

PRELIMINARY AERODYNAMIC DESIGN OF MULTI-STAGE AXIAL FLOW COMPRESSORS USING MATHEMATICA

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Abstract. A Mathematica package is developed for the preliminary aerodynamic design of multi-stage axial flow compressors. For given mass flow rate, pressure ratio and gas properties, the number of stages is first determined. The overall geometry of the compressor is then defined: the hub, shroud and mean diameters. After the compression work is adequately distributed over the stages, radial equilibrium calculation is applied to provide the velocity triangles along the blade height for all stages. Several families of compressor airfoils are implemented for the blade shape design that satisfies the specification of velocity triangles. Suitable output files can be generated providing input for CFD simulation or CAD/CAM. The implemented aerodynamic design procedure agrees reasonably well with the current engineering practice in open literature.

Keywords: turbomachinery aerodynamics, axial-flow compressors, aerodynamic design, preliminary design

1. Introduction

The axial compressor is the one in which the flow occurs in the direction of the rotor axis with the purpose of increasing the pressure of gas fluid through the movement of the rotor blades. It is a device used widely in industrial gas turbines. Preliminary aerodynamic design of a multistage axial flow compressor is the phase of in the design process where the basic outline of the procedure is to iterate through the mean line where the compressor is established, such as the annulus lines, which is in effect solving the mean number of stages and overall length. Glassman (1995) summarized some NASA Lewis computer codes for the determination of design geometry and the prediction of design/off-design performance of compressors and turbines. Gallimore (1999) presented some of the basic principles and rules associated with the design of axial flow compressors, principally for aero-engines, as well as the practical constraints that are inevitably present. Denton e Xu (1999) discussed computational simulation of fully 3D flow effects with particular reference to their use to improve turbomachine's performance. The advent of fully 3D flow field calculation methods has made predictions of these complex effects much more readily available and accurate so that they are now being exploited in design. Turner et al. (2006) described a new turbomachinery design system, T-AXI, that is intended primarily for the use of educators and students, although it is sophisticated enough for actual designs. Iyengar (2007) described a first-principles based multi-objective technique for designing single stage compressors.

In this work, a Mathematica package is developed for the preliminary aerodynamic design of multi-stage axial flow compressors based on the theory of turbomachinery aerodynamics. With the given mass flow rate, pressure ratio, gas properties, and some design parameters, the number of stages is determined. To design the overall geometry of the compressor, i.e. the hub, shroud and mean diameters, it is necessary to choose dimensionless variables (flow coefficient, reaction ratio and stage loading factor) in the range of reference of each stage. The compression work is suitably distributed over the stages. Simple radial equilibrium calculation is applied to provide the velocity diagrams in some points along the blade height for all stages. Consequently, it was possible to calculate all the necessary dimensions to draw a blade satisfying the velocity diagram. The intermediate geometries can be interpolated.

2. Initial Project Parameters and Data

To start the preliminary aerodynamic design of a compressor, it is necessary to specify initially the basic flow parameters, such as the mass flow rate that will work into it, the desired pressure ratio, and, also, under what external environment conditions it will be carried out, such as environmental temperature and pressure. It is also important to know which work fluid will flow in the axial compressor. In this project, the air is chosen as the work fluid, as it is most commonly used for the easiness to be found in nature. The air is treated as an ideal gas, since they provide a reasonable approximation. The equation of state for ideal gas ($Pv = RT$) is used with the constant (Γ) of approximately $287.3J/kgK$ and the ratio between specific heats (γ) of 1.4, which applies for specific heat with pressure constant (c_p) of $1005.55J/KgK$.

There were small losses of pressure with the air input and output in the compressor, which are taken into account with the application of a factor of loss. It should be evaluated the isentropic efficiency of each stage and Mach number at the inlet of the compressor. Another important initial parameter is the preliminary ratio between the diameter of the hub and shroud in the first stage ($d_r = 0.5$), since then will be calculated for all stages. Also it is necessary the higher speed of

all movable components, this location is the nearest blade point of the shroud, as the tip clearance is zero this point is at shroud diameter. Before the general estimates for all stages, it is necessary to choose a radius of reference, and, then, to determine a profile that depends on the diameter of the shroud and the hub. In this project the reference radius (r_c) is given as the mean square radius between the hub and shroud, thus:

$$r_c = \sqrt{\frac{r_h^2 + r_s^2}{2}} \quad (1)$$

Transforming Eq. (1) into terms of speed in the range of reference:

$$U_c = U_s \sqrt{\frac{1 + (d_r)^2}{2}} \quad (2)$$

$$d_r = d_s/d_h. \quad (3)$$

3. Preliminary geometry

Before designing the section of the axial compressor, some calculations were done in the preliminary stages of the extreme end compressor. From the velocity of the reference ratio, it could be calculated the axial velocity on the same radius.

An iterative method is used find the air speed at the entrance of the first rotor. After the convergence of the solution, the values for the temperature, velocity and pressure were established. The density could be calculated by the equation of state for ideal gas. With the mass flow rate initially established in the project, it was estimated the initial section of the area, and consequently obtained the diameter of the shroud. From this, it was calculated the other diameters (reference radius and hub profile) and acquired an angular velocity of preliminary design performed by the compressor.

For the last stage, despite the fact of not knowing the number of stages yet, it is necessary to do another calculation for interactive system, slightly different from before, to get all diameters as in the first stage. In the last stage it occurs in the section of minimum area of all the compressor, and then it is necessary to check the length of the blade to see if it is possible to build the blade. If there is no problem, the other stages can also be constructed. But if there is any problem, it will be necessary to change some initial parameters to be able to get to a possible design.

4. Projecting the number of stages and distribution of the parameters at all stages

To project the number of stages it is necessary to choose an average enthalpy per stage that should be divided by the value of the total isentropic variation of the compressor, which is given by:

$$h_{iso} = \frac{\gamma}{\gamma - 1} T_1 \Gamma \left(\sqrt{\left(\frac{p_n}{p_1}\right)^{\frac{\gamma}{\gamma-1}} - 1} \right) \quad (4)$$

4.1 Flow coefficient, reaction ratio and stage loading factor

If these parameters are known, they can describe the velocity diagram in each stage to the radii of reference. Therefore, it is necessary to choose values for the radii of reference. These parameters are defined below.

Flow coefficient:

$$\phi = \frac{c_x}{U} \quad (5)$$

Reaction ratio:

$$R = \frac{h_2 - h_1}{h_3 - h_1} \quad (6)$$

Stage loading factor:

$$\psi = \frac{h_{03} - h_{01}}{U^2} = \frac{c_{y2} - c_{y1}}{U} \quad (7)$$

which is used to avoid the phenomenon of stall that can have a catastrophic effect on the performance of the compressor and which is important to estimate the surge limit of a compressor (the limit zone in which the compressor can work to avoid inconveniences. In the presence of the blade or the end-wall boundary layer stall, there may be a very abrupt drop in compressor discharge pressure that apparently is neither from the predictions nor the experiment. The best we can do is to attempt to identify stall parameters that can indicate when such an abrupt change is likely to occur. These parameters

are of limited value as a mean of estimating probable stall limits likely to be associate with the compressor's surge limit. A stall criterion proposed by de Haller (1953) is given by:

$$W_2/W_1 < 0.72 \quad (8)$$

This criterion was used in the design. It is extremely conservative, since experiments have found a few points when the rates of speed are well below this limit. Indeed the dimensionless parameters cannot be completely independent. One reason for the reaction and the coefficient of flow is a limit to the coefficient of load, which is given by the following equation:

$$\psi_{lim} = \frac{6\hat{R}}{17} + 0.85\left(\frac{0.5}{\hat{R}}\right)^{1.18} \phi_c^{(2+0.1/\hat{R})} \quad (9)$$

$$\hat{R} = 0.5 + |0.5 - R_c| \quad (10)$$

4.2 Diameter profiles

As previously mentioned, this particular design presents constant shroud radius of the compressor and was calculated previously, so it already has the shroud profile. To design the other two profiles it is necessary to adopt an iterative method, in which we have a system of six equations with six unknowns:

$$T_c^i = T_c^{i-1} - \frac{\psi_c U_c^2}{\frac{\gamma}{\gamma-1} \Gamma \eta_s} \quad (11)$$

$$p_c^i = p_c^{i-1} \left(\frac{T_c^i}{T_c^{i-1}}\right)^{\frac{\gamma}{\gamma-1} \eta_s} \quad (12)$$

$$\rho_c = \frac{p_c}{T_c \Gamma} \quad (13)$$

$$d_h = \sqrt{(d_s)^2 - \frac{4\dot{m}}{\pi \rho_c \phi_c U_c}} \quad (14)$$

$$r_c = \frac{d_s}{2} \sqrt{\frac{1 + (d_r)^{-2}}{2}} \quad (15)$$

$$U_c = \omega r_c \quad (16)$$

It is important to emphasize that the stages depend on each other. For instance, Eqs. (11) and (12) depend on the temperature and pressure of the previous stage. For the first stage it is necessary to use the preliminary calculated values for temperature and pressure involving the external part of the compressor. Figure 1 shows the profiles of hub radius and should radius the generated by the program.

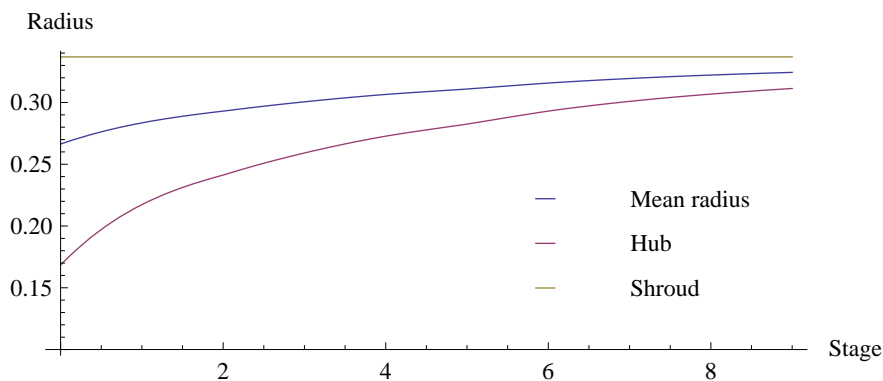


Figure 1. The variation of the radii of hub and shroud along the compressor

5. Velocity diagram

The velocity diagram is extremely important since the blade geometry would not be generated without it. The fluid enters into the rotor with velocity c_1 and direction α_1 . Since the rotor blade translation velocity is U , there is a relative velocity W_1 in the beginning between the flow and the blade. In the outlet, we have the W_2 , which is the flow velocity in relation to the rotor blades, a new velocity and also a flow direction c_2 and α_2 , respectively, and despite losses, it can be considered the same stator diagram in the beginning, as Fig 2 shows. In this project it is considered that the axial velocity is constant through the rotor passage ($c_{x1} = c_{x2}$).

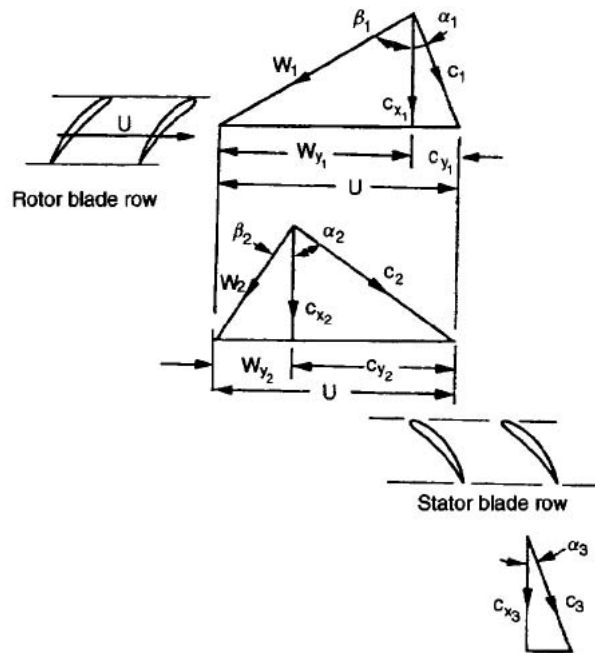


Figure 2. Velocity diagram

Also it is important to determine the velocity diagram in some points in the radial direction to all stages; five points are used to obtain a good blade geometry. The radii have been distributed as follows: one triangle to each profile radius, one to the average between the hub radius and the reference point, and another to the average between the shroud and the reference point ($\{r_h, (r_h + r_c)/2, r_c, (r_c + r_s)/2, r_s\}$). In addition to the performance parameters, it is necessary to provide the vortex type to obtain the c_y distribution. The flow profile to the other radius is obtained to describe the tangential velocity distribution through the radial axis. The equation to general vortex type for the velocity applied inlet and outlet is given respectively:

$$\frac{c_{y1}}{U_c} = (1 - R_c)(r_c/r)^n - \frac{\psi_c}{2}(r_c/r)^m \quad (17)$$

$$\frac{c_{y2}}{U_c} = \frac{c_{y1}}{U_c} - \psi_c(r_c/r) \quad (18)$$

where the exponents n and m are specified and vary depending on the vortex model. To calculate the c_x profile it was used the normal equilibrium equation neglecting the entropy gradient:

$$c_x \frac{\partial c_x}{\partial r} + \frac{c_y}{r} \frac{\partial r c_y}{\partial r} = \frac{\partial H}{\partial r} \quad (19)$$

In this project the constant reaction vortex flow was applied, in which $m = 1$ and $n = -1$. Thus the variation of c_{y1} and c_{y2} is determined, and from Eq. (19) a function that describes the variation of radial velocity (c_{x1} and c_{x2}) is found. Despite the name, the reaction ratio is not constant.

Figure 3 shows some dimensionless parameters depending on the radial dimensionless variation for the first stage of the compressor, generated by the program. It is important to note that the Haller stall criterion is within the limits.

6. Design of the blade for each section 2D

The dimension design needs empirical models based on two-dimensional cascade tests. To begin, it is chosen some geometry parameters of the empirical models that are dependent, as the ratio between the blade maximum thickness and

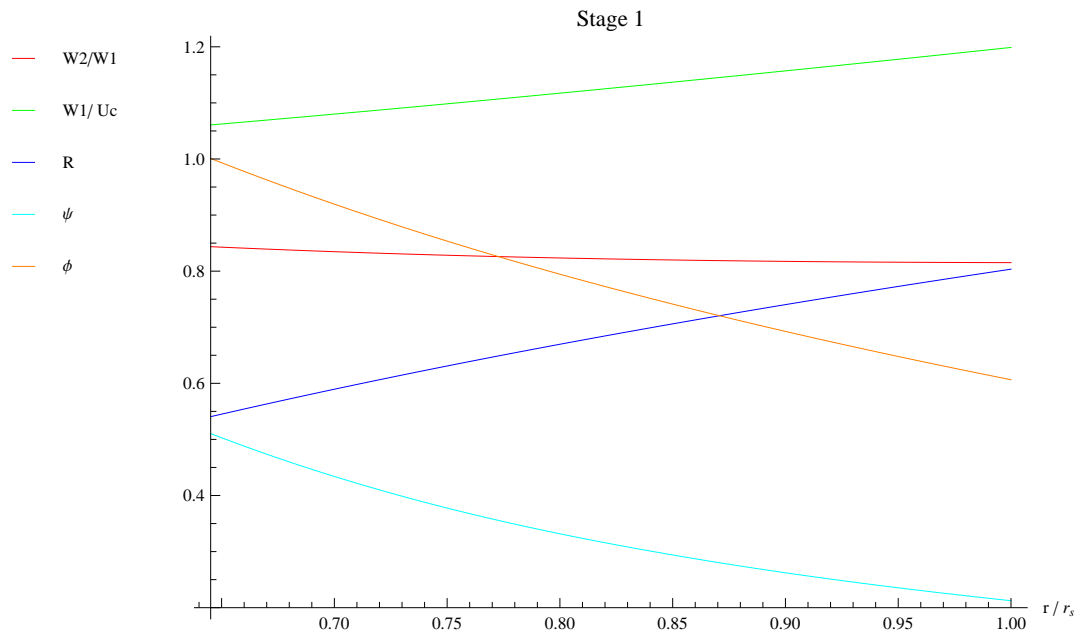


Figure 3. Radial variation of dimensionless parameters of the rotor of the first stage

the chord (t_b/c), the ratio between the point distance of maximum camberline and the section chord (a/c), the number of blade each stage (Z), the superposition of a blade profile that describes its own profile and the thickness of camberline. In this project it was used $t_b/c = 0.1$, $Z = 36$, $a/c = 0.4$ and profile NACA 65-series.

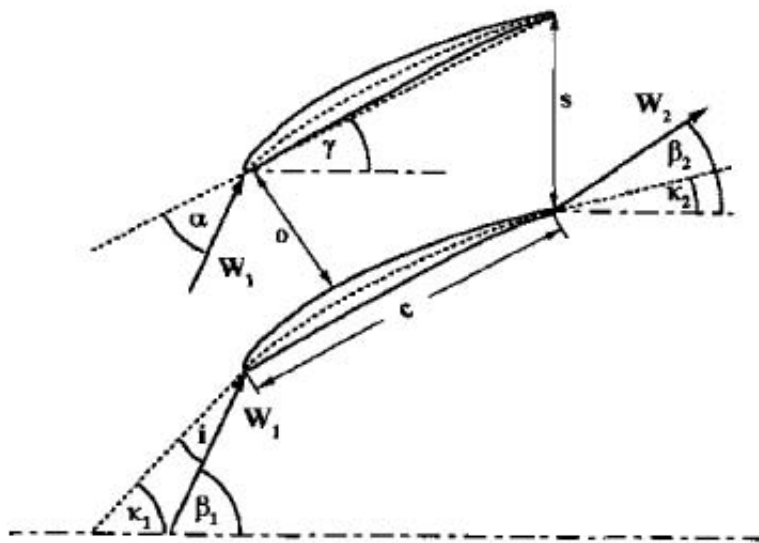


Figure 4. Dimensions for a 2D cascade

It is necessary to calculate, first, the solidity on the sections chosen. For that, this design used a approximation formula which was quoted by Howell (1942) and frequently found to be useful in preliminary performance estimation, the tangent-difference rule:

$$\tan \beta_1 - \tan \beta_2 = \frac{1.55}{1 + 1.5s/l} \tag{20}$$

With the solidity estimation it is possible to calculate the blade chord (c) once the solidity is defined as the ratio between the chord and the blade pitch (s). The blade pitch can be calculated through ratio between the perimeter described by the section and the number of blade in stage ($s = 2\pi r/Z$). Now we must calculate the angles that cover the cascade, initially through geometric calculations to obtain four equations:

$$\alpha = \beta_1 - \gamma \tag{21}$$

$$i = \beta_1 - \kappa_1 \quad (22)$$

$$\delta = \beta_2 - \kappa_2 \quad (23)$$

$$\theta = \kappa_1 - \kappa_2 \quad (24)$$

The empirical model of Lieblein (1960) for incidence angle was used for the solution:

$$i = K_{sh}K_{t,i}(i_0)_{10} + n\theta \quad (25)$$

The first term on the right-hand side of Eq. (25) is the design incidence angle for a camber angle of zero. It is computed from a correlation for NACA 65-series blades with $t_b/c = 0.1$, corrected respectively by K_{sh} and $K_{t,i}$.

$$K_{t,i} = (10t_b/c)^{0.28/[0.1+(t_b/c)^{0.3}]} \quad (26)$$

$$(i_0)_{10} = \frac{\beta_1^{(0.914+\sigma^3/160)}}{5 + 46e^{(-2.3\sigma)}} - 0.1\sigma^3 e^{[(\beta_1-70)/4]} \quad (27)$$

A slope factor, n , from Johnsen e Bullock (1965) along with prediction from the following empirical equation:

$$n = 0.025\sigma - 0.06 - \frac{(\beta_1/90)^{(1+1.2\sigma)}}{1.5 + 0.43\sigma} \quad (28)$$

The modified form of deviation angle by Howell (1942) can be used for all profile types:

$$\delta = \frac{0.92(a/c)^2 + 0.002\kappa}{1 - 0.002\theta/\sqrt{\sigma}} \frac{\theta}{\sqrt{\sigma}} + (K_{sh}K_{t,i} - 1)(\delta_0)_{10} \quad (29)$$

$$(\delta_0)_{10} = 0.01\sigma\beta_1 + [0.74\sigma^{1.9} + 3\sigma](\beta_1/90)^{(1.67+1.09\sigma)} \quad (30)$$

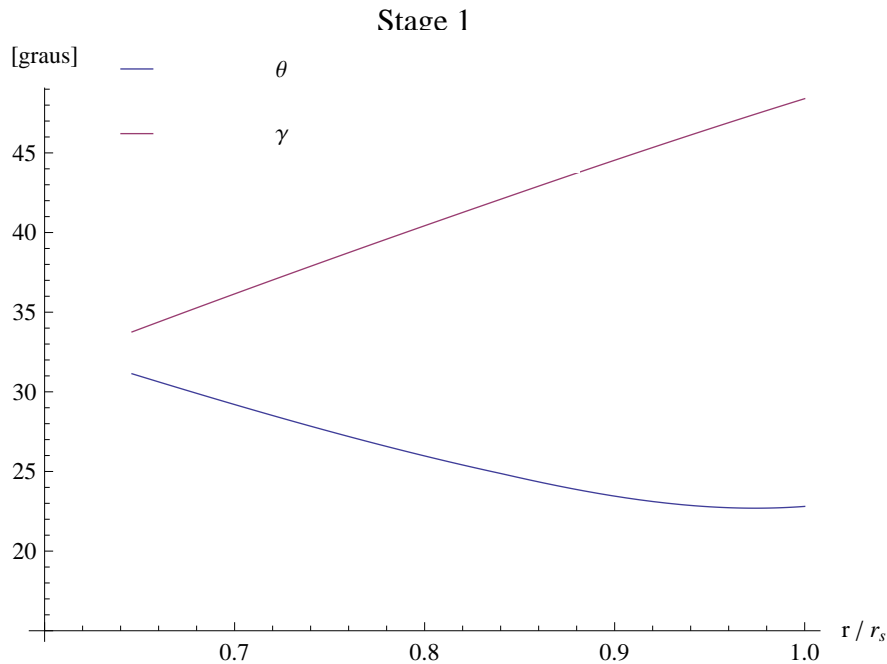


Figure 5. Radial distribution of the camber angle θ and stagger angle γ of the rotor of the first stage

Eqs. (22) and (23), and the empirical models of deviation and incidence angles build an interactive method by which the inlet and outlet angles between the camberline and axial direction (κ_1 and κ_2) are found. In Eq. (24) the camber angle (θ) is defined. The stagger angle (γ) is calculated after determination of the angle of attack. The following angle of attack empirical model was used:

$$\alpha = [3.6K_{sh}K_{t,i} + 0.3532\theta(a/c)^{0.25}]\sigma^{(0.65-0.002\theta)} \quad (31)$$

Thus it was possible to find all the angles necessary for the construction of the blade sections with the thickness profile given by the blade type. Figure 5 shows the radial variation of two important parameters, the camber angle θ and the stagger angle γ , of the first stage for a project using the program. All the steps above can be applied for both the rotor and the stator. And it should be performed for each stage in order to determine the geometry of the blades at all stages.

7. Conclusions

The program to design axial compressors is operating normally, without error. The solutions obtained to some initial data are relatively proportional to the expected. This program achieved by Mathematica Package is being implemented in Visual Basic. From the data obtained by the project (the chord and the angles of the figure 5), the blade geometry of the compressor can be constructed in a CAD tool and then simulated in CFD to study the results with those specified in the design.

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9. References

- de Haller, P. (1953). Das verhalten von tragflügelgittern in axialverdichtern und im windkanal. *Brennstoff-Wärme-Kraft (BWK)*, 5:333–336.
- Denton, J. D. e Xu, L. (1999). The exploitation of three-dimensional flow in turbomachinery design. *Proceedings of the Institution of Mechanical Engineers Part C-Journal of Mechanical Engineering Science*, 213(2):125–137.
- Gallimore, S. J. (1999). Axial flow compressor design. *Proceedings of the Institution of Mechanical Engineers Part C-Journal of Mechanical Engineering Science*, 213(5):437–449.
- Glassman, A. J. (1995). Design geometry and design/off-design performance computer codes for compressors and turbines. (NASA CR-198433).
- Howell, A. R. (1942). The present basis of axial flow compressor design: Part i - cascade theory and performance. Technical Report R&M 2095, British Aeronautical Research Council, London, United Kingdom.
- Iyengar, V. (2007). *A first principles based methodology for design of axial compressor configurations*. PhD thesis, Georgia Institute of Technology.
- Johnsen, I. A. e Bullock, R. O. (1965). Aerodynamic design of axial flow compressors. Technical Report NASA SP-36, NASA, Washington, DC.
- Lieblein, S. (1960). Incidence and deviation-angle correlations for compressor cascades. *ASME J. Basic Engng.*, 82:575–587.
- Turner, M. G., Merchant, A., e Bruna, D. (2006). A turbomachinery design tool for teaching design concepts for axial-flow fans, compressors, and turbines. In *ASME Turbo Expo 2006*, Barcelona, Spain.

10. Responsibility notice

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