CONEM 2010
VI Congresso Nacional de Engenharia Mecânica

ABCM — COMPROMETIDA COM A PROMOÇÃO DO DESENVOLVIMENTO DA ENGENHARIA E DAS CIÊNCIAS MECÂNICAS
ASSOCIAÇÃO BRASILEIRA DE ENGENHARIA E CIÊNCIAS MECÂNICAS

VI CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
VI NATIONAL CONGRESS OF MECHANICAL ENGINEERING
18 a 21 de agosto de 2010 – Campina Grande – Paraíba - Brasil
August 18 – 21, 2010 – Campina Grande – Paraíba – Brazil

Heat dissipation in liquid with magnetic nanoparticles in the presence of a circular polarized field and vorticity

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Abstract. *This work investigates the heating process of magnetic fluids (magnetic nanoparticles embedded in fluids) under the external magnetic field. We adopt the well-established theoretical approach for magnetization of ferrohydrodynamics proposed by Shiliomis et al to describe the motion of free dipoles in a viscous fluid. By taking into account vorticity effects, we show that new features arise when a circular polarized field is considered. The interplay between the Brown and Néel mechanisms of relaxation also presents an important role in the heating process of the liquid. The friction of the particle rotation, due to magnetic torques, with the viscous fluid, constitutes an source of heat. Our analysis allows the determination of the liquid temperature as a function of the model parameters. Such profile is of fundamental interest in the emerging field of research on fuel droplet combustion with magnetic nanoparticles.*

Keywords: *ferrohydrodynamics, magnetic nanoparticles, heating rate*

1. Introduction

The use of alternating magnetic fields on fluids with magnetic nanoparticles (MNPs) allows a production of heat in the host system (Rosensweig (2002)). Such heat source is due to a friction effect mediated by the viscous and magnetic torques on the nanoparticles, being the former opposite to the latter. We propose that this mechanism of ferrofluids heating can be also used to heat fuel droplets with MNPs (see Fig. (1)).

A theoretical study is developed for establishing the basis of future applications. In previous analyses, the determination of the heating process is restricted to quiescent fluids with linearly polarized magnetic fields (Rosensweig (2002); Maenosomo and Saita (2006) and Fachini (2009)). We investigate the heating process for rotating fields in fluids with finite flow vorticities. By taking into account these effects in our calculations of the heating rate function, we show that the heating process is favored and becomes more efficient when performed with circularly polarized magnetic fields.

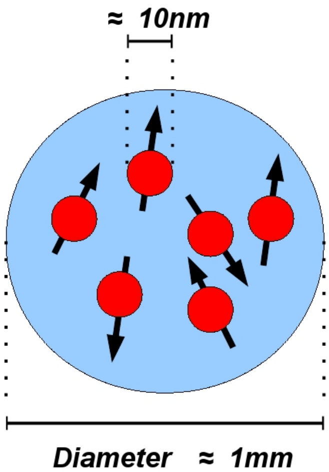

Figure 1. Sketch of the geometry: a fuel droplet with embedded magnetic nanoparticles.

This paper is organized as following. In Sec. 2, the phenomenological model of Shliomis *et al.* (1988); Shliomis and Morozov (1994) and Shliomis (2001) for ferrofluids is discussed, particular attention is given to the relaxation mechanisms of Brown and Néel (Rosensweig (2002)). In Sec. 3, using the formalism of the complex frequency dependent susceptibility (Rosensweig (2002)), we determine the net magnetization in the system due to a circular counterclockwise polarization. The heating rate function for an adiabatic process is derived in Sec. 4, and in Sec. 5, numerical results for the frequency dependence of this function are presented. We summarize our findings in Sec. 6.

2. Theoretical model for ferrofluids

The phenomenological model of Shliomis *et al.* (1988); Shliomis and Morozov (1994) and Shliomis (2001) describes the net magnetization $\vec{M}$ of a viscous fluid with rotating magnetic nanoparticles (MNPs) characterized by a macroscopic angular velocity $\vec{\Omega}_p$ due to an applied magnetic field $\vec{H}$. Magnetic and viscous torques on these particles embedded in a fluid with vorticity flow $\vec{\Omega}$ are considered in this model. The external field interacts with the MNP dipoles $\vec{m}$ forcing an alignement with the field direction. As the particles are in contact with a thermal bath of finite temperature T, the equilibrium magnetization

$$\vec{M_0} \equiv \frac{\varphi}{V} \left|\vec{m}\right| L\left(\left|\vec{\xi}\right|\right) \frac{\vec{\xi}}{\xi} \tag{1}$$

is reached. The constant φ is the volume fraction of the MNPs, V is the MNP single volume and $L\left(\left|\vec{\xi}\right|\right)$ is the Langevin function,

$$L(\xi) \equiv \coth\xi - \xi^{-1}, \tag{2}$$

that depends on the magnitude of

$$\vec{\xi} \equiv \frac{\left|\vec{m}\right| \vec{H}}{k_B T}. \tag{3}$$

This ratio measures the competition between the magnetic energy $\left|\vec{m}\right| \left|\vec{H}\right|$, responsible to orient the MNPs with the magnetic field, and the thermal energy $k_B T$ of the ferrofluid, responsible to disorient the MNPs. Due to a mechanism of relaxation for the dipoles, the magnetization does not reach an instantaneous equilibrium state, there is a characteristic time that imposes a transient period. In the rotating frame of the MNPs, this relaxation process is assumed to be governed by the equation (Rosensweig (2002))

$$\frac{d\vec{M}}{dt} = -\frac{1}{\tau}\left(\vec{M} - \vec{M_0}\right), \tag{4}$$

whose solution is

$$\vec{M} = \vec{M_0}\left[1 - \exp\left(-t/\tau\right)\right]. \tag{5}$$

Note that the equilibrim condition $\vec{M} \sim \vec{M_0}$ is achieved for $t \gg \tau$. In the laboratory frame we have to add to the right hand side of the Eq. (4) the contribution $\vec{\Omega}_P \times \vec{M}$ due to the magnetic torque

$$\vec{\tau}_{mag} \equiv \vec{M} \times \vec{H} \tag{6}$$

to derive

$$\frac{d\vec{M}}{dt} = \vec{\Omega}_P \times \vec{M} - \frac{1}{\tau}\left(\vec{M} - \vec{M_0}\right) \tag{7}$$

known as the equation of motion for the out-of-equilibrium magnetization. The effective relaxation parameter τ consists of two processes in parallel (Rosensweig (2002)), which is expressed as

$$\frac{1}{\tau} \equiv \frac{1}{\tau_B} + \frac{1}{\tau_N}, \tag{8}$$

with

$$\tau_B \equiv \frac{3\eta V}{k_B T} \tag{9}$$

as being the Brownian time of rotational diffusion and

$$\tau_N \equiv \tau_0 \frac{\exp\left(K V_M / k_B T\right)}{\sqrt{K V_M / k_B T}} \tag{10}$$

the Néel relaxation. The fluid and particles properties in Eqs. (9) and (10) are the fluid viscosity η, the hydrodynamic volume V of a single MNP, the constant of proportionality τ_0, the anisotropy constant K and the magnetic volume V_M

of a MNP. For the Brownian relaxation mechanism, the magnetic dipoles rotate together with the particles. For the Néel mechanism, the MNPs are steady ($\vec{\Omega}_p = 0$) and the magnetic dipoles rotate exclusively in respect to the crystal axes of the particles. Thus considering the viscosity η of the fluid, the viscous torque

$$\vec{\tau}_{visc} \equiv \Gamma \left(\vec{\Omega}_P - \vec{\Omega} \right) \tag{11}$$

on the rotating MNPs is opposite to the magnetic torque. According to Eq. (6), the viscous torque is determined by the moment of inertia per unit time $\Gamma = 6\varphi\eta$, the macroscopic angular velocity $\vec{\Omega}_P$ and the fluid vorticity $\vec{\Omega}$. As a result, the macroscopic angular acceleration for the MNPs obeys the following rotational equation

$$I \frac{d\vec{\Omega}_p}{dt} = \vec{\tau}_{mag} - \vec{\tau}_{visc}, \tag{12}$$

where I is the total rotation moment of inertia. The model considered here treats the steady state of Eq. (12), which gives the macroscopic angular velocity for the MNPs

$$\vec{\Omega}_P = \vec{\Omega} + \frac{1}{6\varphi\eta} \left(\vec{M} \times \vec{H} \right). \tag{13}$$

Thus the magnetization rate of change in the laboratory frame is written by

$$\frac{d\vec{M}}{dt} = -\frac{1}{\tau} \left(\vec{M} - \vec{M}_0 \right) + \vec{\Omega} \times \vec{M} - \frac{1}{6\varphi\eta} \vec{M} \times \left(\vec{M} \times \vec{H} \right). \tag{14}$$

Note that the dynamics of Eq. (14) agrees very well with numerical results extracted from a Focker-Planck analysis for the limit $\Omega\tau < 1$ (Shliomis (2001)). To apply this model, this paper investigates the heating process of fluid with MNPs in such limit.

3. The net magnetization and the circular polarization

The heating rate of the system is determined by the magnetization of the ferrofluid due to an alternate magnetic field. In particular, we choose a circular counterclockwise polarization

$$\vec{H} = H_0 \cos(\omega t)\hat{x} + H_0 \sin(\omega t)\hat{y}, \tag{15}$$

where H_0 and ω are the amplitude and the angular frequency of the field, respectively. The solution of Eq. (14) for the magnetization can be obtained numerically. However, we adopt a guessed solution based on the expected behavior of the magnetization. Before applying this method we remind the role of the relaxation process in the system. In the absence of a relaxation mechanism, the magnetization is in-phase with the alternating magnetic field, which is described by the instantaneous version of Eq. (1). Thus, a finite relaxation time introduces a phase shift in the magnetization, i.e.,

$$\vec{M} = M \cos(\omega t - \delta)\,\hat{x} + M \sin(\omega t - \delta)\,\hat{y}. \tag{16}$$

By defining

$$\chi_r \equiv M \cos\delta \tag{17}$$

and

$$\chi_i \equiv M \sin\delta, \tag{18}$$

the magnetization can be expressed in terms of the components of the frequency dependent complex susceptibility $\chi(\omega) = \chi_r - i\chi_i$ (Rosensweig (2002)),

$$\vec{M} = (\chi_r \cos\omega t + \chi_i \sin\omega t)\,\hat{x} + (\chi_r \sin\omega t - \chi_i \cos\omega t)\,\hat{y}. \tag{19}$$

To determine the unknown variables of Eqs. (17) and (18) as functions of the model properties, we consider the low-amplitude approximation for the magnetic field $|\vec{m}| \left|\vec{H}\right| / k_B T \ll 1$, which leads to the instantaneous equilibrium magnetization

$$\vec{M}_0(t) = \chi_0 \vec{H}(t), \tag{20}$$

with $\chi_0 \equiv (\varphi/V) |\vec{m}|^2 / 3k_B T$ as the equilibrium susceptibility. By performing the substitutions of Eqs. (15) and (16) in Eqs. (7) and (13), we show that

$$M = \chi_0 H_0 \cos\delta \tag{21}$$

and

$$\tan\delta = \tau (\omega - \Omega) . \tag{22}$$

The combination of these results leads to the complex susceptibility for the system

$$\chi(\omega) = \frac{\chi_0}{1 + i\tau(\omega - \Omega)} = \chi_r - i\chi_i, \tag{23}$$

and the magnetization

$$\vec{M} = \frac{\chi_0 H_0}{\sqrt{1 + \tau^2 (\omega - \Omega)^2}} \left[\cos(\omega t - \delta)\,\hat{x} + \sin(\omega t - \delta)\,\hat{y}\right] \tag{24}$$

in which the phase shift to the external magnetic field is defined by

$$\delta = \arctan\left[-\frac{\Im\{\chi(\omega)\}}{\Re\{\chi(\omega)\}}\right]. \tag{25}$$

4. The heating rate

According to the first law of thermodynamics $dE = \delta Q - \delta W$ per unit volume, the dissipation energy E per unit volume in a cycle $T = 2\pi/\omega$ of the magnetic field for an adiabatic process $\delta Q = 0$ is the work per unit volume $\delta W = -\vec{H}.d\vec{B}$ done by this field on the ferrofluid. By using the constitutive equation $\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right)$ for the magnetic induction and the cycle condition $\oint \vec{H}.d\vec{H} = 0$, we derive for the limit $\Omega\tau < 1$, the following dissipation energy per unit volume is

$$E = \mu_0 \int_0^T dt\vec{H}.\frac{d\vec{M}}{dt} = -2\pi\mu_0 H_0^2 \Im\{\chi(\omega)\}, \tag{26}$$

where μ_0 is the magnetic permeability of the free space. This function is also called energy density. By taking into account Eq. (23) in Eq. (26), it becomes

$$E = 2\pi\mu_0\chi_0 H_0^2 \frac{\tau\omega(1 - \Omega/\omega)}{1 + \tau^2\omega^2 (1 - \Omega/\omega)^2}. \tag{27}$$

For flow vorticity satisfying the condition $\Omega/\omega \ll 1$, the expression for the density has the same functional dependence as previous result for a magnetic field with a linear polarization (Rosensweig (2002)), except for a factor of two. This factor arises from the degrees of freedom of the alternating magnetic field, while the circular polarization has two components in the xOy plane, the linear case is fixed to only one direction. However, both cases introduce a shift of Ω in the frequency dependence of the energy density. Thus, our expression allows an enhanced heating process for external fields that satisfy the condition $\omega > \Omega$.

The characterization of the system temperature profile for a constant relaxation can be performed defining the heating rate function (Rosensweig (2002) and Maenosomo and Saita (2006))

$$\frac{\Delta T}{\Delta t} = \frac{\omega E}{2\pi\rho c}, \tag{28}$$

where ΔT is the temperature rise in time step Δt during the heating process. For the ferrofluid, ρ is the density and c the specific heat. According to Eq. (26), the heating rate is governed by the behavior of the minus part of the complex frequency dependent susceptibility given by Eq. (23). So this quantity becomes important to investigate the features of the heating process as a function of the model parameters.

To make explicit such dependencies, we prefer to define the dimensionless variables $\Delta = \tau(\omega - \Omega)$ and $\varepsilon = \Omega\tau$ to derive

$$H_C = \left(\frac{\tau\rho c}{\mu_0\chi_0 H_0^2}\right)\frac{\Delta T}{\Delta t} = \frac{\Delta^2}{1 + \Delta^2} + \varepsilon\frac{\Delta}{1 + \Delta^2} \tag{29}$$

as the renormalized heating rate due to the circular polarization. Keeping $\varepsilon < 1$ to ensure the validity of the model, Eq. (29) allows the knowledge of the dependence of the ferrofluid heating rate in terms of the dimensionless relative field frequency and the flow vorticity given by $\triangle$ and ε, respectively. According to Rosensweig (2002), the heating rate due to a linear polarization is the half of a circular,

$$H_L = \frac{1}{2} H_C. \tag{30}$$

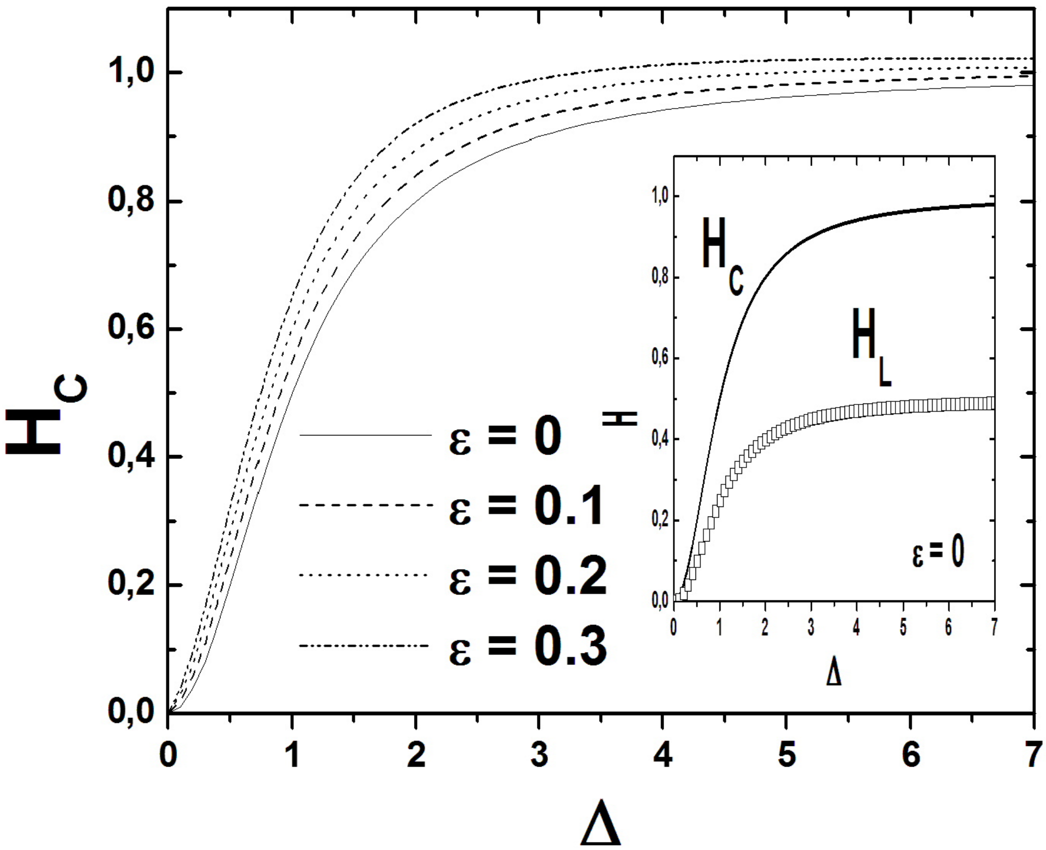

Figure 2. Heating rate H_C due to a circular polarization as a function of the dimensionless relative frequency $\Delta = \tau(\omega - \Omega)$ with different dimensionless vorticities $\varepsilon = \Omega\tau$. For external frequencies near the flow vorticity, the heating rate displays a linear behavior. A crossover region occurs until a saturation value is reached for large relative frequencies. Constant heating rates dependent on the fluid vorticity appear. A comparison between the linear and the circular heating for a quiescent fluid ($\Omega = 0$) is presented in the inset. As seen, the circular case is more efficient.

5. Results

In Fig. (2) we present simulations for the heating rate function. They show a linear behavior for small deviations of the relative frequency $\triangle$, i.e., for values of the magnetic field frequency near to the fluid vorticity. For intermediate deviations, this behavior disappears and the slopes of the curves start to decrease. Such decrease stabilizes and constant heating rates occurs for large deviations, characterized by an external frequency well above to the fluid vorticity. In all these regimes, the heating rate function is vorticity dependent. Efficient heating rates for circular polarization are produced, see the comparison between the circular with linear cases in the inset. Such behaviors are consequence of the Eqs. (29) and (30). The circular case is the double of the linear one.

6. Conclusions

Based on a phenomenological model for ferrofluids, we derived the heating rate function due to a magnetic field with a circular counterclockwise polarization. For a fixed relaxation parameter, we demonstrated that the heating process is more efficient for large differences between the external and vorticity frequencies with a saturation value determined by low vorticities. This mechanism can be applied to heat fuel drops with MNPs.

7. Acknowledgments

This work was supported by the brazilian agency CAPES via PNPD program.

8. References

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