1, \quad R_a \le x \le x_* \end{cases} \tag{12}$$

The values of p and p_1 factors are determined from global equilibrium condition, similar to equation (11), along with the continuity condition at the point $x=R_a$. The analytical integration results in an algebraic equation with no analytically tractable form, therefore, will be needed to perform a numerical iterative procedure for each combination of values of λ and s.

3. RESULTS AND ANALYSIS

3.1. Results of numerical solution of boundary problems

In the effect study of plate proportions, it was found that the ratio $h/b=3$ is sufficient to disregard the effect of boundary conditions on the stress distribution in cross section with crack.

The results for this nodal distribution, obtained by checking convergence as a function of the mesh refinement, for some values of geometrical parameter $\lambda=l/b$, are presented in Table 1, compared with values from the asymptotic solution of equation (1).

Table 1: Nodal values to geometric parameters

λ=0,02				λ=0,05				λ=0,10				λ=0,15			
σ	σ_assint	x	Relative Error	σ	σ_assint	x	Relative Error	σ	σ_assint	x	Relative Error	σ	σ_assint	x	Relative Error
1935,174	∞	0	∞	1944,447	∞	0	∞	1979,401	∞	0	∞	3544,330	∞	0	∞
1067,196	1100,538	1,6624E-05	-3,12%	1072,235	1107,879	1,662E-05	-3,32%	1091,492	1123,966	1,663E-05	-2,98%	1951,057	1999,075	1,662E-05	-2,46%
783,623	778,734	3,3203E-05	0,62%	787,310	783,928	3,32E-05	0,43%	801,496	795,482	3,32E-05	0,75%	1429,821	1414,534	3,32E-05	1,07%
636,882	636,028	4,9774E-05	0,13%	639,855	640,247	4,978E-05	-0,06%	651,375	649,639	4,978E-05	0,27%	1159,657	1155,316	4,977E-05	0,37%
550,051	550,838	6,636E-05	-0,14%	552,657	554,528	6,636E-05	-0,34%	562,615	562,660	6,636E-05	-0,01%	999,458	1000,571	6,636E-05	-0,11%
477,874	450,051	9,9411E-05	5,82%	480,137	453,061	9,941E-05	5,64%	488,780	459,710	9,941E-05	5,95%	865,769	817,496	9,941E-05	5,58%
386,039	367,559	0,00014904	4,79%	387,859	370,011	0,000149	4,60%	394,848	375,446	0,00014904	4,91%	695,090	667,653	0,000149	3,95%
332,263	318,356	0,00019867	4,19%	333,830	320,480	0,0001987	4,00%	339,848	325,187	0,00019867	4,31%	594,629	578,279	0,0001987	2,75%
289,362	260,100	0,00029763	10,11%	290,728	261,835	0,0002976	9,94%	295,971	265,681	0,00029763	10,23%	513,605	472,459	0,0002976	8,01%
236,483	212,422	0,00044623	10,17%	237,600	213,840	0,0004462	10,00%	241,890	216,983	0,00044622	10,30%	412,690	385,855	0,0004462	6,50%
206,370	183,987	0,00059482	10,85%	207,345	185,214	0,0005948	10,67%	211,094	187,935	0,00059482	10,97%	354,343	334,203	0,0005948	5,68%
183,198	150,318	0,00089111	17,95%	184,064	151,321	0,0008911	17,79%	187,397	153,544	0,00089111	18,06%	308,138	273,046	0,0008911	11,39%
155,646	122,765	0,001336	21,13%	156,382	123,584	0,001336	20,97%	159,224	125,400	0,001336	21,24%	251,554	222,997	0,001336	11,35%
140,631	106,331	0,00178088	24,39%	141,297	107,040	0,0017809	24,24%	143,875	108,613	0,00178089	24,51%	219,440	193,146	0,0017809	11,98%
129,784	86,873	0,002668	33,06%	130,399	87,453	0,002668	32,93%	132,783	88,737	0,002668	33,17%	194,713	157,801	0,002668	18,96%
117,710	70,949	0,004	39,73%	118,269	71,423	0,004	39,61%	120,453	72,472	0,004	39,83%	165,160	128,876	0,004	21,97%
111,794	61,429	0,005336	45,05%	112,326	61,838	0,005336	44,95%	114,412	62,747	0,005336	45,16%	149,306	111,582	0,005336	25,27%
108,531	54,952	0,006668	49,37%	109,050	55,318	0,006668	49,27%	111,069	56,131	0,006668	49,46%	139,736	99,817	0,006668	28,57%
106,505	50,169	0,008	52,90%	107,017	50,503	0,008	52,81%	108,962	51,245	0,008	52,97%	133,298	91,129	0,008	31,63%
100,140	15,013	0,089336	85,01%	101,118	26,373	0,029336	73,92%	107,481	47,437	0,009336	55,86%	111,327	55,802	0,021336	49,88%
100,132	14,902	0,090668	85,12%	100,992	25,794	0,030668	74,46%	106,333	44,377	0,010668	58,27%	109,910	54,137	0,022668	50,74%
100,124	14,794	0,092	85,22%	100,851	25,252	0,032	74,96%	105,302	41,842	0,012	60,27%	108,325	52,613	0,024	51,43%
100,115	14,688	0,093336	85,33%	100,686	24,741	0,033336	75,43%	104,216	39,691	0,013336	61,92%	106,489	51,208	0,025336	51,91%
100,105	14,584	0,094668	85,43%	100,493	24,261	0,034668	75,86%	102,900	37,845	0,014668	63,22%	104,330	49,912	0,026668	52,16%
100,093	14,482	0,096	85,53%	100,251	23,808	0,036	76,25%	100,982	36,236	0,016	64,12%	101,597	48,711	0,028	52,06%

As certain models had many nodes because of its dimensions, for a given stretch the results were omitted as indicated in Table 1 from the node at $x = 0{,}008$, indicating only the last 7 nodes in the near of plate edge.

3.2. Results of tension flow study

In tension flow study, in the two regions of the curve of stress diagrams, corresponding the first region to the equation (6) and the second region the equation (7) and equation (8) and their errors, we can observe the deviation of the flow over the theoretical flow expected, as follows in Table.

Table 2: Theoretical and numerical integrations for tension flow

λ	A_1	A_2		A		Flow Deviation
		In	ME	In	ME	
0,02	0,0731077	9,9193372	0,0084225	9,9924449	0,0084225	0,08%
0,05	0,0735933	3,8926744	0,0102852	3,9662677	0,0102852	0,84%
0,1	0,0746763	1,9479854	0,0064163	2,0226616	0,0064163	-1,13%
0,15	0,1327968	3,8375694	0,0156994	3,9703661	0,0156994	0,74%

For the main features of the models, is noted a good relation between the integrations for each region of the curve, beyond compliance in the systematic error on the numerical integration of the second region of the curve (ME).

3.3. Analytical approach of the stress diagram

The analytical integration of equation (11) to the function (10) and simplifying the expression obtained, through division by $\sigma_0 b$ and replacing the formulas (9) and $\lambda = l/b$, permits to isolate the coefficient c_0 as a function of geometrical parameter λ and function $Y(\lambda)$:

$$c_a(\lambda) = \frac{3\lambda}{2-4\lambda}\left(1-\frac{Y^2(\lambda)}{2}\right)\left(1+\frac{\sqrt{1/\lambda-2}}{Y(\lambda)}\right) \tag{13}$$

Replacement of λ values used in numerical analysis (Table 1) in equation (13) shows that $c_0\ (\lambda)$ increases significantly with increasing relative crack length λ (Table 3).

Table 3: Values for c_0 coefficient

λ	$Y(\lambda)$	$c_0(\lambda)$
0,02	1,0033755	0,1226787
0,05	1,0100686	0,2122959
0,1	1,0249068	0,3346946
0,15	1,05227	0,4380178

Based on the values of c_0 (λ), can be constructed stress diagrams according to the formula (10). The results are presented graphically in comparison with diagrams obtained by finite element method and the asymptotic (1) in Figures 6, 7. All diagrams were drawn for the minimum section of the plate, starting from the crack tip to the plate edge; and for experimental stress curve (σ), was omitted the point in crack tip due the high value, as shown in Table 1.

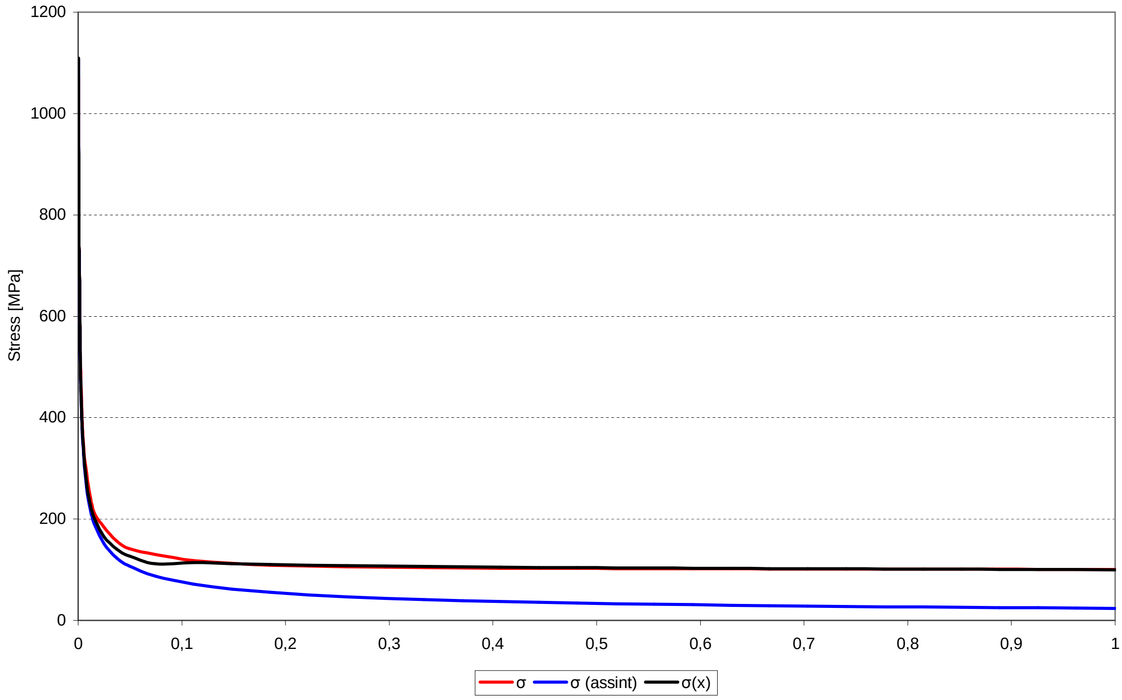

Figure 6: Stress diagrams for λ=0,05

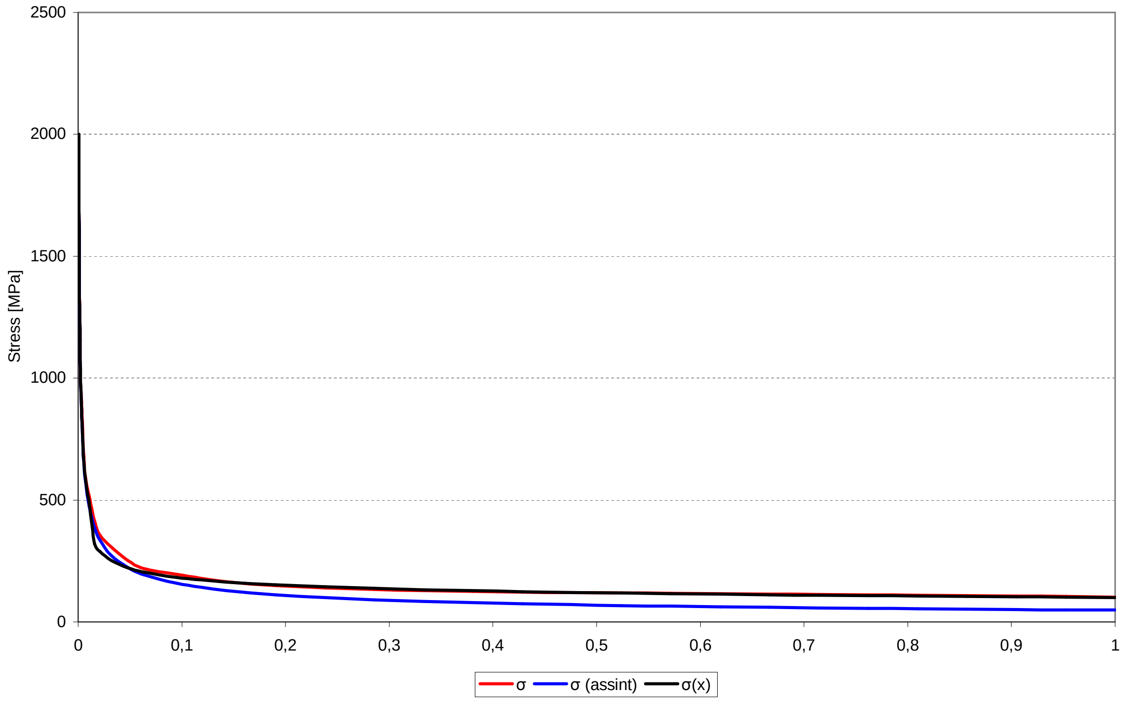

Figure 7: Stress diagrams for λ=0,15

3.4. Results for extension of plastic zone

The analytical integration of the global equilibrium condition for the stress distribution, according to equation (12), results in algebraic equations for parameter $z = R_p/R$. The resolution of this equation requires application of a numerical iterative procedure, whose results, along with values of $c_0\ (\lambda)$ and $s=\sigma_0/\sigma_e$, allow to calculate values of parameters p and p_1. Thus, we can proceed to calculate the extension of plastic zone R_p, matching the first and second line of the formula (12).

Finally we determine the correction factor applicable to the first estimate (4), $q= R_p / R_{p1}$. The obtained values are shown in Table 4.

Table 4: Correction factor of the extent of plastic zone given by eq. (4)

$s=\sigma_0/\sigma_e$	$\lambda=0,02$	$\lambda=0,05$	$\lambda=0,10$	$\lambda=0,15$
0,01	1,006	1,021	1,069	1,187
0,05	1,014	1,034	1,087	1,211
0,10	1,026	1,052	1,115	1,248
0,15	1,040	1,074	1,148	1,296
0,25	1,073	1,131	1,241	1,436
0,35	1,114	1,208	1,382	1,677
0,50	1,199	1,387	1,795	2,707
0,60	1,276	1,583	2,533	not exist
0,70	1,378	1,919	not exist	not exist

Is observed the strong effect of the load factor s on the correction factor, as well as the increase of this effect with increasing relative length of crack. The diagram of elastic-plastic distribution (12), according to one of the combinations of load and geometry, is illustrated in Figure 8, for a particular stretch starting from the crack tip.

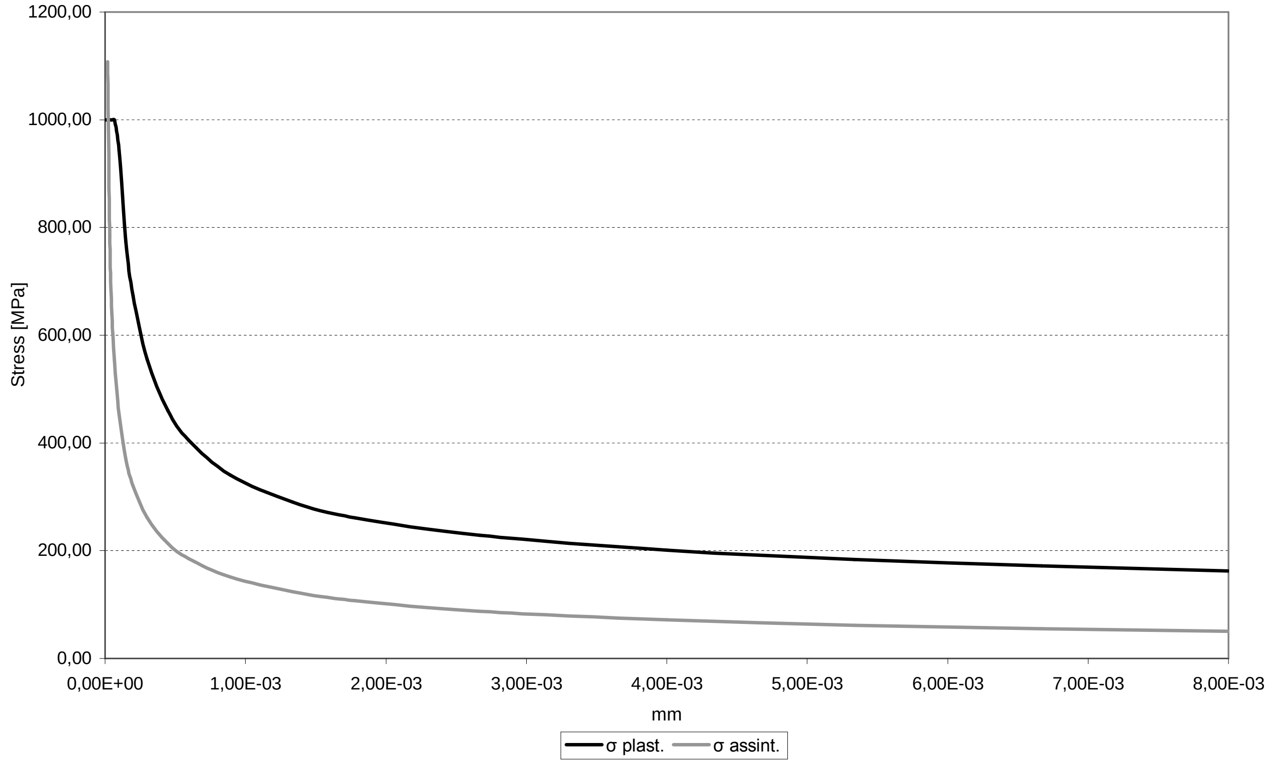

Figure 8: Diagram of elastic-plastic stress in crack tip for λ = 0.05 s = 0.1

4. DISCUSSION AND CONCLUSIONS

The numerical investigation of the boundary problem of linear elasticity for plates with a central crack, subjected to traction orthogonal to the crack surface made it possible establish proportions to model(length/width) and confirmed the validity of the asymptotic analytical solution (1) nearby the cracks. For various values of relative crack length was found that: the normal stress diagram is monotone decreasing, from the crack tip to free edge; the zone of stress concentration is highly localized; in large areas of free edges there is clear trend to the value of external load $\sigma_{0,}$ applied far from the crack. These observations allowed the development of an analytical approach to stress diagrams in the section with a crack. Moreover, since the behavior of the stress diagram near the free edges does not depend on the relative size of stress concentrator in plate center, the local phenomena of plastic deformation near the crack tips should also not affect the state of tension in this area. This fact indicates that the redistribution of stress due to formation of the plastic zone is a phenomenon of limited extent and not completely changes the diagram

The study of tension flow was performed by integration of normal-tension diagram in minimal section. In the region of asymptotic, the stretch of diagram was integrated analytically. In the remaining area, the approximate integration was performed using the Simpson Rule (1/3) and Trapezoid. The values of tension flow for the considered cases showed good correspondence with tension flow applied externally to the plate, with a difference not greater than 1,5%, matching the error margins in numerical integration and numerical solution of boundary problems. Therefore, the law of global equilibrium was applied to determine the tuning parameter in the analytical approach of normal-tension diagram. The formulas of analytical approach, developed in this way, show good correspondence with values of numerical solution and can be used more easily in various tests of structural integrity.

The study of plastic zone extent was performed based on numerical and analytical results obtained in the investigation of theory of elasticity problems. The proposed model of tension flow redistribution considers that the local relief, due to plastic deformation near the crack tip, means increasing the effective load acting on the near region, where the stress distribution is determined primarily by the asymptotic (1). The increased tension in the large region between this zone and the freeboard is secondary; the modeling follows the standard of elastic diagram approach, with tuning parameter necessary for continuity reasons. The results showed that both the relative crack length, as the relative level of loading influence the correction factor, which must be applied to the simple estimative of the extent of plastic zone under plane stress (Eq. (4)).

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6. REFERENCES

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