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EXTENSION OF THE STEGER AND WARMING AND RADESPIEL AND KROLL ALGORITHMS TO SECOND ORDER ACCURACY AND IMPLICIT FORMULATION APPLIED TO THE EULER EQUATIONS IN TWO-DIMENSIONS - RESULTS

Edisson Sávio de Góes Maciel, edissonsavio@yahoo.com.br[1]

[1] Mechanical Engineer/Researcher – Rua Demócrito Cavalcanti, 152 – Afogados – Recife – PE – Brazil – 50750-080

Abstract. *In this work, the second part of this study, the Steger and Warming and the Radespiel and Kroll schemes are implemented following a MUSCL approach, aiming to guarantee second order accuracy and to achieve TVD properties, and employing an implicit formulation to solve the Euler equations in the two-dimensional space. These schemes are implemented according to a finite volume formulation and using a structured spatial discretization. Both schemes are flux vector splitting ones. The MUSCL approach employs five different types of nonlinear limiters, which assure TVD properties, namely: Van Leer limiter, Van Albada limiter, minmod limiter, Super Bee limiter and β-limiter. All variants of the MUSCL schemes are second order accurate in space. The implicit schemes employ an ADI approximate factorization to solve implicitly the Euler equations. Explicit and implicit results are compared, as also the computational costs, trying to emphasize the advantages and disadvantages of each formulation. The schemes are accelerated to the steady state solution using a spatially variable time step, which has demonstrated effective gains in terms of convergence rate according to Maciel. The algorithms are applied to the solution of the physical problem of the moderate supersonic flow along a compression corner. The results have demonstrated that the most accurate solutions are obtained with the Steger and Warming TVD scheme using Van Leer and minmod nonlinear limiters, when implemented in its explicit version. This paper presents the numerical results and the analyses of this study.*

Keywords: *Steger and Warming algorithm, Radespiel and Kroll algorithm, MUSCL procedure, Implicit formulation, Euler equations.*

1. INTRODUCTION

In this work, the second part of this study, the Steger and Warming (1981) and the Radespiel and Kroll (1995) schemes are implemented following a MUSCL approach, aiming to guarantee second order accuracy and to achieve TVD properties, and employing an implicit formulation to solve the Euler equations in the two-dimensional space. These schemes are implemented according to a finite volume formulation and using a structured spatial discretization. Both schemes are flux vector splitting ones. The MUSCL approach employs five different types of nonlinear limiters, which assure TVD properties, namely: Van Leer limiter, Van Albada limiter, minmod limiter, Super Bee limiter and β-limiter. All variants of the MUSCL schemes are second order accurate in space. The implicit schemes employ an ADI approximate factorization to solve implicitly the Euler equations. Explicit and implicit results are compared, as also the computational costs, trying to emphasize the advantages and disadvantages of each formulation. The schemes are accelerated to the steady state solution using a spatially variable time step, which has demonstrated effective gains in terms of convergence rate according to Maciel (2005). The algorithms are applied to the solution of the physical problem of the moderate supersonic flow along a compression corner. The results have demonstrated that the most accurate solutions are obtained with the Steger and Warming (1981) TVD scheme using Van Leer and minmod nonlinear limiters, when implemented in its explicit version. This paper presents the numerical results and the analyses of this study.

For a detailed motivation to study such algorithms, as also their extension to second order spatial accuracy resulting from a MUSCL approach and the use of an implicit formulation, the reader is encouraged to read the first paper of this work, THEORY (Maciel, 2010).

2. RESULTS

Tests were performed in a microcomputer with processor INTEL CELERON, 1.5GHz of “clock”, and 1.0GBytes of RAM memory. Converged results occurred to 3 orders of reduction in the maximum residual value. The entrance angle is equal to 0.0° for the compression corner problem and the freestream Mach number is 3.0. The ratio of specific heats, γ, assumed the value 1.4. The reference to the limiters is abbreviated in this work: Van Leer limiter (VL), Van Albada limiter (VA), minmod limiter (Min), Super Bee limiter (SB) and β-limiter (BL). The explicit formulations of the Steger and Warming (1981) and of Radespiel and Kroll (1995) TVD schemes employ Euler explicit method and time splitting procedure, respectively, to time integration as described in Maciel (2008a,b).

2.1. Compression Corner Physical Problem – Explicit Simulations

The compression corner configuration is described in Fig. 1. The corner inclination angle is 10°. An algebraic mesh of 70x50 points or composed of 3,381 rectangular cells and 3,500 nodes was used and is shown in Fig. 2. The points are equally spaced in both directions.

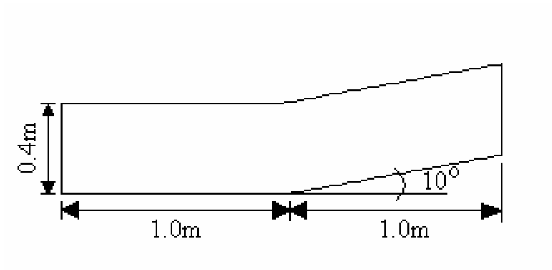

Figure 1. Compression corner configuration.

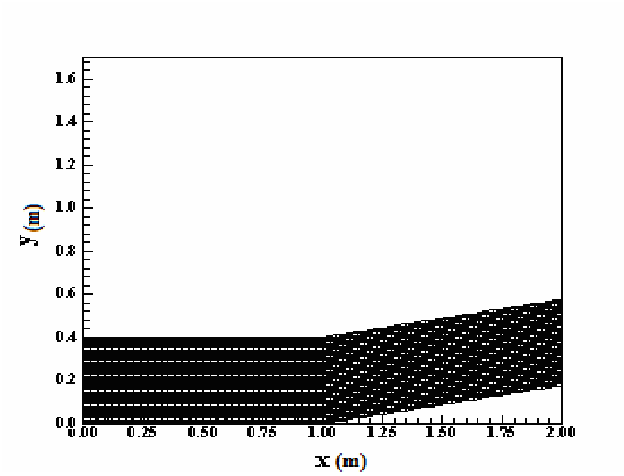

Figure 2. Compression corner mesh.

Figures 3 to 7 exhibit the pressure contours obtained by the Steger and Warming (1981) TVD scheme in its five variants. As can be observed the most severe pressure after the shock is captured by the Steger and Warming (1981) TVD scheme using the SB variant, as also the smallest shock wave thickness is captured by this limiter.

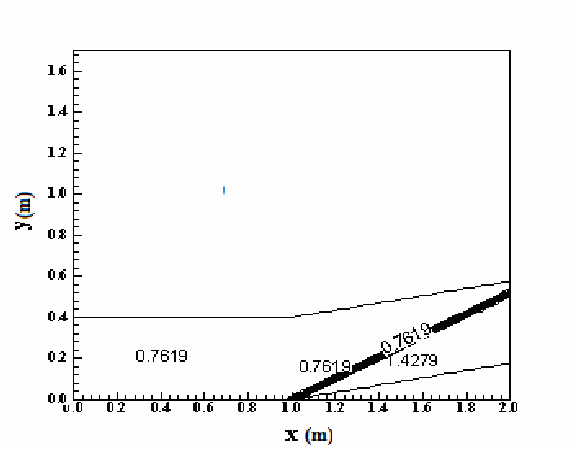

Figure 3. Pressure contours (SW-VL).

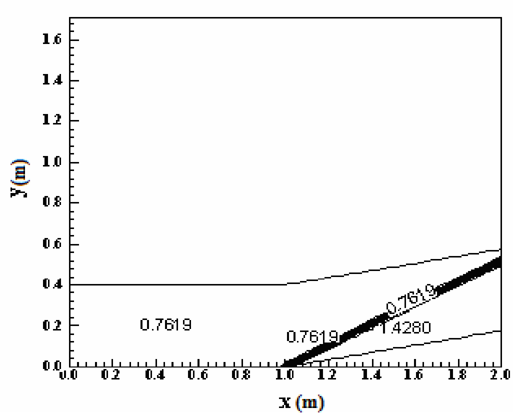

Figure 4. Pressure contours (SW-VA).

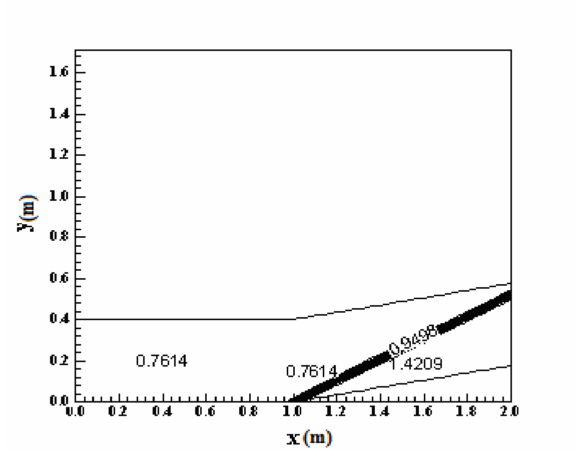

Figure 5. Pressure contours (SW-Min).

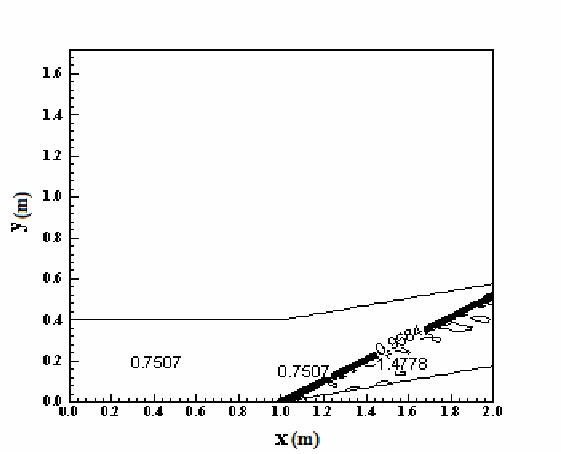

Figure 6. Pressure contours (SW-SB).

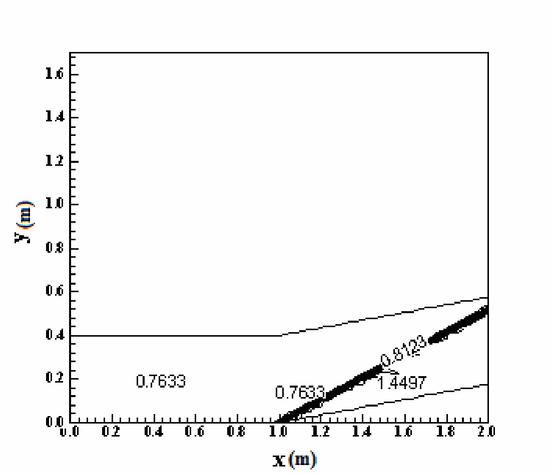

Figure 7. Pressure contours (SW-BL).

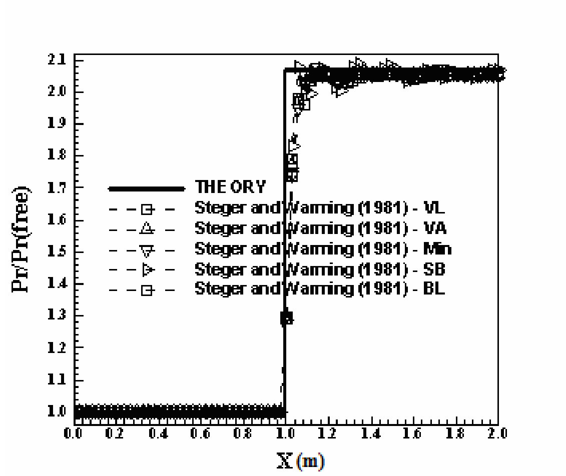

Figure 8. Wall pressure distributions (SW).

Figure 8 shows the wall pressure distributions obtained by all variants of the Steger and Warming (1981) TVD scheme. They are compared with the oblique shock wave theory. As can be observed, some solutions present oscillations at the compression corner, mainly the Steger and Warming (1981) TVD scheme using the SB limiter. Figure 9 exhibits the wall pressure distributions obtained by the Steger and Warming (1981) TVD scheme using VL, VA and Min limiters. As noted, no overshoot or undershoot are observed in the solutions, presenting these ones a smooth behavior. It is also possible to observe that the shock discontinuity is captured within three cells, which is a typical number of cells encountered in high resolution schemes to capture accurately shock waves. So the accuracy of the Steger and Warming (1981) TVD scheme with these three limiters is in accordance with typical results of current high resolution schemes. Figure 10 shows the wall pressure distributions obtained by the Steger and Warming (1981) TVD scheme using the SB and the BL limiters. The SB limiter yields oscillations along the shock plateau, but the shock is also captured in three cells, as is the case with the BL limiter. By the results, the best solutions were obtained with VL, VA and Min limiters, which even capturing a less severe pressure after the shock, detect sharp and smooth pressure distributions at the corner wall.

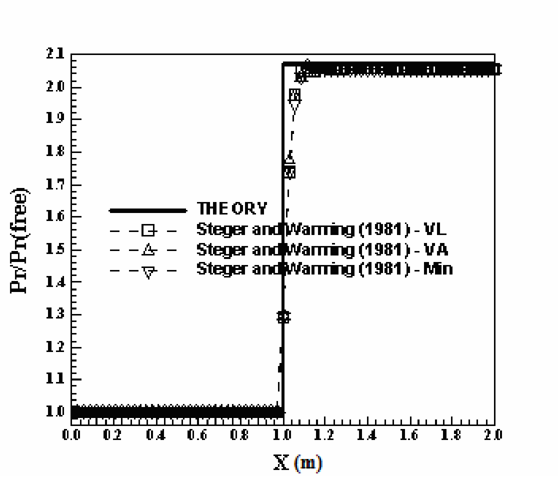

Figure 9. Wall pressure distributions (SW-1).

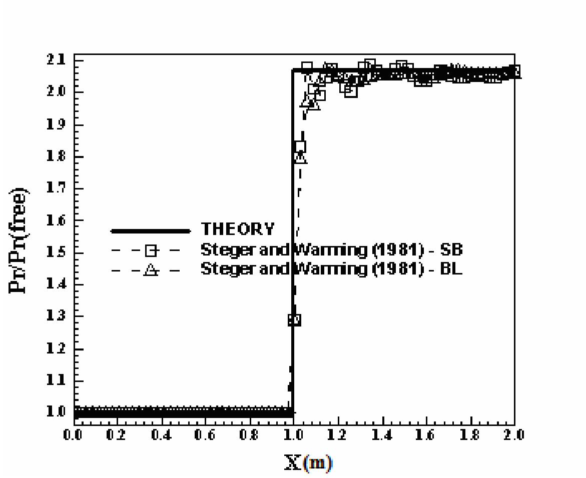

Figure 10. Wall pressure distributions (SW-2).

Table 1. Shock angle and percentage errors to the compression corner problem (SW/Explicit case).

Algorithm:	β (°):	Error (%):
Steger and Warming (1981) TVD – VL	27.5	0.00
Steger and Warming (1981) TVD – VA	27.9	1.45
Steger and Warming (1981) TVD – Min	28.0	1.82
Steger and Warming (1981) TVD – SB	27.2	1.09
Steger and Warming (1981) TVD – BL	27.0	1.82

One way to quantitatively verify if the solutions generated by each scheme are satisfactory consists in determining the shock angle of the oblique shock wave, β, measured in relation to the initial direction of the flow field. Anderson Jr. (1984) (pages 352 and 353) presents a diagram with values of the shock angle, β, to oblique shock waves. The value of this angle is determined as function of the freestream Mach number and of the deflection angle of the flow after the shock wave, φ. To the compression corner problem, φ = 10º (ramp inclination angle) and the freestream Mach number is

3.0, resulting from this diagram a value to β equals to 27.5º. Using a transfer in Figures 3 to 7, it is possible to obtain the values of β to each variant of the Steger and Warming (1981) TVD scheme, as well the respective errors, shown in Tab. 1. The Steger and Warming (1981) TVD scheme using the VL limiter has yielded the best result.

Figures 11 to 15 exhibit the pressure contours obtained by the Radespiel and Kroll (1995) TVD scheme in its five variants. As can be observed the most severe pressure after the shock is captured by the Radespiel and Kroll (1995) TVD scheme using the SB variant, as also the smallest shock wave thickness is captured by this limiter.

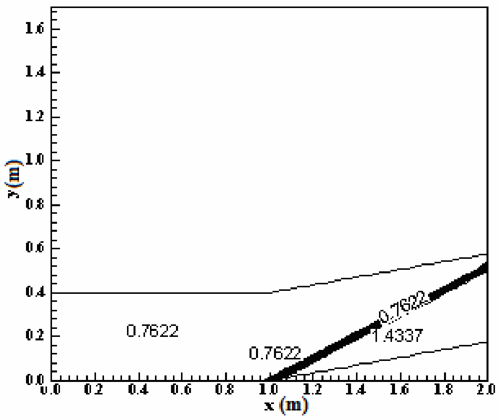

Figure 11. Pressure contours (RK-VL).

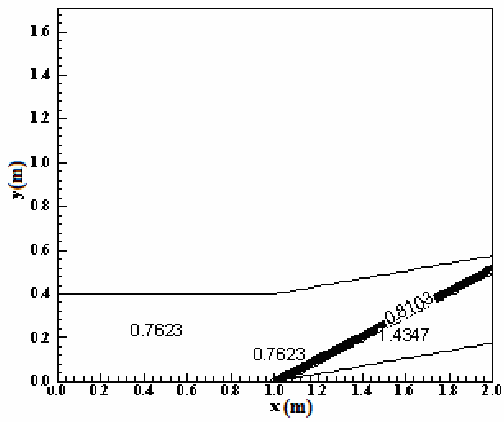

Figure 12. Pressure contours (RK-VA).

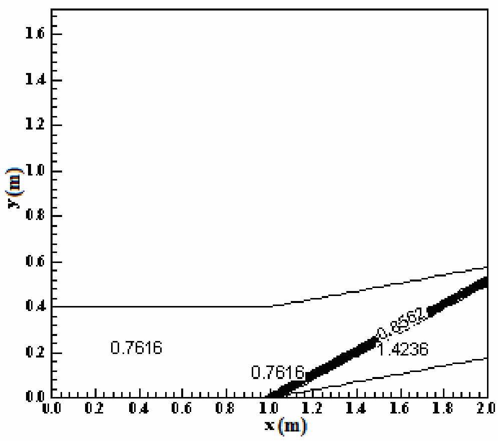

Figure 13. Pressure contours (RK-Min).

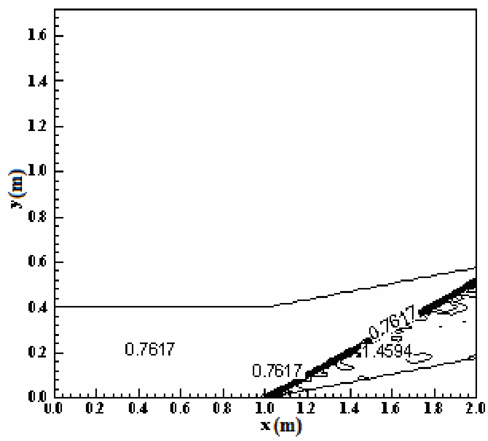

Figure 14. Pressure contours (RK-SB).

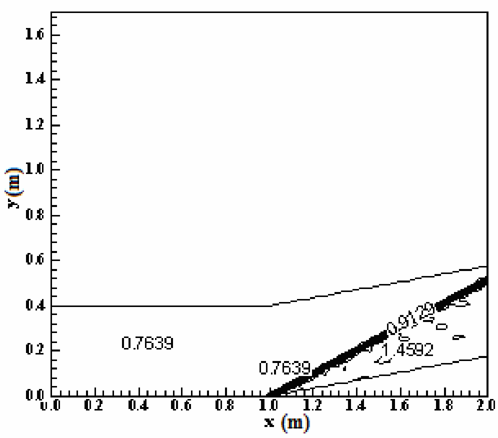

Figure 15. Pressure contours (RK-BL).

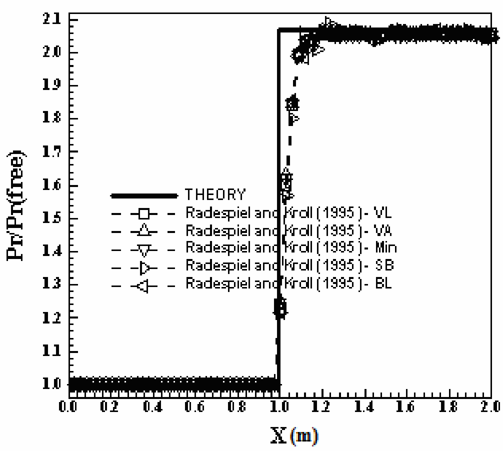

Figure 16. Wall pressure distributions (RK).

Figure 16 shows the wall pressure distributions obtained by all variants of the Radespiel and Kroll (1995) TVD scheme. They are compared with the oblique shock wave theory. As can be observed, some solutions present oscillations at the compression corner, mainly the Radespiel and Kroll (1995) TVD scheme using the SB limiter, but they are in less amount than in the solutions of the variants of the Steger and Warming (1981) TVD scheme. Figure 17 exhibits the wall pressure distributions obtained by the Radespiel and Kroll (1995) TVD scheme using VL, VA and Min limiters. As noted, no overshoot or undershoot are observed in the solutions, presenting these ones a smooth behavior. It is also possible to observe that the shock discontinuity is captured within four cells, which is a typical number of cells encountered in high resolution schemes to capture accurately shock waves. So the accuracy of the Radespiel and Kroll (1995) TVD scheme with these three limiters is in accordance with typical results of current high resolution schemes. Figure 18 shows the wall pressure distributions obtained by the Radespiel and Kroll (1995) TVD scheme using the SB and the BL limiters. The SB limiter yields oscillations along the shock plateau, but the shock is also captured in four cells, as is the case with the BL limiter. By the results, the best solutions were obtained with VL, VA and Min limiters, which even capturing a less severe pressure after the shock, detect sharp and smooth pressure distributions at the corner wall.

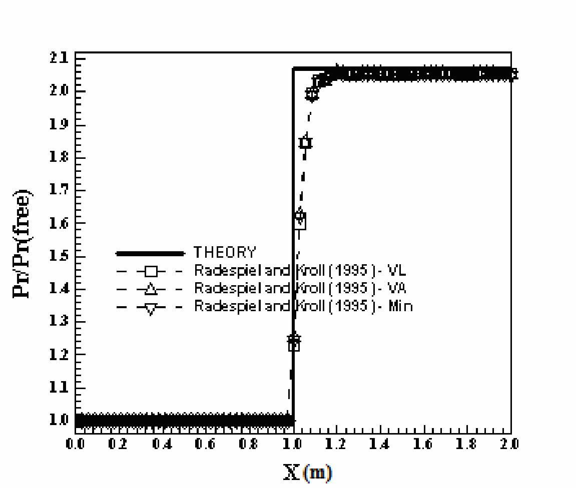

Figure 17. Wall pressure distributions (RK-1).

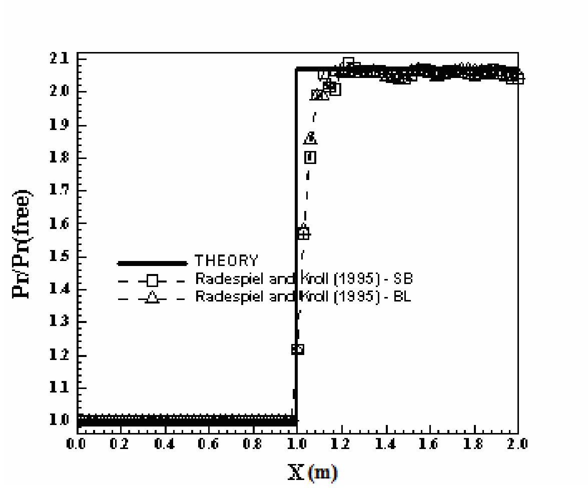

Figure 18. Wall pressure distributions (RK-2).

Analyzing the oblique shock wave angle, using a transfer in Figures 11 to 15, it is possible to obtain the values of β to each variant of the Radespiel and Kroll (1995) TVD scheme, as well the respective errors, shown in Tab. 2. The Radespiel and Kroll (1995) TVD scheme using the VL limiter has yielded the best result.

Table 2. Shock angle and percentage errors to the compression corner problem (RK/Explicit case).

Algorithm:	β (°):	Error (%):
Radespiel and Kroll (1995) TVD – VL	27.5	0.00
Radespiel and Kroll (1995) TVD – VA	28.0	1.82
Radespiel and Kroll (1995) TVD – Min	28.0	1.82
Radespiel and Kroll (1995) TVD – SB	27.6	0.36
Radespiel and Kroll (1995) TVD – BL	27.4	0.36

2.2. Compression Corner Physical Problem – Implicit Simulations

Figures 19 to 23 exhibit the pressure contours obtained by the Steger and Warming (1981) TVD scheme in its five variants. As can be observed the most severe pressure after the shock is captured by the Steger and Warming (1981) TVD scheme using the BL variant, although the smallest shock wave thickness is detected by the SB variant.

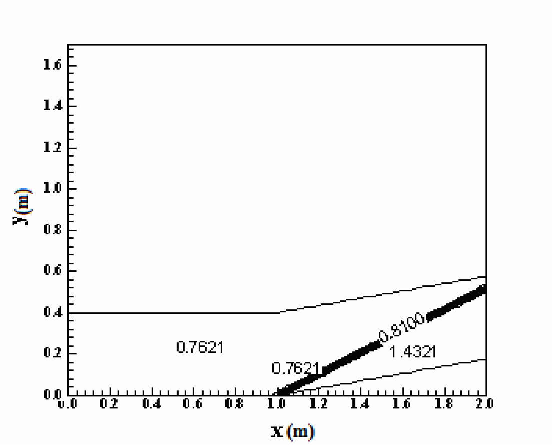

Figure 19. Pressure contours (SW-VL).

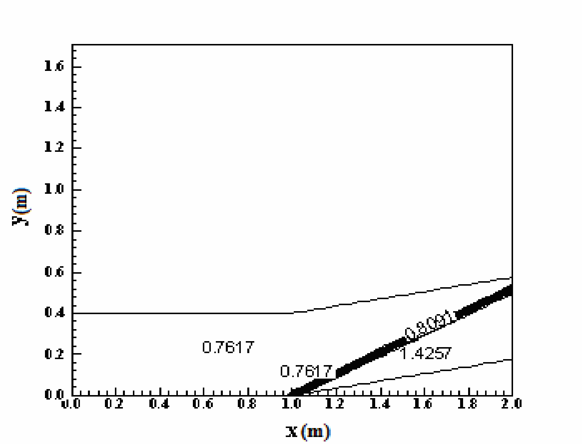

Figure 20. Pressure contours (SW-VA).

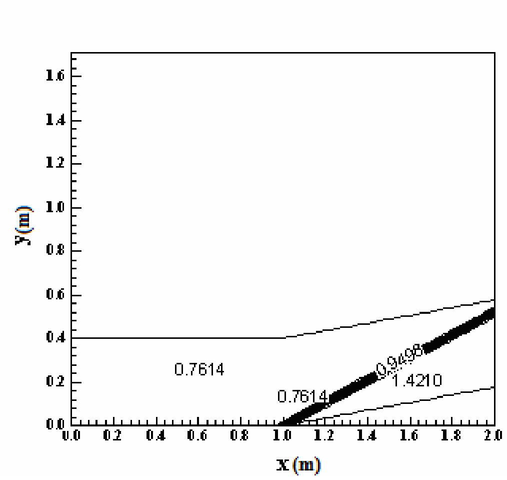

Figure 21. Pressure contours (SW-Min).

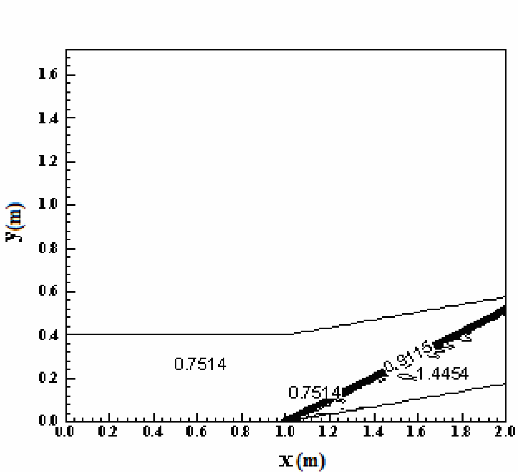

Figure 22. Pressure contours (SW-SB).

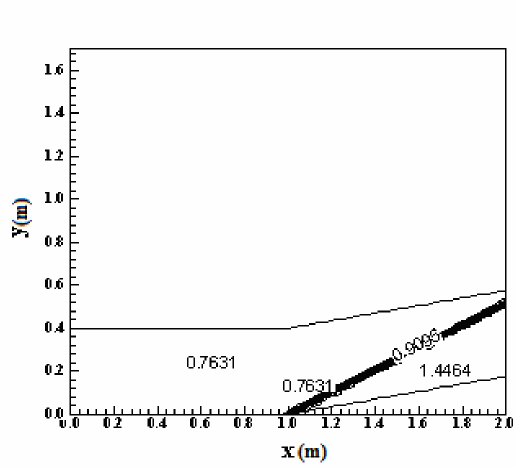

Figure 23. Pressure contours (SW-BL).

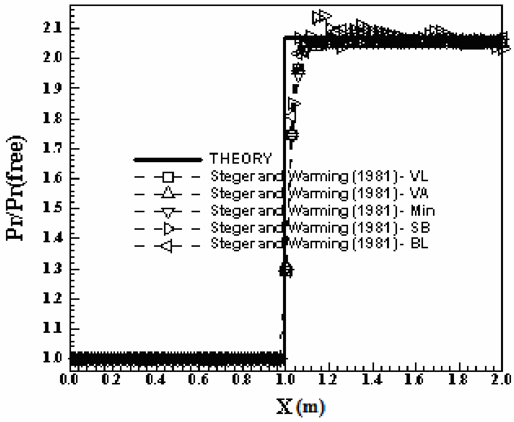

Figure 24. Wall pressure distributions (SW).

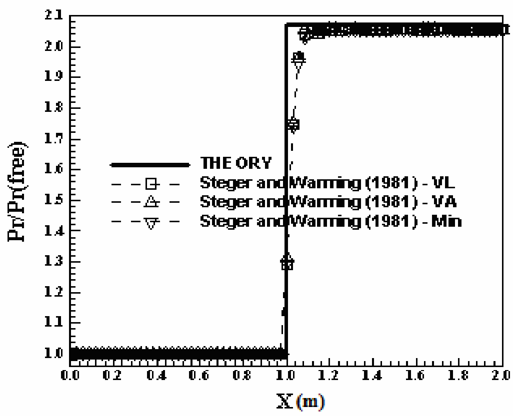

Figure 25. Wall pressure distributions (SW-1).

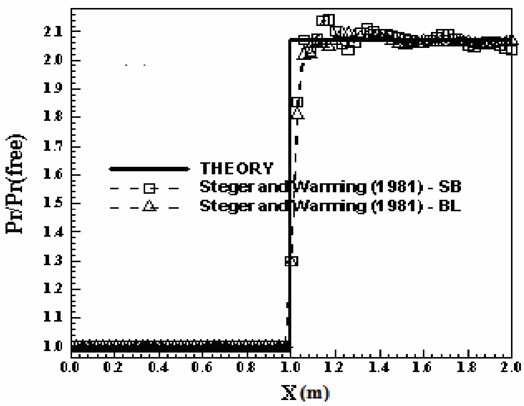

Figure 26. Wall pressure distributions (SW-2).

Figure 24 shows the wall pressure distributions obtained by all five variants of the Steger and Warming (1981) TVD scheme. They are compared with the oblique shock wave theory. As noted, some solutions present oscillations at the compression corner, mainly the Steger and Warming (1981) TVD scheme using the SB limiter. Figure 25 exhibits the wall pressure distributions obtained by the Steger and Warming (1981) TVD scheme using VL, VA and Min limiters. No overshoot or undershoot are observed in the solutions, presenting these ones a smooth behavior. It is also possible to note that the shock discontinuity is captured in three cells, which is also a typical number of cells encountered in high resolution schemes to capture accurately shock waves. Hence, as in the explicit case, the accuracy of the Steger and Warming (1981) TVD scheme with these three limiters is in accordance with typical results of current high resolution schemes. Figure 26 shows the wall pressure distributions obtained by the Steger and Warming (1981) TVD scheme using the SB and BL limiters. The SB limiter yields oscillations along the shock plateau, but the shock is captured in three cells, as is the case with the BL limiter. By the results, the best solutions were obtained with VL, VA and Min limiters, which even capturing a less severe pressure after the shock, detect sharp and smooth pressure distributions at the corner wall.

Analyzing the oblique shock wave angle, using a transfer in Figures 19 to 23, it is possible to obtain the values of β to each variant of the Steger and Warming (1981) TVD scheme, as well the respective errors, shown in Tab. 3. The Steger and Warming (1981) TVD scheme using the VA limiter has yielded the best result.

Table 3. Shock angle and percentage errors to the compression corner problem (SW/Implicit case).

Algorithm:	β (°):	Error (%):
Steger and Warming (1981) TVD – VL	27.8	1.09
Steger and Warming (1981) TVD – VA	27.6	0.36
Steger and Warming (1981) TVD – Min	27.8	1.09
Steger and Warming (1981) TVD – SB	27.0	1.82
Steger and Warming (1981) TVD – BL	27.9	1.45

Figures 27 to 31 exhibit the pressure contours obtained by the Radespiel and Kroll (1995) TVD scheme in its five variants to the implicit case. As can be observed the most severe pressure after the shock is captured by the Radespiel and Kroll (1995) TVD scheme using the BL variant, although the SB limiter yields the smallest shock wave thickness.

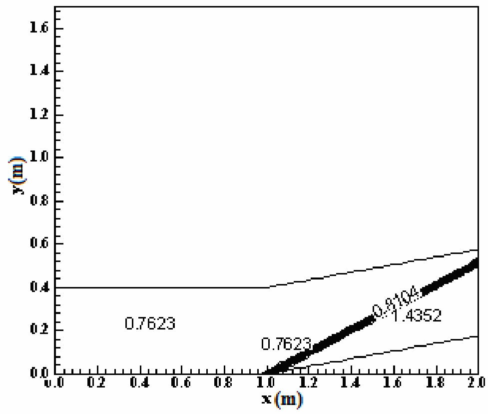

Figure 27. Pressure contours (RK-VL).

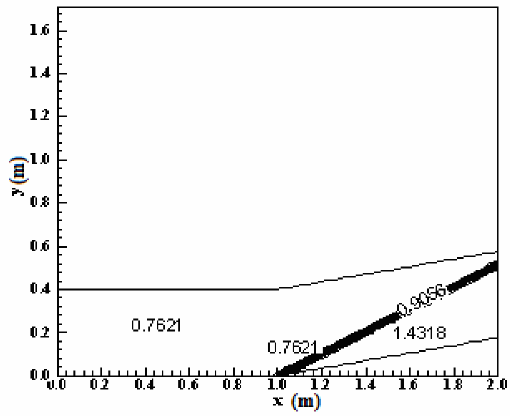

Figure 28. Pressure contours (RK-VA).

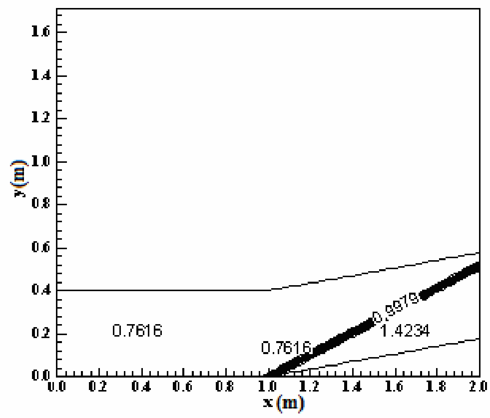

Figure 29. Pressure contours (RK-Min).

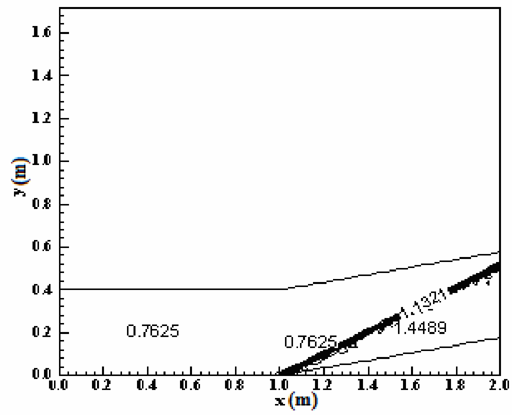

Figure 30. Pressure contours (RK-SB).

Figure 32 shows the wall pressure distributions obtained by all variants of the Radespiel and Kroll (1995) TVD scheme. They are compared with the oblique shock wave theory. As noted, some solutions present oscillations at the compression corner, mainly the Radespiel and Kroll (1995) TVD scheme using the SB limiter, but they are in less amount than in the solutions of the Steger and Warming's variants. Figure 33 exhibits the wall pressure distributions obtained by the Radespiel and Kroll (1995) TVD scheme using VL, VA and Min limiters. As observed, no overshoot or undershoot are noted in the solutions, presenting these ones a smooth behavior. It is also possible to observe that the shock discontinuity is captured in four cells, a typical number of cells to high resolution schemes capture accurately shock waves. Hence, the accuracy of the Radespiel and Kroll (1995) TVD scheme with these three limiters is in accordance with typical results of current high resolution schemes. Figure 34 shows the wall pressure distributions obtained by the Radespiel and Kroll (1995) TVD scheme using SB and BL limiters. The SB limiter yields oscillations along the shock plateau, but the shock discontinuity is also captured within four cells, as is the case with the BL limiter.

By the results, the best solutions were obtained with VL, VA and Min limiters, which even capturing a less severe pressure after the shock, detect sharp and smooth pressure distributions at the corner wall.

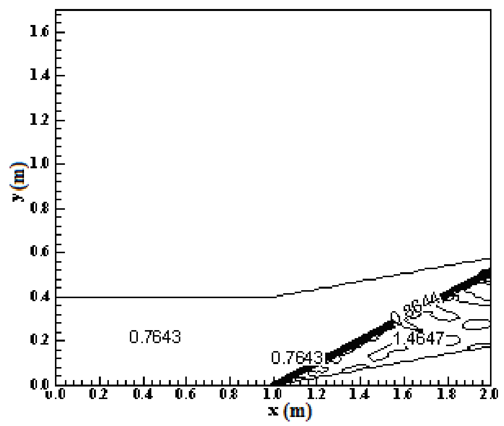

Figure 31. Pressure contours (RK-BL).

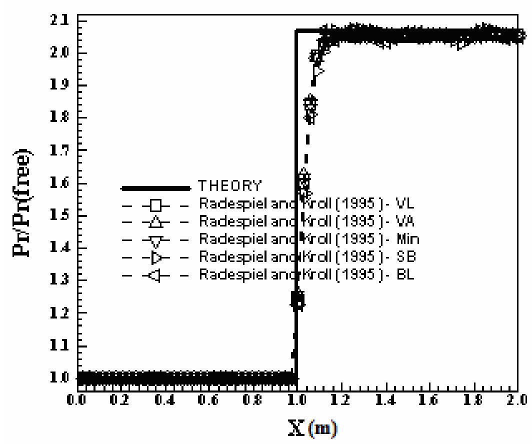

Figure 32. Wall pressure distributions (RK).

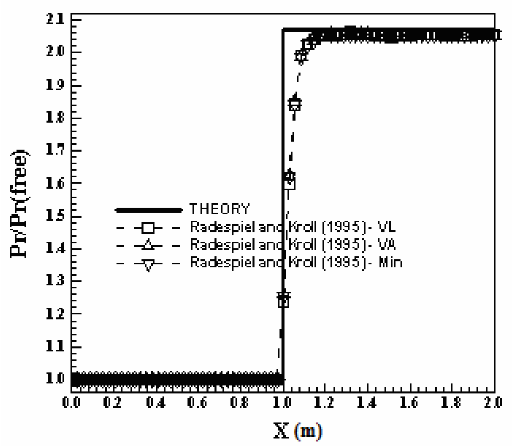

Figure 33. Wall pressure distributions (RK-1).

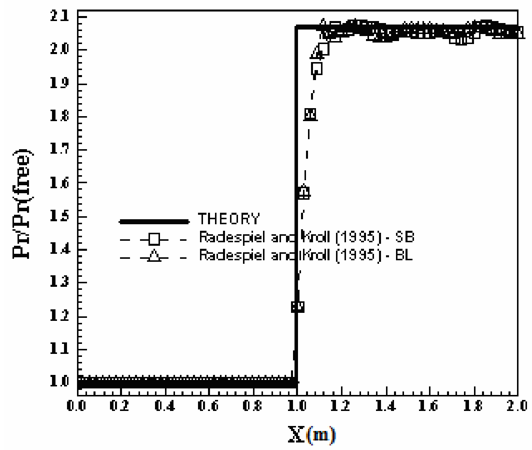

Figure 34. Wall pressure distributions (RK-2).

Analyzing the oblique shock wave angle, using a transfer in Figures 27 to 31, it is possible to obtain the values of β to each variant of the Radespiel and Kroll (1995) TVD scheme, as well the respective errors, shown in Tab. 4. The Radespiel and Kroll (1995) TVD scheme using the BL limiter has yielded the best result.

Table 4. Shock angle and percentage errors to the compression corner problem (RK/Implicit case).

Algorithm:	β (°):	Error (%):
Radespiel and Kroll (1995) TVD – VL	27.8	1.09
Radespiel and Kroll (1995) TVD – VA	28.0	1.82
Radespiel and Kroll (1995) TVD – Min	28.0	1.82
Radespiel and Kroll (1995) TVD – SB	27.7	0.73
Radespiel and Kroll (1995) TVD – BL	27.5	0.00

2.3. Explicit Versus Implicit Comparisons

Figure 35 exhibits the best wall pressure distributions obtained by the variants of the Steger and Warming (1981) TVD scheme and by the variants of the Radespiel and Kroll (1995) TVD scheme in their explicit versions. Only the solutions obtained with the VL, VA and Min limiters of each scheme are presented. The Steger and Warming (1981) TVD variants and the Radespiel and Kroll (1995) TVD variants are in close agreement. All solutions obtained by the variants of the Steger and Warming (1981) TVD scheme in the explicit results capture the shock discontinuity in three cells, as previously emphasized. The variants of the Radespiel and Kroll (1995) TVD scheme, in the explicit case, capture the shock discontinuity in four cells. The best wall pressure distribution in this comparison was obtained by the Steger and Warming (1981) TVD scheme using Min limiter. Figure 36 shows the best wall pressure distributions obtained by the variants of the Steger and Warming (1981) TVD scheme and by the variants of the Radespiel and Kroll (1995) TVD scheme in their implicit versions. Again, only the solutions obtained with the VL, VA and Min limiters of each scheme are presented. Again, all solutions obtained by the variants of the Steger and Warming (1981) TVD scheme in the implicit results capture the shock discontinuity in three cells, which maintain the same behavior in relation to the explicit solutions. The variants of the Radespiel and Kroll (1995) TVD scheme, in the implicit case, capture the shock discontinuity in four cells. The best wall pressure distribution in this comparison was obtained again by the Steger and Warming (1981) TVD scheme using Min limiter.

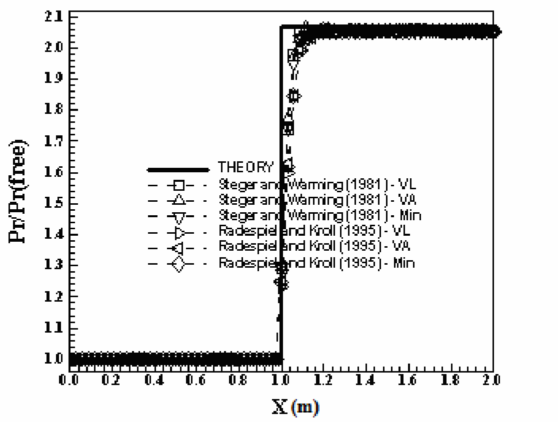

Figure 35. Wall pressure distributions (Exp).

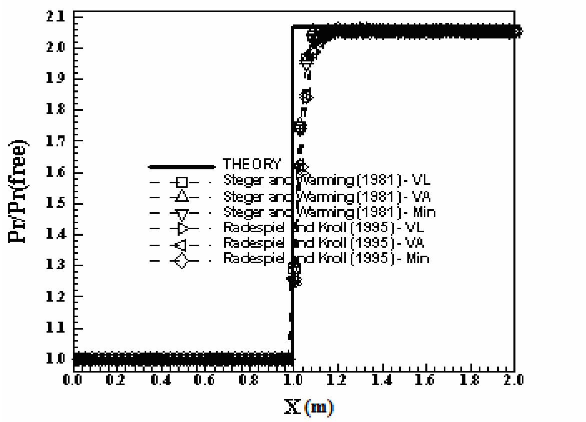

Figure 36. Wall pressure distributions (Imp).

Table 5 presents the best values to the shock angle obtained by the variants of the Steger and Warming (1981) TVD scheme and by the variants of the Radespiel and Kroll (1995) TVD scheme. As can be observed, the best results are obtained with the Steger and Warming (1981) TVD scheme using VL (explicit case) limiter and with the Radespiel and Kroll (1995) TVD scheme using VL (explicit case) and BL (implicit case) limiters. As can be concluded, the explicit results were better than the implicit ones. The Steger and Warming (1981) TVD scheme using Min limiter yielded the best wall pressure distribution in both cases and the best shock angle value in the explicit case using VL limiter. Hence, the Steger and Warming (1981) TVD scheme using VL and Min limiters are the best choice in this study.

Table 5. Shock angle and percentage errors to the compression corner problem (Explicit and Implicit Results).

Algorithm:	β (°):	Error (%):
Steger and Warming (1981) TVD – VL – Explicit	27.5	0.00
Radespiel and Kroll (1995) TVD – VL – Explicit	27.5	0.00
Radespiel and Kroll (1995) TVD – BL – Implicit	27.5	0.00

Table 6. Comparison between explicit and implicit computational costs.

Algorithm:	Exp. Cost(1):	Imp. Cost(1):	Increase (%):
Steger and Warming (1981) TVD – VL	0.0000111	0.0000399	259.46
Steger and Warming (1981) TVD – VA	0.0000114	0.0000401	251.75
Steger and Warming (1981) TVD – Min	0.0000111	0.0000397	257.66
Steger and Warming (1981) TVD – SB	0.0000109	0.0000398	265.14
Steger and Warming (1981) TVD – BL	0.0000109	0.0000395	262.39
Radespiel and Kroll (1995) TVD – VL	0.0000061	0.0000349	472.13
Radespiel and Kroll (1995) TVD – VA	0.0000062	0.0000347	459.68
Radespiel and Kroll (1995) TVD – Min	0.0000058	0.0000353	508.62
Radespiel and Kroll (1995) TVD – SB	0.0000059	0.0000344	483.05
Radespiel and Kroll (1995) TVD – BL	0.0000059	0.0000343	481.36

(1): Measured in seconds/per cell/per iteration.

Table 6 presents the computational costs of the variants of the Steger and Warming (1981) TVD scheme and of the variants of the Radespiel and Kroll (1995) TVD scheme, as also the respective percentage increase in the computational cost when passing from the explicit version to the implicit version. As can be observed the cheapest scheme, in its explicit version, is due to Radespiel and Kroll (1995) TVD scheme using Min limiter and the most expensive are due to the Steger and Warming (1981) TVD scheme using VA limiter. The increase in computational cost involving these two schemes is 96.6%; in other words, the Radespiel and Kroll (1995) TVD scheme using Min limiter is 96.6% cheaper than the VA variant of the Steger and Warming (1981) TVD scheme. In the implicit case, the cheapest scheme is the Radespiel and Kroll (1995) TVD scheme using BL limiter, whereas the most expensive is due to the Steger and Warming (1981) TVD scheme using VA limiter. The increase in computational cost when passing from the Radespiel and Kroll (1995) scheme using BL limiter to the Steger and Warming (1981) scheme using VA limiter is 16.9%.

Another important consideration taking into account Tab. 6 is the great increase in the computational cost involving the same variant of a numerical scheme when passing from explicit to implicit implementation. The Radespiel and Kroll (1995) TVD scheme using Min limiter presents the biggest increase in the computational cost involving all variants (about 508.0%), but it is important to note that all schemes suffer an increase of no minimal 252.0% when passing from explicit to implicit formulation, which means a great penalty to possibly improve the numerical results. In the present study, this penalty was not accepted due to none meaningful improvement in the capture of the shock discontinuity (the same number of cells – three – was detected in both cases with the Steger and Warming, 1981, scheme) was perceptible. The shock angle was also correctly captured by two schemes, in place of only one, when using the explicit formulation. Hence, the author concludes that an explicit formulation, with these algorithms, yields a satisfactory behavior in the numerical results to capture shock discontinuities.

3. CONCLUSIONS

In the present work, second part of this study, the numerical results obtained by the solution of the second order versions of the numerical schemes of Steger and Warming (1981) and of Radespiel and Kroll (1995), which incorporate TVD properties by using a MUSCL approach, is presented. Results with the implicit numerical implementation of these second order schemes are also reported for comparison with their explicit counterparts. The schemes are implemented on a finite volume context, using a structured spatial discretization. First order time integrations like ADI approximate factorization are programmed. The Euler equations in conservation and integral forms, in two-dimensions, are solved. The steady state physical problem of the moderate supersonic flow along a compression corner is studied and compared with theoretical results. A spatially variable time step procedure is also implemented aiming to accelerate the convergence to the steady solution. The gains in convergence with this procedure were demonstrated in Maciel (2005).

The results have demonstrated that the most accurate solutions are obtained with the Steger and Warming (1981) TVD scheme using VL and Min nonlinear limiters, when implemented in its explicit version. The best results in the capture of the shock discontinuity in the corner problem were obtained by the VL, VA and Min variants of the Steger and Warming (1981) and of the Radespiel and Kroll (1995) TVD schemes, without present oscillations, under- or overshoots. This behavior was observed in the explicit and implicit simulations. Both explicit and implicit simulations capture the shock discontinuity in three cells when using the Steger and Warming (1981) TVD scheme, whereas in four cells when using the Radespiel and Kroll (1995) TVD scheme. The best wall pressure distribution obtained in both explicit and implicit cases are due to Steger and Warming (1981) TVD scheme using Min limiter. The shock angle was correctly estimated by the Steger and Warming (1981) TVD scheme using VL (explicit case) limiter. The Radespiel and Kroll (1995) TVD scheme also captures correctly the shock angle using VL and BL limiters in the explicit and implicit cases, respectively. In other words, the explicit formulation provides best results. Hence, as the explicit results are better than the implicit ones and taking into account that the Steger and Warming (1981) TVD scheme using VL and Min limiters yield the best shock angle and wall pressure distribution, respectively, in this case, this scheme with these variants is the best choice in this study.

As can be observed from Table 6 the cheapest scheme, in its explicit version, is due to Radespiel and Kroll (1995) TVD scheme using Min limiter and the most expensive is due to the Steger and Warming (1981) TVD scheme using VA limiter. The increase in computational cost is 96.6%. In the implicit case, the cheapest scheme is the Radespiel and Kroll (1995) TVD scheme using BL limiter and the most expensive is due to the Steger and Warming (1981) TVD scheme using VA limiter. The increase in computational cost is 16.9%.

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