H6DIFERENT ASPECT OF CRITICAL CRACK PROPAGATION IN ELASTIC SOLID STUDY USING THE BAR LIKE DISCRET ELEMENT METHOD

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Abstract. *The simulation of critical crack propagation in Functionally Graded Materials (FGM) is an open problem in the mechanical simulation field nowadays. In the present work this problem is analyzed using the Discrete Element Method (DEM). This method has been used with success in several areas of engineering where the simulation of fracture and fragmentation is relevant. In the present paper several examples are shown. In these examples the results are presented in terms of energetic balance during the fracture process and crack configurations during the whole fracture process.*
The results of the examples that will be presented, show a good correlation with the other author's results, and therefore indicate that the DEM method can be considered as an alternative tool to simulate and help to understand these kinds of problems.

Keywords*: Fracture Mechanics, Critical Propagation, Functionally Graded Materials.*

1. INTRODUCTION

The critical crack propagation in elastic solid materials, particularly when a variable distribution of mechanical properties is presented, has received considerable attention from researchers and engineering communities in the last years. The Functionally Graded Materials (FGM) is a new generation of engineering composites characterized to have a smooth variation of mechanical/thermal, or electromagnetic properties. They are new advanced multifunctional materials, which are tailored to take advantage of their constituents, for example in a ceramic/metal FGM, heat and corrosion resistance of ceramics work together with mechanical strength and toughness of metals. To carry out an application of FGMs, scientific knowledge of fracture and damage tolerance is important for improving their structural integrity.

For this kind of problems, the Finite Element Method with cohesive interfaces is very much used (Xu and Needleman, 1995; Paulino and Zanhg, 2005). The version of the Discrete Element Method (DEM) that will be used in the present work has also been employed in problems where the fracture and fragmentation need to be taken into account during the analysis.

Some examples at this respect are mentioned as follows: In Schnaid et al, 2004 the rupture foundation built with soil bearing a soft sand bed is shown, in Dalguer et. al, (2003) the simulation of the generation and posterior propagation of the seism are taken, in Rios and Riera (2004), the scale effect in concrete is presented and finally the most recent work of Miguel et. al (2008) where the scale effect in rock mechanic is studied. In the previous work carried out by the authors (Barrios D'Ambra et al., 2007; Kosteski et al., 2006 and Kosteski et al. 2008) , it was demonstrated that DEM is capable to measure both the static and dynamic Stress Intensity Factor through different methodologies.

The DEM success to model failure mechanism in brittle materials and the ability of the method to capture the phenomenon as the nucleation of defect motivates its application in problems of Fracture Mechanics.

2. DISCRETE ELEMENT MODEL DESCRIPTION

DEM, in the used version, was developed by Hayashi (1982) and then modified by Rocha (1989) and Iturrioz (1995). It consists essentially in continuous spatial discretization in reticulated regular modules, where the stiffness of bars is defined in such a way to represent the equivalent continuum.

The model mass is discretized and concentrated in the model nodes. Figure 1 shows a module with eight nodes in their vertices and a central node. Each node has three associate degrees of freedom given by the spatial components of the displacement field $\boldsymbol{u}$. Longitudinal and diagonal elements with length Lc and $\sqrt{3}/2\ Lc$ respectively join the masses.

The linear elasticity field Hayashi (1982) checks the equivalence between the cubic array and elastic orthotropic solid with the major axes of the material oriented in the longitudinal element direction. A restriction should be imposed in the Poisson module of $\nu = 0.25$ for perfect equivalence. For other ν values there are slight differences in the shear terms, which can be ignored, especially when our interest is the nonlinear response of the studied model.

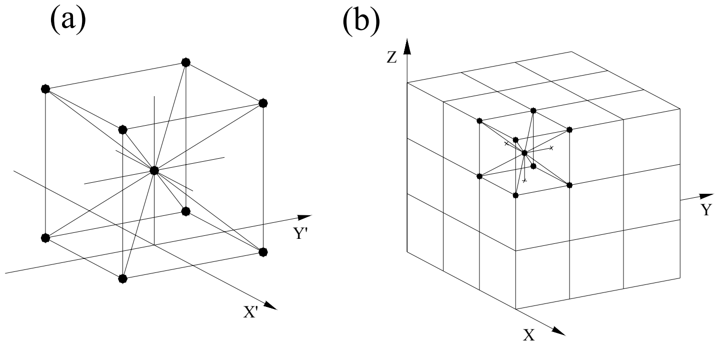

Figure 1. a) Core cubic module detail, b) Prism composed of several cubic modules.

When the materials have linear elastic behavior we can express the N degrees of freedom system motion equation resulting from the spatial discretization as:

$$\mathbf{M}\ddot{\mathbf{u}}+\mathrm{C}\dot{\mathbf{u}}+\mathbf{K}\mathbf{u}=\mathbf{q}(\mathrm{t}) \quad (1)$$

where **M** denotes the diagonal mass matrix, $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ represent displacement and generalized accelerations vectors respectively. **K** represents the stiffness matrix. On the other hand the vector **q**(t) contains the external forces applied. As simplified hypotheses the damping matrix **C** is adopted, proportional to the mass. Equation (1) can be numerically integrated in time domain using a classic scheme of explicit integration (Method of Central Finite Differences).

By means of the nodal coordinate updating in each time step allows to consider the finite displacement as response in natural way i.e. to consider the non linearity geometrics in the analysis without additional cost.

2.1 The elemental constitutive law for the brittle crack modeling

In 1989 Rocha proposed a constitutive bilinear law for the bars based on Hillerborg (1979) model. This law allows DEM to model the brittle failure of the material. In general lines this law can be presented as follows:

Force = function (bar strain) (2)

The constitutive relation is shown in Fig. 2. This figure demontrates that the compression element response is linear elastic, the rupture of the compressed model happened by indirect traction (Poisson effect). In Fig. 2 *Pc* represents the maximum tensile force transmitted by the element, ε_p represents the associated strain to *Pc*, E_A is a constant proportional to the stiffness that links the two before mentioned parameters that is ($Pc = E_A\ \varepsilon_p$) and k_r is a ductility parameter that permits to compute the limit strain ε_r to which the element exhausts its capacity to transmit efforts.

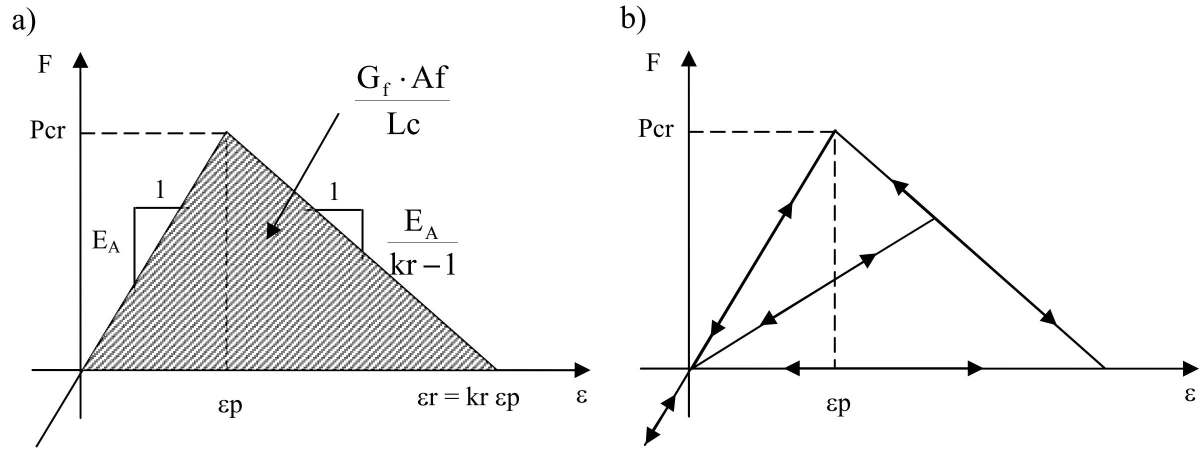

Figure 2. Elemental Constitutive Law of the DEM bars. – a) Adopted constitutive diagram with its control parameters; b) Load and unload scheme. Rocha (1989).

In this way DEM takes into account the nucleation of damaged and posterior material failure, this fact translates itself in deactivation of the bars, which exhaust their strength.

3. APLICATION EXAMPLE

The present example shows a rectangle plate with central notch of Polimetilmetacrilate (PMMA).The dimensions in Fig. 3 are presented. This plate is submitted to imposed constant strain velocity of 5m/s in its extreme borders. Paulino and Zhang (2005) studied the influence that the Functionally Graded Material (FGM) has in the crack dynamic propagation in the same geometry using the Finite Element Method with cohesive zone interface. The cohesive elements were implemented in the region where the crack propagation could occur.

The cited authors studied the influence of three types of different FGM in the crack propagation event. In case 1 homogeneous property was considered. In case 2 a hypothetic FGM material was proposed, where just the cohesive properties graded linearly in y axis direction. Finally, in case 3 not only the cohesive properties, but also the finite element properties were graded.

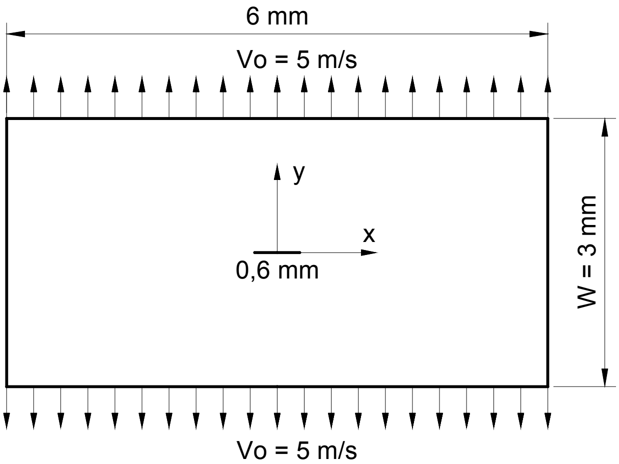

Figure 3. Layout and boundary condition of the rectangular plate with central notch and with constant velocity imposed.

3.1. DEM Model

Only the middle of the plate was modeled due to the geometry and load symmetries of the problem.

The plate was discretized with 60 modules of each side and one module in the thickness direction. The boundary conditions applied permit to represent the left border as a symmetry axis. The plane strain condition was imposed fixing the displacements in the thickness direction; the half of the notch was discretized using 6 cubic modules. The material properties and the main model characteristics are presented in Table 1.

Table 1. Material Properties and parameters used in DEM Model.

Material Properties		DEM Parameters	
E	3.24 GPa	Lc	5.00E-5m
ν	0.35	ν	0.25
ρ	1190 kg/m^3	Δt	1.0E-8 s
G_f	352.3 J/m^2	ε_p	0.025
ε_{YLD}	2-6%	k_r	2.41

Figure 4 shows the constitutive law adopted for the material considered homogeneous in the vertical direction of the plate.

The critical step time is related to the time that an elastic wave takes to pass through the normal elemental bar. In the present case for homogeneous material the step critical time was Δt_{crit} =1.82 E^{-8} s.

The main difference between Paulino and Zhang's (2005) analysis and the present study, is the Poisson coefficient value, in DEM model, when we work with the cubic arrangement (see Fig.1) we get limited to work with Poisson's coefficient of 0.25, if we want to model an elastic isotropic and homogeneous material.

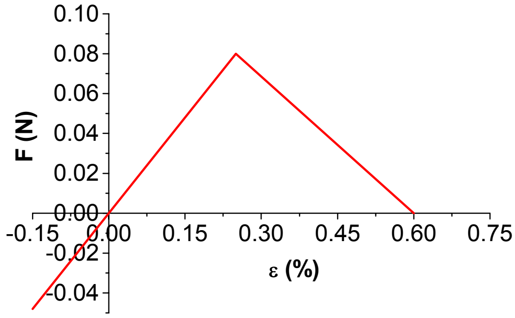

Figure 4. The elemental constitutive law used in DEM model.

If we want to model elastic and isotropic materials with other Poisson's coefficient, it will be necessary to employ another kind of geometric arrangement.

In table 2 the properties of the cases studied by Paulino and Zhang (2005) and the parameters adopted in DEM model are illustrated.

Table 2. Analized cases.

	y position	E [GPa]	G_{Ic} [N/m]
Case 1: homog.	-1/2 W to 1/2 W	3.24	352.3
Case 2: graded G_{Ic}	1/2 W	3.24	528.4
	-1/2 W	3.24	176.1
Case 3: graded E and G_{Ic}	1/2 W	4.86	528.4
	-1/2 W	1.62	176.1

3.2. Obtained Results

3.2.1. Case 1: Homogeneous case

The homogeneous material is considered in this first case. Thus in this case there isn't any variation in the elemental constitutive law in DEM model.

The Figure 5 a) shows a rupture configuration obtained by Paulino and Zhang (2005) that also simulates the middle plate, taking into account the advantage of having symmetry in boundary conditions and in its geometry. Also it is possible to observe in Figure 5 a) that the main crack begins to branch when it reaches a length of 1.05mm with an approximate angle of 29° in relation to the horizontal direction.

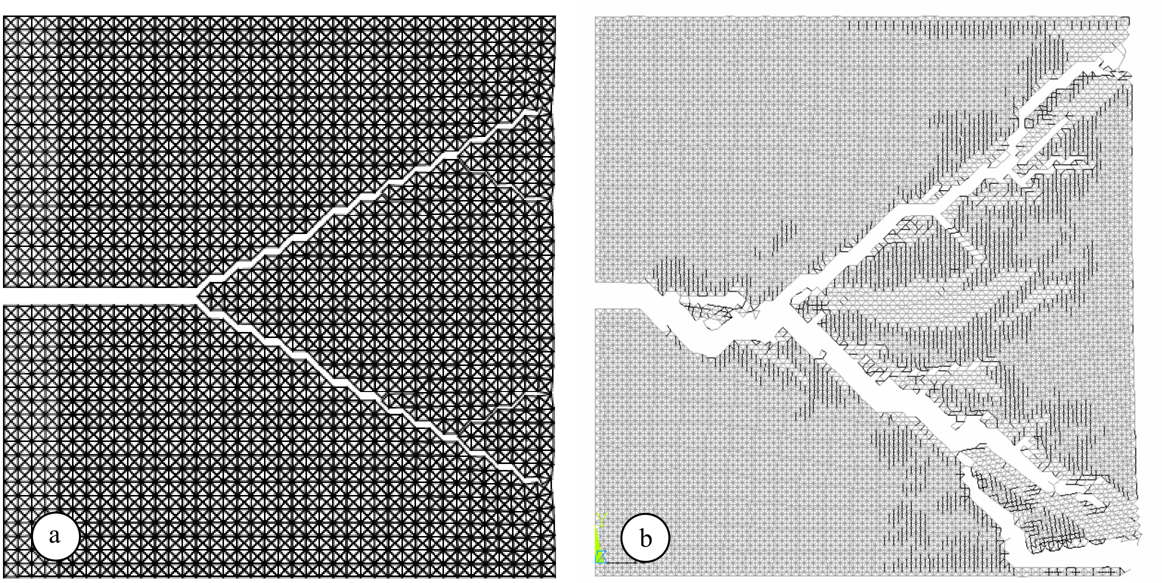

Figure 5. Final configuration for the case 1: Homogeneous material, a) Paulino and Zhang (2005), b) DEM.

In Figure 5b the final configuration obtained with DEM is shown using the parameters indicated in Table 1 with time step in the integration scheme of Δt de 0.10 E^{-8} s. The comparison between Fig. 5a and b shows that in DEM model the length where the bifurcation begins is 1.0mm and the branch angle is 32° in relation to the horizontal direction).

Another characteristic that we see in Fig. 5 (a and b) is the similarity between the two final configurations, for example, it is possible to observe a secondary branching in both figures.

The lack of symmetry in the final configuration of DEM simulation is a characteristic in unstable propagation process. This method has been very sensitive to perturbations that occur during the process. This perturbation could be produced by roundness errors in the calculus and due to little changes in the step integration time.

If these changes are carried out in DEM model, the final configuration is different in details, but shows the same general aspect.

3.2.2. Case 2: Graded G_{Ic}

In the present case only a toughness parameter Gc is graded in vertical direction.

The cohesive interface of the Finite Element Model used by (Paulino and Zhang, 2005) is characterized by three parameters that are: the normal maximum tensile in the interface T_n^{max} , the critical opening displacement δn and the area closed by the curve that is proportional to the toughness Gc. The grade in the interface properties that was implemented modified the curve, as illustrated in Fig. 6(a).

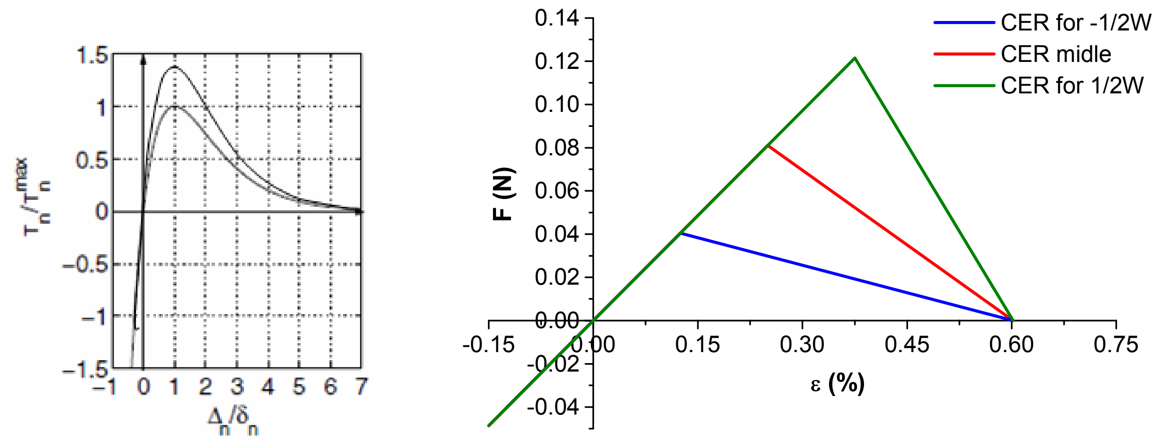

Figure 6. a) The cohesive interface law used by Paulino and Zhang, 2005. b) The elemental constitutive law used in DEM implementation to model the graded material in case 2.

The implementation of the graded material proposed in case 2, with DEM, was carried out through the way indicated in Fig. 6b.

It is possible to observe that Elastic Modulus directly linked with the initial slope is maintained constant. The Gc graduation is obtained modifying the critical strain ε_p , maintaining the ε_r constant to facilitate the comparison with the cohesive model.

The property material in the inferior border is: Gc = 176.1 N/m and ε_p = 0.0125, ε_r = 0.06 , these values grade linearly up to Gc = 528.4 N/m, ε_p = 0.0375 and ε_r = 0.06 in the superior border.

As the toughness minimum occurs in the inferior border and the maximum one occurs in the superior border, it is expected that the crack propagation occurs in the inferior region of the plate, as it is possible to observe in Fig. 7 a and b.

The comparison of Paulino and Zhang, 2005 with DEM in terms of final fractured configuration shows a significant similarity.

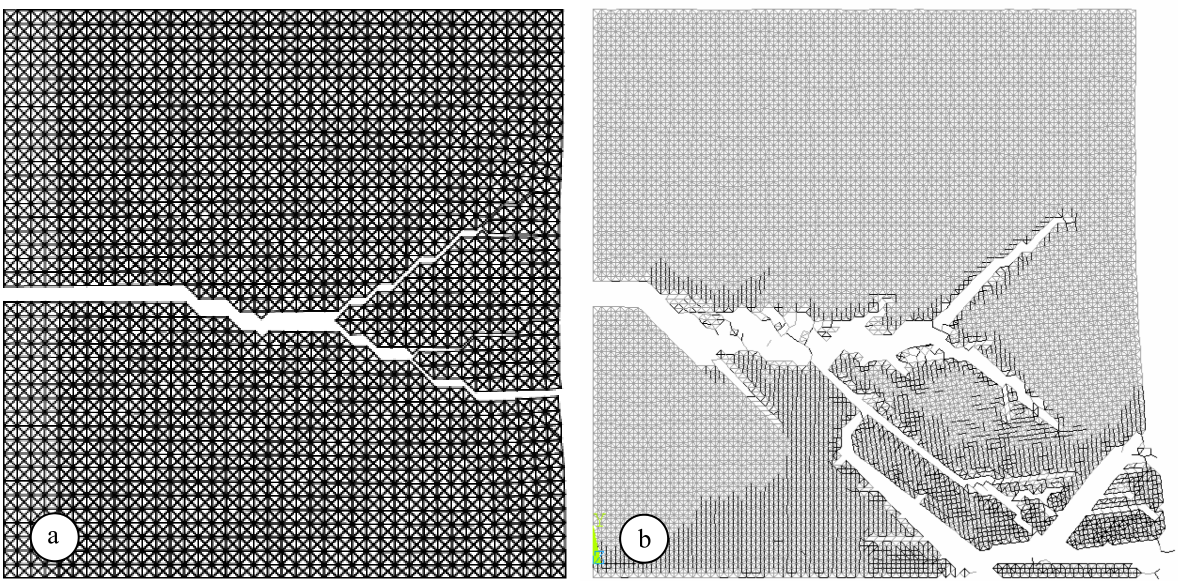

Figure 7. Configuration of rupture obtained for case 2 graded material, a) Paulino and Zhang, 2005. b) DEM.

3.2.3. Case 3: Graded E and G_{Ic}

In case 3, the Elasticity Modulus E and the toughness parameter Gc are changed simultaneously in vertical direction. At Paulino and Zhang (2005) a graduation was proposed to the interface properties in the vertical direction in the same manner of case 2, and the Elasticity Modulus E, in the elements, was modified.

In DEM approach for case 3, the constitutive elemental law was modified as shown in Fig. 8.

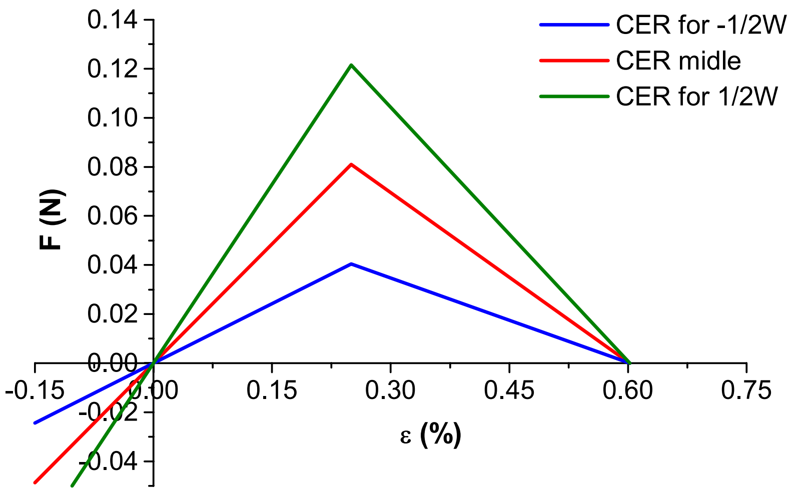

Figure 8. The Constitutive Law used in DEM implementation shows the graduation used in the vertical direction for case 3.

In Figure 9 the comparison between Paulino and Zhang's (2005) implementation and DEM implementation, in terms of final configurations, are shown. In the present case, the similarity of the two configurations is not too clear, but both show the same general tendency.

3.2.4 Comparison in terms of Energy Balance

In the present section we discuss the results in terms of Energy Balance during the whole fracture process. In Figure 10a the complete balance of energy during the whole process for case 1 is shown. Figures 10 b, c, d, the elastic, kinetic and dissipated energy through damage are shown for the three cases, respectively.

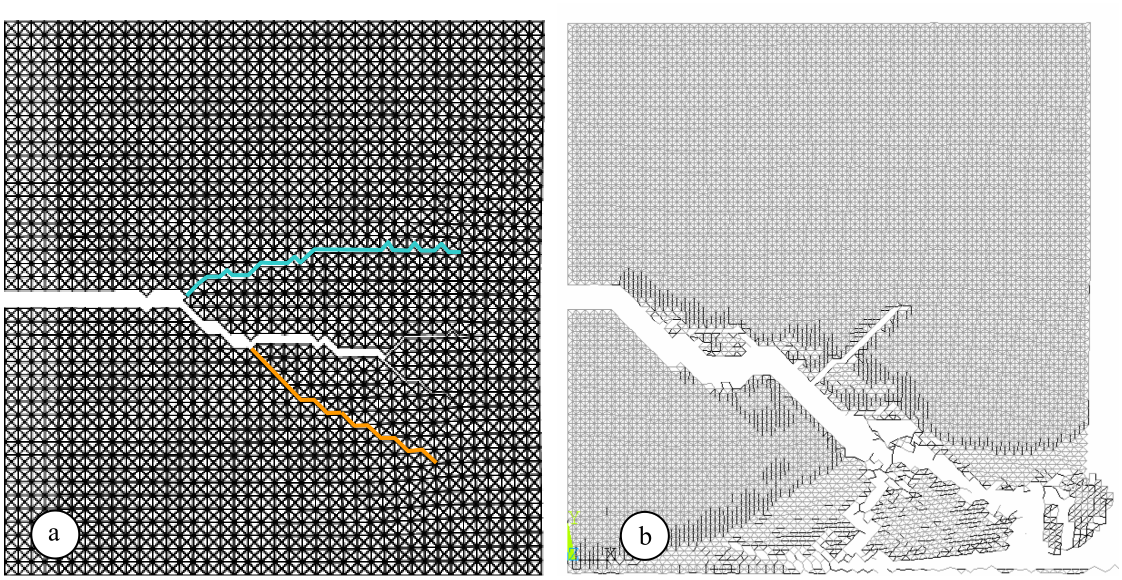

Figure 9: Final configurations obtained in case 3. a) Paulino and Zhang (2005). b) DEM.

As it follows, we mention some observation about the Figure 10 a to d.

- The main differences observed are between case 1 and cases 2 and 3.
- Case 1 shows more dissipated energy through damage due to the more quantity of fractured area generated in the fracture process.
- In Fig. 10 b the minor peak in case 2 and 3, curves are produced by the several secondary branching, that occur in these cases.
- In Fig. 10c the abrupt change in kinetic energy in cases 2 and 3 shows a sensitive change in the crack propagation velocity during the whole process. This is in accordance with the final configuration obtained, where quantities of branches and secondary cracks appear more than in case 1.

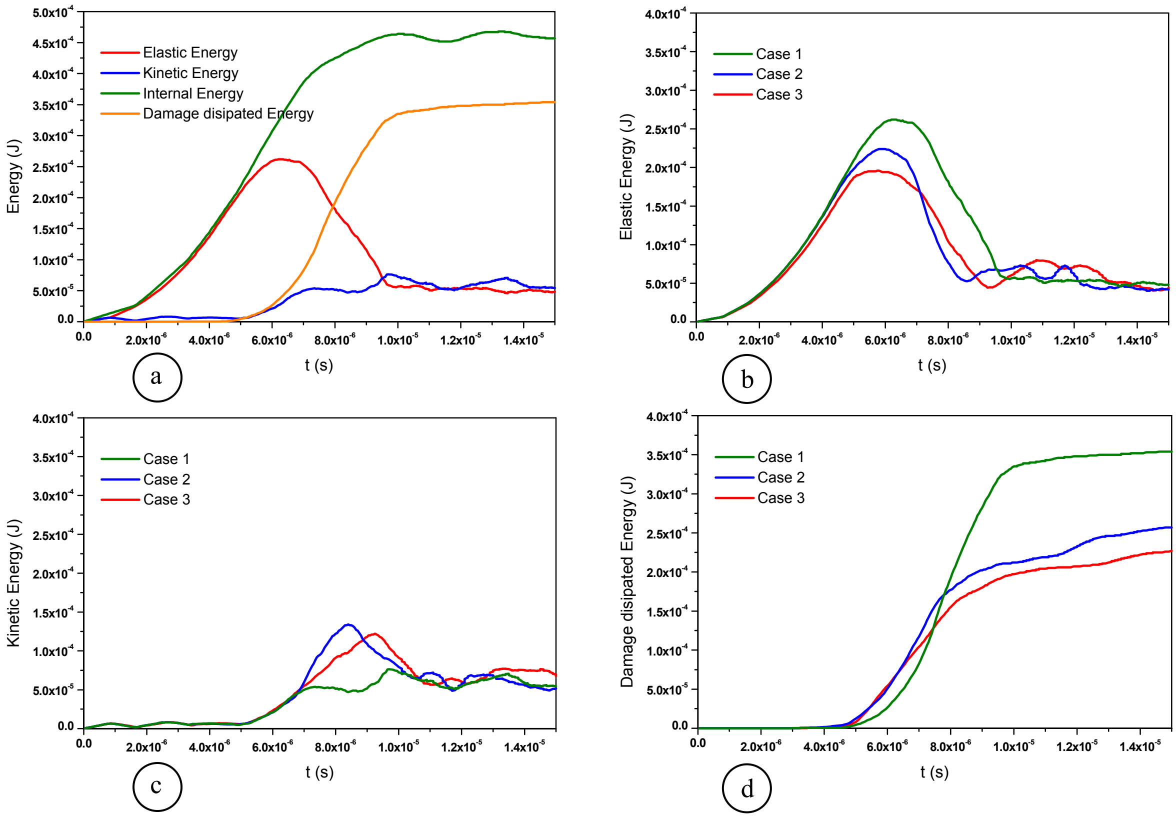

Figure 10: a) Energy balance during the simulated process in case 1, b) The elastic energy, c) Kinetic energy, d) damage dissipated energy for the three cases analyzed.

4. CONCLUSIONS

In the present work the Discrete Element Method to simulate a Graded Material example, analyzed before by Paulino and Zhang (2005) using the Finite Element Method and cohesive interfaces, was employed.

The comparison between two approaches show a good concordance and in consequence verifies that the Discrete Element Method could be used in problems where it is necessary to simulate a critical crack propagation. The results in terms of energy balance permit to explain this kind of process better.

Previous works accomplished by the authors such as Kosteski et. al (2008) the Calculus of Static and Dynamic Fracto-Mechanic parameters in geometric configurations of different levels of complexity using DEM were shown, and the results are satisfactory also in these cases.

5. REFERENCES

Barrios D'Ambra, R., Iturrioz, I., Fasce, L.A., Frontini, P.M., Cisilino, A.P., 2002, “Utilización del método de los elementos discretos en la simulación numérica de ensayos de impacto para caracterización de materiales compuestos poliméricos”, MECOM 2002- Santa Fé - Paraná, Argentina, Octubre 2002. Mecánica Computacional Vol XXI. ISSN 1666-6070. Pp.1121-1134.

Barrios D'Ambra, R., Iturrioz, I., Fasce, L.A., Frontini, P.M., Cisilino, A.P., 2003, “Simulación numérica del ensayo de impacto en probetas de polímeros utilizando el método de los elementos discretos”- Cilamce 2003.

Barrios D’Ambra R.L., Iturrioz I., Coceres H., Kosteski L.E., Tech T.W., Cisilino A., 2007, “Cálculo del factor de intensidad de tensiones utilizando el método de los elementos discretos”. Revista Sul-Americana de Engenharia Estrutural, Passo Fundo, v. 4, n. 2, p. 7-20.

Bocca P., Carpintieri A., 1990, “Snap-back fracture instability in rock specimens: experimenta detection through a negative impulse”, Engng. Fract. Mech, 35:241-250.

Dalguer A., Irikura K., Riera J. D., 2003, “Generations of New Cracks Accompanied by Dynamic Shear Rupture Propagation of the 2000 Tottori (Japan), Earthquake”, Bulletin of the Seismological Society of America, vol. 93, pp. 2236-2252.

Hayashi, Y., 1982, “Sobre um modelo de discretização de estruturas tridimensionais aplicado em dinâmica não linear”. Porto Alegre. Dissertação (Mestrado), Curso de Pós-Graduação em Engenharia Civil, Universidade Federal do Rio Grande do Sul.

Iturrioz, I., Riera, J.D., 2001, “Estudio numérico del efecto de explosivos sobre una superficie plana”, ENIEF XII, Córdoba, Argentina. Mecánica Computacional Vol 20 pp.422-429. Córdoba, Noviembre de 2001.

Kosteski L., Cóceres H., Barrios D'Ambra R., Iturrioz I., Cisilino A., 2006, “Aplicación del Método de los Elementos Discretos para la Determinación del Factor de Intensidad de Tensiones Estático y Dinámico”, Mecánica Computacional Vol XXV, pp. 2109-2123, Alberto Cardona, Norberto Nigro, Victorio Sonzogni, Mario Storti. (Eds.), Santa Fe, Argentina.

Kosteski L., Barrios D'Ambra R., Iturrioz I., 2008, “Determinación de Parámetros Fractomecánicos Estáticos y Dinámicos Utilizando el Método de los Elementos Discretos de barras”, Revista Internacional Métodos Numéricos en Ingeniería (RIMNI), Vol24, Num: 4, pp. 323-343.

Miguel L.F.F., Riera J.D., Iturrioz I., 2008, “Influence of size on the constitutive equations of concrete or rock dowels”, International Journal for Numerical and Analytical Methods in Geomechanics, vol. 32 Issue 15, pp. 1857 – 1881.

Paulino, G.H. y Zhang, Z., 2005, “Cohesive Zone modeling of dynamic crack propagation in functionally graded materials” 5th GRACM International Congress on Computational Mechanics. Limassol, 29 June - 1 July.

Riera, J.D. and Iturrioz, I., 1995, “Discrete element dynamic response of elastoplastic shells subjected to impulsive loading”, Communications in Numerical Methods in Engineering, Vol. 11, pp 417-426.

Riera, J.D. and Iturrioz, I., 1998, “Discrete element model for evaluating impact and impulsive response of reinforced concrete plates and shells subjected to impulsive loading”, Nuclear Engineering and Design, Elsevier, Amsterdam, The Netherlands, 179, 135-144.

Rios R.D., Riera J.D., 2004, “ Size effects in the analysis of reinforced concrete structures”, Engineering Structures, vol. 26, pp. 1115-1125.

Rocha, M.M., 1989, “Ruptura e efeito de escala em materiais não homogêneos de comportamento frágil”, Porto Alegre, 123 p. Dissertação (Mestrado), Curso de Pós-Graduação em Engenharia Civil, Universidade Federal do Rio Grande do Sul.

Spellmeyer, T., Barrios D’Ambra, R., Iturrioz, I., 2001, “Simulación numérica de la propagación de fisuras en sólidos utilizando el método de los elementos discretos”, ENIEF XII, Córdoba, Argentina. Mecánica Computacional Vol 20 pp.506-513. Córdoba, Noviembre de 2001.

Xu, X., Needleman, A., 1995, “Numerical simulations of dynamic crack growth along an interface”, International Journal of Fracture 74, pp. 289-324.