MODELING OF DAMPED NONLINEAR DYNAMIC VIBRATION ABSORBERS BY USING BESSEL FUNCTIONS AND PERTURBATION METHODS

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Abstract. *The present work focuses on the theoretical study and the numerical simulations of two degrees-of-freedom nonlinear damped system, constituted of a primary mass attached to the ground by a linear spring and the secondary mass attached to the primary system by a nonlinear spring (nDVA). In this application, the numerical resolution of the nonlinear equations of motion is made by using two strategies: first, by using the modified Bessel functions, in which the nonlinear spring force is expand resulting in a set of differencial equations of motion more simply to be solved; the second is the use of the perturbation methods thourgh a Taylor series expansion of the nonlinear terms, resulting in a set of linear differential equations. The amplitudes of the harmonic responses of both strategies enables to determine the influence of the nonlinear parameters over the amplitudes of vibration and the suppression bandwith. Moreover, in many applications of nDVAs including design, the optimization procedures based on numerical models is a very useful tool. In this paper, the multiobjective optimization of the nDVA into a frequency band of interest is proposed, with the aim of augmenting the performance of the nonlinear absorber. The multiobjective optimization problem is composed by two objective functions: the first is related to the amplitude of vibration, with the aim of minimizing this amplitude; the second is associated to the suppression bandwidth, where the interest is to maximize this bandwidth. Based on the obtained results, the usefulness of the modeling methodologies in various types of analyses and design of discrete nDVAs is highlighted.*

Keywords*: Nonlinear mechanics, dynamic vibration absorber, Bessel functions, perturbation methods*

1. INTRODUCTION

In its simplest form, DVAs are essentially discrete devices concentrating mass and stiffness that once connected to a given structure, are capable of absorbing the vibratory energy at the connection point, providing a reduction of the vibration level. Much of the knowledge available to date is compiled in the original patent by Frahm (1911), and in the books by Den Hartog (1934) and Koronev and Reznikov (1993), and in some review papers such as those proposed by Steffen Jr. and Rade (2000, 2001).

In the last two decades, a great deal of effort has been devoted to the development of mathematical models for characterizing the mechanical behavior of nDVAs accounting for its typical dependence on parameters that control the nonlinearities (Zhu, 1992). Besides the well-known complexity of the modeling strategy involved in nonlinear dynamics, which constitutes a simple and straightforward means of representing the dynamic behavior of nDVAs, some methodologies have been suggested and have been shown to be particularly suitable to be used in combination with structural systems discretization. This aspect makes them very attractive for the modeling of nonlinear dynamic vibration absorbers. Among these strategies, it should be mentioned the theoretical study proposed by Pipes (1953) and Pai and Schulz (1998), in which some techniques to improve the stability and efficiency of nDVAs into a frequency band of interest have been proposed, leading to refined nDVAs. Also, Rice and McCraith (1987) suggested optimization strategies to be applied to the design of nDVAs through the use of an asymmetric nonlinear Duffing-type element incorporated in the suspension for narrow-band absorption applications.

The present work focuses on the theoretical study and the numerical resolutions of a two d.o.f's nonlinear damped system. Within this context, two strategies are investigated: first, by using the modified Bessel functions. This strategy enables to expand the nonlinear spring forces resulting in a set of differential equations of motion, simplest to be solved; the second is the use of the perturbation methods, by a Taylor expansion series of the nonlinear terms, resulting in a set of linear differential equations, in which the solution of these system is an approximation of the real nonlinear system for the small amplitudes of vibration.

In many applications of nDVAs including design, the optimization procedures based on numerical models is a very useful tool. In this paper, a multiobjective optimization strategy of a nDVA into a frequency band of interest is proposed, with the aim of augmenting the performance of the project of the nDVA. For that, the deterministic multiobjective optimization problem is composed by two objective functions: the first is related to the amplitude of vibration, with the aim of minimizing this amplitude; the second is associated to the suppression bandwidth, where the interest is to minimize this bandwidth. Based on the obtained results, the usefulness of the modelling methodology in various types of analyses and design of discrete nDVAs is highlighted.

2. NONLINEAR DYNAMIC SYSTEM EQUATIONS OF MOTION

Figure 1 illustrates the two degrees-of-freedom (d.o.f's) nonlinear dynamic systems reported in the present work. Figs. 1(a) and (b) show, respectively, the nonlinear undamped and damped systems, composed by a primary mass attached to the ground by a suspension including either linear or nonlinear spring, and a secondary mass coupled to the primary system by a linear or nonlinear spring.

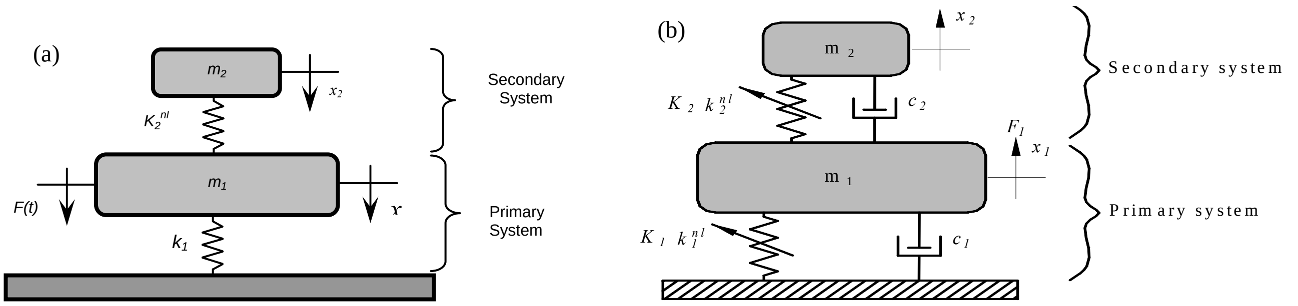

Figure 1. Illustration of the nonlinear two d.o.f's undamped (a) and damped systems (b).

2.1. Modified Bessel functions for undamped system

By considering that $x_1 > x_2$, and the applied force on the primary mass of the form $F(t) = P_0\, sin(\omega t)$, the equations of motion of the two d.o.f's system can be obtained as follows:

$$\begin{cases} m_1\ddot{x}_1 + k_1 x_1 + S_2(x) = P_0\, sin(\omega t) \\ m_2\ddot{x}_2 - S_2(x) = 0 \end{cases} \tag{1}$$

where $S_2(x)$ is the following nonlinear spring force associated to the secondary mass:

$$S_2(x) = \frac{k_2}{a} sinh(B\, sin(\omega t)) \tag{2}$$

where $B = aX$ and $X = X_1 - X_2$.

By considering only the first five terms of the Bessel expansion, Eq. (2) assumes the following form:

$$S_2(t) = \frac{2k_2}{a}\left[I_1(B) sin(\omega t) - I_3(B) sin(3\omega t) + I_5(B) sin(5\omega t)\right] \tag{3}$$

where $I_n(x) = \sum_{k=0}^{\infty} \frac{\left(x/2\right)^{n+2k}}{(k!)\Gamma(n+k+1)}$ and $\Gamma(n) = \int_0^\infty t^{n-1}e^{-t}dt$ is the gamma function.

By considering only the first term in Eq. (3), the equations of motion (1) of the nonlinear undamped system can be obtained as follows:

$$\begin{aligned} &\left(k_1 - \omega^2 m_1\right)(X + X_2) + \frac{2k_2 I_1(B)}{a} = P_0 \\ &-\omega^2 m_2 X_2 - \frac{2k_2 I_1(B)}{a} = 0 \end{aligned} \tag{4}$$

where for $I_1(B) = \frac{B}{2} + \frac{B^3}{16} + \frac{B^5}{384} + \ldots$, Eq. (4) assumes the following forms:

$$
\begin{aligned}
&\left(k_1-\omega^2 m_1\right)(X+X_2)+\frac{2k_2}{a}\left(\frac{B}{2}+\frac{B^3}{16}+\frac{B^5}{384}+\ldots\right)=P_0 \\
&-\omega^2 m_2 X_2-\frac{2k_2}{a}\left(\frac{B}{2}+\frac{B^3}{16}+\frac{B^5}{384}+\ldots\right)=0
\end{aligned} \tag{5}
$$

By adding the expressions (5) and after some mathematical manipulations, the following algebraic equation is obtained in terms of the Bessel term:

$$
\begin{aligned}
&\left\{\frac{k_2\left[\omega^2(m_1+m_2)-k_1\right]}{192}\right\}B^5+\left\{\frac{k_2\left[\omega^2(m_1+m_2)-k_1\right]}{8}\right\}B^3+ \\
&+\left\{k_2\left[\omega^2(m_1+m_2)-k_1\right]+\left(k_1-\omega^2 m_1\right)\left(\omega^2 m_2\right)\right\}B-\omega^2 m_2 a P_0=0
\end{aligned} \tag{6}
$$

The Eq. (6) can be solved numerically by using the "*fsolve*" function of the MATLAB® toolbox.

2.2. Modified Bessel functions for damped system

In this section, the interest is to solve by using the modified Bessel functions, the following nonlinear equation of motion of the damped system show in Fig 1(b):

$$
\begin{aligned}
&m_1\ddot{x}_1(t)+c_1\dot{x}_1(t)+c_2\dot{x}(t)+k_1x_1(t)+S_2(x)=f(t) \\
&m_2\ddot{x}_2(t)-c_2\dot{x}(t)-S_2(x)=0
\end{aligned} \tag{7}
$$

where $x_1>x_2$, $x=x_1-x_2$. $S_2(x)=\frac{2k_2}{a}sinh(ax)$, $x=Xe^{i\omega t}$, $x_1=X_1e^{i\omega t}$ and $x_2=X_2e^{i\omega t}$.

The solution procedure is the same of procedure reported in previously section. Within this aim, the development of Eq. (7) enables to obtain the following expressions:

$$
\begin{aligned}
&\left(k_1-m_1\omega^2+ic_1\omega\right)X_1+ic_2\omega X+\frac{2k_2}{a}I_1(B)=F \\
&m_2\omega^2X_2+ic_2\omega X+\frac{2k_2}{a}I_1(B)=0
\end{aligned} \tag{8}
$$

By making some mathematical manipulations, one can obtain the following amplitudes of vibrations for the primary mass and for the nDVA:

$$
\begin{aligned}
&X_1=-\frac{ic_2\omega}{A_1}X-\frac{2k_2}{aA_1}I_1(B)+\frac{F}{A_1} \\
&X_2=-\frac{ic_2\omega}{m_2\omega^2}X-\frac{2k_2}{am_2\omega^2}I_1(B)
\end{aligned} \tag{9}
$$

where $X=X_1-X_2$, $A_1=\left(k_1-m_1\omega^2+ic_1\omega\right)$. By making some mathematical manipulations and considering that $X=B/a$, Eqs. (9) can be expressed as follows:

$$
\left(-1-\frac{ic_2\omega}{A1}-\frac{k_2}{A1}+\frac{ic_2\omega}{m_2\omega^2}+\frac{k_2}{m_2\omega^2}\right)B+\left(-\frac{k_2}{8A1}+\frac{k_2}{8m_2\omega^2}\right)B^3+\left(-\frac{k_2}{192A1}+\frac{k_2}{192m_2\omega^2}\right)B^3+\frac{aF}{A1}=0 \tag{10}
$$

By varying the excitation frequency ω, the roots of the Eq. (10) that represents the amplitude B can be determined, and consequently, the frequency response f the systems can be obtained.

2.3. Krylov-Bogoliubov perturbation method for nonlinear damped system

In this section, to solve the nonlinear equations of motion of the system represented in Fig. 1(b) one uses the averaging method, by considering that the spring force is expressed as following:

$$r_i(x_i) = k_i x_i + k_i^{nl} x_i^3 ; \quad i = 1 \text{ to } 2 \tag{11}$$

where x_1 and x_2 represent, respectively, the displacements of the primary system and the nDVA.

In the model above, the dampers are linear, however k_i and k_i^{nl} represent, respectively, the linear and nonlinear coefficients of the springs. To obtain the dimensionless normalized equation of motion of the nonlinear system, the displacements are normalized with respect to the length of a given vector x_c, such that, $y_i = x_i / x_c$. By applying the Newton's second law, and after some algebraic manipulations, the following normalized equation of motion of the nonlinear dynamic system can be expressed under the following matrix form:

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\dot{\boldsymbol{y}}(t) + \boldsymbol{K}\boldsymbol{y}(t) = \boldsymbol{f}(t) \tag{12}$$

The normalized mass, damping and stiffness matrices are expressed, respectively, by the following relations:

$$\boldsymbol{M} = \begin{bmatrix} 1+\mu & \mu \\ \mu & \mu \end{bmatrix}; \ \boldsymbol{C} = \begin{bmatrix} \delta_1 & 0 \\ 0 & \mu\delta_2 \end{bmatrix}; \ \boldsymbol{K} = \begin{bmatrix} \eta_1 & 0 \\ 0 & \mu\eta_2 \end{bmatrix} \tag{13}$$

$$\begin{aligned} &\overline{\omega}_i^2 = \frac{k_i}{m_i}, \quad \omega_i = \frac{\omega_i}{\omega}, \quad \zeta_i = \frac{c_i}{2\sqrt{k_i m_i}}, \quad \delta_i = 2\zeta_i\omega_i, \ \mu = \frac{m_2}{m_1}, \quad \eta_i = \omega_i^2, \\ &\varepsilon_i = \frac{k_i^{nl} x_c^2}{m_i \omega^2}, \quad \rho = \frac{\omega_2}{\omega_1}, \quad P_0 = \frac{p}{m_1 \overline{\omega}^2 x_c}, \quad \beta = \frac{P_0}{\eta_1}, \ \Omega = \frac{\omega}{\overline{\omega}_1}. \end{aligned} \tag{14}$$

where the normalized displacement and force vectors are $\boldsymbol{y} = [y_1 \quad y_2]^T$ and $\mathbf{f} = \left[P_0 \cos\tau - \varepsilon_1 y_1^3 \quad -\mu\varepsilon_2 y_2^3\right]^T$.

Various perturbation methods (Nayfeh, 2000) are based on averaging that encompass techniques such as the following: Krylov-Bogoliubov method, Krylov-Bogoliubov-Mitropolsky method, and the method of the generalized average (Thomsen, 2003). In the present work, the Krylov-Bogoliubov method will be used to integrate the matrix equation of motion represented by Eq. (12), leading to an approximate solution of the nonlinear differential equations of motion. In this context, the *Van der Pol Transformation* (Hagedorn, 1988) represented by the following expressions are employed in order to guarantee that the transformation is unique.

$$\boldsymbol{y}(\tau) = \boldsymbol{u}(\tau)\cos\tau + \boldsymbol{v}(\tau)\sin\tau \tag{15}$$

$$\dot{\boldsymbol{y}}(\tau) = -\boldsymbol{u}(\tau)\sin\tau + \boldsymbol{v}(\tau)\cos\tau \tag{16}$$

where $\tau = \omega t$ and $\boldsymbol{u} = (\boldsymbol{u_1}, \ \boldsymbol{u_2})^{\boldsymbol{T}}$ and $\boldsymbol{v} = (\boldsymbol{v_1}, \boldsymbol{v_2})^T$ are assumed to be slow functions of time τ. By making some mathematical manipulations starting from Eqs. (15) end (16), and after that, by substituting the results into Eq. (13), one can obtain the following expression:

$$\left(\boldsymbol{M}\dot{\boldsymbol{v}} - \boldsymbol{M}\boldsymbol{u} + \boldsymbol{C}\boldsymbol{v} + \boldsymbol{K}\boldsymbol{u}\right)\cos\tau - \left(\boldsymbol{M}\dot{\boldsymbol{u}} - \boldsymbol{M}\boldsymbol{v} + \boldsymbol{C}\boldsymbol{u} + \boldsymbol{K}\boldsymbol{v}\right)\sin\tau = \boldsymbol{f}(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{\tau}) \tag{17}$$

The Eq. (17) can then be integrated over the period (0 to 2π). It is worth mentioning that *u* and *v* are taken as constants along this period (it represents a very short time interval). After some algebraic manipulations, one can obtain the first-order ordinary differential system equation corresponding to motions of period 2π for the original system given by Eq. (12). In the case of steady-state periodic vibrations (Borges, 2008), the following condition can be used $\dot{\boldsymbol{u}} = \dot{\boldsymbol{v}} = 0$ into the resulted equations, so that the following nonlinear algebraic system with four equations and four variables u_1, u_2, v_1, v_2 is obtained:

$$\begin{cases} \left(1+\mu-\omega_1^2\right)u_1+\mu u_2-2\zeta_1\omega_1 v_1-\dfrac{3\varepsilon_1\left(u_1^2+v_1^2\right)u_1}{4}+\beta\omega_1^2=0 \\ \mu u_1+\left(\mu-\mu\rho^2\omega_1^2\right)u_2-\mu\left(2\zeta_2\rho\omega_1^2 v_2+\dfrac{3\varepsilon_2\left(u_2^2+v_2^2\right)u_2}{4}\right)=0 \\ \left(\omega_1^2-1-\mu\right)v_1-\mu v_2-2\zeta_1\omega_1 u_1+\dfrac{3\varepsilon_1\left(u_1^2+v_1^2\right)v_1}{4}=0 \\ -\mu v_1+\left(\mu\rho^2\omega_1^2-\mu\right)v_2-\mu\left(2\zeta_2\rho\omega_1^2 u_2-\dfrac{3\varepsilon_2\left(u_2^2+v_2^2\right)v_2}{4}\right)=0 \end{cases} \tag{18}$$

The system of equations (18) can be numerically solved by using the function "*fsolve*" from MATLAB® toolbox. Then, the values of u_1, u_2, v_1, v_2 can be calculated and the vibration amplitudes of the primary and secondary masses of the nonlinear DVA are obtained. The amplitude values are given by r_1 and r_2, according to $r_i=\sqrt{u_i^2+v_i^2}, \quad i=1\,to\,2.$

3. OPTIMIZATION PROBLEM DEFINITION

A general multiobjective optimization problem (MOP) involves the simultaneous optimization of multiple objective functions (Lima, 2007), (Eschenauer, 1990), which may be in conflict with each other, and the goal is to find the best design solutions, which lead to the minimum or maximum values of the various objective functions. In general, in a multiobjective optimization problem there is no single optimal solution and the interaction among different objectives gives rise to a set of compromised solutions, known as the Pareto optimal solutions (Srivinas, 1993). Since none of these Pareto optimal solutions can be identified as better than the others without any further consideration, the interest is to find as many Pareto optimal solutions as possible. A deterministic multiobjective problem includes a set of k parameters (decision variables) and a set of n objective functions ($n \geq 2$), and can be summarized as follows:

$$\begin{cases} \min\limits_{\boldsymbol{x}} \ \boldsymbol{F}(\boldsymbol{x})=\left(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),\ldots,f_n(\boldsymbol{x})\right) \\ g_j(\boldsymbol{x})\leq 0 \quad j=1,\ldots,m \quad \boldsymbol{x}_L\leq\boldsymbol{x}\leq\boldsymbol{x}_U, \quad \boldsymbol{x}\in C \end{cases} \tag{19}$$

where $\mathbf{x}=[x_1, x_2,\ldots,x_k]^T$ is a vector of design variables; $C \subset R^k$ is the design space associated to the equality or inequality constraints $g_j(\mathbf{x})$. For a practical design problem, the vector $\boldsymbol{F}(\mathbf{x})$ is non-linear, multi-modal and not necessarily analytical. In the case of the nonlinear dynamic system, the deterministic optimization problem is composed by two objective functions: the first cost function is the amplitude of the frequency response of the nonlinear system corresponding to the natural frequency of the mode of interest (by minimizing the amplitude of the frequency response at the corresponding resonance peak); the second objective function is the maximization of the suppression band. Fig. 2 shows the definition of the two objective functions considered in the multiobjective optimization problem that can be defined through the following relation:

$$minimize: \{f_1 = amplitude(M1) \quad ; \quad f_2 = -\,suppession\ bandwidth \tag{20}$$

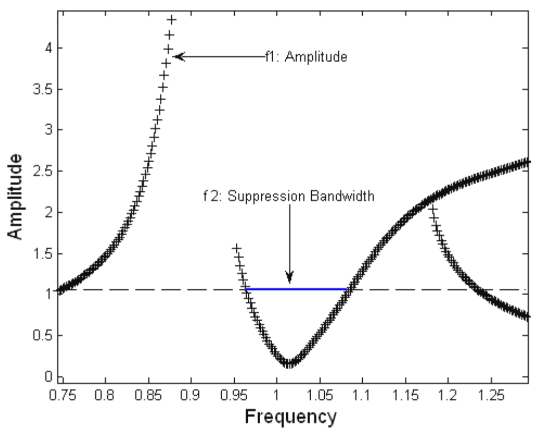

Figure 2. Representation of the objective functions f_1 and f_2.

4. NUMERICAL APPLICATIONS

4.1. Frequency Response of undamped system by using the modified Bessel functions.

This application concerns the same two d.o.f's system reported in Fig. 1(a). The computations performed consist in obtaining the driving point frequency responses associated to the displacements of the primary, x_1 , and secondary, x_2 , systems, respectively. By using the theory presented in Section 2 the nonlinear problem can be solved to obtain the amplitudes values of the frequency response. Figures 3 represent the Frequency Responses of the linear and the corresponding nonlinear systems by considering a nonlinear coefficient $a=20$. Through this figure it is possible to evaluate the influence of the nonlinear coefficient on the response amplitudes and the suppression bandwidth. The dynamic responses shown in Figs. 3 are driving-point related to the vertical displacement of such points. Moreover, it is interesting to note the differences between the linear and nonlinear systems in terms of the amplitudes of vibrations and the response representation by the introduction of the nonlinear parameter *a*. Figure 3(b) show that for very small value of *a,* the frequency response of the nonlinear system is approximately linear.

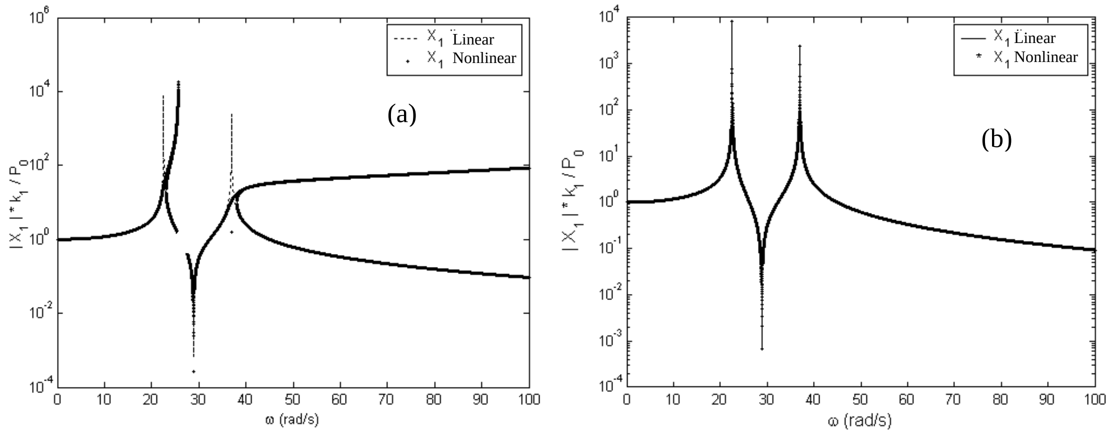

Figure 3. Frequency Response for nominal design of the nonlinear undamped system: (a) $a = 20$; (b) $a = 0.005$.

4.2. Frequency Response for damped system by using the modified Bessel functions.

Figures 4 show the amplitudes of the frequency response for the nonlinear damped system represented in Fig. 1(b) for various damping factors, obtained by the modified Bessel function. By comparing these figures one can note that when the damping factor is augmented the suppression bandwidth becomes larger, demonstrating the significant influence of the damping factor on the dynamic behaviour of the nDVA. Another aspect to be pointed out is that the responses convey valuable information about the influence of the nonlinear parameter and the damping factor on the dynamic behavior of the nonlinear system, being also a very tool for the design, performance analysis, and optimization of nonlinear structures.

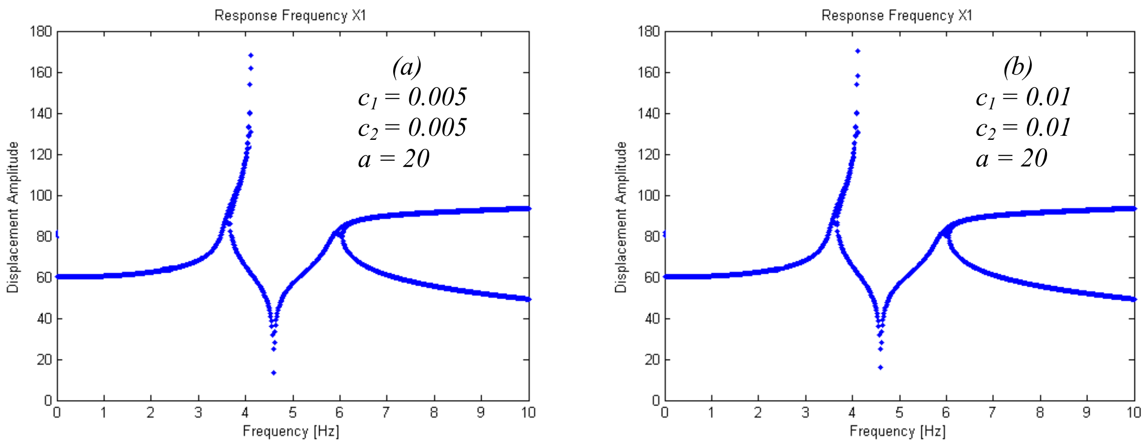

Figure 4. Amplitudes of the Response of the primary mass for different values of damping factor.

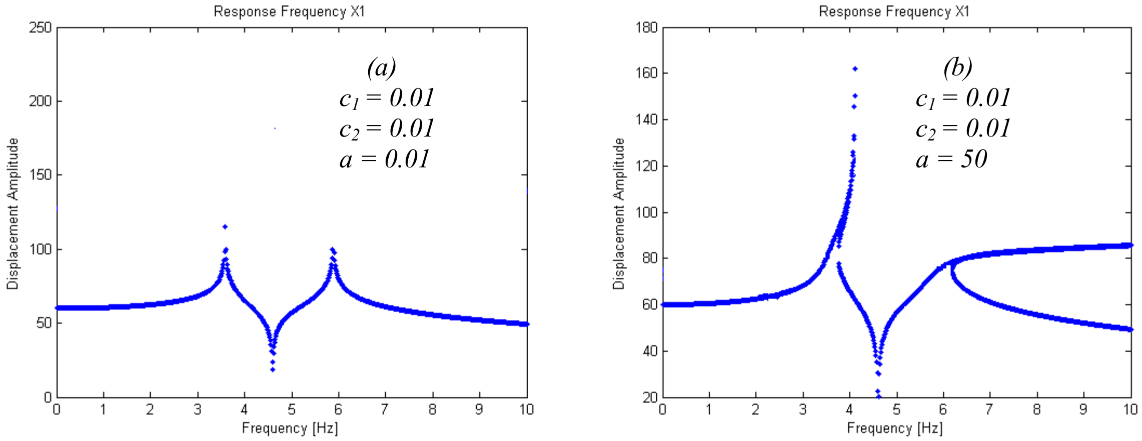

Figure 5. Response of the primary mass for different values of the nonlinear parameter a.

Figure 5 show the frequency response amplitudes of the primary mass for different values of the nonlinear parameter a, putting in evidence the influence of this parameter on the amplitudes of the Frequency Responses. One can note that when a small value for the nonlinearity factor is considered, the responses of the nonlinear system assume the values of the linear case. However, for high value of a, it is an instability region in the frequency band of interest (Borges, 2005).

4.3. Response for damped system by using the Krylov-Bogoliubov averaging method.

With the aim of illustrating the effect of the non-linear coefficient associated to the stiffness of the dynamic vibration absorber, the Response of the system for different values of the nonlinear coefficient $(\varepsilon_1 \text{ and } \varepsilon_2)$ is shown in Fig. 6, for various values of the parameter ε_2, and the other, assuming the following constant values $\beta = 0.1$, $\zeta_1 = \zeta_2 = 0.01$, $\mu = 0.05$ and $\rho = 1$.

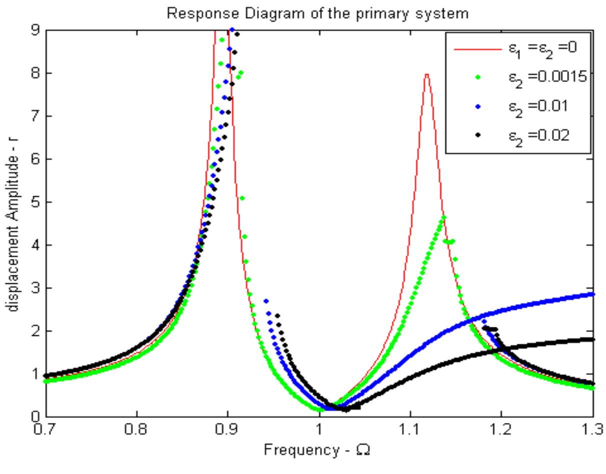

Figure 6. Response of the primary mass for various values of ε_2

One can observe that the principal advantage in using nDVAs is that the amplitude of the dynamic responses in the regions that corresponds to Ω>1, decreases significantly. Moreover, one can observe the instabilities regions, and with the aim of diminish this effects, the system parameters have to be changed, and within this context, the parametric optimization techniques must be used to obtain the best possible results.

5. OPTIMIZATION

The following numerical example is presented to illustrate the application of the optimization technique to obtain a robust design of the nDVA. Figure 1 depicts the test structure considered herein. The design parameters and their corresponding admissible variations are illustrated on Tab. 1. The values of the other variables are assumed as $\varepsilon_1 = 0.001$ and $\xi_1 = \xi_2 = 0.01$. These ranges were chosen according to the sensitivity analysis presented in Borges

(2008), in such large variations with respect to the nominal values are avoided. Only the ranges of the continuous variables are taken as constraints in the robust optimization problem. The computations performed consist in obtaining the driving point frequency responses associated to the displacement x_1 of the primary system.

Table 1. Design variables and their admissible variations

Design parameters	**Nominal values**	**Variations**
ρ	1.0	± 30 %
μ	0.05	± 30 %
β	0.1	± 30 %
ε_2	0.01	± 30 %

The response functions are computed for an excitation force applied at the primary mass, and the responses are acquired at the same point, which is indicated on Fig. 1. The optimization problem is composed by the defined objective functions according to Eq. (20), in which the interest is to minimize the amplitude of vibration and to maximize the suppression bandwidth according to the definition illustrated on Fig. 2.

The parameters of NSGA are defined in Table 2.

Table 2. Definition of NSGA parameters used in the optimization process.

NSGA	
Probability of selection	0.25
Probability of crossover	0.25
Probability of mutation	0.25
Number of generations	100
Number of individus/generation	30
Sharing coefficient (σ)	0.2

Figure 7 represents the evolution of the deterministic solutions. One can conclude that the deterministic solutions represented by the point Pd exhibit a better comprise performance than the others points into the Pareto curve.

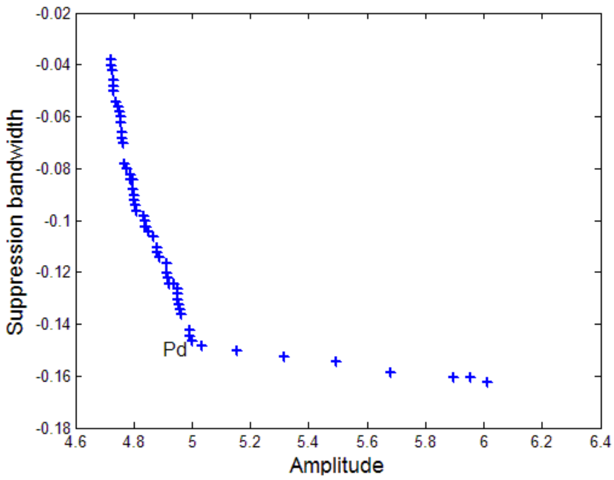

Figure 7. Deterministic solutions – First Front of Pareto

For the deterministic set of solutions corresponding to point Pd indicated in Fig. 8 and represented in Table 3, one can calculate the frequency responses of the nonlinear damped system indicated on Fig. 4.

Table 3: Optimal solutions for point Pd

Optimal point	ρ	μ	β	ε_2
Pd	1.1	0.054959	0.09	0.0091724

Figure 8 compares the amplitudes of the Frequency related to the nominal and optimal designs (indicated on the figure). The comparison permits to evaluate the nDVA effectiveness in terms of the response amplitudes and suppression bandwidth and put in evidence the interest in using optimization procedures with the aim of generate the optimal values of the nDVA.

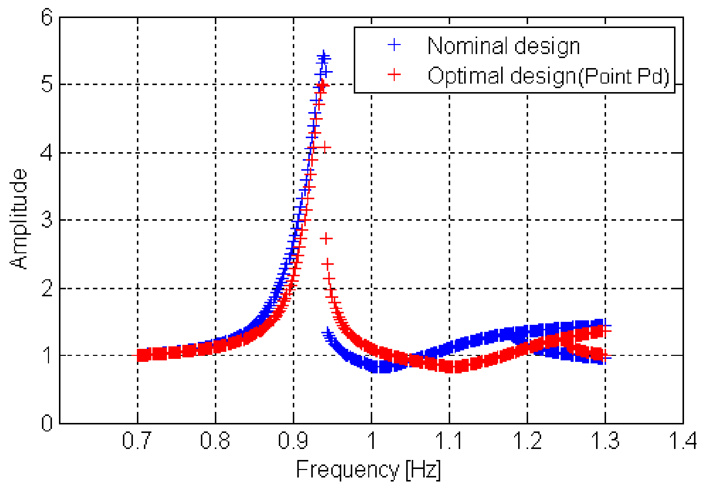

Figure 8. Frequency Response for nominal and optimal designs.

7. CONCLUSIONS

In this paper two methodologies to solve a nonlinear damped and undamped systems, and the optimal design of nonlinear dynamic vibration absorbers was investigated and implemented. The nonlinearities were introduced both in the springs that connect the primary mass to the ground and the absorber to the primary mass.

In the numerical application, an optimization problem involving simultaneously two cost functions was written, aiming at considering relatively complex nonlinear dynamic systems. The choice of the design variables (mass density, mass ratio, force parameter, nonlinearity coefficient) is based on previous knowledge regarding their sensitivities with respect to the amplitude peak and suppression bandwidth. It is worth mentioning that these parameters are directly associated with the effectiveness of the nDVA.

In terms of the system resolution, the equations of motion of the nonlinear two d.o.f's systems were numerically integrated by using the so-called *average method* and the modified *Bessel expansion* that provides an approximate solution to nonlinear dynamic problems. The nonlinear algebraic equations obtained in both cases were numerically solved by using the *"fsolve"* function available in MATLAB® toolbox. This function enables determining the roots of the nonlinear algebraic equations.

As demonstrated by the results, the nonlinearity factor is an important parameter to be investigated during the design procedure of nonlinear dynamic systems, due to its contribution to the reduction of the vibration level and the augmentation of the suppression bandwidth. However, care must be taken with respect to high values of the nonlinearity parameters because of the instabilities that they introduce in the nonlinear systems. This point motivates an important aspect regarding the proposed methodology: obtaining the optimal spring nonlinear coefficient that guarantees the best stable solution for a given system.

Finally, the proposed optimal design strategy demonstrates the importance of considering the design variables to be evaluated during the optimization process in order to obtain the optimal design that guarantees the most effective nDVA for the system considered.

8. ACKNOWLEDGEMENTS

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