OPTIMIZATION OF PUMP-TURBINE USING SEQUENTIAL QUADRATIC PROGRAMMING AND GENETIC ALGORITHMS

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Abstract. *This work presents a computational low cost methodology useful to optimize design of radial pump-turbine. The methodology is based on one-dimensional flow modeling, empiric correlations for energy losses determination and local (SQP) and global (GA and NSGA-II) techniques of optimization. This methodology has the task to search basic geometries that maximize the value of the efficiency for the turbine operating mode as well as for pump operating mode. Due to the small number of design variables (runner blades inlet and outlet angles, wicket gates stagger angle and inlet and outlet stay vanes angles) it was first accomplished an optimization based on mono-objective used to maximize the turbine and pump efficiency but each one at a time. After that a more elaborated multi-objective optimization took place. As an example of application of the developed methodology and tools it is presented and analyzed a radial pump-turbine of specific speed of 30. Such pump-turbine has been previously tested in a laboratory. The results are comparable with the original design of the pump-turbine.*

Keywords*: Turbomachinery, pump-turbine, mean streamline analysis, energy losses, optimization, sequential quadratic programming, genetic algorithms*

1. INTRODUCTION

The optimized design of a hydraulic turbomachinery is necessary in order to minimize the hydraulic losses of each hydromechanics component. Due to the complexity of the flow into the turbomachinery, which is also aggravated by a complex geometry, it turns very difficult to go through a complete calculation. The technique for Fluids Dynamic Computation (CFD) has been used mainly for the flow interaction between some of the main components of a conventional pump, Bert *et al.* (1996), as well as a conventional turbine, Nakamura and Kurosawa (2006). These techniques have also been employed in reversible pump-turbine for the flow interaction calculation with the guide vane and stay vane when the turbomachinery operates as a pump, Ciocan *et al.* (1996), and, between the runner and draft tube, when it operates as a turbine, Kirschner *et al.* (2007). Even when the turbomachinery geometry is not completely defined a methodology is essential for the start of the conceptual design optimization.

This work presents a low computational cost methodology for the optimization of the basic design of a reversible pump-turbine in the design operation point. Such methodology is based on the one-dimensional flow and makes use of several empiric correlations. In a step further the design optimized algorithm including solver and optimization method was implemented in a computational program written in Matlab™. For a given flow, speed, some pre-defined geometric variables and imposed constraints this program searches for basic geometry that allows for the pump-turbine total maximum efficiency operating at both modes. Two optimization methods are used: 1) Local Search Method (mono and multi-objective) by Sequential Quadratic Programming (SQP) and 2) Global Search Method (mono and multi-objective) using Genetic Algorithm (GA) and NSGA-II (multi-objective). This work is structured in the following way: At first it is presented the optimization problem definition and then a brief summary of the optimization techniques. After that it is presented a methodology to get the turbine and pump solver. The last part deals with the results analysis and the conclusions.

2. THE OPTIMIZATION PROBLEM

In the engineering world practical problems are often conflicting. Sometimes more than one objective function has to be simultaneously either minimized or maximized. Problems of that nature are treated as multi-objective optimization also named multi-criteria. The reversible pump-turbine optimization problem brought in this work is solved by the maximization of the total efficiency in both operating modes, either as turbine or as a pump. To solve this problem is appropriate to use the following formulation, Eq. (1).

$$\text{Minimize} \quad f(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}), \quad \text{subject to} \quad g_j(\mathbf{x}) \le 0, j=1,\dots,m, \quad x_i^{\inf} \le x_i \le x_i^{\sup}\ i=1,\dots,n \qquad (1)$$

The functions $f_1(\mathbf{x}) = -\eta_T(\mathbf{x})$ and $f_2(\mathbf{x}) = -\eta_B(\mathbf{x})$ represent the reversible pump-turbine total efficiency in both operating modes. The minimization of $-\eta_T(\mathbf{x})$ and $-\eta_B(\mathbf{x})$ clearly result in the maximization of $\eta_T(\mathbf{x})$ and $\eta_B(\mathbf{x})$. The net head, H_T, and the effective head, H_B, respectively given for both operating modes and resulted from the geometry optimization process must remain between upper and lower bonds previously set. This also applies to the hydraulic efficiency η_{hT} and η_{hB}. These variables together establish the non-linear constraint $g_j(\mathbf{x}), j=1,\dots,m.$ of the problem. For the definition of the design space variables (viable region or search region, *S*) it is used side constraints applied to the design variables. Given some geometric variables and the reversible pump-turbine flow-speed pair of values the search for the optimum design variables begins. They are the runner blades inlet and outlet angles, wicket gates stagger angle and inlet and outlet stay vanes angles.

With the n-dimensional vector x for the decision variables (design variables), $\mathbf{x}=[x_1,x_2,\dots,x_n]^{\mathrm{T}}$, the region *S* is defined by the upper and lower bonds, $x_j^{\inf}$ and $x_j^{\sup}$, $S=\left\{\mathbf{x}\in\Re^n : x_i^{\inf}\le x_i \le x_i^{\sup}, i=1,\dots,n\right\}$, which restricts each decision variable of x. The Solver is based in the Sequential Quadratic Programming (SQP) through the function code `fmincon` of the MATLAB®. It is then intended to find the solution vector $\mathbf{x}^*$ of the problem (optimum solution). The solution given by the method based on the gradient is considered an optimum local solution which can be also a global solution depending on the optimization starting point. It is recommended to use more than one optimization technique to check the convergence of the results, Arora (2004). To overcome the problems faced by the SQP, it is used in this work two population methods. The establishment of the constraints is formulated using quadratic penalization method. This explains how the objective function is penalized. The population method is given by the respective function `ga` and `gamultiobj`, of the MATLAB®. The first function referrers to the standard Genetic Algorithm (GA) as long the second function refers to the NSGA-II algorithm, as described by Deb (2001).

3. OPTIMIZATION METHODS

In the context of the restricted optimization the main idea of the SQP is to obtain a search direction, solving a quadratic programming (QP) sub-problem at each iteration. This means to get a search direction solving a quadratic sub-problem with quadratic objective function and linear constraints. This is anything else than the idea behind the quasi-Newton methods for the unrestricted minimization, Antoniou and Lu (2007). In other words it can be said that an optimization problem with constraint is converted into another sub-problem further simpler. Thus it can be solved with an interactive process without constraint. For the SQP implementation one has to follow three main steps: 1) Hessian matrix actualization; 2) Solve the quadratic problem and new search direction and 3) Merit function. In the Fig. 1 (a) it is shown the optimization process flowchart for the SQP.

The Genetic Algorithm is a stochastic method that allows a parallel search mechanism. It is also adaptive because it is based on the natural selection theory where an organism is more likely to survive long enough to reproduce according to its aptitude, Fonseca and Fleming (1993). There are many others variety of the Genetic Algorithms with no rigorous definition, e.g., some GA can differ as their individuals are moving towards the next generation. As any other evolution algorithm the fundamental idea of the GA rests over 5 main components according to Michalewicz (1996): 1) Genetic representation for potential solution of the problem giving codes for a set of parameters; 2) Creation of the initial population; 3) Evaluation of the objective function by fitness assignment; 4) Genetic operators application during reproduction; 5) Values attribution for the genetic parameters like size of population, operators application probability, etc. In the Fig. 1 (b) it is shown the flowchart for the design optimization using GA. In a simple step the Pareto-front is got with the NSGA-II. This is an algorithm pure elitist in the sense that only the parents and childs which belong to the set of non-dominated individuals are involved in the crossover process and selection. In the Fig. 2 it is shown briefly the mechanism of the NSGA-II, Deb (2001).

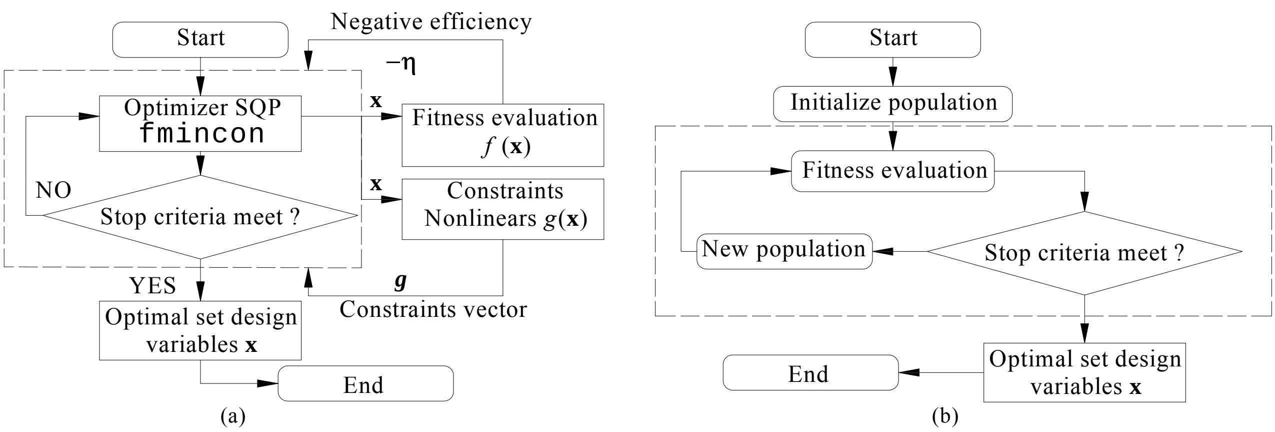


Figure 1. Optimization flowchart for the SQP and GA.

For the case of a general minimization problem there are sets of inferior, superior and non-inferior solutions. A solution vector $\boldsymbol{u}=(u_1,\ldots,u_n)$ is said an inferior solution relative to the solution vector $\boldsymbol{v}=(v_1,\ldots,v_n)$, if $\boldsymbol{v}$ is partially less than $\boldsymbol{u}$, $\forall i=1,\ldots,n, v_i \le u_i$ e $\exists\, i=1,\ldots,n: v_i < u_i$. It means that at least one classified individual exist in $\boldsymbol{v}$ which is less than any other individual in $\boldsymbol{u}$. Following that idea a vector $\boldsymbol{u}=(u_1,\ldots,u_n)$ is said a superior solution relative to $\boldsymbol{v}=(v_1,\ldots,v_n)$ if all $\boldsymbol{v}$ is inferior relative to $\boldsymbol{u}$. Finally the solution $\boldsymbol{u}=(u_1,\ldots,u_n)$ and $\boldsymbol{v}=(v_1,\ldots,v_n)$ are said non-inferiors one relative to the other (Pareto's optimum) if $\boldsymbol{v}$ is nor inferior neither superior relative to $\boldsymbol{u}$, Fonseca e Fleming (1993). Since the first multi-objective technique "Vector Evaluated Genetic Algorithm" a great number of publications about evolutionary algorithm has been represented through of the years. Aiming to solve the engineering problems the Genetic Algorithms have been also suffered important contribution and advancement by all areas. The main aspects and differences of the many types of Evolution Algorithm can be found in the works of Coello (1999) and Konak *et al.* (2006). The format of the viable and unviable regions of the Pareto-optimal set depends on the optimization problem, given constraints and established limits for the design variables. Depending on the optimization problem the Pareto-front can have a format of difficult identification. Li and Zhang (2009) presented several multi-objective problems with continuum functions with Pareto-front of difficult identification. They analyze the approximation of evolution algorithm to get the referred Pareto-front.

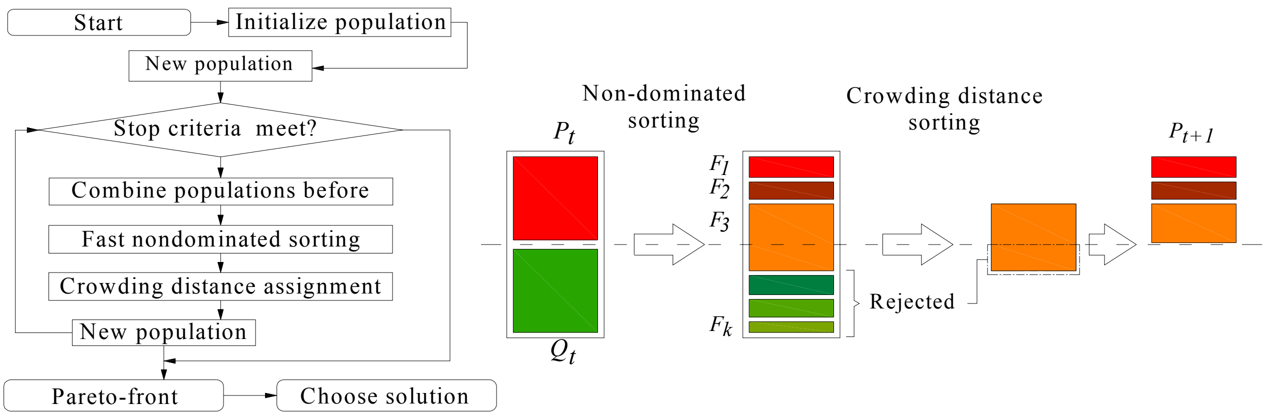


Figure 2. Schematic of the NSGA-II procedure.

4. METHODOLOGY TO OBTAIN PUMP AND TURBINE SOLVERS

To obtain the theoretical hydrodynamic characteristics for the pump and turbine operating modes is necessary to perform separate flow analysis for each hydromechanics component. To begin with this analysis the flow parameters in the exit of one component are calculated taking into consideration its geometry and given parameters of the entrance flow. Therefore the discharge in the exit of one component is used as the entry data for the discharge in the next component and so forth. One should take into consideration when applying this methodology that the runner or impeller discharge Q_{RT} or $Q_{RB,}$ is different from the turbine or pump discharge Q_T or Q_B. These discharges are tied by the efficiency losses η_{fT} or η_{fB}, which also depend on the runner flow characteristic. The Fig. 3 is used to show the main hydromechanics pump-turbine components but does not have the spiral case and the draft tube but both are included in this methodology. The symbols represented in the Fig. 3 (a) calls for the turbine mode and in the Fig. 3 (b) for the pump mode. The Fig. 3 (c) is intended to show the meridian view of the same components of the pump-turbine. The Tab. 1 contains the main dimensions of the hydromechanics components of the pump-turbine. The operating parameters for the best operation point for the turbine as well as for the pump are: $Q_T = 0.454$ (m^3/s), $n_T = 1000$ (rpm) and $H_T = 60.0$ (m) for the turbine mode and $Q_B = 0.335$ (m^3/s), $n_B = 1000$ (rpm) and $H_B = 51.4$ (m) for the pump mode.

4.1. Hydraulic losses

No emphasis will be given such as to help to understand the origin of the several different types of losses. In the works of Denton (1993) and Lakshminarayana (1996) are available the physical description of the mechanisms for several losses. The losses are defined by the increase in entropy. Coefficients are used to express them in terms of average variables.

For the spiral case in turbine mode the following losses are calculated: viscous friction in the reduction (section in the entrance of spiral case) and in the spiral case, losses due to bending of spiral case. For the pump mode besides the previous losses it is added the shock loss in the spiral case. For the guide vane, stay vane and runner in both operating modes it is calculated the following losses: shock loss, losses due to viscous friction and mixture losses. For the draft tube in turbine mode it is calculated the following losses: losses due to vortex in the cone, losses due to viscous friction in the cone, in the elbow and in the exit section, losses due to bending of the elbow and losses in the exit of draft tube. Still considering the draft tube in pump mode it is calculated the following losses: losses due to viscous friction in the reduction (end section), in the elbow and in the reduction (end section) and losses due to the bending of the elbow.

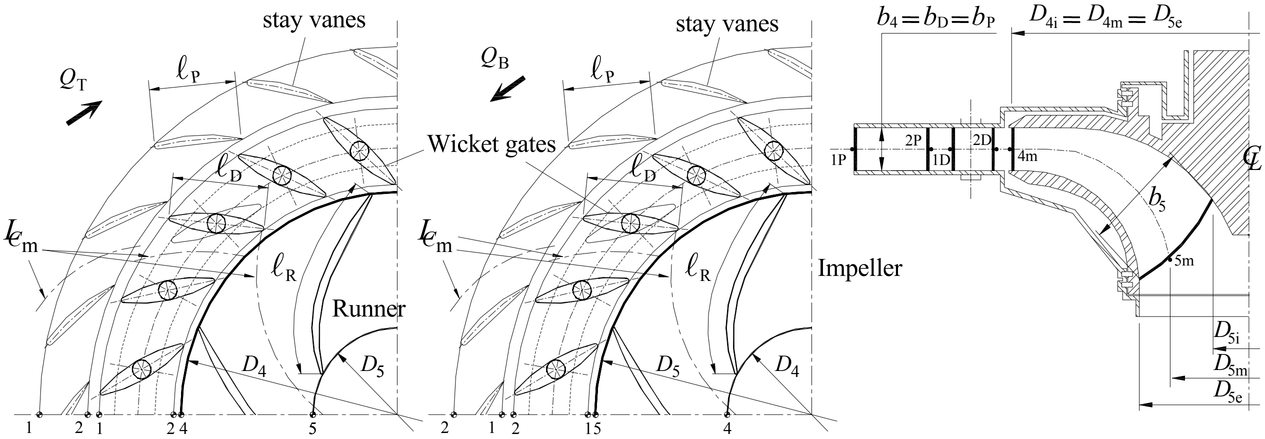


Figure 3. Transversal sections of the runner, guide and stay vanes, (a) Turbine mode and (b) Pump mode, showing the main geometric parameters and (c) Meridional section.

Table 1. Main geometric parameters of the pump-turbine.

Runner	**Symbol**	**Value**	**Unit**	**Pre-distributor**	**Symbol**	**Value**	**Unit**
Inlet diameter	D_4	618.4	mm	Inlet diameter	D_{1P}	1038	mm
Outlet diameter	D_5	265	mm	Outlet diameter	D_{2P}	829	mm
Inlet height	b_4	51	mm	Inlet height	b_{1P}	51	mm
Outlet height	b_5	135	mm	Outlet height	b_{2P}	51	mm
Surface roughness	ε_R	4	μm	Surface roughness	ε_P	6.3	μm
Number of vanes	N_R	6	-	Number of stay vanes	N_P	20	-
Distributor	**Symbol**	**Value**	**Unit**	**Spiral case**	**Symbol**	**Value**	**Unit**
Primitive diameter	D_{pD}	733	mm	Inlet diameter	D_{1C}	740	mm
Inlet diameter	D_{1D}	Variable	mm	Diameter at 180°	D_{2C}	350	mm
Outlet diameter	D_{2D}	Variable	mm	Surface roughness	ε_C	6,3	μm
Inlet height	b_{1D}	51	mm	**Draft tube**	**Symbol**	**Value**	**Unit**
Outlet height	b_{2D}	51	mm	Inlet diameter	D_7	291	mm
Surface roughness	ε_D	6.3	μm	Outlet diameter	D_8	540	mm
Number of wicket gates	N_D	20	-	Surface roughness	ε_T	6.3	μm

The losses due to viscous friction for several components previously mentioned are calculated by Eq. (2),

$$Z_{av} = f \frac{L}{D_h} \frac{V_{ref}^2}{2g} \tag{2}$$

where f is the friction coefficient calculated by the Swamee and Jain (1976) formula, L the length characteristic, D_h the hydraulic diameter, V_{ref} the reference speed and g the gravitational acceleration.

The bending losses calculated by Ueda *et al*. (1980) is given by the Eq. (3),

$$Z_{cur} = K_{cur} \frac{c_{cur}^2}{2g} \tag{3}$$

where K_{cur} is a loss coefficient due to the bending and c_{cur} the circumferential component of the absolute flow speed.

The shock loss for the spiral case, Ida and Kubota (1980), is given by Eq. (4),

$$Z_{ch} = \frac{\zeta_{ch_{car}} \, (c_{m_{2p}} + (c_{u_{2p}} - c_C)^2)}{2g} \tag{4}$$

where $\zeta_{ch_{car}}$ is the shock loss coefficient, $c_{m_{2P}}$ e $c_{u_{2P}}$, respectively, the meridional and circumferential of average absolute speed in the exit of guide vane and c_c the average absolute speed in the spiral case flow. The shock loss for the guide vane, stay vane and runner, Ueda *et al*. (1980), are given respectively by Eqs. (5), (6) e (7),

$$Z_{ch_P} = \zeta_{ch_P} (\cot \alpha_{1P} - \cot \alpha_{1P}^*)^2 \frac{c_{m_{1P}}^2}{2g} \tag{5}$$

$$Z_{ch_D} = \zeta_{ch_D} (\cot \alpha_{1D} - \cot \alpha_{1D}^*)^2 \frac{c_{m_{1D}}^2}{2g} \tag{6}$$

$$Z_{ch_R} = \zeta_{ch_R} (\cot \beta_4 - \cot \beta_4^*)^2 \frac{w_{m_4}^2}{2g} \tag{7}$$

where ζ_{ch} is the shock loss, α_1 e α_1^* the inlet angles, respectively, of the absolute flow and vanes, β_4 e β_4^* the inlet angles, respectively, of the relative flow and blades, c_{m1} the meridional component of the absolute speed in the entrance, and w_{m4} the relative meridional component in the entrance.

The mixture losses for the guide vanes and stay vanes, Ueda *et al*. (1980), is given by Eq. (8),

$$Z_{mw} = \zeta_{mw} \left[\frac{e_2 \, N_{al}}{\pi \, D_2 \sin \alpha_2^*} \right]^2 \frac{c_2^2}{2g} \tag{8}$$

where ζ_{mw} is the coefficient of the mixture losses for the guide vanes and for the stay vanes, e_2, D_2, α_2^* e c_2, respectively, the thickness, diameter, angle and the absolute discharge average speed in the exit of the stay vanes or of the guide vanes, and N_{al} the number of stay vanes or number of guide vanes.

Using the same procedure the mixture losses for the runner, Ueda *et al*. (1980), is given by Eq. (9),

$$Z_{mw_R} = \xi_{mw} \left[\frac{e_5 \, N_{pá}}{\pi \, D_5 \sin \beta_5^*} \right]^2 \frac{w_6^2}{2g} \tag{9}$$

where ζ_{mw_R} is the coefficient of mixture losses for the runner, e_5, D_5, β_5^* and w_6, respectively, the thickness, diameter, angle and average relative speed in the exit of the blade, and $N_{pá}$ the number of blades.

The losses due to vortex which appears in the entrance of the cone of the draft tube comes from the absolute discharge in the exit of the runner when this flow does not have a circumferential component of the absolute speed, c_{u6}, different from zero. According to Ueda *et al*. (1980) these losses can be approximated by Eq. (10),

$$Z_{tur} = \zeta_{tur} \frac{\pi \, D_5 \, b_5 \, c_{m6}}{Q} \frac{c_{u6}^2}{2g} \tag{10}$$

where ζ_{tur} is the vortex coefficient, b_5 and c_{m6}, respectively, the width and meridional component of the absolute speed in the exit of the runner.

The losses in the exit of the draft tube are localized losses which depend on the localized losses coefficient, K, assuming this equal to 1, and from the average speed in the discharge exit on the draft tube, c_8.

4.2. Leakage flow

The leakage flow represented by the volumetric discharge, Q_f, is calculated by $Q_f = Q_T - Q_R$ (turbine mode) or $Q_f = Q_R - Q_B$ (pump mode). The leakage flow is obtained by $Q_f = Q_{fe} + Q_{fi}$, where Q_{fe} is the external leakage flow and Q_{fi} the internal leakage flow. According to Vivier (1966), the leakage flow, discharge Q_{fe} or Q_{fi} can be calculated by Eq. (11),

$$Q_f = \mu\, A_L \sqrt{2\,\Delta p_L/\rho} \tag{11}$$

where μ the empiric coefficient, Vivier (1966), which depends on the seal clearance geometry, A_L the discharge section through the seal clearance geometry, Δp_L is the static pressure difference between the entrance and the exit of the seal clearance geometry and ρ the water specific mass.

4.3. Losses due to side friction

The losses of power due to side friction , P_{al}, is calculated according to Gülich (2003), Eq. (12),

$$P_{al} = \frac{k_{al}}{\cos\delta}\,\rho\,\omega^3\, r^5 \left[1 - \left(\frac{r_i}{r}\right)^5\right] \tag{12}$$

where δ is the bottom cover inclination angle related to radial direction, ω is the runner angular speed, r is the runner larger radius (r_4 for turbine and r_5 for pump) and r_i is the seal average radius. The expression for the factor k_{al} determination is done according to Gülich (2003), which embraces not only the four flow modes in the side spaces (2 laminar e 2 turbulent) as well as the leakage flow, which is given by Eq. (13).

$$k_{al} = \frac{\pi}{2\,s_a^*\,\mathrm{Re}} + \frac{0,0625}{\mathrm{Re}^{0,2}}(1-k_o)^{1,75}\, k_{\varepsilon R} k_f \tag{13}$$

where s_a^* is the relation between side space clearance and the runner radius, Re is the Reynolds number, k_o is the spinning factor of the fluid in the side space with the null leakage flow discharge, $k_{\varepsilon R}$ and k_f , respectively, the factor that embraces the involved surfaces roughness and the leakage flow effect.

4.4. Turbine Solver

The following methodology has been developed for the determination of the turbine solver: 1) Entry data: Geometry values, the discharge and speed pair of point (Q_T, n_T) the hydraulic losses coefficients; 2) Preliminary calculation: discharge sections, narrow down section factors, step and variation calculation of the inlet and outlet diameters for the guide vane; 3) Calculation of the speed in the spiral case, guide vane, stay vane and draft tube; 4) First calculation of the runner speed with the discharge Q_T; 5) Calculation of the friction coefficients; 6) Calculation of the hydraulic losses in the spiral case, guide vanes and stay vanes; 7) Start of the interactive cycle for the calculation of the turbine net head, H_T; 8) Calculation of the hydraulic losses for the runner and draft tube; 9) Calculation of the leakage flow; 10) Calculation of the runner discharge Q_{R_T} ; 11) New calculation of the runner speeds; 12) Calculation of the new coefficient for the viscous friction for the runner; 13) Calculation of the new hydraulic losses for the runner and draft tube; 14) Calculation of the loss of power due to side friction; 15) Calculation of the hydrodynamics characteristics (head and efficiency); 16) Convergence criteria evaluation; 17) Calculation of runner real speeds; 18) Calculation of the new hydraulic losses in the runner and draft tube; 19) New calculus of the leakage flow; 20) New calculation of the loss of power due to side friction; 21) Final results of the hydrodynamics characteristics (η_T).

4.4. Pump Solver

Similar to the turbine solver it was developed a methodology for the pump solver. 1) Entry data: Geometry values, the discharge and speed pair of point (Q_B, n_B) the hydraulic losses coefficients; 2) Preliminary calculation: discharge sections, narrow down section factors, step and variation calculation of the inlet and outlet diameters for the guide vane;3) Calculation of speeds for the draft tube; 4) Calculation of the runner speeds, guide vane, stay vane and spiral case; 5) Calculation of the friction coefficients; 6) Calculation of the hydraulic losses for the draft tube, guide vane, stay vane and spiral case; 7) Start of the interactive cycle for the calculation of pump net head, H_B; 8) Calculation of the runner hydraulic losses; 9) Calculation of the leakage flow; 10) Calculation of the runner discharge Q_{R_B} ; 11) New calculation of the runner speeds; 12) New runner friction coefficients, guide vane, stay vane and spiral case; 13) Calculation of the new runner hydraulic losses and spiral case; 14) Calculation of the loss of power due to side friction; 15) Calculation of the hydrodynamics characteristics (head and efficiency); 16) Convergence criteria evaluation; 17) Calculation of runner real speeds; 18) Calculation of the new runner hydraulic losses; 19) New calculus of the leakage flow; 20) New calculation of the loss of power due to side friction; 21) Final results of the hydrodynamics characteristics (η_B).

The non-guided flow (space between the stay vane and guide vane, and also, the space between guide vane and runner) were modeled according to the following: 1) potential vortex, 2) viscous model according to Whitfield and Baines (1990) and 3) adjusted free-vortex according to Pfleiderer and Petermann (1979). These three models represent a difference lower than 2.35% for the flow speeds. For this work the Whitfield e Baines model was the one chosen.

5. RESULTS

The following intervals for the design variables were established: 20.45° $\le \alpha_{1P_T} = \alpha_{2P_B} \le$ 35.00°, 21.75° $\alpha_{2P_T} = \alpha_{1P_B} \le$ 45°, 22.68° $\le \alpha_{MD} \le$ 50.00°, 18.60° $\le \beta_{4R_T} = \beta_{5R_B} \le$ 40.00° and 14.90° $\le \beta_{5R_T} = \beta_{4R_B} \le$ 35.00°. For the head and hydraulic efficiency it was set the following intervals: 59.35 m $\le H_T \le$ 60.85 m, 89.50 % $\le \eta_{h_T} \le$ 92.89 %, 50.50 m $\le H_B \le$ 51.88 m and 88.78 % $\le \eta_{h_B} \le$ 92.05 %.

For the mono-objective optimization it was initially analyzed the variation of the start point, x_o, using the SQP to turbine and pump operating mode. After some tests it was chosen three possible solutions for both operating modes. The optimality conditions were guaranteed by the tolerances in the directional derivative in the objective function and also for the constraints. Among all possible solutions for the turbine operating mode it was chosen the design optimum SQP-2 (which converged after 34 evaluations of the objective function) and SQP-3 (which converged after 27 evaluations of the objective function) for the pump operating mode, as shown in Tab. 2 (three solutions for different methods).

Using GA it was initially studied the variation of the crossover factor and the variation of the penalty factor for both operating modes. In both operating modes it was used the penalty factor of 300 being that the value used for all penalties. For the turbine operating mode it was used the following parameters: generation number, 60; population size, 20; crossover factor, 0.750; selection, roulette method; and crossover, in two points. For the pump operating mode: generation number, 60; population size, 20; crossover factor, 0.675; selection, roulette method; and crossover, in two points. For both operating modes, the optimizer was called 50 times. Among feasible solutions it was chosen 3 designs for comparison. From those results it was chosen the optimum possible. For the turbine operating mode it was chosen the solution GA-3. For the pump operating mode it was chosen the solution GA-1, as shown in Tab. 2. The stopping of the algorithm was done by the generation limit and after 1200 evaluation of the objective function.

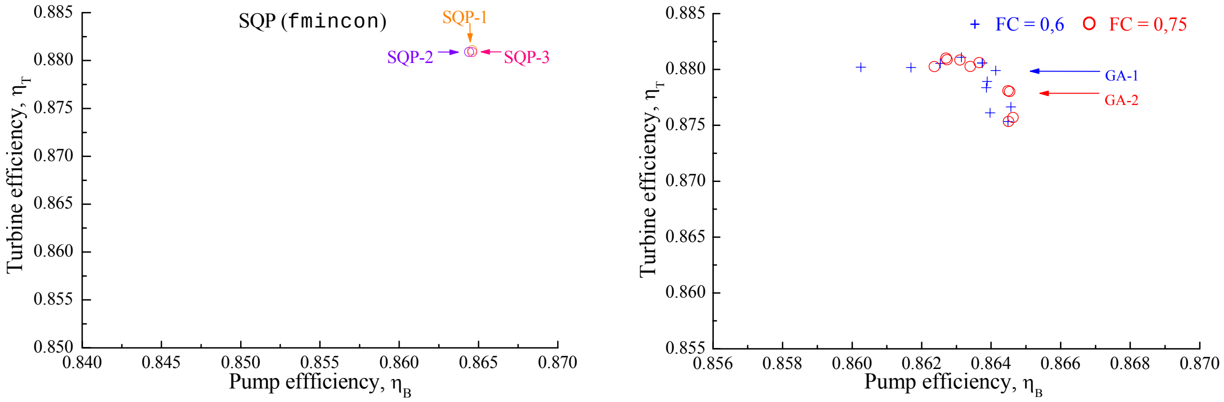
Table 2. Optimum value of the design variables for the mono-objective optimization.

	Turbine mode					Pump mode				
	α_{1P_T} (°)	α_{2P_T} (°)	α_{MD} (°)	β_{4R_T} (°)	β_{5R_T} (°)	SQP-1	α_{2P_B} (°)	α_{MD} (°)	β_{4R_B} (°)	β_{5R_B} (°)
SQP-1	19.31	29.02	35.65	25.00	16.62	SQP-2	21.78	37.60	16.42	22.08
SQP-2	20.01	26.00	35.72	20.80	16.63	SQP-3	26.09	37.02	18.79	22.20
SQP-3	25.03	30.02	36.30	32.00	15.33	GA-1	20.45	37.06	17.67	21.10
GA-1	20.82	23.76	35.68	18.00	16.61	GA-2	20.32	29.91	16.32	20.99
GA-2	21.51	23.74	35.71	19.28	16.62	GA-3	20.28	35.50	19.19	21.00
GA-3	21.34	23.21	35.64	21.20	16.61	24.00	23.29	36.00	19.54	21.00

For the multi-objective optimization it was adopted an optimizing procedure similar to the mono-objective optimization function in the analysis of the start point using the SQP and the crossover factor using the GA and the NSGA-II. For the weight variation in the Eq. (1) it was adopted the values W_{1T} = 0.01 (turbine) and W_{1B} = 0.95 (pump) for the first pair of weights, W_{2T} = 0.05 and W_{2B} = 0.95 for the second pair of weights. From that pair of weights the increment (turbine) and the decrement (pump) was 0.05 resulting in a total of 20 pairs of weights. Again, for the use of SQP, it was chosen 3 values for the start point as possible solutions.

For a given start point, the SQP kept practically over the same solution (premature convergence, as shown by Obayashi *et al.* (2004)) even with variation in the weights. For the GA it was analyzed the crossover factor 0.600 e 0.750 and for the pair of weights the optimizer was called 3 times giving a total of 60 individuals for each factor. The Fig. 4 refers to the solution using the SQP and the GA presenting only the feasible solutions with the following parameters: generation number, 80; population size, 30; crossover factor, 0.600 (GA-1) and 0.750 (GA-2).

The inherent limitation for the use of SQP is its premature convergence in local minimums as presented by Obayashi *et al.* (2004). Besides the summation using weights may not represent very well the Pareto-front, Coello (1999). For the NSGA-II the following parameter as given: population size, 60; crossover factor (CF), 0.650, 0.750 and 0.850; selection, tournament method; and crossover, in two points. For each crossover factor the optimizer was called 50 times and from them 3 Pareto-fronts denominated NSGA-II (1), for CF = 0.650, were chosen got for 141 generations and 8521 objective function evaluation, NSGA-II (2), for CF = 0.750, got for 126 generations and 7621 objective function evaluation and finally NSGA-II (3), for CF = 0.850, got for 108 generations and 6541 objective-function evaluation, according to the Figs. (5), (6a) and (6b) respectively. The choice of the optimum individuals was given by the higher efficiency for the turbine and pump operating modes. The Tab. 3 shows the design variables for the SQP and GA. The Tab. 4 shows the design variables for the NSGA-II.



(a) (b)

Figure 4. Multi-objective optimization using (a) SQP and (b) GA.

Table 3. Optimum value of the design variables for the multi-objective optimization using SQP and GA.

SQP	$\alpha_{1P_B}/\alpha_{2P_T}$ (°)	$\alpha_{2P_B}/\alpha_{1P_T}$ (°)	α_{MD} (°)	$\beta_{4R_B}/\beta_{5R_T}$ (°)	$\beta_{5R_B}/\beta_{4R_T}$ (°)
SQP-1	22.39	21.39	35.93	17.20	21.04
SQP-2	21.75	21.58	36.29	16.10	20.68
SQP-3	23.29	21.27	36.04	17.20	21.04
GA	$\alpha_{1P_B}/\alpha_{2P_T}$ (°)	$\alpha_{2P_B}/\alpha_{1P_T}$ (°)	α_{MD} (°)	$\beta_{4R_B}/\beta_{5R_T}$ (°)	$\beta_{5R_B}/\beta_{4R_T}$ (°)
GA-1	22.26	19.59	35.86	17.13	20.62
GA-2	22.46	21.46	35.93	17.17	21.00

Table 4. Optimum value of the design variables for the multi-objective optimization using NSGA-II.

NSGA	$\alpha_{1P_B}/\alpha_{2P_T}$ (°)	$\alpha_{2P_B}/\alpha_{1P_T}$ (°)	α_{MD} (°)	$\beta_{4R_B}/\beta_{5R_T}$ (°)	$\beta_{5R_B}/\beta_{4R_T}$ (°)
NSGA-II (1)	21.77	21.61	35.78	16.59	20.85
NSGA-II (2)	27.93	26.08	36.01	16.82	20.55
NSGA-II (3)	22.54	29.75	36.08	16.87	20.88

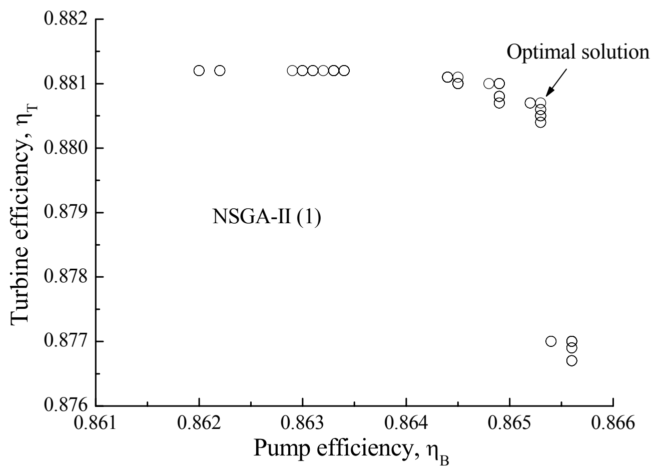


Figure 5. Pareto-front NSGA-II (1).

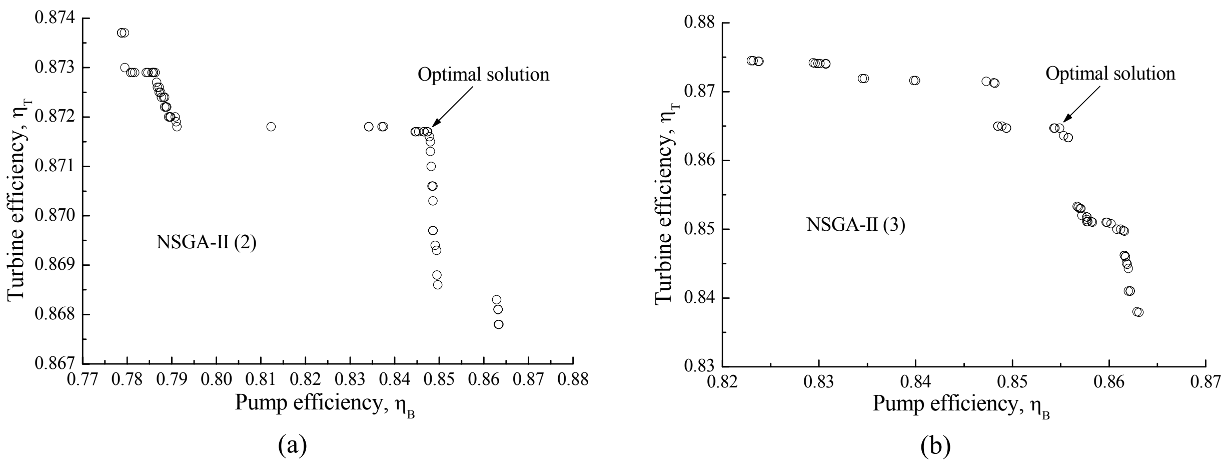


Figure 6. Pareto-front NSGA-II (2) and NSGA-II (3).

Table 5. Experimental results and results obtained by mono-objective optimization.

Experimental results		
Symbol (Unit)	Turbine	Pump
n (rpm)	1000	1000
Q (m^3/s)	0.454	0.335
H (m)	60.0	51.4
η (%)	88.06	86.84
η_h (%)	92.43	91.21
η_f (%)	98.34	97.82
η_m (%)	96.88	97.34
P_e (kW)	235.24	194.45
N_{11} (rpm m$^{0.5}$)	80.04	86.26
P_{11} (W m$^{-3.5}$)	1323.55	1379.83
Q_{11} (m^3/s m$^{-2.5}$)	0.1533	0.1222

	SQP-2	GA-3	SQP-3	GA-1
Symbol (Unit)	Turbine	Turbine	Pump	Pump
n (rpm)	1000	1000	1000	1000
Q (m^3/s)	0.454	0.454	0.335	0.335
H (m)	60.54	60.72	50.88	60.50
η (%)	88.00	88.04	86.42	87.66
η_h (%)	92.79	92.83	92.05	92.66
η_f (%)	98.66	98.66	98.44	98.42
η_{al} (%)	97.58	97.57	96.82	96.88
η_m (%)	98.50	98.50	98.50	98.50
P_e (kW)	237.19	238.09	193.43	236.68
N_{11} (rpm m$^{0.5}$)	79.47	79.36	86.70	79.51
P_{11} (W m$^{-3.5}$)	1316.70	1315.40	1393.7	1315.20
Q_{11} (m^3/s m$^{-2.5}$)	0.1526	0.1524	0.1228	0.1526

Table 6. Results obtained by multi-objective optimization.

	SQP-1	SQP-1	GA-1	GA-1	GA-2	GA-2	NSGA-II (1)	
Symbol (Unit)	Turbine	Pump	Turbine	Pump	Turbine	Pump	Turbine	Pump
n (rpm)	1000	1000	1000	1000	1000	1000	1000	1000
Q (m^3/s)	0.454	0.335	0.454	0.335	0.454	0.335	0.454	0.335
H (m)	60.85	51.88	59.71	51.44	59.50	51.83	60.20	51.16
η (%)	88.11	86.46	87.99	86.41	87.80	86.45	88.07	86.52
η_h (%)	92.89	92.05	92.81	92.02	92.63	92.05	92.88	92.05
η_f (%)	98.66	98.42	98.67	98.43	98.67	98.42	98.66	98.50
η_{al} (%)	97.60	96.88	97.55	96.86	97.53	96.88	97.57	96.88
η_m (%)	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50
P_e (kW)	238.68	197.12	233.91	195.56	232.59	196.95	236.05	194.26
N_{11} (rpm m$^{0.5}$)	79.27	85.85	80.03	86.22	80.17	85.90	79.70	86.46
P_{11} (W m$^{-3.5}$)	1314.90	1379.40	1325.70	1386.10	1325.20	1380.20	1321.50	1388.20
Q_{11} (m^3/s m$^{-2.5}$)	0.1522	0.1216	0.1536	0.1221	0.1539	0.1217	0.1530	0.1225

6. CONCLUSIONS

The methodology presented in this work has been developed to optimize reversible radial pump-turbine design. It was built over three main hypotheses: one-dimensional flow, empiric correlations for losses and optimization techniques based on gradient (SQP) and two population algorithm (GA e NSGA-II). It is a low computational cost methodology and easy to implement. The algorithm was then implemented in a computational program, which had the task to search some basic geometry as the design variables. In a step ahead it maximizes the efficiency of the pump-turbine for both operating modes of the reversible pump-turbine. These basic geometries are the runner blades inlet and outlet angles, wicket gates stagger angle and inlet and outlet stay vanes angles. The results obtained by this methodology are comparables with existing results from reduced scaled pump-turbine model.

The mono-objective optimization was used to indicate a tendency for the design variables values for turbine mode and also for pump mode, but each one at a time. The best solution of the SQP in comparison with the GA solution were in agreement with the reduced scaled model of the pump-turbine and showed good approximation around the optimum global for both operating modes.

The multi-objective optimization has also conducted to results with a tendency for the design variables values in comparison with existing pump-turbine design. All the results for the SQP presented a small variation. The best solution of the SQP compared with the best solution of the GA has also presented agreement with the reduced scaled model results. In a different way, theses results compared with the best solution of the NSGA-II indicated a good approximation around the optimum global. As solution of the problem of optimization conceptual for the radial pump-turbine, the solution NSGA-II (1) is chosen because it has the lowest hydraulic loss among all solutions.

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