

ANALYTICAL SOLUTIONS OF THE HEAT CONDUCTION EQUATION IN STEADY STATE AND TRANSIENT CONDITIONS

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***Abstract.** The governing heat transfer equation for cylindrical pellets in nuclear fuel rods is non-linear when it is expressed in dependence on the temperature. However, all material properties and physical characteristics of crystalline solids are interdependent variables in the topological space, where the interdependences are intrinsic and described by a function of invariant form, with bijective (one-to-one correspondence of values) and convergent behaviors. By taking into account the bijective relationship between the internal energy and material properties, the heat conduction equation is transformed in a linear equation with analytical solutions in steady state or transient conditions. In the present paper, the transient solution is demonstrated for measured evolutions of temperatures at the centerline of fuel rods and at the outlet of the reactor coolant. The solution is general and can be used in other applications.*

***Keywords:** heat conduction; nuclear fuel; topological model*

1. INTRODUCTION

Effective heat removal from the fuel pins of a nuclear reactor poses one of the primary considerations in reactor design. The term of heat conduction defines the energy transportation as a result of atomic interactions under the influence of a non-homogeneous distribution of temperature (Grigull and Sander, 1984). Nuclear fuel vendors and nuclear electricity utilities must demonstrate the safety of the reactor by thorough examination of the outcome of steady state and transient conditions. In special, transient effects may occur in normal operation as well as in accident situations and must be analyzed to assure the fuel pin maintains its integrity during its insertion time in the reactor core. The temperature dependence of the thermal properties makes the thermal analysis of reactor fuel pins to be non-linear. Traditionally, iterative procedure applying finite difference or finite element methods are used to solve this problem in steady state or in transient conditions (Guidotti et al., 1982). However, the design calculations and performance evaluations involve the uncertainties related with these methods but also the uncertainties associated with measured values of physical and thermal properties.

In order to reduce the involved uncertainties, an evaluation methodology in analytical form is being developed. This development considers a topological view of matter and its properties. In this approach are established the formal definitions of convergence, connectedness and continuity to characterize and describe crystalline solids and their properties in a unified notion. All properties (material and physical characteristics) of the crystalline solids are interdependent variables in the topological space. Their interdependences are intrinsic in nature and are described by a function of an invariant form, having bijective (one-to-one correspondence of values) and convergent relationships. The form of the function does not depend on any particular choice of physical quantities to represent a material property – as suggested by Tarantola (2006). The invariant form function is applied to represent the interdependences among the material properties and, in an apparently reverse way, to represent the dependences of physical characteristics on the material properties.

As presented by Dias et al (2007), the relationship H_v/I between fuel volumetric capacity to store heat H_v and the fuel linear capacity to conduct the heat I is demonstrated as the thermal property that makes linear the heat conduction equation. The linearization is obtained by introducing transformations based on material properties.

2. HEAT CONDUCTION EQUATION

The governing heat transfer equation for the fuel pellet is the non-linear, partial differential equation given by

$$Q_f + \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(k \cdot r \cdot \frac{\partial T}{\partial r} \right) - \rho \cdot c_p \frac{\partial T}{\partial t} = 0 \quad (1)$$

that can be rewritten into a form of energy balance by introducing transformations based on material properties:

$$Q_f + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial I}{\partial r} \right) - \frac{\partial H_v}{\partial t} = 0 \quad (2)$$

where r is the radial position inside a fuel pellet; $I = \int k(T).dT$ is the material linear capacity to conduct heat; $H_v = \int \rho.c_p(T).dT$ is the specific volumetric capacity to store heat at constant pressure; Q_f is the volumetric density of the heat generating rate from the fission process; k is the thermal conductivity; ρ is the fuel specific mass and c_p is the specific heat at constant pressure. In a simple consideration of the above equation, we find that the density of the heat generating rate from fission is nullified inside a cylindrical element when the density rates of conducted and stored heats are subtracted.

The relationship among the materials properties

$$\frac{dI}{dH_v} = \frac{k}{\rho.c_p} = \delta \quad (3)$$

is the material property δ (thermal diffusivity) that establishes the consistency required for the values of material properties and for the balance among generated, conducted and stored heats. This consistency is found in the solution of the heat conduction equation.

Material properties I , H_v , k , ρ , c_p amongst others are quantifiable coordinates representing the internal energy state in the fuel and its response capacities to changes in the surroundings. The spatial and/or time distributions of the integrated properties I and H_v are the solution of the Eq. (2). Their bijective relationships (one-to-one correspondence of values) with the temperature conduce to the solution of the Eq. (1):

$$T(r,t) \equiv T(I(r,t)) \equiv T(H_v(r,t)) \equiv T(I(H_v(r,t))) \quad (4)$$

The above space and time solution in terms of temperature $T(r,t)$ can be derived from the solutions for the distributions of conducted or stored heats, $T(I(r,t))$ or $T(H_v(r,t))$, or for their interdependences $T(I(H_v(r,t)))$. The solutions are based on the existence of bijective correlations among the profiles of the generated, conducted and stored heats and the profile of temperature. The distributions of the conducted and stored heats depend on the distributions of thermal conductivity and of heat capacity. A temperature value T in the fuel pellet is related with unique values $I(T)$ and $H_v(T)$. The consistency of experimental values in the Eq. (3) assure that the same temperature T will be obtained in the reverse evaluations $T = T(I)$ or $T = T(H_v)$. To reach this consistency the same set of experimental data is used in the development of the analytical expressions $k = k(T)$, $I = I(T)$ and $T = T(I)$. Additionally, the data $I = I(T)$ and $T = T(I)$ are reduced by the summation of components functions with invariant form, as described below.

2.1. UO₂ Material Properties

In a topological view of the matter, material properties are defined as quantifiable coordinates representing the internal energy state and its response capacities to changes in the surroundings. The variations in the state of internal energy cause (or result from) bijective variations in the values of some material and physical properties. Thus, variations in the internal energy state on dependence of time (transient) and/or space (gradient) as the governing heat transfer equation are also the variations in the material and physical properties. The variations in the internal energy and variations in the material and physical properties are to be concurrently described by the same dependence on time and position. As suggested by Tarantola (2006), all properties (material and physical characteristics) of the crystalline solids are interdependent variables in the topological space; their interdependences are intrinsic in nature and are described by a function of an invariant form, having bijective and convergent relationships. The form of the function does not depend on any particular choice of physical quantities to represent a material property or the internal energy variations.

The Relative Variational Model proposed by Dias and al. (2007) to represent the invariant form function or distribution function is given as:

$$f(y) = \frac{y^3}{1+y^3} \quad (5)$$

where $y = |x-b|/c$ is the relative variation in a coordinate x (as temperature, for instance); b is a reference coordinate (room temperature, for example) and c is the characteristic coordinate related to the median or the variance of the distribution, like the Einstein temperature θ_E in the Einstein's model of the volumetric heat capacity

$\frac{c_v}{3.R} = \frac{(\theta_E/T)^2 . e^{\theta_E/T}}{(e^{\theta_E/T} - 1)^2}$, where R is the gas universal constant (Zemansky, 1968). The derivative and the integral forms of

Eq. (5) are given as:

$$\frac{df}{dx} = \frac{3}{c} \left(\frac{y}{1+y^3} \right)^2 \quad (6)$$

$$\int_b^y f \cdot dx = c \cdot \left(y - \frac{1}{6} \ln \left(\frac{(y+1)^2}{y^2 - y + 1} \right) - \frac{1}{\sqrt{3}} \cdot \left(\arctan \left(\frac{2 \cdot y - 1}{\sqrt{3}} \right) \right) \right) \Big|_0^{x-b/c} \quad (7)$$

In order to represent different effects related with bands of temperature for atomic vibration modes, crystalline phase dissolution, types of atoms or defects and others, the model is generalized as:

$$\frac{F}{F_n} = \sum_{i=1}^{\eta} a_i \cdot \left(\frac{F}{F_n} \right)_i = \sum_{i=1}^{\eta} a_i \cdot f_i(y_i) \quad (8)$$

where F/F_n is the total distribution of F -values in a range F_n (like $c_v/3.R$ of Einstein's model); η is the number of components to represent the property F and a_i are parameters fitting the contribution of the band i in the total property.

The evaluation of 1195 data of UO_2 thermal conductivity is shown in Fig. 1. In the range from 0 K to $T_{melting}=3113$ K, the data are depicted by the summation of 5 components in the form of derivative functions - Eq. (6). The integrated thermal conductivity is given as the summation of components in the form of Eq. (5) - Fig. 2.

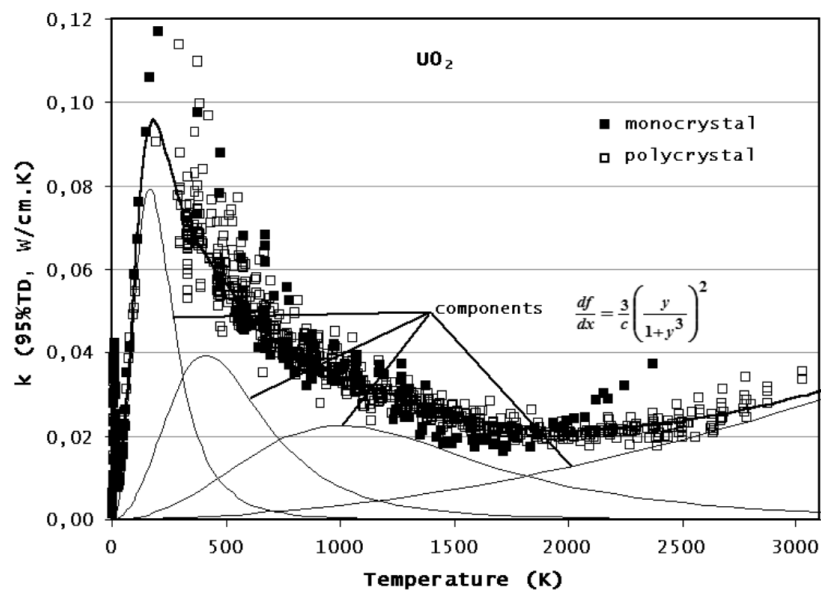


Figure 1. UO_2 thermal conductivity (non-bijective behavior)

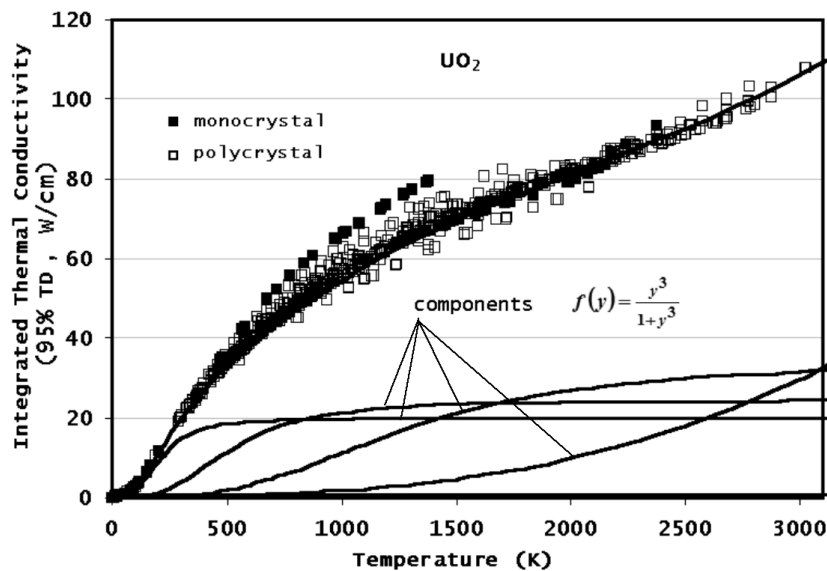


Figure 2. UO_2 integrated thermal conductivity (bijective behavior)

The expression for the reverse function $T(I)$ follows the structure of relative variational model being in this case developed for estimate from room temperature (293,2 K) up to the melting point and in the porosity range of 0 to 10%. The fitting result is shown in the Fig 3. The bijective relationship (one-to-one correspondence of values) and convergence exist between the integrated thermal conductivity and the temperature as shown in Figs. 2 and 3.

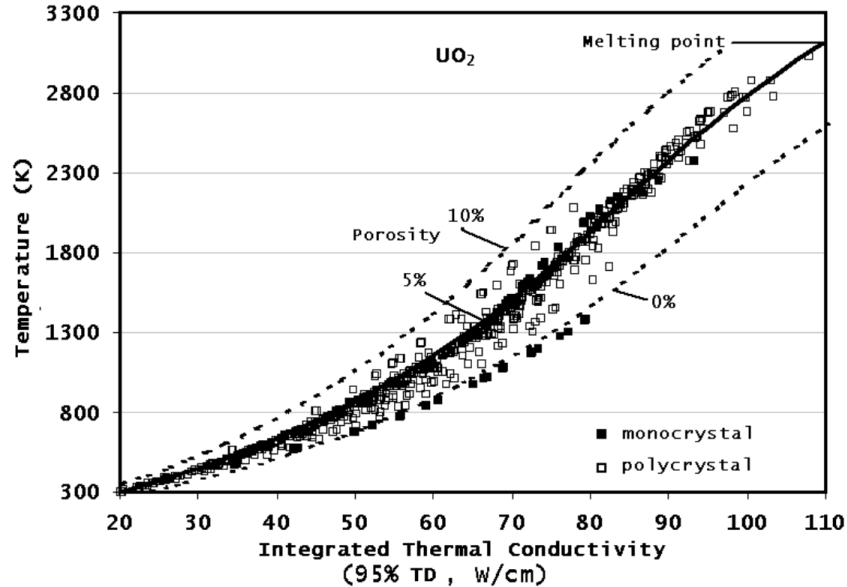


Figure 3. Temperature dependences on the integrated thermal conductivity and porosity (bijective behavior)

The dependences on the temperature of the UO_2 specific heat capacity at constant pressure c_p and thermal expansion coefficient α are described by the summation of components similar to Eq. (5), while the integrals $\int c_p \cdot dT$ (enthalpy variation) and $\int \alpha \cdot dT$ (thermal deformation) are given by the summation of components in form of Eq. 7. The lattice parameter of the $UO_{2\pm y}$ fuel and, consequently, the specific mass and thermal expansion depend on the deviation of the stoichiometry or O-to-U ratio ($y = \left| \frac{O}{U} - 2 \right|$) as the behavior described by Eq. (5). Thus, Eqs. (5) to (8) establish the formal definitions of convergence, connectedness and continuity to characterize and describe the properties in a unified notion. All interdependences of the material properties are described by the summation of a function with invariant form, having bijective and convergent relationships. The function form does not depend on any particular choice of physical quantities to represent a material property. The invariant form function is equally applied to represent the interdependences among the material properties and, in an apparently reverse way, the dependences of physical characteristics on the material properties, as for instance, the temperature dependence on the integrated thermal conductivity – Fig. 3.

2.2. Heat Conduction in Steady State Condition

Under steady state condition Eq. (2) is reduced to:

$$d\left(r \frac{dI}{dr}\right) = -Q_f \cdot r \cdot dr \quad (9)$$

The integral of the thermal conductivity between the surface and the centerline of the fuel pellet is proportional to the linear heat generating rate P_L , as discussed by Robertson (1969):

$$4\pi \int_{T_s}^{T_c} k \cdot dT = 4\pi \cdot (I_c - I_s) = 4\pi \left(\int_0^{T_c} k \cdot dT - \int_0^{T_s} k \cdot dT \right) = P_L \cdot \overline{f_{P_L}} = \pi \cdot s^2 \cdot Q_f \cdot \overline{f_{P_L}} \quad (10)$$

where I_c and I_s are the integrals of the thermal conductivity to the temperatures in the centerline and surface of the fuel pellet; and the factor $\overline{f_{P_L}}$ is the mean value of the heat source distribution in the fuel, that is in the region $0 \leq r \leq s$ (s = fuel pellet radius).

The radial distribution of the integrated thermal conductivity in the fuel pellet is then given by

$$4.\pi \int_{T_s}^{T_r} k.dT = 4.\pi.(I_r - I_s) = P_L \cdot f_{P_L}(r) = \pi.s^2.Q_f \cdot f_{P_L}(r) \quad (11)$$

that is

$$I_r = I_s + \frac{P_L}{4.\pi} \cdot f_{P_L}(r) = \int_0^{T_s} k.dT + \frac{P_L}{4.\pi} \cdot f_{P_L}(r) = \int_0^{T_s} k.dT + \frac{s^2.Q_f}{4} \cdot f_{P_L}(r) \quad (12)$$

where $I_s = \int_0^{T_s} k.dT$; $f_{P_L}(r)$ is the radial distribution of the heat source factor and T_s is the fuel surface temperature evaluated from the heat transfer condition to the coolant channel.

As the I_r -values are determined by means of the Eq. (12), the temperature distribution in the fuel is the unknown T_r in the equation:

$$I_r = \int_0^{T_r} k.dT = \int_0^{T_s} k.dT + \frac{P_L}{4.\pi} \cdot f_{P_L}(r) = \int_0^{T_s} k.dT + \frac{s^2.Q_f}{4} \cdot f_{P_L}(r) \quad (13)$$

$$\text{that is } T_r = T(I_r) \quad (14)$$

where $T(I_r)$ plotted in Fig. 3 is the reverse function to determine the temperature from the value of the integrated thermal conductivity.

In terms of the integrated thermal conductivity, the heat conduction equation in steady state condition is rewritten as:

$$\frac{d}{dr} \left(r \frac{dI}{dr} \right) + \frac{I}{r} = 0 \quad (15)$$

and the solutions of this equation result from assumptions concerning the relationships between coordinates I and Q_f , as discussed by Robertson (1969), Weismann (1977), Guidotti et al. (1982) and Pontedeiro et al. (2005).

Considering the proportionality of relative variations in the heat source and in the geometry as $dQ_f/Q_f \equiv dr/r$ and the heat source factor $\mathfrak{S} = Q_f(s)/\overline{Q}_f$ between the fission density on the pellet surface $Q_f(s)$ and the mean fission density \overline{Q}_f , the density of the heat generating rate as a function of volume or position is given by:

$$Q_f(r) = \frac{Q_f(s)}{2.\mathfrak{S}} \left(3 - \mathfrak{S} - 3(1 - \mathfrak{S}) \left(\frac{r}{s} \right)^2 \right) \quad (16)$$

and the distribution of the linear heat generating rate is described as

$$P(r) = \pi.r^2.Q_f(r) = V(r).Q_f(r) \quad (17)$$

where $V(r) = \pi.r^2$ is the fuel cylindrical volume at the radial position r .

The integrated thermal conductivity and linear heat generating rate are defined in the volume range $0 \leq r \leq s$, that is:

$$\frac{I(r) - I(s)}{I(0) - I(s)} = \frac{\int_{T_s}^{T_r} k(T).dT}{\int_{T_s}^{T_c} k(T).dT} = \frac{P_L(s) - P_L(r)}{P_L(s)} \quad (18)$$

where T_s and T_c are the surface and centerline temperatures of the fuel pellet respectively. This equation represents the relative variation in the material property I (similar to $c_p/3R$ or F/F_n), which has a bijective relationship with the temperature: $I = \int_0^T k(T).dT = f_I(T)$ and $T = f_T(I)$. In fact, the $P_L(s)$ -value is the fuel rod linear heat rate as determined by the enthalpy variation in the coolant, the \mathfrak{S} -factor is evaluated by neutronic calculations and the temperature on the pellet surface T_s is obtained from linear calculations. Therefore, the above equation establishes a bijective relationship among the radial position, linear heat generating rate, integral of the thermal conductivity and temperature. The temperature distribution is determined by the relationship $T_r = f(I(r))$, where $f(I)$ is the analytical, reverse and bijective relationship between the temperature and the integrated thermal conductivity shown in Fig.3.

2.3. Heat Conduction in Transient Condition

Under transient condition - Eq. (2), the variations are taken in relation with time t , in a transient process $0 \leq t \leq t_M$, and also in relation with position r inside the cylindrical pellet $0 \leq r \leq s$, where t_M is the transient duration and s is the pellet radius. As shown in the previous section there is a linear relationship between the distributions of the integrated thermal conductivity I and of the radial heat sources Q_f . The non-linearity in the Eq. (2) depends on the relationship of the material properties I and H_v . The ratio between the material properties I and H_v defines the thermal diffusivity as:

$$\delta = \frac{I}{H_v} = \frac{\int k(T).dT}{\int \rho.c_{pm}(T).dT} = \frac{\Delta I/\Delta T}{\Delta H_v/\Delta T} = \frac{dI}{dH_v} = \frac{k}{\rho.c_{pm}} \quad (19)$$

where ρ is the specific mass, c_{pm} is the specific heat at constant pressure and per unit of mass.

The equations $H_v = I/\delta$ or $dH_v = dI/\delta$ describe the volumetric capacity of a material to store heat in accordance with its linear capacity to conduct it. This relationship is a functional form similar to the relation between potential and electric current in electricity: $V = R.I$.

The relative variations are rewritten as follows:

$$\frac{dH_v}{\rho.c_{pm}} - \frac{dI}{k} = 0 \quad \text{or} \quad (\Delta T)_{H_v} - (\Delta T)_I = 0 \quad (20)$$

$$\frac{dH_v}{\rho.c_{pm}.dT} + \left(1 - \frac{dI}{k.dT}\right) = 1 (=100\%) \quad (21)$$

$$\frac{dI}{dH_v} = \frac{I}{H_v} = \delta \quad (22)$$

where the Eq. (20) implies the existence of only one possible solution as the temperature variation determined by the relative variation in the enthalpy must be the same as the one determined by the relative variation in the heat conduction. The differentials in both forms of energy (stored and conducted) can be taken as being relative to the differentials in space, time or any other coordinate. Equation (21) deals with the complementary variations and represents the thermal heat balance, where $(1 - dI/(k.dT))$ is the conducted heat fraction, and $dH_v/(\rho.c_{pm}.dT)$ represents the stored heat fraction in relation with the generated total (=100%). Finally, the Eq. (22) defines the thermal diffusivity as the superficial rate constant used to convert the stored heat H_v into conducted heat I .

It should be emphasized that properties and their interdependences (also properties) are mean values measured as material responses resulting from changes in the internal energy states of the fuel.

The stored heat in a steady state condition was established during the reactor startup transient. This transient is characterized by three competitive processes, the volumetric heat generating rate resulting from fission, the heat accumulation and the heat conduction. In fact, the heat conduction must be considered in two aspects: the heat conduction inside the fuel pellet and the heat loss on its surface. In the startup transient, the UO_2 capacity to store heat increases with the temperature, but its capacity to conduct heat is reduced simultaneously. In spite of this behavior, the steady state condition is reached when the heat loss equals the inner heat generating rate.

Considering the Bernoulli's solution $I(r,t) = \phi(r). \varphi(t)$ we find the heat conduction equation as:

$$Q_f + \frac{\phi}{r} \frac{d}{dr} \left(r \frac{d\phi}{dr} \right) - \frac{\phi.d(\phi/\delta)}{dt} = 0 \quad (23)$$

When this equation is multiplied by an infinitesimal volume $2.\pi.r.dr$ and integrated, we obtain:

$$P_g^S - \phi.P_l^S = \frac{d\phi}{dt} . \Delta P^S \quad (24)$$

where $P_g^S = 2.\pi.\int_0^r r.Q_f.dr$ is the linear heat generating rate, and $P_l^S = -2.\pi.r.d\phi/dr$ is the linear heat loss on the surface $2.\pi.r$, and $\Delta P^S = 2.\pi.\int_0^r \frac{r.\phi}{\delta}.dr$ is the linear heat accumulation. In the above equations, the superscript S denotes the coordinates in the steady state condition.

The energy balance during the transient of the reactor startup is given as:

$$P_g^S . \tau = P_l^S . \int \phi.dt + \Delta P^S . \int \frac{d\phi}{dt} . dt = \quad \text{or} \quad P_g^S = P_l^S . \frac{\int \phi.dt}{\tau} + \Delta P^S . \frac{\phi}{\tau} \quad (25)$$

$$= P_l^S . \int \phi.dt + \Delta P^S . \phi$$

where τ is the transient characteristic time.

In a given cylindrical element of the fuel pellet, the excess of enthalpy H , and the heat flux I on the surface r are connected by:

$$\frac{I(r)}{H_v(r)} = \tilde{\alpha}(r) = \frac{r^2}{t_c} \quad (26)$$

where $\tilde{\alpha}(r)$ is the thermal diffusivity on the surface r of the element and t_c is the characteristic time for the conduction of the heat amount $r^2 \cdot H_v$ at rate I . Under a steady state condition, the linear heat accumulation ΔP^S is related to the linear heat loss rate by the equation $\Delta P^S = P_l^S \cdot t_c$. Thus, Eq. (25) is rewritten as $P_g^S = P_l^S \left(\int \varphi \cdot dt + t_c \cdot \varphi \right) / \tau$ and from Eq. (24)

we have: $P_l^S \cdot t_c \cdot \frac{d\varphi}{dt} + P_l^S \left(1 - \frac{t_c}{\tau} \right) \cdot \varphi - \frac{P_l^S}{\tau} \cdot \int \varphi \cdot dt = 0$.

Applying the time derivative, the linear homogenous second order equation comes to $\frac{d^2\varphi}{dt^2} + \left(\frac{\tau - t_c}{\tau \cdot t_c} \right) \cdot \frac{d\varphi}{dt} - \frac{\varphi}{\tau \cdot t_c} = 0$ or the general form is depicted as:

$$\frac{d^2\varphi}{dt^2} + \frac{1}{t_1} \cdot \frac{d\varphi}{dt} + \frac{\varphi}{t_2^2} = 0 \quad (27)$$

where $1/t_1$ and $1/t_2$ are rate constants for the heat accumulation and heat conduction.

The heat conduction equation is the transient equation $I = \phi(r) \cdot \varphi(t)$, where $\varphi(t)$ is the solution to Eq. (27). The transitory function for the modeling of the transient condition is:

$$\varphi(t) = K_1 \left(1 + \frac{\frac{t-t_0}{t_1} - \frac{t-t_0}{t_2}}{t_2 - t_1} \right) \quad (28)$$

where $K_1 = \left(\frac{|t-t_0|}{t-t_0} + 1 \right) / 2$ is the constant factor ($= 0$ to $t \leq t_0$ and $= 1$ to $t > t_0$); t_0 is the initial time for the variation in the heat flow (inertial time, since the heat accumulation capacity of the materials is higher than the heat conduction capacity); and $1/t_1$ and $1/t_2$ are rate constants of the heat accumulation and heat loss processes.

When the fuel pellet undergoes a transition from a steady state $\phi_1(r)$ to another steady state $\phi_2(r)$ with the thermal diffusivity governing the heat conduction equation, the linear heat generating rate is described by the following transitory equation

$$I(r, t) = \phi_1(r) \cdot [1 - \varphi(t)] + \phi_2(r) \cdot \varphi(t) \quad (29)$$

where ϕ_1 and ϕ_2 are the steady state solutions in the form of Eq. (18) and φ given by Eq. (28) is the transitory function between states 1 and 2.

2.4. Model Application to the Transient Condition

The temperature response of a fuel rod during reactor shutdown has been measured for many years in the OECD-Halden Reactor Project (HRP, 1997). It has been shown that the shutdown processes can be characterized by time constants which depend on different fuel design, operational parameters, and, among others material properties, the irradiation induced degradation of the fuel thermal conductivity. Concurrently, the outlet temperatures of the reactor coolant are also measured. The fuel rods in the reactor core are cooled by nucleated boiling and the coolant flow is boosted according with the natural convection process. In the scram transient, the stored heat in the fuel rods is the heat source and the radioactive decay enhances the asymptotic behavior of the temperature evolution. The transient of the coolant temperature in the reactor outlet is given by the equation:

$$\frac{\Delta T_c^t}{\Delta T_c^0} = 1 - K_o \cdot K_1 \cdot \varphi(t) \quad (30)$$

where $\Delta T_c^t = T_c^t - T_c^\infty$ and $\Delta T_c^0 = T_c^S - T_c^\infty$; T_c^t is the coolant temperature at time t ; T_c^∞ is the convergence temperature of the coolant; $T_c^S = T_c^0$ is the initial temperature of the coolant; $K_o = (\Delta T_c^0 - \Delta T_c^\infty) / \Delta T_c^0$ is the convergence limit (lower than one as the radioactive decay is an additional heat source), and the reference is at 20 °C: $\Delta T_c^0 = (T_c^0 - 20)_{t \rightarrow 0}$ and $\Delta T_c^\infty = (T_c^\infty - 20)_{t \rightarrow \infty}$

As indicated by the results in Fig 4 and Table 1, the transitory model fits in well with the temperature of the coolant in a fast scram transient.

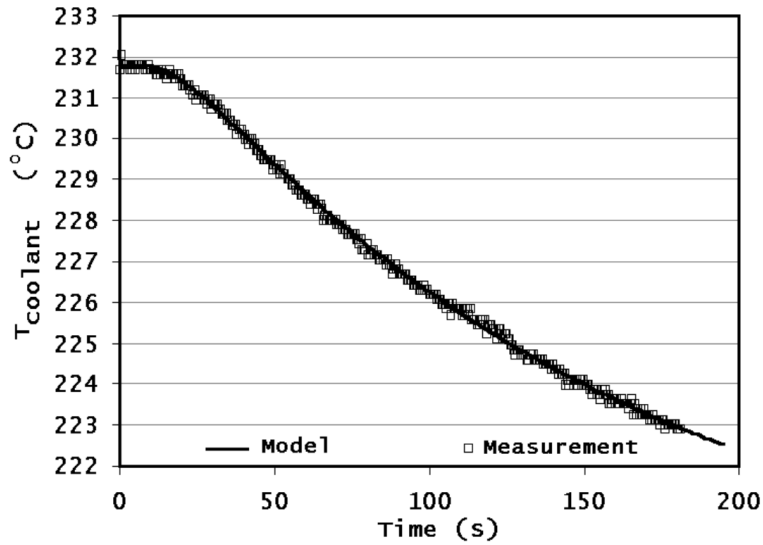


Figure 4. Temperature of the coolant during the reactor scram transient

The transitory model applied to measurements of the centerline temperature of the fuel pellet is shown in the Fig. 5 and the fitting constants are also given in Table 1. In this case the referential is the outlet temperature of the coolant.

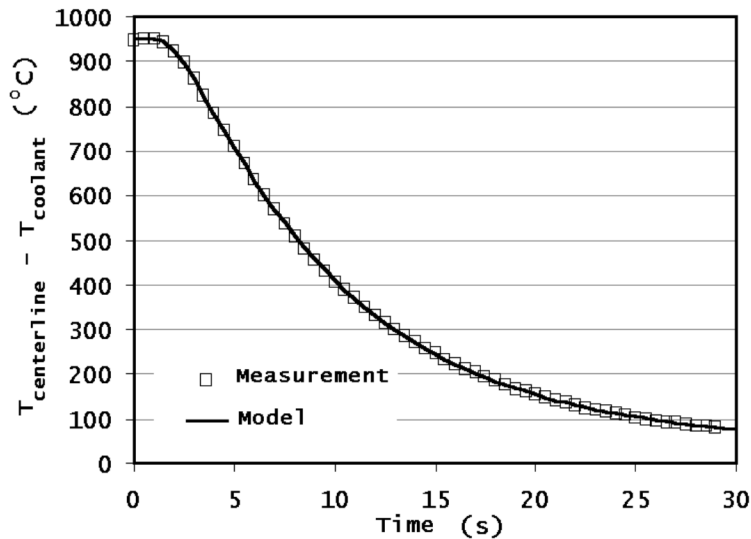


Figure 5. Centerline temperature of the fuel pellet during the reactor scram transient

Table 1 –Fitting constants in the reactor scram transient

Variables	Coolant Temperature		Fuel Centerline Temperature	
	Determ. Coef. R^2	0.9990	Determ. Coef. R^2	0.99999
	Fitted St. Error (°C)	0.09	Fitted St. Error (°C)	1.4
	Fitting Constants	Standard Error	Fitting Constants	Standard Error
ΔT_c^0 (°C)	211.7	0.02	950.1	0.9
ΔT_c^∞ (°C)	13.8	0.19	44.4	0.9
t_0 (s)	10.5	0.81	1.18	0.03
t_1 (s)	8.97	1.0	8.31	0.04
t_2 (s)	158.2	3.9	1.21	0.04
Number of points:	360 measurements in 180 s		56 measurements in 28 s	

The fitted K_0 -values represent the dependence of the radioactive heat decay on the reactor operational power as shown in Fig. 6. Values calculated from the rod centerline temperatures and from coolant temperatures of the IFA-597 experiment are congruent. These results show the fuel rods and, of course, the reactor core as the residual heat source. The complementary values $(1-K_0)$ have variations from 3% up to 5% that are consistent with the estimates of residual heat decay of the reactor.

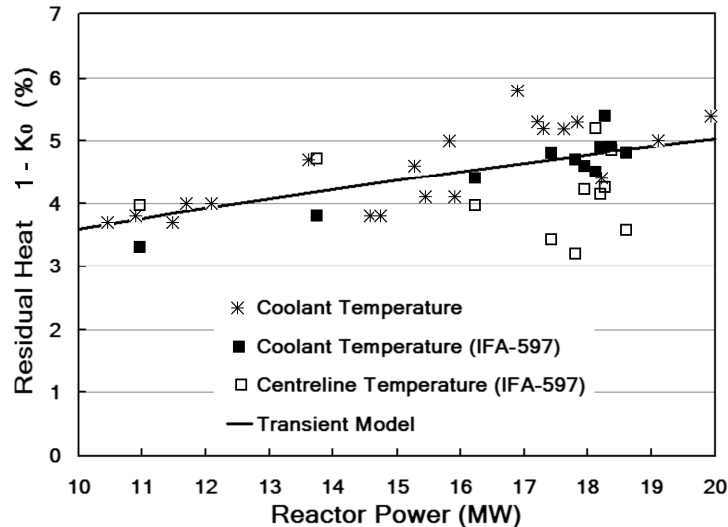


Figure 6. Residual heat decay and reactor power

3. CONCLUSIONS

The characterization and understanding of the interaction between matter and energy have been developed from quantifications of material and physical properties. As quantifications of variations occurred in internal energy states, properties and physical characteristics of crystalline solids are interdependent variables of the topological space. Their relationships and interdependences are described by means of an intrinsic function in the relative variational model. This function and the model characterize and describe the solid in a unified notion. The form of the invariant function does not depend on any particular choice of coordinates for the description and characterization of solids because their variations have an intrinsic dependence on the internal energy variations.

The linear relationships between the integrated thermal conductivity I and the density of the heat generating rate Q_f as well as between the integrated thermal conductivity I and the material capacity to store heat H_v , $\delta = dI / dH_v = k / \rho.c.p_m$, make possible to solve analytically the heat conduction equation in terms of time and spatial balance in the conducted heat: $I(r, t) = \phi_1(r) \cdot [1 - \phi(t)] + \phi_2(r) \cdot \phi(t)$. The description of time and spatial variations of the temperature derives from the bijective relationships $T = T(I)$ and $I = I(T)$.

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